

# STELLAR ANGULAR DIAMETERS AND VISUAL SURFACE BRIGHTNESS—I

## LATE SPECTRAL TYPES

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### SUMMARY

Numerous stellar angular diameters found at occultation permit the relationship between surface brightness and colour index to be defined for stars as late as M8. The best relationship is found for the index ( $V-R$ ) and is well defined for the entire range of stellar temperatures, without dependence on luminosity class. The relationship is valid for M, S and C stars and almost independent of interstellar extinction. It applies to a wide range of variable stars. Angular diameters determined at occultation are closely consistent with those found by other methods and have certain advantages of ease and simplicity. The relationship is compared with other calibrations of effective temperature and bolometric correction. It is conjectured that the relation remains valid during the cycle of each type of variable to which it applies. A first discussion of an application to a Mira is given with promising results. If correct, the conjecture has extremely wide applicability and intensive work on this is in progress.

### I. INTRODUCTION

Using the luminosity formula  $L \sim D^2 T_e^4$ , where  $D$  is the stellar linear diameter, and with the Sun as a calibration point, it is easy to derive the equation

$$\log T_e + 0.1 C = 4.2207 - 0.1 V_0 - 0.5 \log \phi', \quad (1)$$

in which  $T_e$  is the effective temperature,  $C$  the bolometric correction,  $V_0$  the unreddened apparent magnitude in the  $UBV$  system, and  $\phi'$  the stellar angular diameter in arc milliseconds. We define a quantity  $F_v$  as given by the right-hand side of equation (1).  $F_v$  is then linearly related to the visual surface brightness and can be calculated for stars of known angular diameter.

Discussion of a similar parameter by Wesselink (1969) demonstrated a tight correlation between the visual surface brightness and the unreddened colour index  $(B-V)_0$ . For values of  $(B-V)_0$  which were not very large, a parameter like  $F_v$  could be represented as linear in  $(B-V)_0$  to good accuracy. However, only four of the 19 angular diameters available to Wesselink were for stars cooler than F5, so the use of this relation was restricted to early-type stars. On the basis of four additional diameters of cool stars, Warner (1972) argued that the linearity with  $(B-V)_0$  persisted at least to spectral type M2, though Warner omitted some cases ( $\mu$  Gem,  $\alpha$  Her) which would have proved discordant. This was apparently confirmed by Harwood *et al.* (1975) using several recent angular diameters from lunar occultations.

TABLE I  
Observed angular diameters

HR	Star	MK	UT date	$\phi'$ (arc ms)	se (arc ms)	$F_V$	se	Ref.
(a) McDonald occultation diameters								
867	RZ Ari	M6 III	1974 Nov. 28	9.5	2.0	3.146	0.047	1
2286	$\mu$ Gem	M3 IIIa	1974 Feb. 4	13.5	1.4	3.369	0.023	2
	V Cnc	Soe-S4,9e	1973 Apr. 11	2.8	0.8	2.847	0.065	3
3541	X Cnc	C5,4	1971 Nov. 9	9.0	0.8	3.105	0.026	4
3882	R Leo	M6.5e-M9e	1972 May 19	76	5	2.482	0.015	5
4432	87 Leo	K4 III	1973 May 12	3.7	0.4	3.459	0.021	6, 23, 24
5301		gM3	1975 Mar. 29	3.6	0.5	3.449	0.030	7
5824	42 Lib	K4 III	1972 Sep. 13	2.5	0.3	3.525	0.026	22
6134	$\alpha$ Sco	M1 Ib	Numerous	41	2	3.323	0.015	8, 17
6861		M3	1972 Sep. 16	3.6	0.8	3.322	0.049	22
6913	$\lambda$ Sgr	K2 III	1972 Apr. 6	4.4	0.3	3.618	0.015	21
7150	$\xi^2$ Sgr	K1 III	1974 Aug. 28	3.0	1.0	3.631	0.072	1
7900	$\nu$ Cap	M2 III	1972 June 29	4.7	0.5	3.366	0.023	9
8318	47 Cap	M3	1972 Dec. 11	3.2	0.6	3.368	0.042	22
8698	$\lambda$ Aqr	M2 III	1969 Dec. 15	8.2	0.4	3.385	0.011	10
9004	TX Psc	C6, 2	1975 Jan. 18	9.8	0.5	3.225	0.015	11, 19, 20
(b) Other angular diameters								
	Sun	G2 V		$1.92 \times 10^6$		3.753	0.003	12
681	$\circ$ Cet	M5e-M9e	1972 June 25	52	5	3.013	0.029	13
681	$\circ$ Cet	M5e-M9e	1972 Sep. 29	58	5	2.639	0.035	13
1457	$\alpha$ Tau	K5 III	1972 Dec. 21	24	2	3.445	0.018	14
2061	$\alpha$ Ori	M2 Iab	Numerous 1972	54	3	3.313	0.032	13, 14
	YY Gem	Mo.5 V		0.458	0.016	3.407	0.009	15
4127	46 Leo	gM2	1966 May 26	5.6	1.1	3.302	0.043	16
5340	$\alpha$ Boo	K2 IIIp	Numerous 1971-72	24	3	3.535	0.027	14, 17
6406	$\alpha$ Her	M5 II	Numerous 1971	31	3	3.157	0.021	17, 18
8775	$\beta$ Peg	M2 II-III	Numerous 1971-72	18	2	3.351	0.024	14, 17
8834	$\phi$ Aqr	M2 III	1969 Oct. 21	4.9	0.8	3.454	0.035	16

