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Published on: 01 Sep 2017 - Journal of Applied Nonlinear Dynamics (L & H Scientific Publishing, LLC.)

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Steering Control for a Rigid Body with two Torque Actuators using Adaptive Back Stepping

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Submission Info

Communicated by A. C. J. Luo
 Received 13 April 2016
 Accepted 29 August 2016
 Available online 1 October 2017

Keywords

Feedback stabilization
 Lyapunov Function
 Systems with drift
 Nonholonomic systems
 Adaptive Backstepping

Abstract

This paper presents a simple steering control algorithm for a rigid body model, which is a famous example of non-holonomic control systems with drift. The controllability Lie Algebra of a rigid body model contains Lie brackets of depth two. We propose a back-stepping-based adaptive controller design under the strict-feedback form. We analyze two cases for continuous steering. In the first case, the parameters of the model are assumed to be known while in the second case these are estimated by considering them unknown. This approach does not necessitate the conversion of the system model into a “chained form”, and thus does not rely on any special transformation techniques. The practical effectiveness of the controller is illustrated by numerical simulations and graceful stabilization.

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1 Introduction

The design of feedback control laws for systems with nonholonomic constraints has been a topic of interest for researchers over the last few years. The problem statement was to find control laws that can stabilize these systems about an equilibrium point. These systems with non-holonomic constraints often arise in the form of mobile robots and robot manipulators that are either designed with fewer actuators than the degree of freedom or they must be able to function in the presence of actuator failures. There is considerable challenge in the stabilization of such systems as pointed out in a famous paper by Brockett [1] that these systems cannot be stabilized by continuously differentiable, time invariant, state feedback control laws.

A number of approaches have been proposed for the stabilization of these systems to overcome the limitations that are imposed by Brockett. A complete survey of the field can be found in [2]. The solutions that have been presented can be divided into three types. Smooth time varying controllers [3,4], discontinuous or piecewise smooth controllers [5,6] and hybrid controllers [7]. All the discontinuous stabilization control strategies resulted in rough tracking and stabilization control over the states with time.

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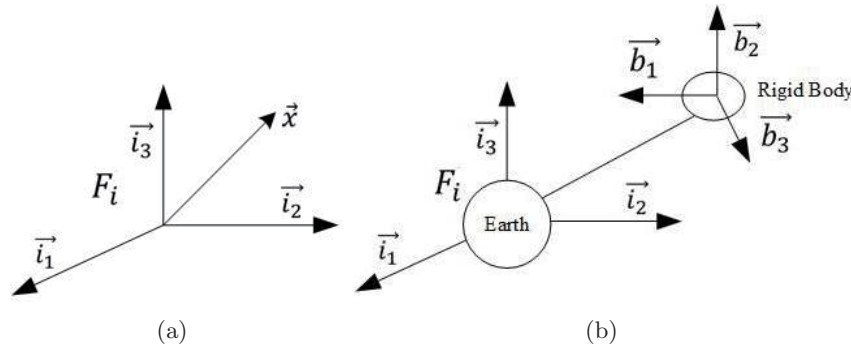


Fig. 1 (a) generic reference frame and vector and (b) the inertial and body fixed frames of rigid body.

In this paper a steering control algorithm for the rigid body model is presented. The method is based on the adaptive backstepping technique originally proposed in [8–10]. The objective is to steer the system from any initial state to any desired state. In our work we propose continuous steering control algorithm in order to achieve more graceful control. The proposed scheme also controls the plants with unknown parameters, therefore helpful in smooth tracking and adaptation. This method does not require alteration of the system model into a “chained form”, and so it does not rely on any particular transformation techniques.

2 A kinematics model of rigid body with two torque actuators

Let us consider a frame F_0 attached to the rigid body and whose axis correspond to the principal inertia axes of the body, and a fixed frame F_1 whose attitude is the desired one for F_0 . Let us also denote X the angular velocity vector of the frame F_0 with respect to the frame F_1 , expressed in the basis of F_0 . J the diagonal of the principal moments of inertia $J = \text{Diag}(j_1 + j_2 + j_3)$ and $S(X)$ represents the matrix representation of the cross product called the Rotation matrix.

$$S(X) = \begin{pmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{pmatrix}.$$

