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STELLAR PULSATIONS ACROSS THE HR DIAGRAM: Part II

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ABSTRACT

Stars over essentially the whole mass domain can become pulsationally unstable during various stages of their evolution. They will appear as variable stars with characteristics that are of much diagnostic value to astronomers. The analysis of such observations provides a challenging and unique approach to study aspects of the internal constitution and evolutionary status of these objects that are not accessible otherwise. This review touches on most classes of known pulsating variable stars and tries to elucidate connections to stellar physical aspects. To aid future investigations, we stress questions and problems that we believe are yet to be resolved satisfactorily.

1. INTRODUCTION

Cox (1975), in his introductory report at the 19th Liège conference, concluded that " . . . overall, the [pulsation] theory and its application are in a fairly satisfactory state, except for a few disturbing problems. . . ." Among these were the excitation mechanism for the β Cephei variables, the mass discrepancy for Cepheids, and the influence of physical mechanisms such as convection, rotation, and magnetic fields. Some of these problems remain with us to this day.

This article on stellar pulsations deals with the application of the theory presented in the first part of the review, published in Volume 33 of this series

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(Gautschy & Saio 1995; GS95 in the following). We have tried to find a few distinct but conceptually connected paths across the Hertzsprung-Russell (HR) diagram along which the classes of pulsating variables can be linked to their evolutionary states. First, we deal with the pulsating stars on or near the main sequence. Next, we follow the evolution of low-mass stars and discuss the domains of pulsational instabilities that they encounter. Finally, we turn to massive stars and their variabilities and review the pulsation-based explanation attempts.

2. PULSATIONS CLOSE TO THE MAIN SEQUENCE

The traditional association with pulsating variables on or close to the main sequence has been confined either to β Cephei stars or pulsators at the intersection of the classical instability strip with the main sequence. This picture has had to be revised during the past decade. In the following subsections, we outline the present state of understanding of pulsating variables in the evolutionary phase of mainly core-hydrogen burning. Discussions of rapidly oscillating Ap (roAp) stars and more generally of the seismological aspects of solar-type stars are kept concise as there are numerous recent and comprehensive reviews available (Kurtz 1990, Gough & Toomre 1991, Matthews 1991, Shibahashi 1991, Brown & Gilliland 1994).

2.1 *Lower Main-Sequence Stars*

Our state of knowledge of pulsational instabilities in red dwarfs (main-sequence stars that are essentially fully convective and have masses below about 0.25 $M_{\odot}$) has not evolved far from what Cox (1974) described in his review. No new observational evidence indicating oscillations has emerged for low-mass stars on the main sequence. Gabriel (1969) performed quasi-adiabatic stability analyses on low-mass stellar models and found radial pulsation instabilities. The treatment of convection was, however, rather rudimentary. As there is no observational evidence yet for variability in such stars we might suspect that a fully nonadiabatic treatment and/or more sophisticated handling of convection might change the theoretical picture.

In the very low mass domain, the giant planet Jupiter is believed to oscillate nonradially (Deming et al 1989, Magalhães et al 1990, Mosser et al 1993). It is unclear whether this kind of behavior can also be expected in other low-mass objects.

In solar-type stars, observational data gathered to date does not prove the presence of oscillations unambiguously [see GONG92 (p. 599 below) for recent observational experiments]. The general theoretical belief is that solar-type stars should show oscillations with similar signatures as found in the sun itself.

Christensen-Dalsgaard & Frandsen (1983) estimated the amplitudes of such oscillations to be largest for stars with masses around 1.5 $M_\odot$. The photometric variability would remain in the μmag regime, and radial-velocity variations are expected to remain bounded by a few meters per second. Information from the many stochastically excited modes in such stars should provide constraints for the internal constitution similar to what low-ℓ modes do for the sun (Gough & Toomre 1991). A very promising approach to detecting solar-type oscillations was recently presented by Kjeldsen et al (1995). They observed changes of the equivalent widths of Balmer lines of η Boo with a low-dispersion spectrograph. The basic idea of their approach is that any solar-type oscillations cause temperature fluctuations that dominate over radial-velocity variability. The temperature fluctuations are monitored by the time variation of the equivalent width of selected spectral lines. Kjeldsen et al (1995) claimed to detect 13 distinct oscillation frequencies. The deduced frequency separations seem to agree reasonably well with theoretical expectations for a G0 IV star comparable with η Boo.

When solar-type oscillations are found in distant stars they will provide a new and accurate tool to determine fundamental stellar parameters. Basic asteroseismological aspects and their potential benefit for stellar astronomy were discussed recently in Brown & Gilliland (1994), Brown et al (1994), and in contributions in GONG92 and GONG94.

Near the intersection of the red edge of the classical instability strip with the main sequence—but usually at somewhat lower temperatures—a newly recognized class of variable stars is observed. Of the order of a dozen early F-type stars (Krisciunas & Handler 1995) exhibit photometric and radial-velocity variations with periods ranging from about 5 hours to roughly 2 days. The low-amplitude light variability of particular examples is described in Balona et al (1994a,b) (γ Dor), Krisciunas et al (1993) (9 Aur), and Lampens (1987) and Matthews (1990a) (HD 96008). The physical cause of the variability is unclear. The length of the periods of some of the F-type variables, those with rather high rotation velocities, could be compatible with rotationally induced phenomena such as starspots. Chromospheric activity is, however, generally restricted to spectral types later than about F7 (Radick et al 1983). Furthermore, the variable F-type stars themselves are not chromospherically active, and the persistence of the same periods over at least several years makes it difficult to attribute them to starspots. Multiperiodicity, as claimed for 9 Aur and γ Dor, suggests that indeed stellar oscillations are involved. The long periods observed can only be explained by high-order *g*-modes. Simple modeling shows that either the partial He II ionization zone or the rather weak Z-bump in the opacity data (cf GS95, Section 3.3.2) could potentially contribute to the excitation of such modes. But since very high overtones must be unstable, a large number of g-modes should be excited simultaneously if the classical κ-mechanism were the destabilizing agent. Observations, however, do not show the persistence of a large number of modes.

Close to the main sequence, within the boundaries of the classical instability strip, at masses of $\approx 2\ M_\odot$, lies the class of rapidly oscillating Ap (roAp) stars. Their light variation ranges from a few to roughly 50 mmag in the blue. The multiple periods are confined to between about 5 and 15 minutes. Many of the period multiplets with separations of a few μHz are believed to be rotational splittings; even when these components are removed, multiple periodicities often remain. The short periods of the observed pulsations must be due to high-order, low-degree p-modes. The oscillation amplitudes are modulated in accordance with the temporal variation of the stars' strong magnetic fields (some hundred to several thousand Gauss on the surface). The timescales of the modulation agree with the rotation periods of the stars. Mostly, roAp stars are slow rotators for their spectral type (rotation periods between 2 and 12 days); this is attributed to efficient braking due to their strong magnetic fields. The oscillations must be influenced by these strong fields. Phenomenologically, the basic pulsational behavior can be described successfully by the oblique pulsator model (Kurtz 1982). The pulsational axis is assumed to be aligned with the star's magnetic axis, which, by itself, has a nonvanishing obliquity relative to the rotation axis. For an external observer, such a geometrical setup leads to rotationally induced frequency splittings. For theoretical purposes, sophisticated perturbation schemes have been developed during the past few years. Using the relative amplitudes of frequency multiplets, they allow deductions about the alignments of the different axes on the star relative to the observer and estimates of averaged magnetic fields inside the star (Dziembowski & Goode 1985, 1986; Kurtz & Shibahashi 1986; Shibahashi & Takata 1993; Takata & Shibahashi 1995a). An alternative phenomenological model, which did not catch on so successfully, was the spotted pulsator model proposed by Mathys (1985). He assumed the pulsation axis to be aligned with the rotation axis. The oscillation patterns are hence always seen from the same aspect angle. In the spotted pulsator approach, the amplitude modulation results from the inhomogeneous distribution of the flux on the stellar surface, which is a function of the magnetic phase during a stellar rotation period.

The oblique pulsator model seems to more favorably account for the observed properties (Kurtz 1990). From a theoretical point of view, however, it is not clear how the oblique pulsation is maintained. The angular dependence of such an oblique pulsation of say $\ell = 1$ is expressed by a linear combination of spherical harmonics $Y_1^m(\theta, \phi)$ with $m = 0$ and $m = \pm 1$, where θ and ϕ are the spherical coordinates associated with the rotation axis. These components have frequencies $\sigma_m = \sigma_0 + m\Omega C_{n,1}$ in the corotating frame (cf GS95). To maintain the oblique pulsation, $C_{n,1}$ must vanish; otherwise the three components will have different frequencies in the corotating frame and the pulsation pattern would drift about the magnetic axis on a timescale of $1/C_{n,1}\Omega$. Nonmagnetic spherical stellar models predict $C_{n,1}$ to be of the order of 10^{-2} to 10^{-3} for high-order *p*-modes whereas observations, obtained from HR3881, lead to an upper limit of about 10^{-5} (Kurtz 1990). Dolez & Gough (1982) argued that it is difficult for the magnetic fields to suppress any drifting of pulsational patterns.