## References to Table I:

- (1) Africano *et al.* (1975).
- (2) Dunham *et al.* (1975).
- (3) McGraw & Angel (1974).
- (4) Bartholdi *et al.* (1972).
- (5) Nather & Wild (1973).
- (6) Dunham *et al.* (1974).
- (7) Unpublished, private communication (DSE).
- (8) Evans (1957).
- (9) Dunham *et al.* (1973).
- (10) Nather *et al.* (1970).
- (11) Dunham *et al.* (1976).
- (12) Allen (1973).
- (13) Bonneau & Labeyrie (1973).
- (14) Currie *et al.* (1974).
- (15) Kron (1952).
- (16) Poss (1971).
- (17) Gezari *et al.* (1972).
- (18) Knapp *et al.* (1974).
- (19) Lasker *et al.* (1973).
- (20) de Vegt (1974).
- (21) Nather (1972).
- (22) Harwood *et al.* (1975).
- (23) Unpublished, private communication from N. M. White,  $\phi' = 3.3$  ms of arc.
- (24) Unpublished, private communication from W. H. Sandmann. His data tape analysed by us gives  $\phi' = 0.0$ , but shows evidence of limb distortion. We have not used the value.

Largely as a result of the activities of the lunar occultation programme at the McDonald Observatory, there is now available a considerable number of measures of angular diameters of red stars. In the present paper we investigate the relations between  $F_v$  and the *UBVRI* colour indices for stars cooler than the Sun. In Paper II (Barnes, Evans & Parsons 1976) we revise Wesselink's work in the light of additional angular diameters of early-type stars.

## 2. OBSERVATIONS

The angular diameters are listed in Table I, where we have shown separately results by the Texas group (both present and former members, some now far afield) and results from other sources. The table shows quite dramatically the importance of the occultation technique. Its power is even more apparent when the telescope apertures are considered. The great majority of the occultation diameters were measured with telescopes of less than 1-m aperture, whereas (apart from the Sun) all but three of the other determinations required the Mt Palomar 5-m telescope.

Following a decision discussed elsewhere (Dunham, Evans & Vogt 1975), we have in every case adopted the hypothesis of complete darkening to the limb in interpreting the occultation measures. This is supported by the limb-darkening calculations of Ridgeway, Wells & Carbon (1974).

For the coolest stars in Table I there is some indication that the diameter may be wavelength dependent. Bonneau & Labeyrie (1973) find a decrease in diameter with increasing wavelength out to  $\lambda \sim 1.04 \mu\text{m}$  for  $\alpha$  Cet and  $\alpha$  Ori. The latter case is disputed by Currie, Knapp & Liewer (1974). Lunar occultation measures

TABLE II  
Photometric data

HR	Star	MK	V	se	B-V	se	V-R	se	R-I	se	Ref.
	Sun	G2 V	-26.74		0.65		0.52		0.29		1
681	o Cet	M5e-M9e	3.5	0.2	1.45		2.55	0.10	2.20	0.10	2, 3, 4
681	o Cet	M5e-M9e	7.0	0.3	1.55		4.00	0.20	3.15	0.20	2, 3, 4
867	RZ Ari	M6 III	5.86	0.10	1.44		2.25	0.18	2.24	0.10	5
1457	$\alpha$ Tau	K5 III	0.86		1.54		1.23		0.94		6
2061	$\alpha$ Ori	M2 Iab	0.42	0.30	1.84		1.64		1.28		6
2286	$\mu$ Gem	M3 IIIa	2.87		1.64		1.57		1.38		6
	YY Gem	Mo.5 V	9.83		1.49		1.39		1.02		7
	V Cnc	So-S4.9e	11.5	0.2	—		3.1	0.2	2.3	0.2	4, 8
3541	X Cnc	C5.4	6.39	0.17	3.09	0.20	2.15	0.20	1.41	0.10	5, 6, 9
3882	R Leo	M6.5-M9e	7.98		1.71		4.2	0.3	3.25	0.3	5, 10, 11, 12
4127	46 Leo	gM2	5.45		1.70		1.37		1.10		5
4432	87 Leo	K4 III	4.78		1.52		1.09		0.85		13
5301		gM3	4.94		1.67		1.44		1.10		13
5340	$\alpha$ Boo	K2 IIIp	-0.05		1.23		0.97		0.65		6
5824	42 Lib	K4 III	4.96		1.33		0.96		0.68		1
6134	$\alpha$ Sco	M1 Ib	0.91	0.10	1.84		1.55		1.23		6
6406	$\alpha$ Her	M5 II	3.18		1.54		2.19		2.16		6
6861		M3	6.20	0.10	1.96		1.92		1.97		13
6913	$\lambda$ Sgr	K2 III	2.81		1.04		0.75		0.56		6
7150	$\xi^2$ Sgr	K1 III	3.51		1.18		0.80		0.59		6
7900	$\nu$ Cap	M2 III	5.19		1.64		1.43		1.14		13
8318	47 Cap	gM3	6.00		1.66		1.41		1.38		13
8698	$\lambda$ Aqr	M2 III	3.79		1.65		1.42		1.19		6
8775	$\beta$ Peg	M2 II-III	2.42		1.67		1.50		1.32		6
8834	$\phi$ Aqr	M2 III	4.22		1.55		1.28		1.08		6
9004	TX Psc	C6, 2	5.00	0.10	2.52	0.10	1.87	0.10	1.35		14, 15