The Rotation matrix R from frame F_0 to F_1 is denoted by $R_{F_1}^{F_0}$ and is element $SO(3)$ which is defined as:

$$SO(3) = \{R | R \in \mathcal{R}^{3 \times 3}, R^T R = 1 \text{ and } \det R = 1\}.$$

Where I is a 3×3 matrix. If R is the Rotation matrix representing the attitude of F_1 with respect to F_0 (and whose column vectors are the basis vectors of F_1 expressed in F_0). We get the well known equations

$$\dot{R} = S(x)R,$$

$$J\dot{x} = S(x)Jx + B(\tau_1, \tau_2, 0)^T.$$

where τ_i are the torques applied to the rigid body and B represents the directions in which these torques are applied where the above equation is the control system with two scalar inputs and state space $SO(3) \times \mathcal{R}^3$. We make assumption that $B = I_3$ that means the torques are applied in the direction of principal inertia axes. However, our result can be easily extended to any location of the actuators from which the rigid body is controllable, after an adequate change of state and control variables, similar to

one proposed in [11]. When the rigid body moves in local frame with velocity V , the components of the velocity along X, Y, Z axes are given by

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} j_{23}YZ \\ J_{31}ZX \\ J_{12}XY \end{bmatrix}.$$

Where the Euler angles are α, β and γ , the relation between the time rate of Euler angles and torque τ is $\tau = (\tau_1, \tau_2, 0)^T$ is given by

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{bmatrix}.$$

Combining the equations and introducing new set of coordinates $(X, Y, Z = x_1, x_2, x_3)$ and $(U_1, U_2, 0 = \tau_1, \tau_2, 0)$ we get the following equations as given in [11].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} j_{23}x_2x_3 \\ J_{31}x_3x_1 \\ J_{12}x_1x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U_1 + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} U_2. \tag{1}$$

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2, \quad x \in \mathfrak{R}^3. \tag{2}$$

where $f(x) = \begin{bmatrix} J_{23}x_2x_3 \\ J_{31}x_3x_1 \\ J_{12}x_1x_2 \end{bmatrix}$ $g_1(x) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ & $g_2(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

The kinematics model (2) has the following important properties:

- (P1) The vector fields $g_1(x)$ & $g_2(x)$ are linearly independent.
- (P2) System (2) satisfies the LARC (Lie algebra rank condition) for accessibility, namely that $L(g_1, g_2)$, the Lie algebra, $L(g_1, g_2)(x)$ spans \mathfrak{R}^3 at each point $x \in \mathfrak{R}^3$.

To verify property P2, it is sufficient to calculate the following Lie brackets of $f(x), g_1(x)$ & $g_2(x)$:

$$g_3(x) \stackrel{def}{=} [f, g_1](x) = \begin{bmatrix} 0 \\ -J_{31}x_3 \\ -J_{12}x_2 \end{bmatrix} \quad \& \quad g_4(x) \stackrel{def}{=} [f, g_2](x) = \begin{bmatrix} -J_{23}x_3 \\ 0 \\ -J_{12}x_1 \end{bmatrix} \quad \& \quad g_5(x) \stackrel{def}{=} [f, g_4](x) = \begin{bmatrix} 0 \\ 0 \\ -J_{12} \end{bmatrix}.$$

which satisfy the LARC condition: $span\{g_1, g_2, g_5\}(x) = \mathfrak{R}^3, \forall x \in \mathfrak{R}^3$.

3 The control problem

- (SP): Given a desired set point $x_{des} \in \mathfrak{R}^3$, construct a discontinuous feedback strategy in terms of the controls $u_i : \mathfrak{R}^3 \rightarrow \mathfrak{R}, i = 1, 2$ such that the desired set point x_{des} is an attractive set for (2), so that there exists an $\varepsilon > 0$, such that $x(t; t_0, x_0) \rightarrow x_{des}$, as $t \rightarrow \infty$ for any initial condition $(t_0, x_0) \in \mathfrak{R}^+ \times B(x_{des}; \varepsilon)$.

Without the loss of generality, it is assumed that $x_{des} = 0$, which can be achieved by a suitable translation of the coordinate system.