An important step towards identifying the instability mechanism is to map the roAp stars into a physically meaningful parameter space. Establishing a two-color diagram, for example, that clearly outlines and separates the instability region of roAp stars photometrically from stable stars has proven elusive (Matthews 1990b). The instability region of the roAp stars seems to at least partly overlap that of δ Sct variables (Section 2.2) on the HR diagram. The δ Sct pulsations are known to be driven by the partial He II ionization zone. In roAp stars, diffusion, which is believed to be responsible for the chemical anomalies (Michaud 1970, 1980), is thought to drain the He abundance in regions where He II ionization would otherwise be the driving mechanism. The excitation mechanism responsible for the oscillating Ap stars remains to be identified. Shibahashi (1983) and Cox (1984) contemplated magnetic overstable convection as a possible driving mechanism; in their picture, the restoring force results from the magnetic field, which resists being dragged along by convective motion. Matthews (1988), on the other hand, based on a local stability analysis, suggested that the κ-mechanism due to partial ionization of Si IV was the destabilizing agent. Vauclair & Dolez (1990) and Vauclair et al (1991) speculated that a very weak stellar wind (removing $\approx 10^{-14}\ M_\odot$ year^{-1}) might transport enough He along magnetic field lines into the polar regions, where it settles and could excite oscillations. Detailed, self-consistent calculations for any of the above mechanisms are not available. Furthermore, observational evidence for prerequisites for the proposed mechanisms to work has not yet been collected. Not only does the excitation mechanism remain to be identified, but also, the physics of the mode selection remains unclear. Some of the roAp stars pulsate in multiple modes that are not consecutive radial orders. Many oscillation modes show very good long-term stability (at least over a decade), whereas others decay on the order of days.

As the oscillation frequencies are believed to be of high order, asymptotic mode analysis can be applied (see Tassoul 1980; Unno et al 1989, section 16). The often observed frequency differences (of the order of 20–80 μHz) between the multiple periods can be caused either by differences in radial order *n* of equal-ℓ modes or by alternating even and odd ℓ values. The first case allows one to measure the asymptotic quantity $\Delta\nu_0 \propto (M_*/R_*^3)^{1/2}$. The latter case provides a measure of $\Delta\nu_0/2$, however. Lines of equal $\Delta\nu_0$ on the HR diagram show that smaller values translate a star of known effective temperature to a later evolutionary phase and a higher mass (Shibahashi & Saio 1985, Gabriel et al 1985). A later evolutionary stage implies faster evolution as the stars would then be burning hydrogen already in a thick shell. Heller & Kawaler (1988) computed period changes that develop during and shortly after the main-sequence evolution. If roAp stars are indeed in their subgiant phase, evolutionary period changes should be detectable within a few years of monitoring. Martinez & Kurtz (1990) accumulated such data for HD 101065, a roAp star undergoing period changes, whose $dP/dt \equiv \dot{P}$ turns out to be about ten times larger than the maximum predicted by Heller & Kawaler (1988). Additionally, the observed sign of $\dot{P}$ is wrong compared with theory. Whether the observations are contaminated by binary motion is unclear. In any case, looking for evolutionary period changes promises to be a suitable method to solve the frequency-spacing dilemma.

Recently, Kurtz et al (1994) and Kurtz (1995) reported frequency variations in roAp stars on the timescale of $\sim$ 100 days. Such variations are considered now to be common among roAp stars. The signature is cyclic, and it is considered to be intrinsic to the pulsations and not of evolutionary origin. Kurtz (1995) attributed the frequency modulation to a changing magnetic field strength. Such a variation would modify the structure of the acoustic cavity and hence the magnitude of the eigenfrequencies. It is interesting that, based on our present understanding of magnetic fields in stars, magnetic cycles similar to the solar one are not expected for upper main-sequence stars.

2.2 *δ Scuti Stars*

Stars with masses of $1.5 \lesssim M/M_\odot \lesssim 2.5$ enter the lower instability strip (see Figure 1 of GS95) either in their core hydrogen-burning phase or when they evolve towards the base of the giant branch burning hydrogen in a shell. Such stars are commonly accepted as candidates for δ Sct-like oscillations. Very recently, some δ Sct variables were identified to be pre-main-sequence objects (Kurtz & Marang 1995). The pulsation periods of δ Sct variables range from about 0.02 to 0.25 days, indicating low-order radial or nonradial *p*-modes of low spherical degree. Some of the recent observational data hint at the presence of even *g*-modes (Breger et al 1995). We know of single/double mode δ Sct stars with large amplitudes (some tenths of a magnitude); the majority, however, are multiperiodic with small amplitudes (10^{-3}–10^{-2} mag). The lack of sufficiently long temporal baselines of observing runs prevented clear statements about the richness of the frequency spectrum in the past. Thanks to recent multisite campaigns we know now of multiply periodic δ Sct variables that show up to about a dozen oscillation modes (Belmonte et al 1993, Breger et al 1995). Results from Doppler imaging of rapidly rotating δ Sct stars indicate that high-degree modes ($12 \lesssim \ell \lesssim 16$) might also be excited (Kenelly et al 1992, Matthews 1993).

When considering the stars populating the lower instability strip, only a fraction ($\lesssim 50\%$) of them are observed to be photometrically variable, at least with amplitudes above the presently discernible limit of a few mmag. Because the number of known pulsators increases steeply towards low amplitudes (Baglin et al 1973, Breger 1979), we might suspect the putative stable stars to be pulsating with very low amplitudes. The presence of stable stars in the δ Sct variability domain, the different naming of allegedly differing subclasses of short-period pulsators, and the presence of both Population I and II objects in the lower instability strip make any survey rather confounding. Discussions of the classification issue from the observational viewpoint can be found in Eggen (1979), Breger (1979), Nemec & Mateo (1990), and references therein. For simplicity, we call variable stars with properties as mentioned above δ Sct variables—this is sufficient for the discussion of basic theoretical issues. On the observational side, examples of instructive reviews of δ Sct stars are those by Baglin et al (1973), Breger (1979), Wolff (1983), and Kurtz (1988) (who concentrated on those stars for which detailed frequency spectra were available).

The reviews by Petersen (1976) and Cox (1983) contain comprehensive accounts of early theoretical work on δ Sct variables, addressing in particular linear stability analyses and radial nonlinear simulations. Lee (1985b) reanalyzed the oscillatory stability of low-order nonradial modes of δ Sct-like stars in their subgiant phase. A generalization of Osaki's (1977) WKB treatment for the interior to a quasi-adiabatic treatment (Dziembowski 1977a, Lee 1985a) allowed the solution of the eigenvalue problem over the complete stellar models, including the condensed central parts of the star, which enforce short spatial wavelengths in the eigenfunctions. The very small local wavelength is caused by the large magnitude of the bump in the Brunt-Väisälä frequency in the deep interior (cf Figure 2 in GS95). This peak in the Brunt-Väisälä frequency close to the center lets the eigenmodes with δ Sct-like periods adopt dual character. They behave like *p*-modes in the envelope and turn locally into *g*-modes in the deep interior. If the evanescent zone (white area between the *p*- and *g*-mode propagation domains in the inlet of Figure 2 in GS95) between the two propagation regions is thin and the Brunt-Väisälä frequency in the inner regions is very large, then the deep interior constitutes a substantial energy sink. Dissipation in the deep interior will possibly stabilize *p*-modes that are driven in the He II ionization zone by the κ-mechanism. Nonetheless, Lee (1985b) found a large number of unstable modes in his subgiant model (see Figure 2 in Lee 1985b), a result very much like that of the earlier studies dealing with main-sequence-type stars (e.g. Dziembowski 1977a). The number of destabilized low-degree modes in these linear stability analyses is much higher than the number of modes presently observed. Dziembowski & Królikowska (1990) speculated that trapping of a fraction of the eigenmodes in the envelope could serve as a selection mechanism. Trapped modes with their reduced amplitudes in the deep interior and hence with reduced dissipation in that region would be favored to reach observable amplitudes. For low-degree modes, i.e. modes that are in principle photometrically detectable, the trapping efficiency was most pronounced for $\ell = 1$.

The amplitude-limiting process acting in δ Sct stars and other low-amplitude variables is thought to be different from the one that is relevant in large-amplitude variables like Cepheids or RR Lyrae stars. The observed frequency spectra of low-amplitude pulsators differ significantly from what simple linear pulsation theory predicts. Only a fraction of the linearly unstable modes reach an observable level. Besides the selection by trapping as mentioned above, nonlinear mode interactions between two or three modes, where one of them is linearly unstable, are proposed as efficient amplitude limiters (Dziembowski 1982). In a later investigation (Buchler & Goupil 1984), nonadiabaticity and nonlinearities in the growth rates were also accounted for by stepping up to second- and third-order nonlinearities. In case of δ Sct-like stars, the coupling of low-order, low-degree *p*-modes with pairs of *g*-modes limited amplitudes efficiently (Dziembowski & Królikowska 1985). Pairs of *g*-modes can be parametrically excited if the acoustic mode amplitudes exceed a threshold. This opens the possibility for a cascade-like spread of coupling of the now unstable *g*-modes with initially stable higher-order pairs of *g*-modes. For the case of three-mode interaction, the final amplitudes turn out to be too large compared with observation. By introducing rotation and associated frequency splitting, the chances for additional resonances, and hence for further reduction of the amplitudes, are increased (Dziembowksi et al 1988). Three-mode interactions result in steady amplitude solutions only in exceptional cases. More frequently, amplitude modulations in time are predicted (Moskalik 1985). Constraints for the theory of nonlinear mode coupling might hence come from observed long-term amplitude variations of oscillation modes in δ Sct stars.

The seeming coexistence of variable and stable stars in the same region of the HR diagram was attributed to the action of gravitational settling of helium and levitation of suitable metallic elements by radiation pressure in the stellar envelopes (Baglin 1972, 1976). This works only for slow pulsators since rapid rotation tends to destroy this effect. According to Kurtz (1989), diffusion and radiative levitation might indeed account for the mostly—but not exclusively—stable spectroscopically peculiar Am stars that are encountered in the lower instability strip. Physically, the disappearance or weakening of the He II ionization zone stabilizes the stars. Nonetheless, even for significantly He-depleted envelopes Cox et al (1979) found pulsational instabilities due to residual He II and some enhanced driving in the partial H I ionization zone close to the red edge of the classical instability strip. Not only can peculiar A and F stars be pulsationally stable in the δ Sct region, but also nonvariable stars with "normal" spectra were identified (Breger 1979). Because many of the stable candidates might be pulsating with low amplitudes, high-precision photometry needs to confirm the advocated ratio of stable to variable stars in the lower instability strip. The recent discovery of large-amplitude δ Scuti-like pulsation in a Am star casts doubt on the levitation mechanism being responsible for the abundance anomaly (Kurtz et al 1995).

