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Stellar angular diameters from infrared photometry. Application to Arcturus and other stars; with effective temperatures

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Summary. A method for determining stellar angular diameters from absolute infrared photometry is described, and an application made to Arcturus and 27 other stars. The accuracy of the method in the best conditions using present observations is about 5 per cent, although higher accuracy is possible. The diameter deduced for Arcturus is 0.0201 ± 0.0010 arcsec, corresponding to an effective temperature of 4410 ± 88 K. For earlier type stars there is good agreement with the results of intensity interferometry. Effective temperatures are calculated for six other stars.

1 Introduction

Atomic oscillator strengths are now being measured at Oxford to an accuracy of at least 5 per cent (Blackwell *et al.* 1975b, 1976a, b). In order to use these accurate oscillator strengths effectively for stellar spectroscopy effective temperatures of improved accuracy for cooler stars are needed, for these are known to little better than 200 K at 4500 K, even for well-studied bright stars. Ideally, an accuracy of 13 K at 4500 K, is needed to match an oscillator strength accuracy of 5 per cent for low-excitation lines (Blackwell & Willis 1977). This order of accuracy is beyond any present techniques, but in this note we suggest that an improvement can be made in the measurement of effective temperature through the use of an angular diameter derived from infrared photometry. Given the angular diameter θ, the effective temperature is derived from the integrated flux from the star at the earth, $\mathscr{F}_E$, through the defining relation $\sigma T_e^4 = 4\mathscr{F}_E/\theta^2$. We now consider the measurement of θ.

2 Measurement of angular diameter

There are many ways of measuring or deducing the angular diameter of a star, but the difficulty lies in determining it to a useful accuracy. If we set an arbitrary target accuracy of 1 per cent for T_e, corresponding to an accuracy of 45 K at 4500 K, then we seek an accuracy of 2 per cent in θ.

Methods for determining angular diameters may be divided into direct (chiefly interferometric) and indirect methods (chiefly photometric). The direct methods, with the

Table 1. Direct methods for measurement of angular diameter.

Method	Authors	Claimed accuracy (per cent)
Michelson interferometer	Pease (1931)	* 8
	Currie, Knapp & Liewer (1974)	27 (for α Boo, 0.020 arcsec)
Intensity interferometer	Hanbury-Brown (1974)	
	Hanbury-Brown, Davis & Allen (1974)	2 (for β Orionis, 0.0024 arcsec)
Speckle interferometry	Gezari, Labeyrie & Stachnik	12 (for β Peg. 0.016 arcsec)
	Bonneau & Labeyrie (1973)	9 (for α Orionis, 0.055 arcsec)
	Worden (1976)	32 (for α Boo, 0.020 arcsec)
Lunar occultations	Nather & Evans (1970)	

* estimated by present authors

accuracy claimed for them, are summarized in Table 1. With the exception of the intensity interferometer, none of these techniques is so far capable of giving results of the required accuracy. There can be little doubt about the generally high order of accuracy of the intensity interferometer, but it is unsatisfying that its results have not so far been confirmed by an independent method of comparable accuracy. The method suffers from the disadvantage that it is suitable only for bright stars of type earlier than about F8. Speckle interferometry does not yet appear to give a sufficient accuracy, even for stars of large angular diameter.

The photometric methods have been summarized by Wesselink, Paranya & Devorkin (1972) in an introduction to a catalogue of some 2400 stellar radii determined photometrically and based on existing measurements obtained for 16 stars by the Michelson interferometer and intensity interferometer. Of the various methods discussed there the most relevant to our purpose is that of Gray (1967, 1968). He points out that the radius of a star is given by $R = D\sqrt{F_{\mathrm{E},\lambda}/F_{\mathrm{S},\lambda}}$ in which D is its distance, $F_{\mathrm{S},\lambda}$ is the monochromatic flux from the star and $F_{\mathrm{E},\lambda}$ the monochromatic flux received at the Earth. He makes use of this expression by calculating the flux $F_{\mathrm{S},\lambda}$ using a model atmosphere with an effective temperature based on measures of spectral distribution of flux, but instead of obtaining an angular diameter, equal to $2R/D$, he uses the relation to derive R from the known distance D.

3 Proposed method: test on the Sun

Our prime aim has been to devise an independent and accurate method for determining angular diameters, and subsequently effective temperatures, that does not need a calibration. Qualitatively, our method is as follows. If the monochromatic flux from a star and the corresponding flux received at the Earth are both known, the angular diameter follows from the relation $\theta = 2\sqrt{F_{\mathrm{E},\lambda}/F_{\mathrm{S},\lambda}}$. In the visible region, the stellar surface flux depends strongly on effective temperature so that an accurate calculation of surface flux is possible only if the temperature is already known accurately. But in the infrared region the surface flux is insensitive to effective temperature. Hence, even a poor approximation to the effective temperature will lead to a good value for the angular diameter. Having obtained a first value for the angular diameter, using a reasonable first approximation to the effective temperature, we now obtain an improved value for the effective temperature using a measurement of the total integrated flux from the star, $\mathscr{F}_{\mathrm{E}}$, and then an improved value of θ from the infrared flux. The total flux is used because the effective temperature is defined by the total flux.

Although the use of a colour may give a greater sensitivity we wish to avoid colours because they are less fundamental. They depend on an uncertain calibration made in the visible region through the use of a model atmosphere, and Blackwell & Willis (1977) have shown that visible, and ultraviolet continuum fluxes cannot be computed accurately using current model atmospheres. Other advantages of working in the infrared are that the blanketing is smaller and the stellar opacity sources more certain, and the interstellar extinction is smaller. The method does require the absolute flux calibration to be independent of a model atmosphere for the standard stars, which would contain assumptions that could lead to a circularity of argument.