Notes to Table II: Unless otherwise indicated, se =  $\pm 0.03$  mag.

References to Table II

- (1) Allen (1973) (4) Barnes (1973) (7) Veeder (1974) (10) Nather & Wild (1973) (13) Eggen & Stokes (1970)  
 (2) Mattei (private comm.) (5) Eggen (1967) (8) McGraw & Angel (1974) (11) Landolt (1967) (14) Johnson & Mendoza (1965)  
 (3) Mendoza (1967) (6) Johnson *et al.* (1966) (9) Landolt (1966) (12) Eggen (1975) (15) Eggen (1972)

of R Leo (Neugebauer *et al.* 1972) and IRC + 10011 (Zappala *et al.* 1974) disclosed much larger diameters at, and beyond,  $\lambda \sim 10 \mu\text{m}$  than at shorter wavelengths. This is presumably attributable to a circumstellar particle cloud transparent to visible light.

As a precaution against wavelength dependent diameters, we have selected angular diameters as observed near 7000 Å when multiple observations of a star are available. In this manner, the angular diameters drawn from other sources become consistent with the occultation measures, which were generally made near 7000 Å for the coolest stars.

The photometric data are collected in Table II. All the data are on the *UBVRI* system of Johnson (1966) and we have given priority to the values quoted by Johnson *et al.* (1966). In some cases it has been necessary to transform Eggen's narrow-band (102), (102, 65) and (65, 62) system to the *RI* system, in which cases the transformations given by Eggen (1969) were employed. The list includes a considerable number of variable stars for which it has been difficult to establish photometric data at the time of the angular diameter measurement. The reasoning which led to the values adopted in Table II is included in the remarks to certain individual stars which follow.

*o Cet.* Angular diameters at visual phases 0.05 and 0.33 are given by Bonneau & Labeyrie (1973) for a variety of wavelengths. We have interpolated their diameters to  $\lambda = 7000 \text{ Å}$ . Visual magnitudes on the appropriate dates were kindly supplied by Janet Mattei from the records of the American Association of Variable Star Observers. Colour indices were read from phase curves constructed from the photometry by Barnes (1973) and Mendoza (1967). Uncertainties quoted in Table II are estimates based upon the cycle-to-cycle repeatability of the quantities.

*RZ Ari.* This semiregular variable has a small amplitude. Therefore we have simply averaged all the available data to obtain the values and uncertainties in Table II.

$\alpha$  *Ori.* Although the diameters measured by Bonneau & Labeyrie (1973) show a marked trend with wavelength, those of Currie *et al.* (1974) do not. Combining the measures from both groups, we find no significant trend with wavelength to the red of 5000 Å. The value in Table I is therefore the mean of six diameter measurements in the interval 5000–7000 Å. The quoted uncertainty in the visual magnitude is the standard deviation of the values given by Johnson *et al.* (1966).

*YY Gem.* We have adopted Kron's (1952) solution to this eclipsing, spectroscopic binary and a parallax of 0".066 from Jenkins (1963). The photometric data refer to the 'mean component' of the binary, not to the combined light of the nearly identical stars.

*V Cnc.* The colour indices of this extreme S-type Mira variable are very difficult to establish for the phase (0.75) at which the occultation occurred, due to the lack of photometry. McGraw & Angel (1974) estimated ( $V-I$ ) from Wing's (1967) narrow-band, red photometry of V Cnc at other phases and in other cycles. Their value of  $5.1 \pm 0.3$  can be used with Barnes' (1973) data on S-type Mira variables (his Fig. 6) to infer  $(R-I) = 2.1 \pm 0.2$  and  $(V-R) = 3.0 \pm 0.2$ . An alternative method makes use of the similarity of the  $V$  vs  $(R-I)$  and  $V$  vs  $(V-R)$  curves for Mira variables of similar spectral type. By shifting these curves for individual variables to a common origin, defined by the values at visual maximum, a mean relation can be defined for  $\Delta V$  vs  $\Delta(R-I)$  and for  $\Delta V$  vs  $\Delta(V-R)$ . We constructed such relations from Barnes' photometry of R Cam (S2,9e–S8,7e), R Cyg (S3,9e–



S6,8e), R Lyn (S3,9e–S6,8e), and S UMa (So.5,9e–S5,9e), which are similar to V Cnc in spectral type. According to McGraw & Angel, V Cnc had  $V = 11.5 \pm 0.2$  at occultation, which indicates that it was 3.6 mag fainter than at mean visual maximum. Adopting the  $\Delta V$  vs colour curves, we obtain  $\Delta(R-I) = 1.30 \pm 0.20$  and  $\Delta(V-R) = 1.45 \pm 0.20$ . Barnes (1973) gives  $(R-I) = 1.20$  and  $(V-R) = 1.75$  at visual maximum for V Cnc, so we estimate for phase 0.75,  $(R-I) = 2.5 \pm 0.2$  and  $(V-R) = 3.2 \pm 0.2$ . These values agree with the independent estimates from McGraw & Angel; therefore in Table II we give the means of the two methods.