4 Controller design

The model of a rigid body (2) can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= J_{23}x_2x_3 + u_1 & (a), \\ \dot{x}_2 &= J_{31}x_1x_3 + u_2 & (b), \\ \dot{x}_3 &= J_{12}x_1x_2 & (c). \end{aligned} \quad (3)$$

Assuming that the parameters are known. Consider the equation (3a), and choose $u_1 = x_2 - J_{23}x_2x_3$, equation (3a) becomes:

$$\dot{x}_1 = x_2. \quad (4)$$

Now by considering x_2 as the virtual control, α_1 as the stabilizing function and $z_1 = x_2 - \alpha_1$ be the error variable, equation (4) can be rewritten as:

$$\dot{x}_1 = z_1 + \alpha_1. \quad (5)$$

To work out α_1 , consider the Lyapunov function: $V_0 = \frac{1}{2}x_1^2$ for (5). Then,

$$\dot{V}_0 = x_1\dot{x}_1 = x_1\{z_1 + \alpha_1\}.$$

By choosing $\alpha_1 = -x_1$, the above equation becomes:

$$\dot{V}_0 = -x_1^2 + x_1z_1.$$

Equation (5) becomes,

$$\dot{x}_1 = z_1 - x_1. \quad (6)$$

Consider the equation (3b), and choose $u_2 = x_3 - J_{31}x_1x_3 + \theta(t)$ where $\theta(t) = \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t)$. $\hat{\theta}(t)$ and $\tilde{\theta}(t)$ are time varying functions which will be determined independently, while θ_{ss} is the steady state value of $\hat{\theta}(t)$. Then equation (3b) becomes:

$$\dot{x}_2 = x_3 + \theta(t). \quad (7)$$

Now by considering x_3 as the virtual control, α_2 as the stabilizing function and $z_2 = x_3 - \alpha_2$ be the error variable, equation (7) can be rewritten as:

$$\dot{x}_2 = z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t).$$

Since $z_1 = x_2 - \alpha_1 = x_2 + x_1$ its dynamics can be written as:

$$\dot{z}_1 = \dot{x}_2 + \dot{x}_1 = z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t) + z_1 - x_1. \quad (8)$$

To work out α_2 , consider the Lyapunov function: $V_1 = V_0 + \frac{1}{2}z_1^2$ for (5) & (8). Then,

$$\dot{V}_1 = -x_1^2 + z_1(z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + z_1) + z_1\tilde{\theta}(t).$$

By choosing $\alpha_2 = -2z_1 - \hat{\theta}(t) + \theta_{ss}$

$$\dot{V}_1 = -x_1^2 - z_1^2 + z_1z_2 + z_1\tilde{\theta}(t).$$

Equation (8) becomes:

$$\dot{z}_1 = z_2 - z_1 - x_1 + \tilde{\theta}(t). \quad (9)$$

Consider the equation (3c): $\dot{x}_3 = J_{12}x_1x_2$.

Since $z_2 = x_3 - \alpha_2 = x_3 + 2z_1 + \hat{\theta}(t) - \theta_{ss}$ its dynamics can be written as:

$$\dot{z}_2 = \dot{x}_3 + 2\dot{z}_1 + \dot{\hat{\theta}}(t) = J_{12}x_1x_2 + 2z_2 - 2z_1 - 2x_1 + 2\tilde{\theta}(t) + \dot{\hat{\theta}}(t). \tag{10}$$

Consider the Lyapunov function: $V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}^2(t)$ for (5), (8) & (10). Then,

$$\dot{V}_2 = -x_1^2 - z_1^2 + z_2(J_{12}x_1x_2 + 2z_2 - z_1 - 2x_1 + \dot{\hat{\theta}}(t)) + \tilde{\theta}(t)(z_1 + 2z_2 + \dot{\hat{\theta}}(t)).$$

By choosing

$$\begin{aligned} \dot{\hat{\theta}}(t) &= -J_{12}x_1x_2 - 3z_2 + z_1 + 2x_1, \\ \dot{\tilde{\theta}}(t) &= -z_1 - 2z_2 - \tilde{\theta}(t). \\ \dot{V}_2 &= -x_1^2 - z_1^2 - z_2^2 - \tilde{\theta}^2(t). \end{aligned}$$

Equation (10) becomes,

$$\dot{z}_2 = -z_2 - z_1 + 2\tilde{\theta}(t). \tag{11}$$

The closed loop system becomes:

$$\begin{aligned} \dot{x}_1 &= z_1 - x_1, \\ \dot{z}_1 &= z_2 - z_1 - x_1 + \tilde{\theta}(t), \\ \dot{z}_2 &= -z_2 - z_1 + 2\tilde{\theta}(t). \end{aligned} \tag{12}$$

Since $x_1, z_1, z_2 \rightarrow 0$ & $\hat{\theta}(t) \rightarrow \theta_{ss}$

$$\begin{aligned} x_2 &= z_1 - x_1 \rightarrow 0, \\ x_3 &= z_2 - 2z_1 - \hat{\theta}(t) + \theta_{ss} \rightarrow 0. \end{aligned}$$

Now assuming J_{23}, J_{31} & J_{12} are unknown parameters.