For many years, it proved impossible to fit simultaneously the values observed for δ Sct and Cepheid variables to the theoretical Petersen diagrams(P_1/P_0 versus P_0) when standard stellar-evolution assumptions were invoked in the modeling. For such analyses, the periods of the radial fundamental mode P_0 and of the first overtone P_1 are usually calculated in the adiabatic approximation. Andreasen et al (1983) realized that the period ratios depend sensitively on the He content of the stellar envelopes. Indeed, noncanonically high values for He were suggested not only to accommodate unusually high period ratios (e.g. VZ Cnc, Cox et al 1984) but also to account for the theoretical, radial blue edge of δ Sct stars, which was considered too hot compared with observations. When Andreasen (1988) artificially increased the opacity at a few times 10^5 K he was able to reconcile observation and theory in the Petersen diagrams for double-mode δ Sct stars and for Cepheids. The new generation of opacity tables (OPAL and OP) fully support Andreasen's findings as was shown in preliminary calculations by Christensen-Dalsgaard (1993) (see Figure 1).

The location of the blue edge for radial instabilities in δ Sct-like models does not shift considerably when invoking the new opacities (Li & Stix 1994). The particular choice of the outer boundary in the computations seems, however, to influence the stability properties considerably. If the boundary conditions are formulated close to the photosphere, the eigenfunctions are sufficiently quenched in the H I/He I partial ionization zone to render it ineffective. When extending the eigensolutions further into the atmosphere, the H I/He I ionization zone is assigned more weight and it contributes to the driving so that the blue edge is shifted towards higher temperatures and fits the observations better.

The increase of the Brunt-Väisälä frequency (cf Figure 2 in GS95) that occurs in the central region when the star leaves the main sequence forces the

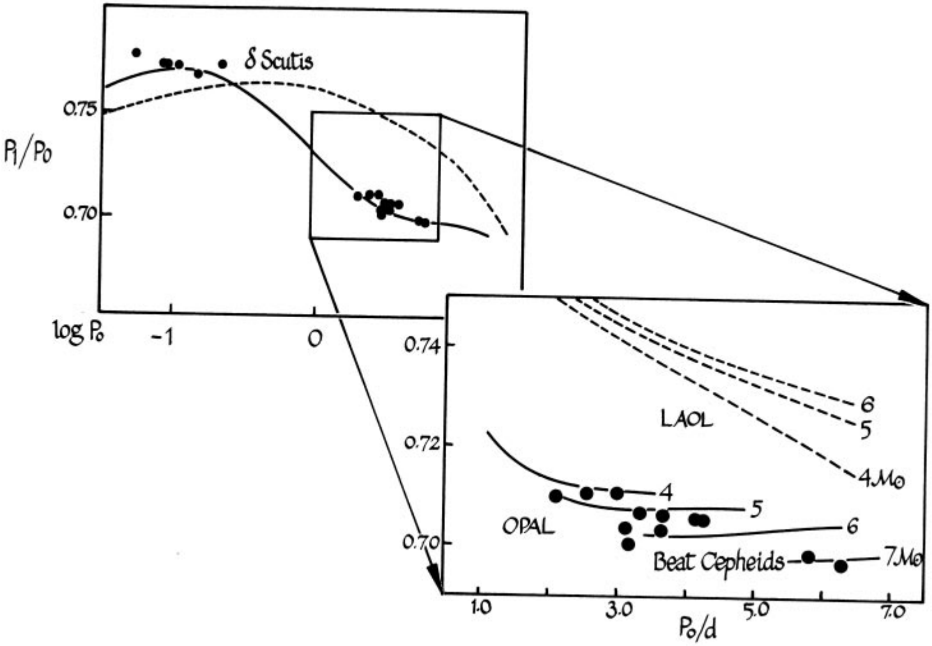

Figure 1 Petersen diagram for population II pulsators. The period ratios of the first to the fundamental mode are plotted against the logarithm of the fundamental mode in the upper diagram to accommodate δ Sct as well as beat Cepheids. The solid curve shows the convincing agreement resulting from calculations with new opacity data (OPAL). The broken curve displays the long-standing discrepancy due to the Los Alamos (LAOL) opacity data. The inset in the lower right allows a more detailed view of the beat Cepheid results, in particular the dependence of the period ratios on the stellar mass.

eigenfrequencies of some *g*-modes to penetrate the frequency domain of the *p*-modes. Assuming that those modes experiencing an avoided crossing (see GS95, Section 3.2.3) at some evolutionary phase—when a *g*-and a *p*-mode happen to be close to each other—are excited and also are observable, then we can potentially probe the very deep interior of such an evolved star. At the inner boundary of the radiative region, just outside the convective nuclear-burning core, the form of the Brunt-Väisälä frequency is governed by the detailed structure of the core/envelope (convection/radiation) transition. Dziembowski & Pamyatnykh (1991) proposed frequencies and irregularities in frequency separations of the core *g*-modes of δ Sct variables that proved useful for assessing their evolutionary state and unraveling properties of overshooting from the convective core.

Before structural and/or evolutionary aspects can be confidently constrained with δ Scuti variability, i.e. before asteroseismology is possible, the observed modes must be reliably identified. This is far from easy because the typical δ Scuti power spectrum is rather sparse. Presently, stellar evolution modeling is accompanying the mode identification to constrain the interpretations. Improvements can be expected from multicolor or simultaneous spectroscopic observations. Different degrees and orders of modes cause different signatures in color indices and in line variability that can contribute to positive identifications. Another promising approach is to search for δ Sct variables in clusters so that ensemble analyses can be performed. Common quantities that enter the modeling, such as age and maybe convection parameterization, can be determined for the whole ensemble of variables (cf Brown & Gilliland 1994).

Blue stragglers are stars whose positions in the HR diagram render them too young compared with the age of the agglomeration (often globular and old open clusters) with which they are associated. For a recent review of these objects see Stryker (1993) and Bailyn (1995) and references therein. Some of the blue stragglers happen to fall into the instability strip and exhibit δ Sct-like variability. Mateo (1993) reviewed the observational status of variable blue stragglers in old stellar systems (see also Nemec & Mateo 1990). The 24 stars Mateo (1993) mentioned are all attributed a single pulsation frequency, and they all seem to have rather large amplitudes, which is likely to be an observational bias. Gilliland et al (1991) discovered two multiperiodic blue stragglers in M67 from high-precision differential CCD photometry. The pulsational properties, in particular of multiperiodic blue stragglers, might prove important for constraining possible formation scenarios (Gilliland & Brown 1992). Should these mysterious stars be the result of coalesced binaries, then structure and chemical abundances can be expected to be peculiar compared with evolved single stars. Such abnormalities are likely to leave their traces in oscillaton periods and period spacings.

2.3 *Slowly Pulsating B Stars*

The slowly pulsating B (SPB) stars are multiperiodic variable mid-B-type stars (B3–B8) with periods between about 1 and 3 days. The prototype of this group, 53 Per (B4 IV, $v \sin i = 17$ km s^{-1}), was discovered as a line-profile variable star by Smith (1977). Its line-profile variations are well explained by velocity fields from low-ℓ nonradial pulsations on a rotating star (Smith & McCall 1978). The photometric variability of 53 Per was described in Percy & Lane (1977) and Africano (1977). Buta & Smith (1979) determined two periods (of about 2 days) for the light variation. Later, Waelkens & Rufener (1985) found a number of other mid-B-type stars showing small-amplitude light and color variations with periods between 1 and 3 days. In some of these stars line-profile variability could be detected (Waelkens 1987). Furthermore, Waelkens (1991) reported that all 7 of the then-known SPB stars were multiperiodic, and he confirmed *g*-mode pulsations as the cause of their variability. These findings suggested that variable mid-B stars, including 53 Per itself, form a distinct group of pulsators. The term "slowly pulsating B stars" was proposed by Waelkens (1991).

The SPB pulsations are recognized as *g*-modes because observed periods are longer than the expected period of the radial fundamental mode. Figure 1 displays the location of the SPB instability strip relative to the one of the β Cepheids and the blue part of the classical strip on the HR diagram. The low ℓ values were inferred from their photometric variability. Waelkens (1991) realized that the photometric amplitudes decrease towards longer wavelengths for his whole sample of stars, indicating that photospheric temperature modulation influences the light variation significantly.

Physically, the excitation of *g*-modes in SPB stars can be understood as an extension of the β Cephei instability (Gautschy & Saio 1993, Dziembowski et al 1993) (cf Section 2.4 below) towards longer periods. For main-sequence stars less massive than β Cepheids, high-order *g*-modes are excited. Because the modes populate the frequency domain rather densely, an increasing number of *g*-modes become excited simultaneously for lower masses. Hence, the multiperiodicity of the SPB stars (cf Smith et al 1984, Waelkens 1991) is in accordance with our theoretical understanding. Figure 2 shows the large number and the different radial orders of excited *g*-modes (enclosed in the dotted area of the inlet in the upper right) during the main-sequence evolution of a 5 $M_{\odot}$ star. The first turn-around of the evolutionary tracks of the appropriate stellar masses determines the width of the instability strip. During this evolutionary phase the hydrogen exhaustion in the stellar core induces a contraction of the central region, which then leads to a strong growth of the Brunt-Väisälä frequency. The large magnitude of the Brunt-Väisälä frequency is responsible for short spatial wavelengths—and enhanced dissipation—in the eigenfunctions of oscillation modes with periods relevant for SPB stars. The accompanying dissipation in the deep interior eventually overcomes the driving in the Z-bump region of the envelope (cf Figure 4 in GS95). Along the main sequence, the SPB instability strip terminates at around 2.5 $M_{\odot}$. The exact value depends somewhat on metallicity. At these low masses, the radial orders of the excitable *g*-modes become so high that strong radiative dissipation eventually overcomes the excitation by the κ-mechanism. The theoretical low-mass boundary for the *g*-mode instability agrees with the null result of a search for line-profile variations in late B (B8–B9.5) stars conducted by Baade (1989).