*X Cnc.* This carbon star is a small amplitude semiregular variable. We have averaged all the available photometric data to obtain the values and uncertainties in Table II.

*R Leo.* The uniform disk diameter given by Nather & Wild (1973) was increased by 13 per cent to give an estimate of the fully darkened diameter (Nather, McCants & Evans 1970). The  $V$  and  $(B-V)$  values refer to observations two days after occultation. Unfortunately the  $VRI$  colour indices of R Leo at visual maximum have not been reported in the literature, so the method employed for V Cnc to estimate colour indices at the phase (0.27) of the occultation cannot be used. Instead we have used the less reliable stratagem of adopting the colour index variations of another Mira variable as representative for R Leo. Among the variables with  $VRI$  photometry adequate for this purpose, the most suitable is T Cep.

TABLE III

*Properties of T Cep and R Leo*

	Spectral- type range	Visual amplitude (mag)	Period (days)
T Cep	M5e–M9e	4.3	388
R Leo	M6.5e–M9e	4.2	312

Table III compares the relevant parameters of the two stars as given in the *General catalogue of variable stars* (Kukarkin *et al.* 1969). The colour curves given by Barnes (1973) for T Cep were examined at phase 0.27 after visual maximum to obtain the values in Table II. To check this procedure, we also determined the colour indices for phase 0.83 at which Mendoza (1967) quotes  $UBVRI$  data for R Leo. Our estimates from T Cep agreed with Mendoza's values for R Leo to within 0.20 mag in both colour indices. (One objection which could be raised to this method concerns differential reddening between the two stars. However, using the method discussed in the Appendix to Barnes' paper, we find nearly identical reddening for the two.)

$\alpha$  Her. The two angular diameters determined by modern techniques are discordant. Gezari, Labeyrie & Stachnik (1972) give  $0''.031 \pm 0''.003$  by means of speckle interferometry; whereas Knapp, Currie & Liewer (1975) give  $0''.058 \pm 0''.009$  from amplitude interferometry. We have adopted the former because it agrees with an (uncertain) value determined by Pease (1931) from Michelson interferometry. The photometric data in Table II have been corrected for the effects of the G5 III secondary by assuming Johnson's (1966) colour indices for G5 III and Hoffleit's (1964) magnitude difference between the components, 2.3 mag in  $V$ .

*TX Psc.* According to Eggen (1972) the visual amplitude is quite small for this carbon star. We have averaged all available photometry to obtain the values and uncertainties in Table II. As an aside, we can discount the suggestion by Lasker, Bracker & Kunkel (1973) that *TX Psc* has anomalous light variations, because of the discrepancy between Eggen's visual magnitude of 4.94 and the range quoted in the *General catalogue of variable stars* (Kukarkin *et al.* 1969), 6.6–7.7 mag. As noted in that Catalogue, the range refers to *photographic magnitude*, which is expected to be  $\sim 2.5$  mag fainter than the visual magnitude for this star and to have a larger amplitude.

### 3. THE DEPENDENCE OF $F_V$

Values of  $F_V$  computed from angular diameters and visual magnitudes are shown in Table I where the quoted errors include contributions due to both parameters. In the calculation of  $F_V$  effects of interstellar extinction have been ignored. Both Warner (1972) and Dunham *et al.* (1973) have argued that these effects are likely to be slight even for large extinction values. None of the stars listed in Table I is likely to have  $A_V$  larger than a few tenths of a magnitude and, to put the