Let \hat{J}_{23} be the estimated value of J_{23} and $\tilde{J}_{23} = J_{23} - \hat{J}_{23}$ be the parameter error. Consider the equation (3a), and choose $u_1 = x_2 - \hat{J}_{23}x_2x_3$, equation (3a) becomes:

$$\dot{x}_1 = x_2 + \tilde{J}_{23}x_2x_3. \tag{13}$$

Now by considering x_2 as the virtual control, α_1 as the stabilizing function and $z_1 = x_2 - \alpha_1$ be the error variable, equation (13) can be rewritten as:

$$\dot{x}_1 = z_1 + \alpha_1 + \tilde{J}_{23}x_2x_3. \tag{14}$$

To work out α_1 , consider the Lyapunov function: $V_0 = \frac{1}{2}x_1^2$ for (14). Then,

$$\text{Then, } \dot{V}_0 = x_1\dot{x}_1 = x_1\{z_1 + \alpha_1\} + \tilde{J}_{23}x_1x_2x_3.$$

By choosing $\alpha_1 = -x_1$, the above equation becomes:

$$\dot{V}_0 = -x_1^2 + x_1z_1 + \tilde{J}_{23}x_1x_2x_3.$$

Equation (14) becomes,

$$\dot{x}_1 = z_1 - x_1 + \tilde{J}_{23}x_2x_3. \tag{15}$$

Consider the equation (3b), and choose $u_2 = x_3 - \hat{J}_{31}x_1x_3 + \theta(t)$ where $\theta(t) = \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t)$. $\hat{\theta}(t)$ and $\tilde{\theta}(t)$ are some time varying functions which will be determined independently, while θ_{ss} is the steady state value of $\hat{\theta}(t)$. Let \hat{J}_{31} be the estimated value of J_{31} and $\tilde{J}_{31} = J_{31} - \hat{J}_{31}$ be the parameter error. Then equation (3b) becomes:

$$\dot{x}_2 = \tilde{J}_{31}x_1x_3 + x_3 + \theta(t). \tag{16}$$

Now by considering x_3 as the virtual control, α_2 as the stabilizing function and $z_2 = x_3 - \alpha_2$ be the error variable, equation (16) can be rewritten as:

$$\dot{x}_2 = z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t) + \tilde{J}_{31}x_1x_3.$$

Since $z_1 = x_2 - \alpha_1 = x_2 + x_1$ its dynamics can be written as:

$$\dot{z}_1 = \dot{x}_2 + \dot{x}_1 = z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + \tilde{\theta}(t) + \tilde{J}_{31}x_1x_3 + z_1 - x_1 + \tilde{J}_{23}x_2x_3. \tag{17}$$

To work out α_2 , consider the Lyapunov function: $V_1 = V_0 + \frac{1}{2}z_1^2$ for (14) & (17). Then,

$$\dot{V}_1 = -x_1^2 + z_1(z_2 + \alpha_2 + \hat{\theta}(t) - \theta_{ss} + z_1) + \tilde{J}_{23}(x_1x_2x_3 + z_1x_2x_3) + \tilde{J}_{31}z_1x_1x_3 + z_1\tilde{\theta}(t).$$

By choosing $\alpha_2 = -2z_1 - \hat{\theta}(t) + \theta_{ss}$

$$\dot{V}_1 = -x_1^2 - z_1^2 + z_1z_2 + \tilde{J}_{23}(x_1x_2x_3 + z_1x_2x_3) + \tilde{J}_{31}z_1x_1x_3 + z_1\tilde{\theta}(t).$$

Equation (17) becomes:

$$\dot{z}_1 = z_2 - z_1 - x_1 + \tilde{\theta}(t) + \tilde{J}_{31}x_1x_3 + \tilde{J}_{23}x_2x_3. \tag{18}$$