A simple example shows that the proposed method is practicable. An error of 210 K in a first approximation to a true temperature of 5500 K (i.e. 4 per cent) would lead to an error in the calculated flux at 4 μm of 4.3 per cent for a normal main sequence star. Hence, neglecting the effect of error in the absolute flux at 4 μm at the Earth, we immediately deduce an angular diameter for the star that is accurate to 2.2 per cent as a first approximation. Supposing, for the sake of example, that the error in the integrated flux is also negligible, the resulting error in the deduced temperature is only 1.1 per cent. The next iteration will give an apparent accuracy of 0.3 per cent, but of course the final accuracy on iteration will be determined by the accuracies of the measured fluxes and the assumption that blanketing in the infrared region may be neglected.

We have tested the method in part by applying it to the determination of the angular diameter of the Sun. In Table 2 we give the flux from the Sun at the Earth at several wavelengths in the infrared, based on the compilation of Allen (1973), which in turn is based chiefly on the work of Labs & Neckel (1968, 1970). This compilation is in reasonable agreement with measurements by Saiedy (1960), Kondratyev *et al.* (1965) and Murcray, Murcray & Williams (1964). The table also gives the solar continuum fluxes calculated using the HSRA model (Gingerich *et al.* 1971), which are assumed to be a good approximation to the observed line blanketed fluxes. The last column gives the calculated angular diameter corresponding to the fluxes at each wavelength. The mean calculated angular diameter agrees with the observed angular diameter to within 1 per cent, with a standard deviation over the eight wavelengths of only 0.9 per cent. Other solar models give a comparable accuracy: for example, the model of Holweger & Muller (1964) gives a diameter of 1921.4 arcsec, which deviates from the true diameter by 0.1 per cent only. This order of agreement is the best that can be expected considering the accuracy with which the infrared solar fluxes are

Table 2. Deduction of angular diameter of the sun from absolute infrared flux.

Wavelength (μm)	Measured flux at Earth (W m^{-2} Hz^{-1})	Calculated flux at Sun (W m^{-2} Hz^{-1})	Calculated solar diameter (HSRA model) (arcsec)
2	1.55 (−12)	7.13 (−8)	1923
2.5	1.08 (−12)	4.92 (−8)	1933
3.0	7.80 (−13)	3.58 (−8)	1926
4.0	4.80 (−13)	2.14 (−8)	1954
5.0	3.22 (−13)	1.41 (−8)	1971
7.0	1.62 (−13)	7.45 (−9)	1924
10.0	8.23 (−14)	3.73 (−9)	1938
13.0	4.93 (−14)	2.23 (−9)	1940

Mean angular diameter = 1939 arcsec.

Observed solar diameter 1919 arcsec (Allen 1973).

known. The solar line blanketing in this region is small (Migeotte, Neven & Swensson 1956, 1957), a principal contributor being the CO molecule at 4.7 μm (Goldman *et al.* 1973), and it does not significantly affect our results.

This solar test does not illustrate the insensitivity of the method to the assumed effective temperature, and we present a test of this kind later in the paper. Meanwhile we assess the usefulness of the method by applying it to Arcturus. Later we apply it to other stars for which infrared data are available.

4 Application to Arcturus: angular diameter and effective temperature

The star Arcturus is relevant to the present work because of the many measurements of its angular diameter and effective temperature that have been made using various methods. It is also currently of great astrophysical interest. We list in Table 3 the primary measures of angular diameter that have been made, together with the authors' stated errors, and the equivalent effective temperatures, supposing that $\mathscr{F}_E = 5.08 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ (Blackwell *et al.* 1975a).

Table 3. Measures of the angular diameter of Arcturus with the corresponding effective temperatures.

Technique	Author	Wavelength (nm)	Diameter (arcsec)	Notes	Effective temperature (K)
A	Pease (1931)	575	0.020	(1)	4419
A	Pease (1931)		0.022	(2)	4213
A	Beavers (1965)		0.024		4034
B	Currie, Knapp & Liewer (1974)	500	0.026 ± 0.007	(3)	3876^{+658}_{-436}
C	Gezari *et al.* (1972)	500	0.022 ± 0.030	(4)	4213^{+321}_{-260}
C	Worden (1976)	420	0.027 ± 0.010	(5)	3803^{+990}_{-554}
C	Worden (1976)		0.019 ± 0.006	(4)	4534^{+974}_{-582}

Technique A Michelson interferometer.
B Amplitude interferometry.
C Speckle interferometry.

Notes:

(1) Probably uniform disk value.
(2) Corrected for limb darkening by present authors (Section 4).
(3) Total uncertainty.
(4) For uniform disk (UD).
(5) For limb darkened disk (LD).

The interferometer measures require a correction for limb darkening, but it is probable that Michelson & Pease (1921) did not apply this correction in their use of the Michelson stellar interferometer, and there is no mention of any correction having been applied to the measures of α Boo reported by Pease (1931). The correction has been evaluated by Dünnweber (1936) in terms of the limb-darkening coefficient β in the expression for the disk intensity at $\mu = \cos\theta$, $I_\mu = I_{0.0}\ (1 + \mu\beta)$. In his study of the stellar temperature scale, Kuiper (1938) followed Pannekoek (1935), and calculated a final limb-darkening correction to the angular diameter of 1.09. We have used model atmospheres of Bell *et al.* (1976) with parameters (T_e = 4500 K, $\log g$ = 1.50, [A/H] = −0.5) to obtain a limb-darkening correction of 1.12, giving a corrected diameter of 0.022 arcsec for α Boo. This diameter has been included in Table 3 with the corresponding effective temperature.