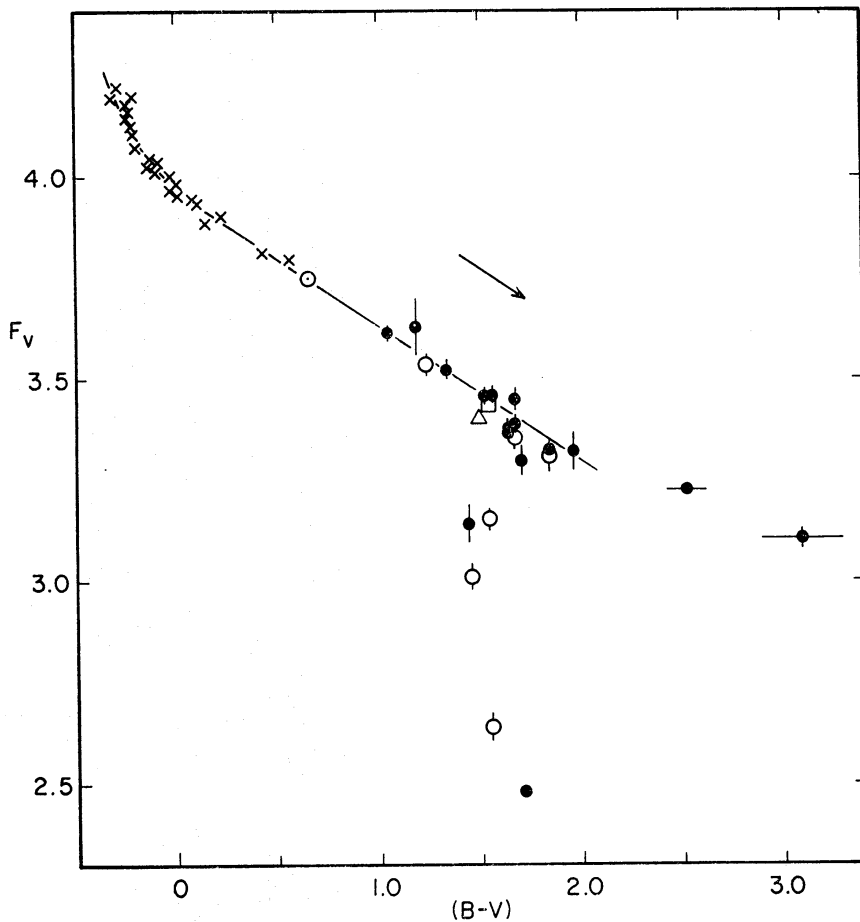


FIG. 1. The surface brightness parameter  $F_V$  as a function of colour index  $(B-V)$ . Symbols indicate the sources of the angular diameters: ●, lunar occultations; ×, intensity interferometry; ○, speckle interferometry; □, amplitude interferometry; △, *YY Gem*; and ○, the Sun. The arrow shows the displacement which would result from 1 mag of visual interstellar extinction.

matter roughly, no star near enough to have a measurable angular diameter is likely to be remote enough to be greatly affected by interstellar extinction. An even more cogent reason is the fortuitous coincidence that (assuming  $R = 3.0$ ) the reddening lines shown in Figs 1–3 are almost exactly parallel to the distributions of points shown there. In establishing the relations discussed below uncertainties in interstellar extinction should present no problem. In the *use* of the relationships for stars at great distances care will be necessary to remove such effects.

Fig. 1 shows  $F_V$  vs  $(B-V)$  from Tables I and II with (by way of illustration) data for stars earlier than the Sun which will be discussed in detail in Paper II. The linearity of the relation discussed by Wesselink (1969) and by Warner (1972) continues to  $(B-V) \sim 1.5$  and thereafter breaks down since  $(B-V)$  ceases to be a significant index of the energy distribution in very cool stars due particularly to TiO blanketing. Nather & Wild (1973) brought the data for R Leo on to the linear relation by applying large (and necessarily uncertain) TiO blanketing corrections to their data. This is not a generally satisfactory procedure though our discussion below may offer a way of computing these blanketing corrections in any given case. The best linear relation derivable from Fig. 1 for solar spectral types is

$$F_V = 3.964 - 0.333 (B-V) \quad -0.88 \leq (B-V) < \sim 1.5 \quad (2)$$

which compares with the corresponding coefficient of 0.365 given by Warner (1972).