The low-ℓ, high radial order g-modes that are excitable in SPB stars let them assume essentially asymptotic modal properties (cf Section 3.2.2. in GS95). Deviations from the naively expected equal separations between adjacent periods of the g-modes of like degree were found in theoretical modeling (Dziembowski et al 1993). The periodic patterns in the period differences are induced by partial trapping of the modes around the outer edge of the nuclear-burning core. Only in variable white dwarfs are such signatures also encountered, which we can take advantage of for seismic analyses (see Section 3.2). The long periods of the SPB stars make it extremely challenging to obtain sufficiently accurate frequency data to perform seismological studies of value.

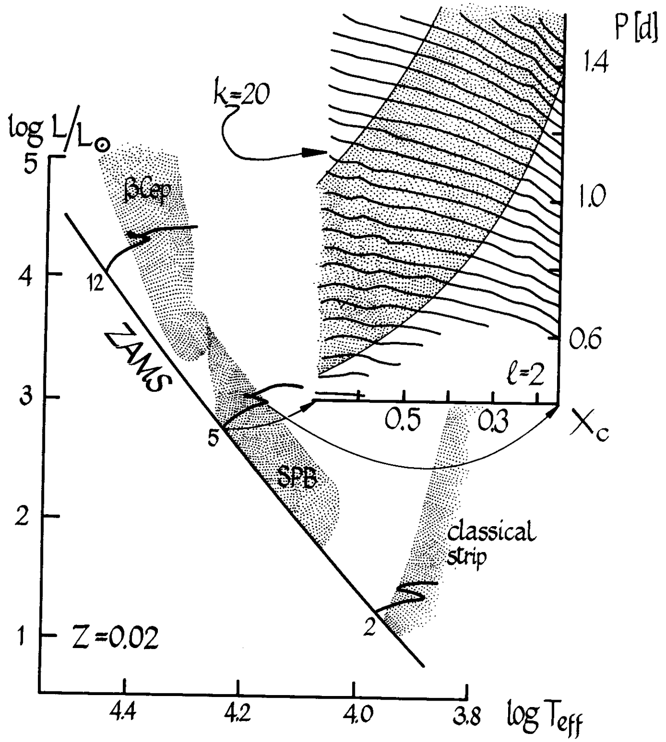

Figure 2 HR diagram of the upper main-sequence region. Evolutionary tracks of 2, 5, and 12 $M_\odot$ are shown leaving the ZAMS. Dotted areas mark pulsational instability regions. The finger at the upper left contains the β Cepheids. The area denoted by SPB contains the slowly pulsating B stars. On the lower right we show the blue part of the classical instability strip where δ Sct stars are found. The inset on the upper right shows the evolutionary variation of $\ell = 2$ g-mode periods of a 5 $M_\odot$ star. The temporal evolution is parameterized by the central hydrogen content X_C. The dotted strip indicates the location of the unstable modes.

All SPB stars seem to be slow rotators (Smith 1977, Waelkens 1987, Waelkens et al 1991). Since most SPB stars were discovered photometrically, the low rotation speeds can hardly be attributed to a selection effect. No SPB stars are found in the young open clusters NGC 3293 (Balona 1994) and NGC 4755 (Balona & Koen 1994) despite their containing many β Cephei stars. Balona & Koen (1994) suspected that rapid rotation in these stars suppresses the *g*-mode instability. The effect of rotation on the stability of high-order *g*-modes has not yet been tackled theoretically.

The presently known sample of SPB stars contains the star 53 Per and the 13 other stars listed by North & Paltani (1994). Sometimes, other members of the class of "53 Per variables"—defined spectroscopically as variable line-profile B stars by Smith (1980a)—are considered as members of the SPB group. However, a blind merging of the two groups, defined by differing aspects of their variability, might be inadequate.

2.4 *β Cephei Stars*

The β Cephei stars are a group of short-period ($\lesssim$ 0.3 d) variables. Their oscillations are detectable in radial-velocity as well as in light variations. A list of the presently known β Cephei stars is given in Sterken & Jerzykiewicz (1993). The excitation of the pulsations of β Cephei stars and the slowly pulsating B stars was recently, after many years of futile attempts, found to be due to classical κ-mechanism driving in the Z-bump encountered in the new generation of opacity data (see Section 3.2.2 in GS95) (Kiriakidis et al 1992, Moskalik & Dziembowski 1992).

The κ-mechanism tends to excite pulsations with periods comparable to the thermal timescale of the excitation zone; this applies to radial as well as nonradial pulsations. In the region of β Cephei variables, oscillation modes with periods between $\approx$ 0.1 and 0.3 day are potentially excited. As effective temperature decreases, the depth of the driving zone increases and so do the periods of excited modes. On the main sequence, for models with 7–8 $M_{\odot}$ ($\log L/L_{\odot} \approx$ 3.5) the length of the most favored period lies in the transition region between *p*-modes and *g*-modes (see Figure 2). Towards lower masses, long-period *g*-modes (with a dense frequency spectrum) are excited and observed in slowly pulsating B stars. A phenomenological distinction between the β Cephei stars and the SPB stars is introduced most naturally between 7 and 8 $M_{\odot}$ (the periods there are of the order of a few tenths of a day). Physics, though, does not enforce a formal subdivision of the two groups of variable stars.

The theoretically determined instability region (cf Figure 1) on the HR diagram for $Z = 0.02$ (Dziembowski & Pamyatnykh 1993), determined using a recent OPAL opacity release, comprises most of the observed β Cephei variables;

exceptions are some metal-rich stars. Waelkens et al (1991) found that the blue edge of the instability region is bluer (i.e. a more extended instability region on the HR diagram) for more metal-rich β Cephei stars. Another effect of the heavy-element dependence of the β Cephei pulsations is the lack of β Cephei variables in the Magellanic Clouds (Balona 1992, 1993). These facts are consistent with Z-bump driving. Theoretically, it was realized (see Moskalik 1995) that the detailed physical treatments of heavy elements in the opacity calculations, such as Ni, Cr, and Mn, influence the pulsational driving and hence the extent of the instability region, in particular in the low-mass region of the β Cephei stars.

The paucity of β Cephei variables at luminosities exceeding $\log L/L_\odot \approx 4.5$ is explained by the shifting of the instability region into the post-main-sequence phase (Dziembowski & Pamytnykh 1993). The evolutionary timescale is faster there; thus the probability of finding variable stars in this domain is much lower.

Pulsational instability regions deduced from linear theory and from nonrotating spherical star models do not yet coincide in all cases with observed domains of β Cephei variables. The β Cephei stars in the open cluster NGC 3293 (Balona & Engelbrecht 1981, Balona 1994) are confined to $4.4 < \log T_{\rm eff} < 4.44$, and all stars in this temperature range seem to pulsate. Comparing this instability domain with the theoretical result for $Z = 0.03$ obtained by Dziembowski & Pamyatnykh (1993) indicates that despite the observed high-$T_{\rm eff}$ boundary being roughly consistent with the theoretical blue edge, the observed low-$T_{\rm eff}$ boundary is much bluer than the theoretical one. Moreover, Balona & Koen (1994) have found a few constant stars within the β Cephei instability strip in NGC 4755.

The majority of β Cephei variables are multiperiodic (see e.g. Figures 22 and 23 in Sterken & Jerzykiewicz 1993). Some of the identified frequencies can be attributed to rotational m-splitting. These variables also show line-profile variations, which are mostly line broadening and narrowing (rather than traveling bumps and dips). Osaki (1971) showed that such line-profile variations are produced by the combination of rotation and nonradial pulsation with low $|m|$-values. Smith (1980b, 1983) and Campos & Smith (1980) compared theoretical line-profile variations with observed ones to determine pulsation modes and periods. These studies, performed by a trial-and-error method, were difficult because of the many parameters involved in the computation of the theoretical line profiles. Additionally, temperature variation, which was not accounted for, affects the line profiles of β Cephei stars markedly (Balona 1987, Cugier 1993). To overcome the difficulties, Balona (1986a,b, 1987, 1990a) proposed a moment method to identify pulsation modes. This approach is useful for nonradial pulsations with low $|m|$ in slowly rotating stars. Aerts et al (1992) and Mathias et al (1994) extended and applied the method to the β Cephei variables δ Ceti and α Lupi.

Pulsation modes may also be estimated by using the amplitudes of the variability in the visual and in the UV. This method makes use of the light variation resulting both from oscillatory variations in temperature (surface brightness) and from geometrical effects. The relative contributions vary with wavelength and with spherical degree of the oscillation mode (Dziembowski 1977b). Applying this method, Watson (1988) and Cugier & Boratyn (1992) concluded that large-amplitude, single-periodic stars pulsate in their fundamental mode. Recently, Cugier et al (1994) extended the method by including the amplitude of the radial-velocity variation.