The results of speckle interferometry also need correction for limb-darkening. According to Worden (1976), the value for the limb-darkened disk is 0.027 arcsec against 0.019 arcsec for the uniform disk. Gezari, Labeyrie & Stachnik (1972) give only the value for a uniform disk, and we give the effective temperature corresponding to this in the table. The value for α Boo used in the calibration of the Wesselink Catalogue (Wesselink *et al.* 1972) is 0.025 arcsec.

Table 4. Measures of the temperature of Arcturus by methods that are independent of knowledge of its angular diameter.

Technique	Author	Effective temperature (K)
Continuum distribution	Blackwell & Willis (1977)	* 4400
Photometry	Mäckle *et al.* (1975)	4260
Colours, excitation	van Paradijs & Meurs (1974)	4350
Colours	Williams (1971)	4338
Colours	van Paradijs (1976)	† 4290

* Revision of value of 4500 K proposed from continuum scans (Blackwell *et al.* 1975a).

† Authors' interpolation in $T_e/R - I$ calibration of van Paradijs using $R = -1.02, I = -1.67$ (Gillett *et al.* 1968).

Table 4 shows some values of temperature T derived by methods that are independent of knowledge of the angular diameter. The angular diameter adopted by Wesselink *et al.* for their catalogue, namely 0.025 arcsec, corresponds to $T_e = 3925$ K. If we take $T = 4300$ K as representative of the values in Table 4 we see that there is a discrepancy between this value and the results of Table 3. This discrepancy has been discussed by Blackwell (1975) on a less elaborate basis, and we now apply the proposed method of angular diameter determination in an attempt to resolve it.

We have calculated values for the angular diameter by the method of Section 3 using observed fluxes $F_{E,\lambda}$ at discrete wavelengths between 2 and 13 μm. These fluxes are given in Table 5, together with the surface fluxes for Arcturus, calculated for $T_e = 4000$ and 4500 K using the model atmospheres of Bell *et al.* (1976) with log $g = 1.50$, $[A/H] = -0.5$. The resulting angular diameters are given in the table. It will be seen that the sensitivity of calculated angular diameter to the assumed temperature is very small, being only about 1.2 per

Table 5. Measured and calculated fluxes from Arcturus, and deduction of angular diameter for various assumed temperatures.

Wavelength (m)	Measured flux at earth ($\mathrm{W\,m^{-2}\,Hz^{-1}}$)	Calculated flux at Arcturus ($\mathrm{W\,m^{-2}\,Hz^{-1}}$)		Calculated angular diameter (arcsec)			
		4000 K	4500 K	4000 K	4500 K	4400 K	4260 K
2	1.19 (−22)	4.51 (−8)	5.24 (−8)	0.0212	0.0197	0.0200	0.0204
3	6.21 (−23)	2.34 (−8)	2.69 (−8)	0.0212	0.0198	0.0201	0.0205
4	3.69 (−23)	1.42 (−8)	1.62 (−8)	0.0211	0.0197	0.0200	0.0204
5	2.40 (−23)	9.44 (−9)	1.03 (−8)	0.0208	0.0194	0.0197	0.0202
7	1.36 (−23)	5.05 (−9)	5.73 (−9)	0.0214	0.0201	0.0204	0.0207
10	6.67 (−24)	2.56 (−9)	2.90 (−9)	0.0210	0.0198	0.0201	0.0204
13	4.08 (−24)	1.54 (−9)	1.74 (−9)	0.0212	0.0200	0.0203	0.0206

cent/100 K; this corresponds to a gradient of deduced T_e of only 0.6 per cent/100 K. The last two columns give interpolated values of the angular diameter for the two assumed values of T_e of 4400 K (Blackwell & Willis 1977) and 4260 K (Mäckle *et al.* 1975). The mean diameters at these two temperatures, 0.0201 and 0.0205 arcsec respectively, differ by two per cent only.

The deduced values are also insensitive to the details of the model atmosphere used. For example, use of the Mäckle atmosphere (Mäckle *et al.* 1975) having T_e = 4260 K and log g = 0.9 gives the same value, 0.0205 arcsec, as an interpolation between results from the Bell atmospheres having T_e = 4000 and 4500 K. Use of a Peytremann atmosphere having T_e = 4500 K and $\log g = 1.0$ (Cayrel 1977, private communication) gives 0.0196 arcsec against the corresponding value of 0.0198 arcsec for a Bell atmosphere with the same value of T_e but with $\log g = 1.5$.

The angular diameters interpolated for T_e = 4400 K using the Bell atmospheres are plotted against wavelength in Fig. 1, which also shows the diameters given in Table 3. The constancy of diameter with wavelength is remarkable, showing not only that the observed relative spectral energy distribution is reliable, but the model atmosphere approach adopted here is correct. The average value is 0.0201 ± 0.0010 arcsec, the probable error of 5 per cent corresponding to an assumed probable error of 10 per cent in the absolute flux measured at the Earth. This value differs greatly from most of the directly measured angular diameters corrected for limb darkening given in Table 3. The nearest is the original measure of Pease, 0.020 arcsec, although it is not known whether this value has been corrected for limb-darkening. The effective temperature corresponding to our value of θ is 4410 ± 88 K. In estimating the probable error we note that T_e is calculated from the relation

$$T_e^4 = \frac{1}{\sigma}\,\frac{\mathscr{F}_E}{F_{E,\lambda}}\cdot F_{S,\lambda},$$

where $\mathscr{F}_E$ is the observed integrated flux. For a cool star, where the infrared flux predominates, $\mathscr{F}_E$ and $F_{E,\lambda}$ are equally affected by any error in absolute calibration. We judge that the probable error in the ratio $\mathscr{F}_E/F_{E,\lambda}$ is no more than 8 per cent so that the corresponding error in T_e is about 2 per cent, i.e. 88 K. The effective temperature deduced in the present work is in good agreement with the value of 4400 K obtained from the continuum distribution by Blackwell & Willis (1977).