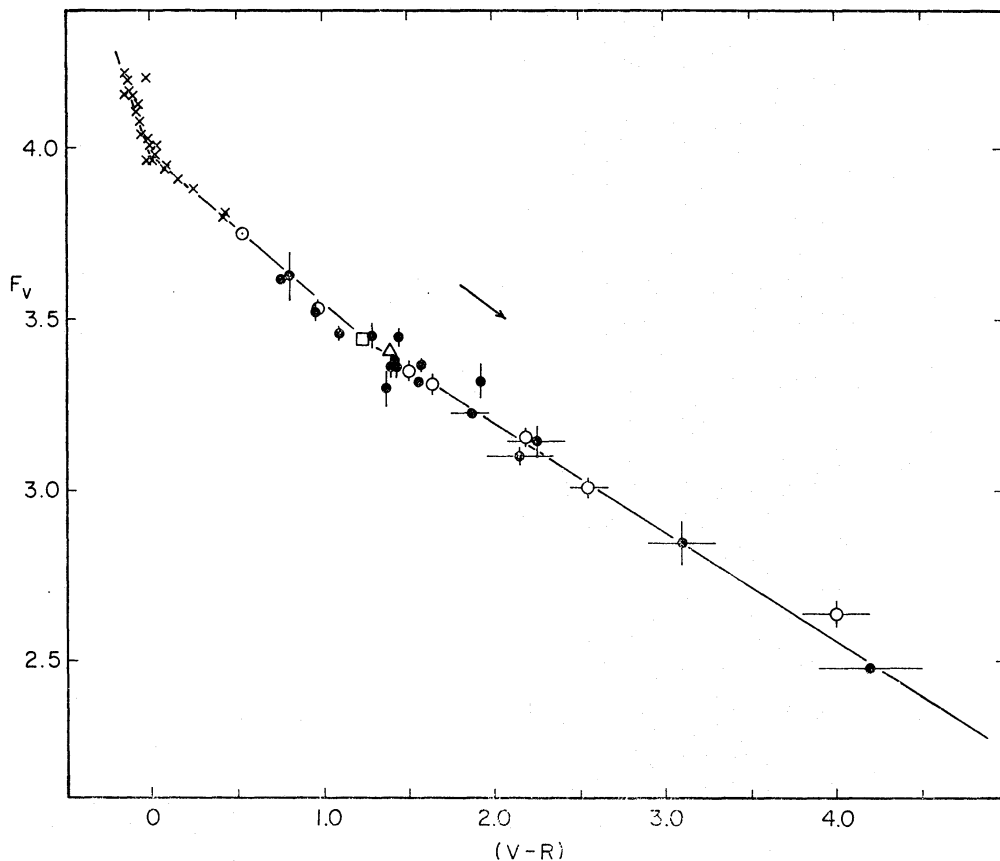


FIG. 2. The surface brightness parameter  $F_V$  as a function of colour index  $(V-R)$ . Symbols are the same as in Fig. 1.



In a search for a relation having a wider range of validity we turn to other colour indices among which we quickly reject ( $U-B$ ) as being not only a worse index of the radiation distribution than ( $B-V$ ) but also liable, especially among Mira variables (Smak 1964) to pronounced contamination by Balmer emission. The function  $F_v$  is plotted against ( $V-R$ ) and ( $R-I$ ) in Figs 2 and 3. These show, especially in the former case a relationship with vastly extended range of linearity and greatly diminished scatter. There is a remarkable accord among the various methods of determination of angular diameters which goes to show that the criticisms of the occultation method by Gold (1953) are quite without foundation.

For stars cooler than the Sun we may fit the data of Figs 2 and 3 by straight line segments:

$$F_v = 3.977 - 0.429(V-R) \quad 0.00 \leq V-R \leq 1.26 \quad (3)$$

$$F_v = 3.837 - 0.320(V-R) \quad 1.26 \leq V-R \leq 4.2 \quad (4)$$

or

$$F_v = 3.824 - 0.386(R-I) \quad 0.42 \leq (R-I) \leq 3.25. \quad (5)$$

The scatter in the ( $R-I$ ) relation, as distinct from the ( $V-R$ ) lines, is not wholly attributable to observational uncertainty. The rms residuals in  $F_v$  are 0.040 for ( $V-R$ ) and 0.082 for ( $R-I$ ). There is no trend with luminosity class in the ( $R-I$ ) residuals which are probably interpretable in terms of molecular blanketing.

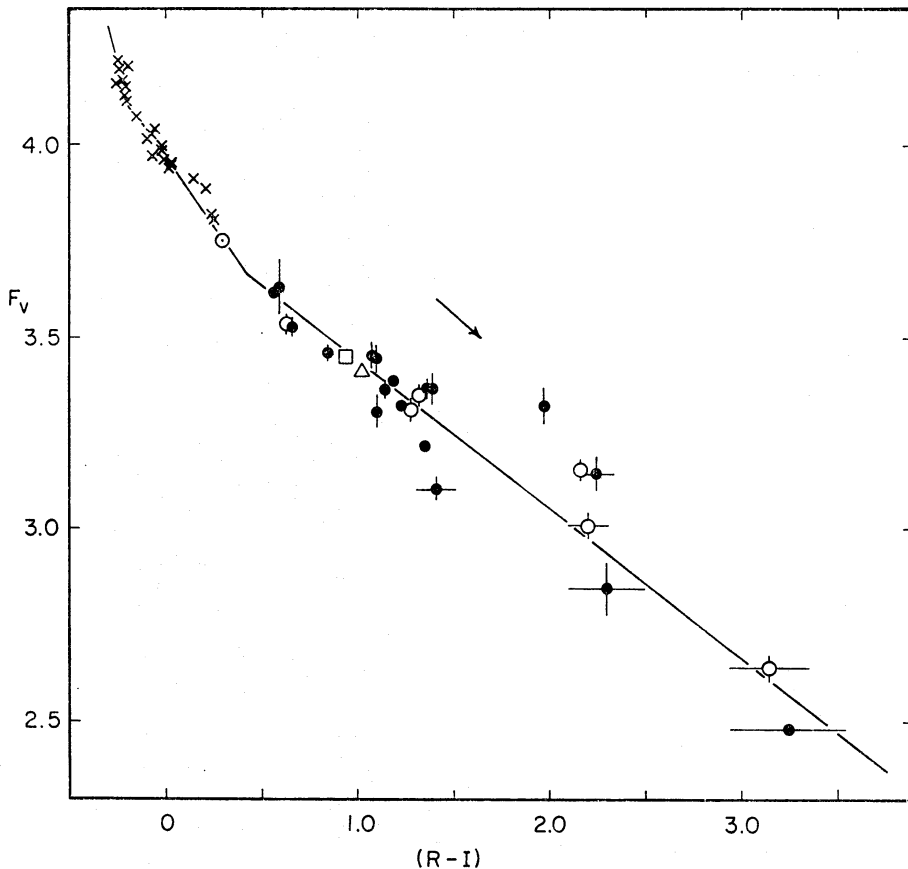


FIG. 3. The surface brightness parameter  $F_v$  as a function of colour index ( $R-I$ ). Symbols are the same as in Fig. 1.