Consider the equation (3c) and let \hat{J}_{12} be the estimated value of J_{12} and $\tilde{J}_{12} = J_{12} - \hat{J}_{12}$ be the parameter error. The equation (3c) becomes:

$$\dot{x}_3 = \hat{J}_{12}x_1x_2 + \tilde{J}_{12}x_1x_2. \tag{19}$$

Since $z_2 = x_3 - \alpha_2 = x_3 + 2z_1 + \hat{\theta}(t) - \theta_{ss}$ its dynamics can be written as:

$$\dot{z}_2 = \dot{x}_3 + 2\dot{z}_1 + \dot{\hat{\theta}}(t) = \hat{J}_{12}x_1x_2 + \tilde{J}_{12}x_1x_2 + 2z_2 - 2z_1 - 2x_1 + 2\tilde{\theta}(t) + 2\tilde{J}_{31}x_1x_3 + 2\tilde{J}_{23}x_2x_3 + \dot{\hat{\theta}}(t). \tag{20}$$

Consider the Lyapunov function: $V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}^2(t) + \frac{1}{2}\tilde{J}_{23}^2 + \frac{1}{2}\tilde{J}_{31}^2 + \frac{1}{2}\tilde{J}_{12}^2$ for (14), (17) & (20). Then,

$$\begin{aligned} \dot{V}_2 = & -x_1^2 - z_1^2 + z_2(\hat{J}_{12}x_1x_2 + 2z_2 - z_1 - 2x_1 + \dot{\hat{\theta}}(t)) + \tilde{\theta}(t)(z_1 + 2z_2 + \dot{\hat{\theta}}(t)) \\ & + \tilde{J}_{23}(x_1x_2x_3 + z_1x_2x_3 + 2z_2x_2x_3 + \dot{\tilde{J}}_{23}) + \tilde{J}_{31}(z_1x_1x_3 + 2z_2x_1x_3 + \dot{\tilde{J}}_{31}) + \tilde{J}_{12}(z_2x_1x_2 + \dot{\tilde{J}}_{12}). \end{aligned}$$

Replacing $\dot{\tilde{J}}_{23} = -\dot{\hat{J}}_{23}$, $\dot{\tilde{J}}_{31} = -\dot{\hat{J}}_{31}$ & $\dot{\tilde{J}}_{12} = -\dot{\hat{J}}_{12}$.

$$\begin{aligned} \dot{V}_2 = & -x_1^2 - z_1^2 + z_2(\hat{J}_{12}x_1x_2 + 2z_2 - z_1 - 2x_1 + \dot{\hat{\theta}}(t)) + \tilde{\theta}(t)(z_1 + 2z_2 + \dot{\hat{\theta}}(t)) \\ & + \tilde{J}_{23}(x_1x_2x_3 + z_1x_2x_3 + 2z_2x_2x_3 - \dot{\hat{J}}_{23}) + \tilde{J}_{31}(z_1x_1x_3 + 2z_2x_1x_3 - \dot{\hat{J}}_{31}) + \tilde{J}_{12}(z_2x_1x_2 - \dot{\hat{J}}_{12}). \end{aligned}$$

By choosing

$$\begin{aligned} \dot{\hat{\theta}}(t) &= -\hat{J}_{12}x_1x_2 - 3z_2 + z_1 + 2x_1, \\ \dot{\tilde{\theta}}(t) &= -z_1 - 2z_2 - \tilde{\theta}(t), \\ \dot{\hat{J}}_{23} &= x_1x_2x_3 + z_1x_2x_3 + 2z_2x_2x_3, \\ \dot{\hat{J}}_{31} &= z_1x_1x_3 + 2z_2x_1x_3, \\ \dot{\hat{J}}_{12} &= z_2x_1x_2, \\ \dot{V}_2 &= -x_1^2 - z_1^2 - z_2^2 - \tilde{\theta}^2(t). \end{aligned}$$

Equation (20) becomes,

$$\dot{z}_2 = -z_2 - z_1 + \tilde{J}_{12}x_1x_2 + 2\tilde{\theta}(t) + 2\tilde{J}_{31}x_1x_3 + 2\tilde{J}_{23}x_2x_3. \tag{21}$$