We have assumed in this analysis of Arcturus that the observed fluxes can be represented

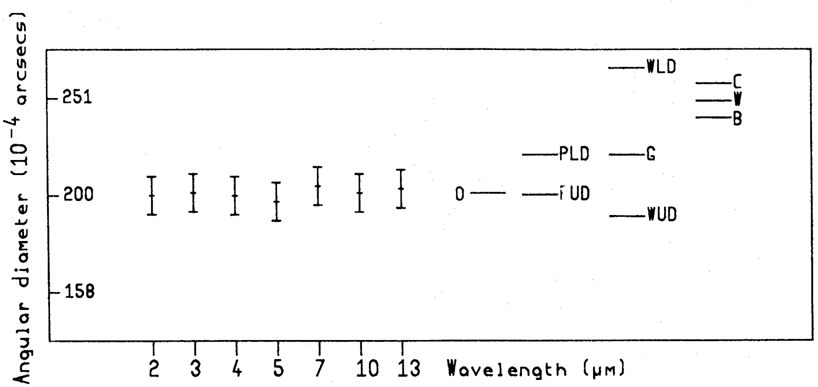

Figure 1. Deduction of angular diameter of Arcturus at a set of infrared wavelengths from measured fluxes. Other measurements of angular diameter are marked on the diagram: PUD, PDL, Pease (1931), uncorrected and corrected for limb darkening; W. Wesselink *et al.* (1972); WUD, WLD, Worden (1976); G, Gezari, Labeyrie & Stachnik (1972); C, Currie *et al.* (1974); Beavers (1965). The angular diameter scale is logarithmic.

by calculated continuum fluxes from a model atmosphere, and that there is no significant disturbance by atomic or molecular absorption. The constancy of angular diameter with wavelength, and the absence of an increase near 10 μm which could be attributed to a circumstellar dust cloud and possibly a chromospheric contribution (Lambert & Snell 1975), both confirm this assumption. It is also confirmed by direct observation of spectra, by Johnson & Mendez (1970) as far as 2.5 μm and by Geballe, Wolbman & Rank (1972) in the region of 4.7 μm. The spectra obtained by Gillett, Low & Stein (1968) over the range 3–13 μm do not show any significant features.

5 Application to other stars and comparison with the results of intensity interferometry: effective temperatures

We have checked the angular diameters obtained using the present method against diameters given by the intensity interferometer (Hanbury-Brown, Davis & Allen 1974) for seven of the stars in their list for which infrared data are available. These stars are listed in Table 6, together with adopted measures of infrared flux. α Vir is omitted from the final list of results (Table 8) because it is a spectroscopic binary, and the present method fails for a composite star.

For an initial working value of the temperature of each star we have used the measures of Schmidt (1972), Schild, Peterson & Oke (1971) and Morton & Adams (1968), and the scale of Johnson (1966). The adopted values of T_e are given in Table 6. These values are not claimed as accurate; they are first working values only to start the iteration process. To calculate the surface infrared fluxes we have used chiefly the models of Bell *et al.* (1970) supplemented by those of Carbon & Gingerich (1969), Klinglesmith (1971) and Peytremann (1974). Measured values of infrared flux at the Earth have been taken from the work of Johnson (1964), Gillett *et al.* (1968), Gillett, Merrill & Stein (1971), Gehrz & Woolf (1971) and Thomas, Hyland & Robinson (1973), and from the absolute calibration of Johnson (1965, 1966) which is not based on model atmospheres. Adopted values of observed flux are given in Table 6.

As the deduced angular diameter is insensitive to the adopted temperature, the first working temperature already gives diameters accurate to a few per cent, assuming that the infrared fluxes are error-free. For example, for α Lyr a change in T_e of 500 K leads to a 2.5 per cent change in angular diameter, and the same temperature change for β Ori leads to only 0.3 per cent change in angular diameter. An improved angular diameter is now derived by combining the first approximation with the measured integrated flux to give an improved value for the effective temperature. This effective temperature then replaces the first working value to give an improved diameter.

Table 6. Adopted effective temperatures and infrared fluxes for those stars whose angular diameters obtained by the present method are compared with diameters measured by the intensity interferometer.

* B.S.No.	Adopted T_e(K)	Adopted fluxes (W m^{-2} Hz^{-1}) 2μm	3μm	4μm	5μm	7μm	10μm
1713	12 300	6.28 (−24)	3.01 (−24)	1.81 (−24)	1.20 (−24)	6.61 (−24)	3.91 (−25)
2326	6 904	1.61 (−23)	1.23 (−23)	7.83 (−24)	5.45 (−24)	3.07 (−24)	1.52 (−24)
2491	10 200	2.59 (−23)	1.34 (−23)	8.26 (−24)	5.59 (−24)	2.94 (−24)	1.44 (−24)
2693	6 199	–	2.84 (−24)	1.94 (−24)	1.42 (−24)	8.28 (−25)	4.38 (−25)
2943	6 300	1.29 (−23)	7.42 (−24)	4.50 (−24)	3.03 (−24)	1.70 (−24)	9.17 (−25)
5056	22 600	–	–	5.86 (−25)	4.14 (−25)	1.96 (−25)	7.98 (−26)
7001	9 650	7.11 (−24)	3.84 (−24)	2.40 (−24)	1.72 (−24)	9.23 (−25)	4.37 (−25)

* Star names are given in Table 7.