## 4. DISCUSSION

The  $(F_V, V-R)$  relation has a remarkable range of applicability and appears to be valid for carbon stars, S-type stars, M giants including Mira variables, and M supergiants, and the inclusion of the early-type non-degenerate stars in Fig. 4 indicates that there is a smooth relationship between  $F_V$  and  $(V-R)_0$  which applies all the way from O5 ( $\zeta$  Pup) to M8 (R Leo).

The plot includes data obtained at random phases for a considerable variety of types of variable stars and since the relation is so well defined we now conjecture that because of this randomness of phase we may conclude that *these variable stars during their cycles of variation continue to obey the  $(F_V, V-R)$  relation*. The conjecture is supported by the inclusion of two separate points for Mira. Other investigations now under way strengthen the belief that the conjecture is valid, and will be published in due course. For the moment we turn our attention to two topics, namely, the comparison of the values of  $F_V$  and/or angular diameter computed from the empirical relation with previous discussions of the effective temperature and bolometric correction scales, and secondly, assuming the conjecture to be correct, an introduction to the extremely wide field of application which it may open up.

*Comparison with other sources*

Fig. 4 compares values of  $F_V$  from equations (3) and (4) with those derived from  $T_e$  and  $C$  as given by Johnson (1966), Dyck, Lockwood & Capps (1974) and Veeder (1974). The plot includes all stars from Tables 2 and 3 of Dyck *et al.* (1974) for which  $VRI$  magnitudes are available. All the M dwarfs from Veeder (1974) for

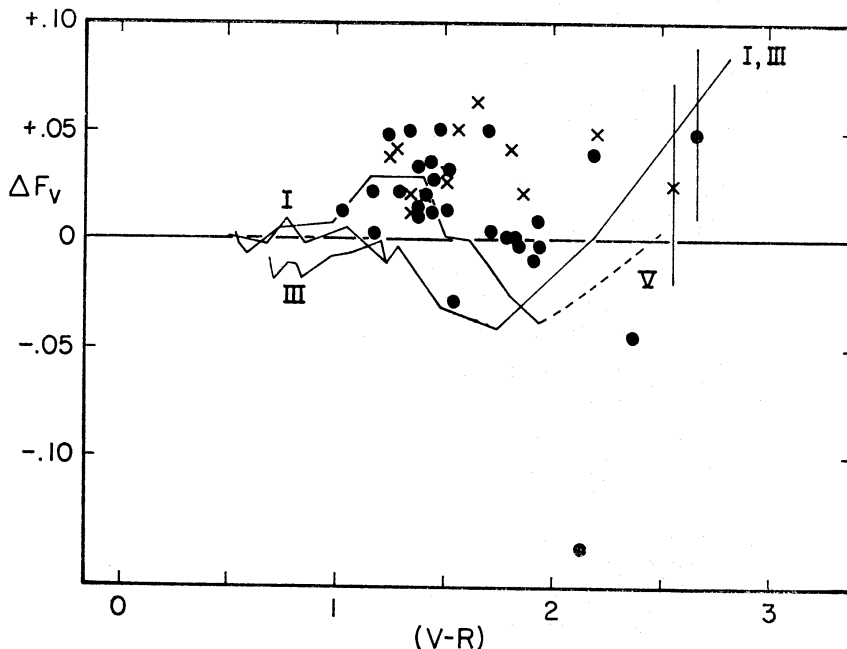


FIG. 4. The surface brightness parameter  $F_V$  computed from  $\log T_e + 0.1C$  minus  $F_V$  computed from equations (3) and (4) plotted against colour index  $(V-R)$ . The solid lines are from Johnson (1966), the crosses from Dyck *et al.* (1974), and the dots from Veeder (1974). Typical uncertainties are shown for the two coolest stars.

which he gives  $VRI$  magnitudes are included, but photometrically unresolved binaries are not, nor are stars for which  $R$  and  $I$  magnitudes were obtained by transformation from the Kron system. The residuals are almost all small, the only systematic trend noticeable being for the  $M_0$ – $M_2$  stars ( $(V-R)_0 \sim 1.3$ ) for which our values of  $F_V$  are about 0.035 smaller than the others. The intersection of the two lines of equations (3) and (4) occurs here and the residuals would be reduced if a smooth transition curve were drawn. On the other hand since nearly half the  $(V-R)$  values for early  $M$  stars come from transformations of narrow-band photometry by Eggen and since he himself (Eggen 1969) has pointed out that uncertainties as large as 0.1 mag can be generated in the process it is best to await photometry on the Johnson system before modifying our presentation. In the same