Most β Cephei variables vary regularly; the amplitudes of a few of them change, however, drastically on short timescales. One example is Spica (α Vir), a double-lined spectroscopic binary with an orbital period of 4.01 day and a pulsation period of 0.1738 day discovered by Shobbrook et al (1969). The existence of the β Cephei-type variation was traced back to about 1890 (Smak 1970, Shobbrook et al 1972, Dukes 1974). The amplitude decreased in time and became undetectable by 1972 (Lomb 1978). In contrast, Smith (1985) observed traveling bumps and dips in the rotationally broadened Si III line profiles, indicating the existence of nonradial pulsations with high $|m|$. Balona (1985) suggested that the amplitude changes as the angle between the line of sight and the axis of nonradial pulsations (i.e. rotation axis) varies, due to the precession of the rotation axis around the orbital axis of the binary system. The period of precession is of the order of the period of apsidal motion, estimated to be $\approx$ 130–140 year (Shobbrook et al 1972, Dukes 1974). This hypothesis fails, however, if the light variations were caused by radial pulsations. Another example of a rapid change of the pulsation amplitude is the Be star 27 CMa, in which the β Cephei-type light variation grew from zero amplitude in less than two years (Balona & Rozowsky 1991). Since some Be stars are located in the domain of β Cephei variables, the existence of a β Cephei-type variability in a Be star is not surprising, but the cause for the rapid growth of the amplitude remains mysterious.

2.5 *Be-Type and Related Stars*

The Be stars are characterized by rapid rotation and by the occurrence of emission features in spectral lines. The emission components are attributed to a circumstellar disk (see Sletteback 1988 for a review). Two kinds of short periodic ($\lesssim$ day) variations are observed in Be stars (see e.g. Baade 1987 for a review): 1. high-order line-profile variations (e.g. ζ Oph (O9.5Vne), Walker et al 1979) in which bumps or dips are traversing rotationally broadened absorption lines from the blue to the red and 2. a photometric variability with periods of the order of a day (see Balona 1990a), which seems to be associated with low-order line profile (line asymmetry) variations (Baade 1982).

At least some of the high-order line-profile variations (LPVs) of Be stars are explained by nonradial pulsations of high azimuthal order $|m|$ (ζ Oph, Vogt & Penrod 1983; μ Cas, Baade 1984; γ Cas, Yang et al 1988; ζ Tau, Yang et al 1990). The line-profile variability of ζ Oph, the prototype of this group, was attributed to nonradial pulsations with $m = -8$ by Vogt & Penrod (1983), while Kambe et al (1990, 1993a) concluded that two modes ($m = -4$ with a period of 3.3 h and $m = -7$ with a period of 2.4 h) are simultaneously excited. In these investigations, nonradial pulsations were assumed to be sectoral ($\ell = |m|$) spheroidal modes. If this assumption is dropped, however, then the pulsation modes can no longer be identified from the LPVs alone (Osaki 1986a, Kambe & Osaki 1988).

The angular dependence of amplitude distribution of nonradial oscillation of a rotating star cannot be expressed by a single spherical harmonic. A spheroidal mode of given ℓ and m is accompanied by toroidal components of degree $\ell \pm 1$ and spheroidal components with $\ell \pm 2$. (Components with different m do not appear because axisymmetry is preserved in a rotating star.) The importance of these components is proportional to $|m|\Omega/\omega$ and $(\Omega/\omega)^2$, respectively, where ω is the oscillation frequency in the corotating frame and Ω stands for the angular frequency of rotation. Therefore, to fit theoretical LPVs to observed ones for a rapidly rotating star, theoretical modeling should account at least for the effect of toroidal components (Kambe & Osaki 1988). Aerts & Waelkens (1993) calculated LPVs including the effect of toroidal components; their results show that toroidal components induce additional bumps in the line profiles when Ω/ω exceeds 0.2. It is desirable, however, to use self-consistently derived relative amplitudes of toroidal components (which depend on $|m|\Omega/\omega$). Also, the influence of temperature variations during the pulsational cycle, which are not negligible in variable B stars (Balona 1987), should be included. For low-frequency *g*-modes, a somewhat simplified approach is possible (Berthomieu et al 1978). Lee & Saio (1990a,b,c) and Lee et al (1992) obtained theoretical line-profile variability due to low-frequency oscillations thus accounting in their analysis for temperature variations.

On the HR diagram, Be stars lie within the β Cephei or in the SPB instability regions, so it is reasonable to expect the high-$|m|$ modes in Be stars to be also excited by the Z-bump. Kambe et al (1993a) argued that the detected modes in ζ Oph are prograde modes with periods between 7 and 15 h in the corotating frame; they correspond to Q values[1] in the range 0.06–0.12. These periods are

[1] $Q \equiv P \cdot (\bar{\rho}/\bar{\rho}_\odot)^{1/2}$ where P is the period of the pulsation in days and $\bar{\rho}$ is the mean density of the star.

longer than those of β Cephei variables. Theoretically, the stability properties of corresponding oscillation modes in rapidly rotating stars are not yet clear (cf Lee & Baraffe 1995). But there is more to the LPVs in Be stars than only regular variability. In λ Eri, the high-order LPVs are irregular (Smith 1989, Kambe et al 1993b, Gies 1994). These variations are thought to be caused by stochastic small-scale activity above the photosphere (Smith 1989, Smith & Polidan 1993).

The other type of periodic variation, photometric variability, is ubiquitous in Be stars. Those Be stars exhibiting photometric variability are occasionally referred to as λ Eri stars (Balona 1990a). Numerous λ Eri stars were recently identified in open clusters of both the Small and Large Magellanic Clouds (SMC and LMC) (Balona 1992, 1993). The κ-mechanism associated with the Z-bump clearly cannot to be responsible for the light variations of these objects, even if their variability is attributable to oscillations.

The periodic light variations are strongly correlated with the rotation speed. The physical origin of the light variability, which is probably associated with low-order ($|m| \approx 2$) LPV, is not yet certain. Three possible mechanisms are proposed, two of which are rotational modulations caused by an inhomogeneous brightness distribution on the stellar surface and an asymmetric structure of the circumstellar envelope (Balona 1990a). The origin of these inhomogeneities was supposed to be connected with magnetic fields; there is, however, no clear observational evidence for the existence of strong magnetic fields in Be stars. The other mechanism involves a nonradial pulsation with frequency, in the frame corotating with the stellar surface, which is so small that observed periods are correlated with rotation. An excitation mechanism for such slow nonradial pulsations was proposed by Lee & Saio (1986, see also Lee 1988). They argued that oscillatory convection in the rotating convective core couples with a high-order g-modes in the envelope and so induces an overstable nonradial mode.

Because periodic light variations are common among Be stars, they are suspected to be related to the mechanism for the observed episodic mass loss. [Balona et al (1991) found that all rapid rotators in the open cluster NGC 3766 that showed low-order—i.e. small $|m|$—line-profile variations were in fact Be stars.] Assuming the periodic light variations to be induced by magnetic fields, Balona (1990b) proposed the episodic mass loss to be associated with giant flares on these stars. Alternatively, angular momentum transport by nonradial pulsations in rotating stars was proposed as an alternative mechanism for episodic mass loss (Ando 1986, Osaki 1986b, Lee & Saio 1993). In this picture, interior angular momentum is transported to the surface by the action of nonradial oscillations, eventually forcing the surface region to rotate supercritically, leading to a subsequent ejection.

2.6 *Very Massive Main-Sequence Stars*

The stability properties of very massive main-sequence stars were discussed in the past mainly in connection with the most massive stable stars expected to be encountered. Observational evidence exists for stars having masses exceeding 100 $M_\odot$ but probably below 200 $M_\odot$ (de Jager & Nieuwenhuijzen 1991). The numbers are estimated by comparing positions on the HR diagram with standard stellar evolution tracks, usually neglecting rotation, so presently stated values are likely to change in the future.

Ledoux (1941) investigated the possibility of nuclear burning to destabilize massive stars pulsationally through the action of the ϵ-mechanism (cf Section 3.3.1 in GS95). With very simple input physics he derived a mass of about 100 $M_\odot$ on the main sequence, above which stars pulsate due to the ϵ-mechanism of CNO burning. For most stars, the relative pulsation amplitudes derived from comparing the deep interior with the surface regions are small throughout the thermonuclear burning region so that the ϵ-mechanism is inefficient for pulsational destabilization. Above a critical mass, radiation pressure becomes large enough for the displacement to achieve sufficient amplitude also in the central region. The ϵ-mechanism can then contribute efficiently to the work integral and eventually overcome the radiative damping in the envelope. Hence, radiation pressure is an important side aspect for the action of the ϵ-mechanism. Cox (1974) reviewed attempts made before the mid-1970s to determine the upper mass limit on the main sequence due to ϵ-destabilization. This question turned out to be a delicate quantitative problem because the driving rates due to nuclear-burning terms are very low, usually of the order of $|\sigma_{\rm I}/\sigma_{\rm R}| = 10^{-6}$ (such expressions are obtained when the time dependence of the pulsation equations is parameterized by $\exp({\rm i}\sigma \cdot t)$, a ratio of 10^{-6} means that it takes $10^6/2\pi$ pulsation periods for the amplitude to grow by a factor of 2.72). Small nuclear driving has to compete with comparable radiative damping in the envelope. Hence, slight inaccuracies, e.g. in treating opacities and their derivatives, have serious consequences for the final outcome. Discussions in the literature of numerical results on this issue were therefore accordingly heated and involved in the past.

After some years of quiescence, the question of the upper mass limit received renewed attention after the advent of the new generation of opacity tables (OPAL and OP). Stothers (1992) concluded, on the basis of OPAL opacities, that an improved temperature dependence of the opacity leads to a higher central concentration in the stars, which reduces the efficiency of the ϵ-mechanism. The radial fundamental mode turned unstable for higher masses than in earlier studies (e.g. Ziebarth 1970), namely at 121 $M_\odot$ for $Z = 0.02$. In a more elaborate study, which included a larger mass range and which also involved analyzing a number of low-order overtones, Glatzel & Kiriakidis (1993a) arrived essentially at the opposite of Stothers' conclusions. The instability of the radial fundamental mode was computed to be of only minor importance because a much stronger instability (with $|\sigma_I/\sigma_R| \approx 0.1$) developed due to strange modes at lower masses. This mode-resonance instability appears at masses above about 60 $M_\odot$. Increasing the stellar mass, the instability extended from high frequencies towards the radial fundamental mode. However, even when considering only ϵ-destabilization, Glatzel & Kiriakidis (1993a) concluded that the critical mass was smaller than that determined from older studies.