The integrated fluxes that we have used are based on the work of Code *et al.* which combines ultraviolet fluxes obtained using *OAO-2* data (Code *et al.* 1976), visual fluxes measured by Davis & Webb (1974) taking the flux from α Lyr at λ 550 nm as 3.47×10^{-9} erg cm^{-2} s^{-1} Å^{-1} (3.50×10^{-23} W m^{-2} Hz^{-1}) and infrared fluxes from sources similar to our own. We have modified the Code *et al.* integrated fluxes by combining them with more recent data in the UV obtained from the Sky Survey Telescope (32/68) on the European Astronomical Satellite *TD-1* (*Jamar et al.* 1976) over the region 130–180 nm and adopting the new absolute calibration of α Lyr in the visible region by Hayes & Latham (1975) giving the flux at λ 550 nm of 3.39×10^{-9} erg cm^{-2} s^{-1} Å^{-1} (3.42×10^{-23} W m^{-2} Hz^{-1}). The modified fluxes are given in Table 8. They are all about 2 per cent smaller than the fluxes of Code *et al.*

The iteration procedure may be illustrated by the example of δ CMa. An observed infrared flux combined with the first approximation for temperature (6199 K) gives an angular

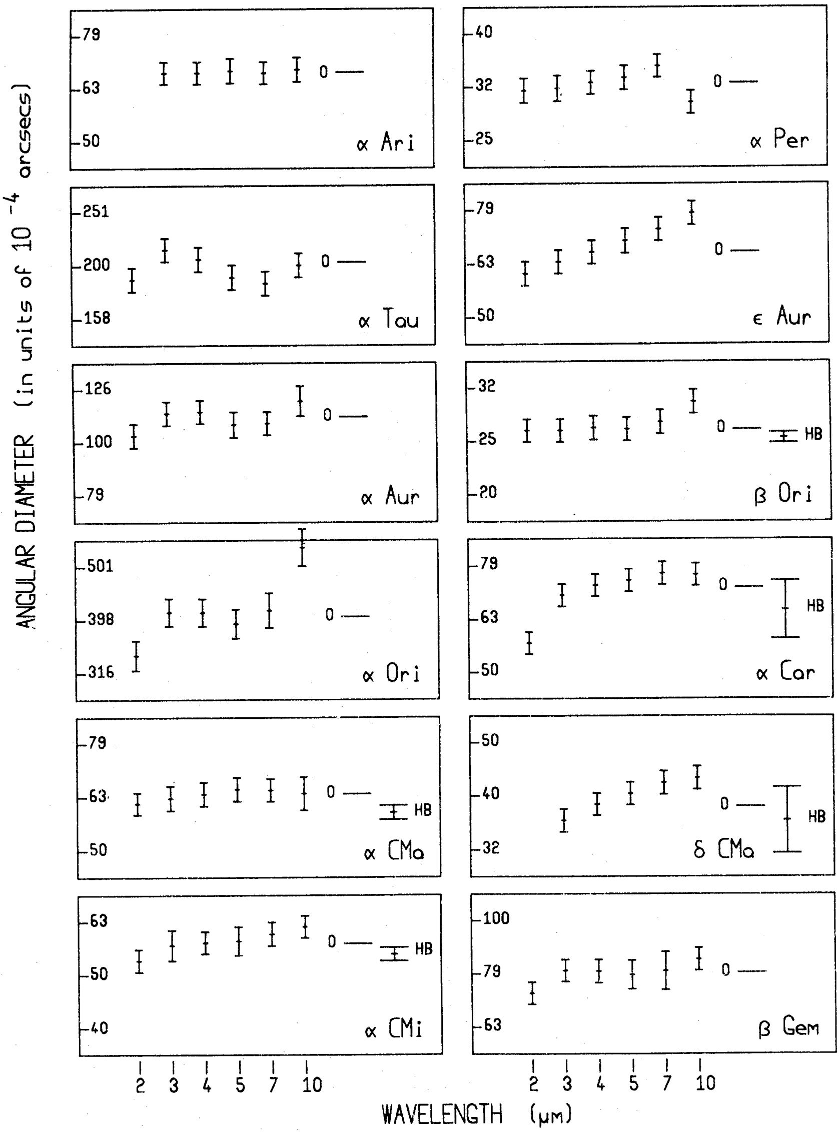

Figure 2 (a)

Figure 2 (a and b). Deduction of angular diameters of a group of stars at various infrared wavelengths. The mean diameters for wavelengths 3, 4 and 5 μm are denoted by O (Oxford), and where available the intensity interferometer values are also given, denoted HB (Hanbury-Brown), with their stated errors. The angular diameter scales are logarithmic.

diameter of 37.9 x 10^{-4} arcsec. The observed integrated flux of 6.0 x 10^{-9} W m^{-2} Hz^{-1}, combined with this angular diameter gives an improved temperature of 5954 K. This temperature when combined with the observed infrared flux gives a diameter of 38.7 x 10^{-4} arcsec. The fourth approximation to the temperature is 5877 K, and the corresponding approximation to the diameter is 38.9 x 10^{-4} arcsec.

Values of angular diameter calculated in this way for each wavelength for which there is an infrared flux are displayed in Fig. 2. The diagrams for these stars are of variable quality. If, as we suppose, the infrared flux has a thermal origin, the deduced angular diameter should be independent of wavelength, as indeed it is for Arcturus. Any change with wavelength is attributable to: (1) a circumstellar dust shell giving an apparent increase in angular diameter in the region of 10 μm, (2) blanketing due to line absorption, which will give an apparent decrease in angular diameter in the blanketed region, (3) an incorrect model atmosphere, (4) interstellar reddening, which will give an upward slope towards longer wavelengths. Also, errors in the adopted values of the infrared fluxes could give some irregularity especially as fluxes at different wavelengths have sometimes been taken from different sources of data.