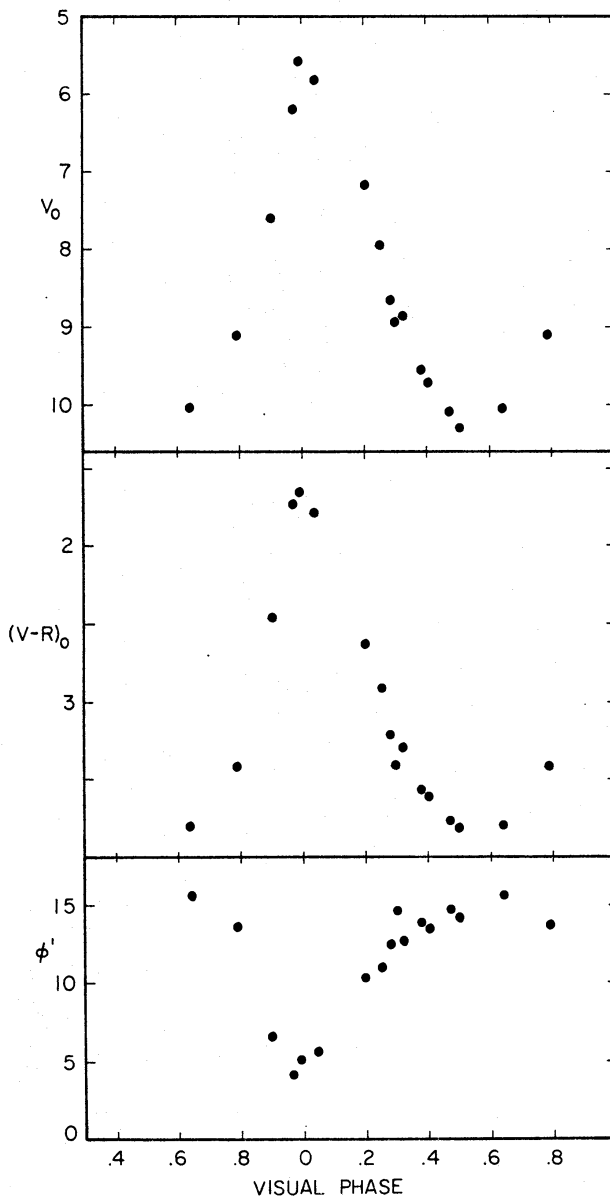


FIG. 5. The visual magnitude, colour index, and angular diameter, in arc milliseconds, of  $R$  Tri plotted against the visual phase.

region and to the same degree of accuracy there is agreement between Veeder's (1974) work on M-dwarfs and our relation.

For stars with  $(V-R) \geq 1.7$  the  $F_v$  values from Dyck *et al.* are higher than ours consistent with their claim that their effective temperature scale is several hundred degrees hotter than previously thought for stars cooler than 3000 K. Their scale is heavily dependent on angular diameters and total fluxes for the two stars  $\alpha$  Cet and R Leo. In the latter case they used the uniform disk angular diameter and the total flux from a cycle during which the star was brighter than that in which the occultation occurred. Both features tend to raise the estimated effective temperature.

#### *Application to cyclic variation of a Mira star*

As an example of the application of the relationship to a single variable, if the conjecture is correct, we take the case of R Tri for which Barnes (1973) has given rather complete photometric data. Corrections for a visual extinction of 0.57 mag have been made, but even if they were omitted the computed diameters would decrease by only 3 per cent. The rms scatter in  $F_v$  about the regression lines of 0.040 leads to a computed angular diameter uncertainty of 18 per cent.

The results for an average light curve derived from several cycles are shown in Fig. 5. The phase of the computed diameter variation relative to the visual light curve agrees with that deduced from near-infrared photometry (Pettit & Nicholson 1933). Minimum diameter occurs at phase 0.05 before visual maximum, and hence about phase 0.15 before bolometric light maximum. The phase of maximum diameter is less well defined in Fig. 5, but occurs near visual minimum.

The diameter semi-amplitude is 57 per cent of the median, considerably larger than the range found by Pettit & Nicholson (1933) for most Mira variables. Because of the large range in  $(V-R)_0$  covered during a cycle, the computed angular diameter range is sensitive to any curvature in the  $(F_v, V-R)$  relation and we may later have to include a slow change of slope at large  $(V-R)_0$  which would decrease the maximum computed angular diameter attained. If we adopt an absolute magnitude for this star of  $-1.8$  from the period-luminosity relation of Clayton & Feast (1969) and an apparent magnitude at mean maximum of 5.7 (after correction for interstellar extinction) from Kukarkin *et al.* (1969), we derive a distance of 300 pc. The median stellar radius is  $317 R_\odot$ . On the crude assumption that the radius varies linearly from minimum to maximum in one-half of the 266-day period, the amplitude of the radial velocity curve is  $34 \text{ km s}^{-1}$ , including limb effect corrections by Parsons (1972). This is slightly larger, but only slightly, than the value expected for a Mira variable. The application of the relationship in this case, assuming the truth of the conjecture, gives very reasonable results. Further work tending to support this position and of very wide application is now in progress.

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