

STEFFENSEN'S INEQUALITY FOR POSITIVE MEASURES

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Abstract. We generalize Steffensen's inequality for positive measures. We obtain conditions for these inequalities which are invariant in form to Steffensen's inequality for absolutely continuous measures. Further, we produce linear functionals which generate exponential convexity and Cauchy means.

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