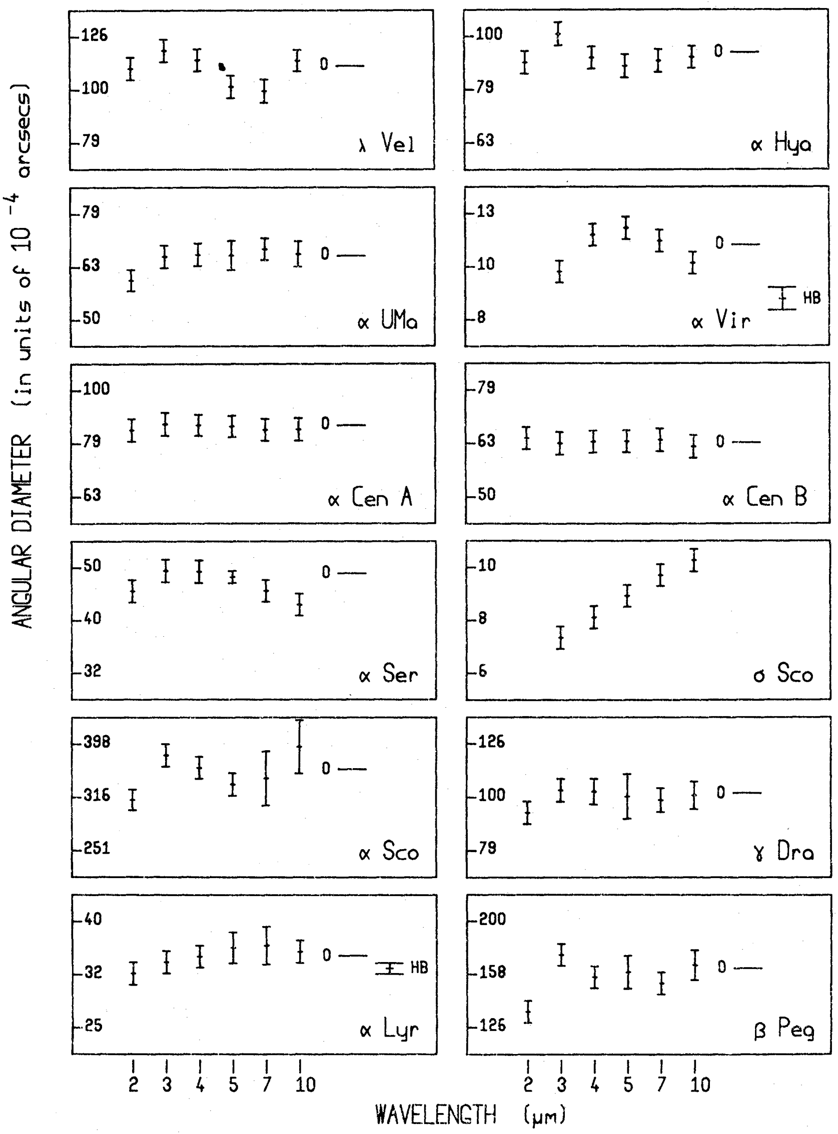

Figure 2 (b)

Of the six stars in common with Hanbury-Brown *et al.* the one with the most straightforward diagram is that of β Orionis, which shows a constant value of θ with a slight rise in the 7–10 μm region, presumably due to circumstellar dust. The next is α CMa which gives a clearly defined angular diameter with no indication of circumstellar dust. The remaining four stars of this group, in order of ease of interpretation, are α CMi, α Lyr, α Car, δ CMa. In order to obtain an objective value of the angular diameter, we have chosen to average data for the wavelengths 3, 4 and 5 μm; data at 2 μm are rejected in taking a mean because of possible blanketing and interstellar reddening, and those at 7 and 10 μm are also rejected because of possible circumstellar dust. Studies of stellar infrared spectra (e.g. Johnson & Mendez 1970; Gillett *et al.* 1968; Woolf, Schwarzschild & Rose 1964; Geballe *et al.* (1972) and Wing & Spinrad 1970) indicate that, except in some extreme cases, there will not be large errors due to line absorption, but at 2 μm we might expect some absorption due to CN and C_2 for cooler stars. The diagrams of Fig. 2 suggest that this is so because, apart from α Car which is probably exceptional, the largest decrease at 2 μm is associated with late-type stars. The resulting diameters are given in Table 7. The estimated errors are based on the spread of points on the diagrams of Fig. 2 and an absolute calibration error of 5 per cent. This table also contains measures made with the intensity interferometer, and measures taken from the Wesselink Catalogue. A plot giving the relationship with the interferometer measures is shown in Fig. 3. This plot and Table 7 show a good agreement between the results from the two techniques, the average discrepancy between the two being 7.5 per cent. The claimed accuracies are comparable. However, a systematic effect may arise because Hanbury-Brown *et al.* (1974) have used a limb-darkening correction calculated for continuum only without absorption lines. The accuracy of the present method depends primarily on the accuracy of the absolute flux measures, and it could be that the systematic differences between the two techniques are due to an error in the absolute calibration of the fluxes. If the flux measures were reliable absolutely to 5 per cent then the accuracy of the present method would rival that of the intensity interferometer, or be even more accurate. More importantly, this comparison shows that the method gives good results for hotter stars, for which a direct comparison with the intensity interferometer is possible, and it is very likely therefore that it will give accurate results for cooler stars, for which no comparable method is available.

Table 7. Stellar angular diameters derived by present technique compared with those obtained by the intensity interferometer and other methods.

B.S. No.	Name	Spectral type	Angular diameter (unit 10^{-4} arcsec) Oxford *	intensity interferometer †	Wesselink ‡	Gray §
1713	β Ori	B8Ia	26.7 ± 0.8	25.5 ± 0.5		
2326	α Car	F0Ib-II	70.8 ± 1.9	66 ± 8	60	
2491	α CMa	AIV	65.7 ± 1.8	58.9 ± 1.6		61.2
2693	δ CMa	F8Ia	38.9 ± 5	36.0 ± 5	39	
2943	α CMi	F5IV-V	57.3 ± 1.6	55.0 ± 1.7	57.6	
7001	α Lyr	AOV	34.9 ± 1.0	32.4 ± 0.7		33.6

* Present work.
† Hanbury-Brown *et al.* (1974).
‡ Wesselink *et al.* (1972).
§ Gray (1967, 1968).