The closed loop system becomes:

$$\begin{aligned} \dot{x}_1 &= z_1 - x_1 + \tilde{J}_{23}x_2x_3, \\ \dot{z}_1 &= z_2 - z_1 - x_1 + \tilde{\theta}(t) + \tilde{J}_{31}x_1x_3 + \tilde{J}_{23}x_2x_3, \\ \dot{z}_2 &= -z_2 - z_1 + \tilde{J}_{12}x_1x_2 + 2\tilde{\theta}(t) + 2\tilde{J}_{31}x_1x_3 + 2\tilde{J}_{23}x_2x_3. \end{aligned} \tag{22}$$

Since $x_1, z_1, z_2 \rightarrow 0$ & $\hat{\theta}(t) \rightarrow \theta_{ss}$

$$\begin{aligned} x_2 &= z_1 - x_1 \rightarrow 0, \\ x_3 &= z_2 - 2z_1 - \hat{\theta}(t) + \theta_{ss} \rightarrow 0. \end{aligned}$$

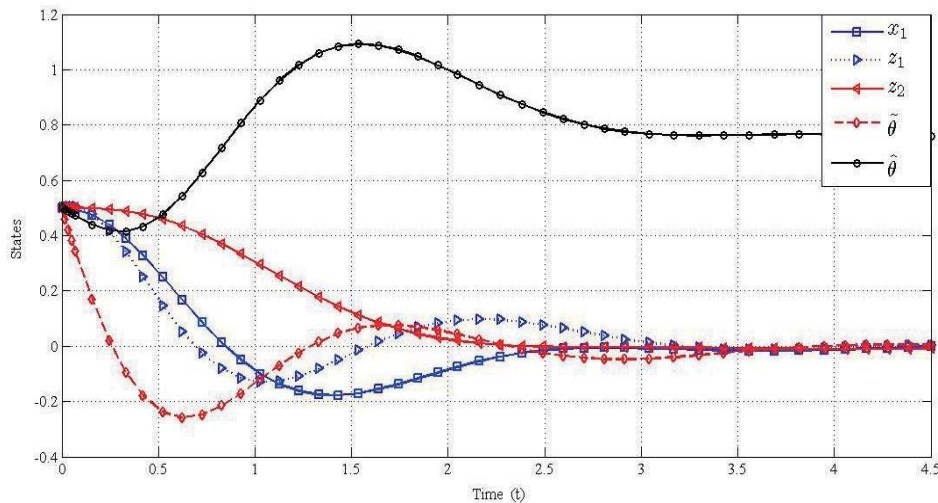


Fig. 2 Simulation results indicating stability of 3 states of two torque actuator system starting with initial condition 0.5 with error function approaching to zero with time.

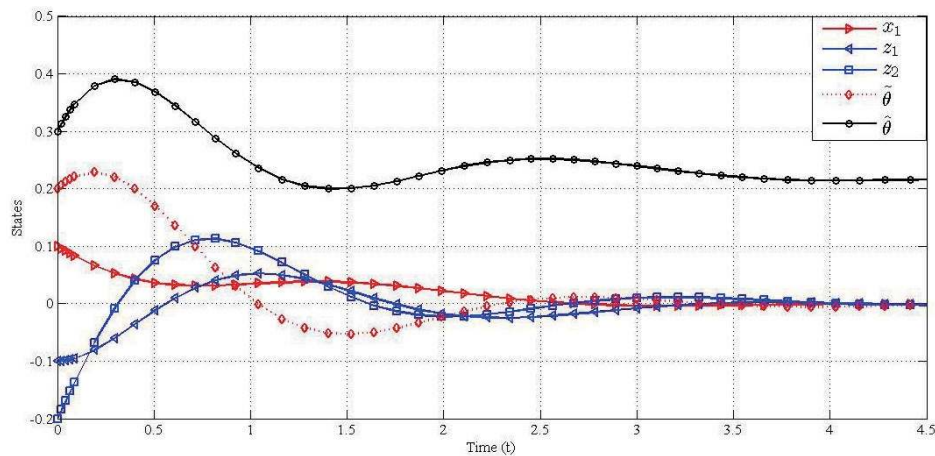


Fig. 3 Simulation results indicating stability of 3 states of two torque actuator system starting with varying initial conditions with error function approaching to zero with time.

5 Results

The model of the rigid body with two torque actuators has been transformed into a closed loop system (12) and (22) for known and unknown parameters respectively using the adaptive backstepping technique. The simulations of the models are given below. It can be seen that all the states of the system are going to zero. The aim was to steer them to a desired value which was assumed to be zero. It is evident from the simulations that the objective has been achieved.