The repeatedly recovered discrepancies in the magnitude of the critical mass determined by ϵ-destabilization may well be connected with details of the numerical treatment of the opacity data. Nonetheless, the existence of strange modes is undoubtedly a secure feature induced by the pronounced Z-bump and has an important impact on the stability of massive stars (see also Section 4). The growth of the strong strange-mode instabilities into their nonlinear regime has still to be thoroughly investigated. Glatzel & Kiriakidis (1993a) reported large mass-loss rates to have occurred in preliminary computations.

Whether ϵ-driven pulsational instabilities are crucial for terminating the massive end of the mass function remains unclear. Instabilities induced by nuclear burning are most efficient when the stars are essentially homogeneous, i.e. when they have settled on or close to the zero-age main sequence (ZAMS). The growth times of ϵ-induced instabilities are comparable with the main-sequence lifetimes and might hence be irrelevant for stellar evolution issues. The strange modes, on the other hand, show growth times of the order of the dynamical timescale of the stars; these envelope oscillations are much better candidates for shedding mass.

3. PULSATIONS IN EVOLVED LOWER-MASS STARS

First, lower-mass stars—understood as those climbing the asymptotic giant branch (AGB) and starting carbon burning in a degenerate state ($\lesssim 8\ M_\odot$ on the ZAMS)—are discussed with respect to the pulsational instabilities they encounter during their evolution. All stars originating from regions hotter than the classical instability strip on the ZAMS (i.e. $M_* \gtrsim 2\ M_\odot$) are prospective pulsation variables as they cross the HR diagram and pass through the instability strip on their way to the base of the giant branch. However, the evolutionary timescale is so short (therefore the notion of the Hertzsprung *gap*) that the probability of observing them therein is very low. The triple-mode pulsator AC And is presently considered as one of those rare objects in this transition phase (Fernie 1994). In the following, we restrict our attention to phases after the onset of central helium burning.

Combined photometric and radial-velocity observations of classical pulsators—such as Cepheids and RR Lyrae stars—are frequently used to derive radii and associated quantities by means of the Baade-Becker-Wesselink method. The basic principle underlying the approach is simple: From photometry and spectroscopy, radius ratios and radius differences are determined at suitably chosen phases during the pulsation cycle so that in principle absolute radii can be deduced. The detailed application to obtain accurate physical calibrations of pulsating stars is, however, neither very transparent nor easy. A number of assumptions must be introduced (e.g. on limb darkening, asymmetries of line profiles during pulsation, physical calibration of color indices, etc). The degree to which these assumptions are realized in stars restricts the level of accuracy achievable, and it may change from one type of pulsator to another. Gautschy (1987) and Moffett (1989) discussed the Baade-Becker-Wesselink method and its variants comprehensively; Moffett (1989) emphasized more recent developments.

3.1 *Through the He-Burning Stage*

When stars ascend the giant branch the more massive ones may reach the low-luminosity domain of red variables (see the upper right in Figure 1 in GS95) already during the first ascent, before the onset of central He burning. The general belief is, however, that the red variable stars are already in the asymptotic giant branch stage. During their first ascent stars spend only a short time in the topmost region of the giant branch. Hence, the probability is rather low of finding a red variable in that evolutionary stage.

3.1.1 RR LYRAE STARS[2] After the onset of degenerate central He burning, stars with masses between roughly 0.5 and 2.0 $M_\odot$ settle on or close to the horizontal branch (HB), but only stellar masses below about 0.75 $M_\odot$ are potential RR Lyrae pulsators during some phase of central helium burning. Accurate numbers depend on details of the assumed stellar physics (Dorman 1992a). The periods of this class of variable stars are around half a day. Stars appear as RR Lyrae variables either when they are close to the zero-age HB or else later upon their evolving to the blue or to the red (depending on the mass of the star). Observed HB stars are deduced to have had main-sequence masses above about 0.8 $M_\odot$ to reach the HB stage within a Hubble time.

The basic instability mechanism responsible for RR Lyrae pulsations is well understood on the basis of linear pulsation theory. Cox (1974, 1975) provided extensive reviews of the theoretical situation up to the early 1970s. Later,

[2]Three subclasses of RR Lyr pulsators, denoted by *a*, *b*, and *c*, exist. Types *a* and *b* have asymmetric light curves and are fundamental modes pulsators. The light curves of the first overtone type *c* RR Lyrae stars are essentially sinusoidal. For details see Cox (1974).

attention focused on nonlinear modeling (Stellingwerf 1975, 1982; Kovács & Buchler 1988a), the question of mode selection (Simon et al 1980, Buchler & Kovács 1986), and the redward extension of the instability strip including the role played by time-dependent convection. In an early attempt, Deupree (1977a,b) simulated two-dimensional convection and its interaction with pulsations of RR Lyr stars. He found convection to quench pulsations in low-$T_{\rm eff}$ models. Considerable efforts were invested in developing one-dimensional descriptions of time-dependent convection, which were coupled with hydrodynamical codes (Xiong 1981; Stellingwerf 1982, 1984a,b,c; Stellingwerf & Bono 1993; Gehmeyr 1992a,b, 1993). Such simulations find concurringly that the convective flux is enhanced during the compressed phases, diminishing the efficiency of the κ-mechanism, which couples to the radiative flux. Convection tends to reduce the amplitude of the pulsation; the actual amount depends on free parameters of the particular convection descriptions. In low-$T_{\rm eff}$ models with extended convection zones the variation of the convective efficiency during the pulsation cycle is advocated to cause a bump in the ascending branch of the light curve (Stellingwerf 1984c, Gehmeyr 1992b). At present it is unclear whether such a bump prevails in observed light curves of RR Lyrae stars. Accurate photometry of red RR Lyr stars should be useful to constrain the role played by convection theory in such pulsators.

RR Lyrae variables constitute easily identifiable and rather bright members of many globular clusters. As a consequence, RR Lyrae stars are used extensively as standard candles to determine distances to their host clusters. Based on the absolute magnitude of the RR Lyrae variables, ages of globular cluster systems are determined that bear important clues for galactic evolution and even for cosmology (Sandage 1982a,b or Sandage 1993a,b,c). To address questions of globular-cluster ages and distance scales the presently required accuracy of the physical calibration of RR Lyrae variables is very high. Delicate and complex issues of location and morphology of the horizontal branch (Buonanno et al 1989, Caputo et al 1989, Dorman 1992a, Lee et al 1994) and the range of acceptable evolutionary stages of RR Lyr variables (Lee et al 1990) are being investigated. On the pulsation-theoretical side, extensive numerical simulations (Kovács & Buchler 1988a, Simon 1989, Guzik & Cox 1993, Feuchtinger & Dorfi 1994) including the latest improvements in constitutional physics were performed. These attempts concentrate on the influence of envelope physics; modified input physics in the deep interior (such as opacity sources or elemental mixing processes) seems to affect mostly the structural properties (Dorman 1992b). Extensive grids of combined calculations for which pulsation properties are derived from stellar evolution models, both using the same input physics, do not exist. Such a procedure might at least eliminate the worry of how much disagreement between stellar evolution and pulsation theory is induced by inconsistent modeling procedures. Additional uncertainties are introduced by the lack of fully self-consistent temperature–color-index relations over the whole range relevant for RR Lyrae stars and by the choice of mean values of the periodically varying color indices (Sandage 1990a). Roughly speaking, the ultimate goal is eventually to reach a proper understanding, and therewith a correct quantification, of the Oosterhoff-period-shift (the mean period of RR Lyrae stars in Oosterhoff-type I clusters is 0.1 day shorter than that of Oosterhoff-type II clusters when determined by an ensemble mean over the periods) or of the Sandage-period-shift (the RR Lyrae periods decrease as the cluster metallicity increases, determined on a star-by-star basis at fixed $T_{\rm eff}$) (cf Sandage 1982a, 1990b; Bono et al 1994). This is done by arriving at acceptable RR Lyrae masses and luminosities over the whole parameter domain ([Fe/H], Y, [O/Fe],...) covered by globular clusters. A general agreement has not been reached, and the body of literature on that issue is considerable. The review of Rood (1990) pointed out the complexity associated with horizontal branch evolution, and he has sketched the large parameter space that must be dealt with correctly.

The elusive origin of the *Blazhko effect* is of much interest for stellar pulsation theory. This effect is a secular variation in form and amplitude of the fundamental-mode oscillation on a timescale between 20 and 100 days and is found in about 15–30% of the RR*ab* pulsators. A comprehensive review of the observational aspects was presented by Szeidl (1988). It seems, as also pointed out by Gloria (1990), referring to field RR Lyrae data from the 4th edition of the *General Catalogue of Variable Stars*, that the mean period of the Blazhko effect RR*ab* stars is shorter than the mean period of steadily pulsating RR*ab* variables. The theoretical meaning of this correlation, should it indeed not be an observational bias, is unclear. The Blazhko effect has been confirmed only for RR*ab* stars but not for RR*c* variables. One theoretical model—the oscillating oblique magnetic rotator (Cousens 1983)—attributed the Blazhko effect to a stellar magnetic field and its interaction with radial pulsation. The Blazhko period would then essentially be the stellar rotation period. At least for the star RR Lyrae itself a significant magnetic field that varies with the Blazhko period as well as with the pulsation period is observed (Babcock 1958, Romanov et al 1987). For other variables with a Blazhko effect, reliable data do not exist. The observed tertiary period of RR Lyrae, a modulation of the Blazhko effect itself, on a timescale of about four years, is preferably attributed to the magnetic cycle of the star. Very recently, Takata & Shibahashi (1995b) revisited and rederived the oscillating oblique magnetic rotator model. The method is very similar to the modeling of roAp stars, but in RR Lyrae stars the radial fundamental mode is self-excited and this mode picks up quadrupole components due to Lorentz and Coriolis forces. In contrast to Cousens (1983), Takata & Shibahashi (1995b) found a Blazhko amplitude that depends on the magnetic field strength. Moskalik (1985) attempted an explanation by looking for possible internal mode resonances in RR*ab* stellar models. His amplitude equations, including only lowest-order nonlinear couplings, revealed a resonance between the fundamental mode and the third overtone as the most likely way to produce a light variation similar to the Blazhko effect. The long-term evolution, however, could not be studied in his model. Since no clear discriminants emerge from either the theoretical or the observational side, none of the suggested explanations can yet be disqualified.