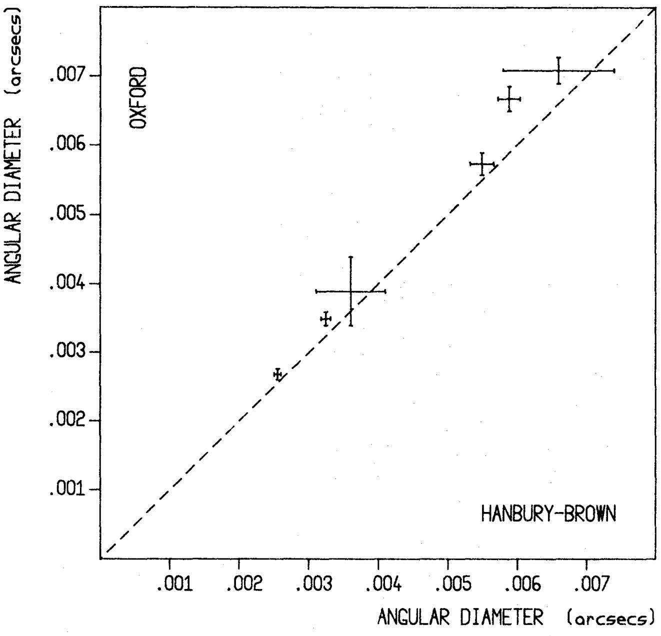

Figure 3. Relation between angular diameters of six stars as determined by the intensity interferometer (Hanbury-Brown), and by the present technique (Oxford).

Effective temperatures derived from the angular diameters and the revised integrated fluxes are given in Table 8, together with comparisons with the effective temperatures derived by Code *et al.* (1976) from the diameters determined with the intensity interferometer. The new temperatures are lower than the comparisons, partly because of the downward revision of the integrated flux but chiefly because the Oxford diameters are larger than the interferometer diameters.

6 Application to cooler stars

We now proceed to cooler stars for which there are no universally accepted standard diameters. The infrared flux data and deduced diameters are given in Table 9 and 10, although iteration is not possible because of the lack of integrated flux data. A selection of diagrams is included in Fig. 2. Some of these, e.g. α Ari and α Cen A & B, are of good quality

Table 8. Effective temperatures derived from Oxford angular diameters compared with temperature, derived by Code *et al.* (1976) from angular diameters determined with the intensity interferometer. The integrated fluxes are revisions of those of Code *et al.*

Star	Integrated Flux (unit 10^{-9} W m^{-2})	Effective temperature Oxford *	Code *et al.*
β Ori	37.1 ± 1.7	11 180 ± 206	11 550 ± 170
α Car	45.0 ± 1.8	7 206 ± 173	7 460 ± 460
α CMa	113.2 ± 4.4	9 349 ± 160	9 970 ± 160
δ CMa	6.0 ± 0.3	5 877 ± 390	6 110 ± 430
α CMi	18.0 ± 0.8	6 359 ± 113	6 510 ± 130
α Lyr	30.1 ± 1.2	9 282 ± 232	9 660 ± 140

* Present work.

Table 9. Adopted effective temperatures and infrared fluxes.

B.S.No. *	Adopted T_e(K)	Adopted fluxes (W m^{-2} Hz^{-1}) 2μm	3μm	4μm	5μm	7μm	10μm
617	4400	–	–	4.30 (–24)	2.90 (–24)	1.52 (–24)	7.80 (–25)
1017	6520	4.55 (–24)	2.34 (–24)	1.46 (–24)	1.01 (–24)	5.92 (–25)	3.82 (–25)
1373	4760	1.65 (–24)	1.00 (–24)	6.97 (–25)	5.50 (–25)	3.96 (–25)	3.27 (–25)
1457	3820	8.90 (–23)	5.98 (–23)	3.32 (–23)	1.90 (–23)	9.66 (–24)	5.73 (–24)
1577	4340	1.14 (–23)	8.51 (–24)	6.15 (–24)	4.49 (–24)	2.54 (–24)	1.51 (–24)
1605	6904	1.82 (–23)	1.02 (–23)	6.62 (–24)	4.79 (–24)	2.81 (–24)	1.62 (–24)
1708	4850	3.63 (–23)	2.19 (–23)	1.32 (–23)	7.91 (–24)	4.31 (–24)	2.62 (–24)
2061	3600	2.77 (–22)	2.05 (–22)	1.25 (–22)	7.44 (–23)	4.57 (–23)	3.99 (–23)
2990	4760	1.86 (–23)	1.16 (–23)	6.91 (–24)	4.47 (–24)	2.45 (–24)	1.38 (–24)
3634	3820	3.02 (–23)	1.81 (–23)	1.03 (–23)	5.36 (–24)	2.81 (–24)	1.86 (–24)
3748	3960	2.08 (–23)	1.39 (–23)	6.89 (–24)	4.24 (–24)	2.36 (–24)	1.24 (–24)
4301	4760	1.20 (–23)	7.45 (–24)	4.54 (–24)	2.97 (–24)	1.67 (–24)	8.83 (–25)
5459	5800	2.95 (–23)	1.56 (–23)	9.25 (–24)	6.11 (–24)	3.10 (–24)	1.58 (–24)
5460	4580	1.35 (–23)	6.59 (–24)	3.97 (–24)	2.65 (–24)	1.43 (–24)	6.71 (–25)
5854	4400	6.24 (–24)	3.91 (–24)	2.30 (–24)	1.45 (–24)	6.81 (–25)	3.03 (–25)
6084	28 600	–	5.13 (–25)	3.42 (–25)	2.55 (–25)	1.50 (–25)	8.39 (–26)
6134	3600	2.32 (–22)	1.74 (–22)	9.52 (–23)	5.35 (–23)	3.10 (–23)	2.09 (–23)
6705	3820	2.19 (–23)	1.38 (–23)	8.34 (–24)	5.29 (–24)	2.77 (–24)	1.46 (–24)
7924	8480	–	–	1.19 (–24)	7.59 (–25)	3.63 (–25)	3.20 (–25)
8775	3600	4.68 (–23)	3.56 (–23)	1.81 (–23)	1.23 (–23)	6.04 (–24)	3.68 (–24)

* Star names are given in Table 10.