The controller designed above guarantee that in the presence of uncertain bounded nonlinearities the closed loop systems (12) and (22) remains bounded. Simulation results demonstrates that in our proposed method the uncertainties are more specific. They consist of unknown constant parameters which appear linearly in the system equations (3). In the presence of such parametric uncertainties we have achieved both boundedness of the closed loop states and convergence of the tracking error to zero.

Case 2: With unknown parameters

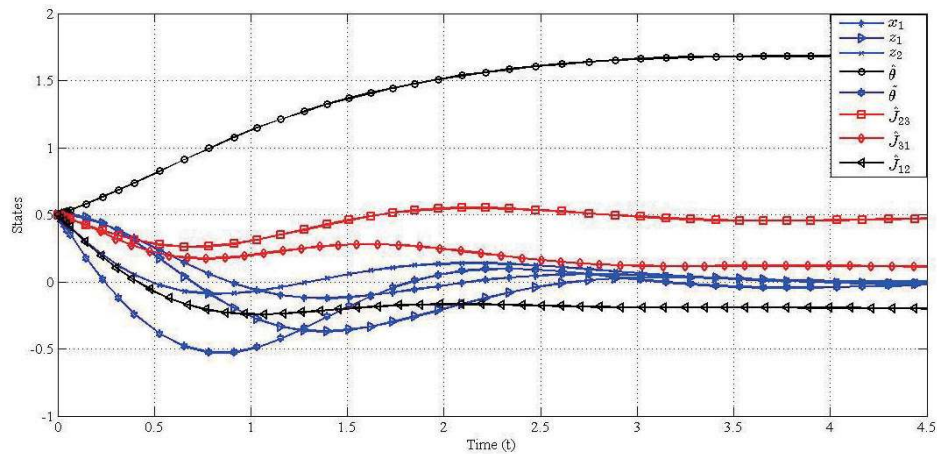


Fig. 4 Simulation results indicating stability of 3 states of two torque actuator system with 4 unknown parameters starting with initial condition 0.5 with error function approaching to zero with time.

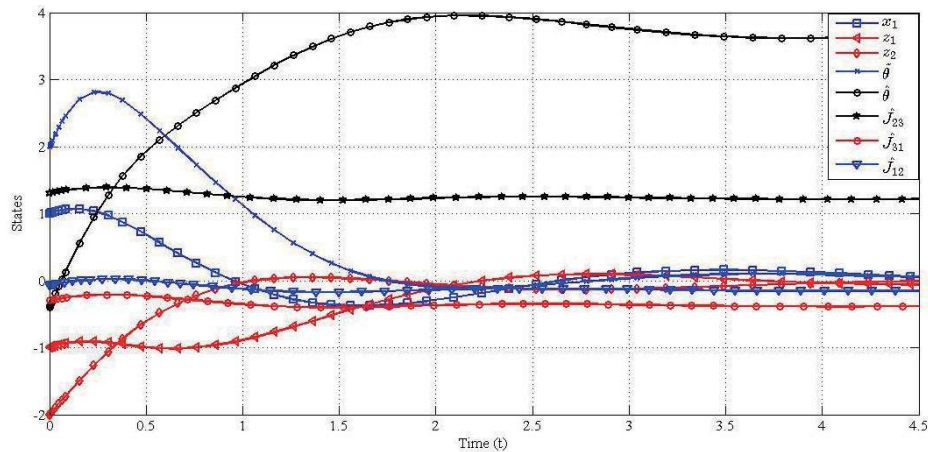


Fig. 5 Simulation results indicating stability of 3 states of two torque actuator system with 4 unknown parameters starting with varying initial conditions with error function approaching to zero with time.

6 Conclusion

In this paper, a systematic method for the construction of steering control for the rigid body model with two torque actuators is introduced without transforming the system into “chain form” using adaptive backstepping technique. The main objective was to steer the system from any initial state to a desired state. The method has been successful in dealing with the control difficulties caused by the uncertainties present in the system. The designed controllers effectively accommodate the parametric changes by processing the output, since the output carries the information of the system’s states hence helpful in achieving the boundedness of the plant state and in tracking problem. The effectiveness of the approach is general and can be applied to any nonholonomic control system with drift. The proposed controller has achieved the desired purpose which is evident from simulation results.

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