Despite RR Lyrae stars having usually low heavy-element abundances the new opacity generation (OP and OPAL) had some impact on the period ratios of double-mode RR Lyrae (RR*d*) stars. Kovács et al (1991) discussed the period ratios resulting from newly constructed pulsation models that were assumed to be appropriate for double-mode RR Lyrae variables in Oosterhoff I or II type clusters. Acceptable masses (compared with stellar evolution) could only be obtained when requiring $Z < 0.001$; within the parameter Z the distribution of the heavy elements was assumed to be solar. When allowing for nonsolar heavy-element ratios relative to Fe in the stellar material, an unexpected ambiguity in the results emerged due to competing effects from different chemical species (Kovács et al 1992).

In the Galactic field, only three RR*d* stars are presently known (Jerzykiewicz & Wenzel 1977, Clement et al 1991). Even in globular clusters, the RR*d* phenomenon was only recently appreciated (Sandage et al 1981). Szeidl (1988) estimated up to about 15% of RR Lyrae stars to be of RR*d* type. These variables are believed to be pulsating simultaneously in the fundamental and in the first overtone mode. Indeed, the colors or temperatures of RR*d* stars are confined to the transition region between the RR*c* and the RR*ab* instability domain. Observationally, the first overtone always has a higher amplitude than the fundamental mode. Kovács et al (1986) found the relative contributions of the fundamental and first overtone mode to have remained constant in the RR*d* stars of M15 over two decades. This result is not compatible with the simple mode-switching scenario of a star evolving accidentally through the transition region. Also, the observed number of RR*d*s appears to be too high for such a picture to apply. Bono & Stellingwerf (1993) pointed out that for their calculations the timescale for mode switching agrees better with predictions from stellar evolution. Nonetheless, the physical mechanism for long-term maintenance of double-mode pulsations is far from understood. Nonlinear pulsation calculations proved to have severe problems in simulating stably pulsating double-mode models (Stellingwerf 1974, Kovács et al 1992). Despite much effort, observed properties of double-mode RR*d* stars and double-mode Cepheids have not been reproduced satisfactory. For Cepheids, no stably pulsating double-mode models are known to exist. Some persistent double-mode pulsations have recently been constructed for RRd-type models (Stellingwerf & Bono 1993, Kovács & Buchler 1993). In most of these models, however, the amplitude of the fundamental mode is larger than the one of the first overtone—contradicting the observational evidence. The three-mode resonance $\sigma_F = \sigma_1 + \sigma_2$ seems to play an important role for persisting double-mode pulsations. The subscripts at the oscillation frequencies σ refer to the fundamental and the first two overtone modes. Kovács & Buchler (1993) noticed that the artificial viscosity required for the numerical codes to work had to be reduced below a certain threshold for the RR*d* models to reach the correct periods.

Observationally, the first overtone pulsators, the RR*c* variables, have light curves that are sinusoidal and very distinct from the asymmetric light curves of the fundamental-mode RR*ab* pulsators. This behavior is well reproduced by nonlinear simulations. The physical determinants of the light curve form have never been explained satisfactorily though. An attempt, based on one-zone models, was published by Stellingwerf et al (1987).

3.1.2 POPULATION II CEPHEIDS As the evolution after He core burning proceeds, stars with masses above approximately 0.51 $M_\odot$ evolve off the HB to approach and ascend the AGB. Either during the early evolution away from the HB or during shell flashes along the AGB, some stars can enter the instability strip again and appear as so-called population II or Type II Cepheids. The periods of population II Cepheids range from about 0.8 days—at the transition to RR Lyrae stars—to about 30 days, above which the stars in the classical instability strip are classified as RV Tau stars (see Section 3.1.3). For observational and theoretical reviews see Wallerstein & Cox (1984) and Gingold (1985), respectively. Linear pulsation calculations reveal that the combination of He II and H/He I ionization drives the pulsations. The fundamental radial or the first overtone mode seems to be excited (Nemec et al 1994). The instability strip of the metal-poor pulsators is considerably broader than that of population I variables at the same luminosity. In particular the red edge is shifted to lower temperatures. Wallerstein & Cox (1984), referring to Deupree & Hodson (1977), argue that this downward shift in temperature is connected with a reduced efficiency of convection. No recent, and in particular no systematic, studies are available in the literature that clarify what pulsation modes are potentially excited and where, in detail, the borders of the instability region of population II Cepheids lie on the HR diagram.

The short-period population II Cepheids or AHB1 (above horizontal branch; Diethelm 1990, Sandage et al 1994) are post-HB stars that pass through the instability strip during their evolution towards the AGB as they exhaust the helium in their cores. The period range of AHB1 stars is about 0.8–5 days. The observed properties and theoretical interpretations are thoroughly discussed in Sandage et al (1994). (Although these variables were sometimes called BL Her stars, the name is not suitable because the star BL Her itself is not metal deficient.) Since the timescale of evolution in the core helium exhaustion phase is much faster (of the order of 100 times) than during the core He burning phase, the number of AHB1 stars is much smaller than that of RR Lyrae stars. Because of the fast evolution, period changes for some of the AHB1 stars are actually observed (Wehlau & Bohlender 1992, Diethelm 1996). These data show that their periods are increasing on a timescale of 1–10 days / 10^6 years, in accordance with theoretical prediction. Below about $M = 0.51\ M_\odot$ (depending on the uncertainties of chemical composition) the stars do not evolve back to the AGB and hence do not cross the instability strip anymore; they instead turn towards high temperatures at some early phase of their post-HB evolution.

There is a group of population II variables called AC (anomalous Cepheids) that have periods similar to those of AHB1 stars. Their period-luminosity relation is, however, different from that of AHB1 variables, indicating that they are more massive than AHB1 stars (see e.g. Wallerstein & Cox 1984, Nemec et al 1994). The AC stars are thought to result from the coalescence of close binaries.

The period distribution of the population II Cepheids shows a gap between 5 and 10 days. The lack of stars in this period, and hence luminosity range, is thought to have evolutionary origin. Stars with short periods are leaving the HB to subsequently approach the AGB. Stars with long periods are either on a blueward excursion during late He-shell flashes or are already on their final departure from the AGB on their way towards the white-dwarf cooling region (Gingold 1976). The luminosity differences within the group of longer-period population II Cepheids (W Vir variables) can be attributed to a mass difference of the pulsators or to a different evolutionary status. Evolutionary calculations indicate that only below a critical remaining envelope mass do the He-shell flashing objects perform blueward loops. The number and distribution of periods of long-period population II Cepheids and their comparison with the estimated evolutionary timescales within the instability strip are not in satisfactory agreement (Gingold 1976, 1985). Therefore, a good statistical sample of the luminosity distribution of population II Cepheids should help considerably in clarifying the issue of their evolutionary status. Also, if the long-period population II pulsators have indeed terminated their AGB evolution and if they are not merely in an unstable He-shell burning episode, their periods should exclusively decrease in the long run as they evolve to higher temperatures.

3.1.3 RV TAU VARIABLES The longest-period W Vir stars seem to continuously change into what are classified as RV Tau variables, so they might in principle be considered as long-period population II Cepheids. Their pulsations are also driven by partial H and He ionization. The light curves show regularly alternating deep and shallow minima (double-wave form). Based on the luminosities ($\log L/L_{\odot} > 3$), the application of the coremass–luminosity relation, and the length of periods (50–150 d) larger masses are deduced for RV Tau variables than for the lower-luminosity W Vir stars. Jura (1986) saw indications in *IRAS* data of RV Tau stars leaving the AGB and evolving towards the white-dwarf domain; he inferred a very short duration—of the order of 500 years—of the pulsation phase. The double-wave light curves, which are a defining characteristic for RV Tau variables, are attributed to an internal resonance effect (similar to the Hertzsprung progression in Cepheids; see Section 3.1.5). Based on linear adiabatic theory, no satisfactory mode resonances could be identified by Takeuti & Petersen (1983). Fadeyev & Fokin (1985) reported a 2:1 resonance between the fundamental and the first overtone mode in their nonlinear modeling of RV Tau-like stars. Linear nonadiabatic pulsation calculations led Worrell (1987) to conclude that a single resonance is unlikely to be sufficient to understand the double-wave light curve of the RV Tau variables over the whole relevant $T_{\rm eff}$ and L range. The opposite conclusion was reached by Tuchman et al (1993) based on their linear nonadiabatic study.

In recent years, a number of nonlinear pulsation simulations have been performed to study the dynamical properties of population II pulsators. As the luminosity-to-mass ratio of the models was increased the oscillatory motions underwent transitions towards a low-dimensional chaotic behavior (Buchler & Kovács 1987, Aikawa 1987, Kovács & Buchler 1988b, Moskalik & Buchler 1990, Aikawa 1993). Period doublings, suggesting the so-called Feigenbaum sequence towards chaotic dynamics, occurred in a series of models with high luminosity-to-mass ratios upon reducing the effective temperatures of the equilibrium models. When increasing the L/M ratios, tangent bifurcations, leading to intermittency in the dynamics, were encountered. Based on such numerical studies, Kovács & Buchler (1988b) concluded that the RV Tau light curves represent early phases in a naturally occurring period-doubling bifurcation sequence for which the L/M ratio serves as a control parameter. In the simulations, the increasing degree of irregularity of the pulsations with rising luminosity (at constant mass) due to continuing period-doublings finds some observational correspondence.