Table 10. Comparison between stellar angular diameters obtained using the present technique with those obtained by use of other techniques.

B.S.No.	Name	Spectral type	Angular diameter (unit 10^{-4} arcsec) Oxford	Wesselink	Others		
617	α Ari	K2III	67.8 ± 0.8	87			
1017	α Per	F51	32.2 ± 1.8	29			
1373	δ Tau	K0III	26.2 ± 18	29			
1457	α Tau	K5III	203 ± 14	–	240(C)	200(P)	
1577	ι Aur	K3II	80.3 ± 3.0	110			
1708	α Aur	G8III	112 ± 5.8	–			
2061	α Ori	M21	405 ± 30	–	530(C)	410(P)	
2990	β Gem	K0III	80 ± 2.9	–			
3634	λ Vel	K51	111 ± 8.0	–			
3748	α Hya	K4III	93.1 ± 3.9	140			
*4301	α UMa	K0III	66.4 ± 2.1	–			
5459	α CenA	G2V	86.2 ± 2.3	–			
5460	α CenB	K1III	63.5 ± 1.6	–			
5854	α Ser	K2III	49.1 ± 3.0	–			
*6134	α Sco	M1I	357 ± 27	–	400(P)	420(S)	390(E)
6705	γ Dra	K5III	102 ± 4.3	–			
7924	α Cyg	A2I	25.4 ± 8.0	–			
8775	β Peg	M2II–III	163 ± 12	–	210(P)	160(S)	210(C)

Oxford: present work.

(C) Currie *et al.* (1974).
(P) Pease (1931).
(S) Speckle interferometry.
(E) Evans (1955).

* Binary system.

yielding accurate diameters. Others, for example ϵ Aur and σ Sco, show a steep slope which is probably due to an extinction decreasing with wavelength, especially as the Palomar Sky Survey shows ϵ Aur to be on the edge of a region of heavy obscuration and σ Sco is in a region of strong nebulosity. This obscuration precludes reliable diameters being obtained for these two stars and we do not include them in Table 10. The star δ CMa shows a weak gradient, and it is also situated in a weakly obscuring region. Other stars such as α Sco show a large variation of θ with wavelength, probably due to inaccurate flux measurements.

We have in general excluded very-low-temperature stars ($T_e \leqslant 3600$ K) because of the likelihood of line blanketing and absence of precise information about it, and also because of the lack of reliable model atmospheres for these stars. For the three stars α Boo, α Tau and β Peg, that are discussed by Tsuji (1976), we find qualitative agreement with his model atmosphere analysis, in that our angular diameters are smaller than those measured by other techniques, which leads to higher values of T_e as Tsuji suggests.

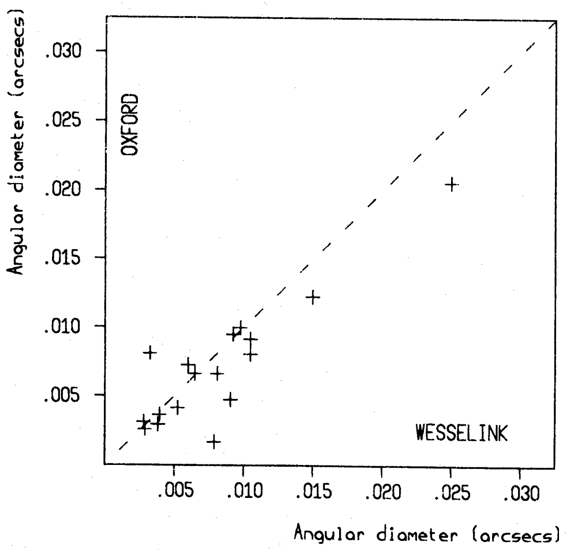

Figure 4. Relation between angular diameters of stars as deduced from the present method with their values taken from the catalogue of Wesselink (Wesselink *et al.*, 1972).

A comparison between all Oxford diameters with those in the Wesselink Catalogue is shown in Fig. 4. In view of the good agreement with the results for intensity interferometry it is likely that the large spread on this diagram is attributable to errors in the Wesselink diameters.

We do not extend this analysis beyond 10 μm, although a large body of data exists, because of the possibility of thermal emission from circumstellar dust at these wavelengths.

Conclusions

Judging by the comparisons that we have made with the results of intensity interferometry the method appears to be capable of a good accuracy, the chief limitation arising from the reliability of the infrared fluxes. We suggest that the ultimate accuracy attainable is at least equal to that of the present intensity interferometer and probably better. Our method as applied in this paper seems greatly superior in accuracy to speckle interferometry. The method has the advantage that it is applicable to cool stars if a reasonably good model atmosphere is available, and it is useable down to the brightness limit at which an accurate infrared flux is obtainable. As we have applied it in this paper the method will suffer from the effect of interstellar extinction, but such extinction will usually be small in the infra-red and if necessary an allowance could be made for it. An important advantage is that, unlike all direct methods, this method is almost independent of knowledge of limb darkening because it is so small in the infrared region. Its calculation is implicit in the analysis because it is contained in the prediction of flux by the use of a model atmosphere.

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