The fully radiative pulsation models studied in the previously mentioned analyses, which adopt chaos-like properties, all approached low-dimensional chaotic attractors. A major deficiency of the modeling was the omission of time-dependent convection with its feedback on the pulsations. Whether chaotic dynamics still develops along the same routes and whether low-dimensional attractors will persist even in the presence of extensive convective envelopes remain unclear (Perdang 1991). To ensure the correct phase-space behavior, in particular for the long-term evolution, the quality of the numerical methods has to be very high; for critiques of numerical methods used for computing of chaotic phenomena see Miller (1991) and Yee et al (1991).

3.1.4 MIRA AND SEMIREGULAR VARIABLES The low-mass, long-period ($P \gtrsim$ 80 d) variables located at very low temperatures and luminosities above about $10^3\ L_\odot$ (cf Figure 1 in GS95) are known under a variety of names in the literature. These long-period AGB stars play an important and still controversial role in our understanding of strong mass loss, the formation of planetary nebulae, and the structure of the white-dwarf mass-function. From the point of view of pulsation theory we do not distinguish between the various observationally motivated classes that are conceivably superimposed on the same basic processes in these stars. The different behavior of the various families of cool variables can be caused by different masses, chemical compositions, or evolutionary stages along the AGB. For example, the observational distinction between Mira and semiregular (SR) variables is not necessarily a deep physical one. Often the distinction is based on the amplitude of the light variation. Large-amplitude variables are attributed to the class of Miras whereas low-amplitude pulsators are considered as SR variables. Both categories have, however, roughly the same regularity of their pulsational cycles (Whitelock 1990).

The driving mechanism of the pulsations is probably the combined action of partial H and He I ionization. Over a large fraction of the envelopes of these variables, energy transport by convection dominates and the timescale of convective overturn is of the same order as the pulsation cycle. Hence, any statements on the pulsational driving and on the extension of the instability region depend crucially on our understanding of the coupling of pulsation and convection and of the influence of convection on the equilibrium structure of the stars. Both aspects are not well comprehended at present. Balmforth et al (1990) showed that the inclusion of turbulent pressure in their models alters the equilibrium structure so that the excitation rates and also the periods of radial pulsation modes are considerably influenced. In pulsation calculations, not only should the perturbed convective energy flux be accounted for but also the perturbation of the turbulent pressure to fully describe the coupling of pulsation and convection in these envelopes. For their supposedly low-luminosity AGB model Balmforth et al (1990) obtained the first overtone as the dominantly destabilized pulsation mode; this agrees with the results of Fox & Wood (1982), who included the time dependence of the convective flux in a kind of flux retardation model. Only at luminosities leading to periods of about 320 days does the fundamental mode grow faster. Based on these calculations, most Mira variables were deduced to pulsate in the first overtone mode. Ostlie & Cox (1986) concluded from their linear pulsation calculations (in which turbulent pressure was included in some of the equilibrium models but the perturbation of the convective flux was neglected in the stability analyses) that Mira-type variability is consistent with fundamental mode pulsations. Using nonlinear initial-value simulations of Mira-type pulsations, Wood (1990a) concluded that, after artificially suppressing the growth of the fundamental mode in his models to force them into the first overtone, the velocity fields that built up were incompatible with observations. Hence, Miras were considered to pulsate in the fundamental mode. Tuchman (1991), applying his "acceleration analysis" to observed CO molecular lines in Mira variables, rejected the possibility of a fundamental-mode variability in his sample.

Spatial high-resolution observations of Mira (*o* Cet) itself (Haniff et al 1992) and of R Leo (Tuthill et al 1994), together with parallax estimates, allowed a direct estimate of the radius. Rather independent of the particular choice of the mass, a pulsation constant Q resulted that pointed to an excited first overtone. If, however, radii were derived from the excitation temperature of CO molecular lines the pulsation of *o* Cet was assigned to the fundamental mode (Hughes 1993). It must be kept in mind that the radii of Mira-type stars change by more than a factor of two when going from optically thin regions to the stellar photosphere. Thus, care has to be taken when radii derived by different techniques are compared.

Bessell et al (1989) attempted to reduce uncertainties in determining temperatures of Mira variables at different pulsation phases. Uncertain temperatures corrupt the accurate determination of Q values. They based their detailed radiative transfer calculations on density and temperature profiles obtained from nonlinear simulations.The resulting spectra are not yet fully satisfactory, possibly because detailed thermodynamic and radiative processes in the shock regions need to be dealt with in the hydrodynamic simulations. Furthermore, the quantifications of the pulsation constant Q rely on linear stability analyses (cf Wood 1995). To conclude, the debate on which mode is excited in Mira variables cannot presently be considered as settled (for a detailed recent discussion see Wood 1995).

Nonlinear pulsation simulations of Mira envelopes (incorporating with radiative energy transport only) show complicated multiple shock structures in their atmospheres (Bowen 1988, Wood 1979). Mostly, such calculations were not based on self-excited pulsations but on piston-driven motion of the stellar matter in the outermost layers. Although Feuchtinger et al (1993) presented promising preliminary results of high-quality numerics and coupled radiation-hydrodynamics, they still based their calculations on piston-driven pulsations in purely radiative model envelopes. Höfner et al (1995), using an extension of the same Viennese radiation-hydrodynamics code, presented interesting results of time-dependent dust formation in the atmospheres of long-period variables. They obtained quasi-periodic dust formation/destruction cycles and associated variable mass-loss rates even with static inner boundary conditions for certain abundance ratios of carbon and oxygen.

Observations indicate that the mass-loss rates of Mira variables are correlated with the pulsation period. Longer-period stars, which are also more luminous, tend to have higher mass-loss rates (Whitelock 1990 and references therein). The available simple nonlinear simulation models cannot reproduce the high values observed (Wood 1990b). Considerable improvements can be expected from properly dealing with convection in the envelopes of the long period variables and/or from the dynamical influence of grain formation and destruction (Höfner et al 1995). Pijpers & Habing (1989) estimated that dissipation of acoustic energy flux due to convection would be able to induce mass-loss rates between 10^{-7} and 10^{-4} $M_\odot$ year^{-1}.

A simple evolutionary scenario assumed Mira variables to evolve along the AGB towards higher luminosities and longer periods with an accompanying increase of mass loss until a critical luminosity was reached where the envelope would be shed. Thereafter, the Mira variables would be obscured by an optically thick envelope and become possibly observable as variable OH/IR sources that eventually evolve into a planetary-nebula system. Both observational and theoretical evidence speak against such a simple picture (Wood 1990a, Vassiliadis & Wood 1993, Whitelock et al 1994), however. Comparing the number densities of Mira stars with those of planetary nebulae or clump giants suggests that the Mira phase lasts for only about 5×10^4 years. For stars with masses of about 1 $M_\odot$ evolution on the AGB during a Mira lifetime does not result in a significant luminosity increase nor in any appreciable period change. The kinematic properties of Miras change with the length of their pulsation period, indicating that only small changes of the pulsation period occur during the lifetime of a Mira variable (Whitelock et al 1994). Additionally, Wood (1990a) argues that it is not possible for stars with masses below about 1.5 $M_\odot$ to ascend sufficiently high on the AGB to explain the very long periods (1000–2000 days) and the very low temperatures observed in OH/IR sources. Hence, the OH/IR variables are assumed to stem from a subgroup of stars that are more massive than what we see as shorter-period Miras. Under such circumstances the OH/IR sources were to be considered as late AGB stars rather than as post-AGB and pre–planetary nebulae objects. In terms of period, the variable AFGL objects, stars with strong

IR excesses, lie between the optically identified Mira variables and the radio-luminous OH/IR sources. The same applies with regard to the amount of mass loss and the degree of obscuration by circumstellar material (Jones et al 1990).

According to the interpretation of the Mira instability region mentioned above, the observed period-luminosity (PL) relation established for LMC Mira variables (Feast et al 1989 and references therein) would not represent an evolutionary sequence of stars but indicates a range of stellar masses occupying the Mira domain at different luminosities. Because red variables belong to the most luminous stars in a stellar system, the existence and robustness of a PL relation is of relevance for any kind of distance determination. Hence, the quality of the actual PL relation and estimates of the intrinsic scatter (either a stochastic one or one due to a hidden mass or color dependencies) are essential. Feast et al (1989) found that the Mira variables, in particular the O-rich Miras, obey well-determined period-luminosity-color relationships. This indicates that Mira variables occupy an instability strip of finite width on the HR diagram.

Because of their comparable kinematic properties the SR variables in the Galaxy are believed to emerge from the same populations as do the Mira variables (Jura & Kleinmann 1992). The periods of the SR variables are (at least in the sample of Jura & Kleinmann) usually shorter than those of Miras. Also, for the SR variables, the determination of the pulsation mode is controversial. Presently the short-period SR ($P \lesssim 150$ d) stars are assumed to be first or second overtone pulsators. For longer-period semiregular variables, on the other hand, pulsations in the fundamental mode are preferred.

Irregular variables show spectra with clear giant or supergiant characteristics. Jura & Kleinman (1992) referred also to the kinematical properties that led them to assume an affiliation of these stars with the same population as the long-period Mira variables with $300 < P < 400$ d. The evolutionary state of the irregulars seems, nevertheless, to be unclear in particular due to the uncertainties in assigning reliable luminosities to them. If the irregulars are situated at the low-luminosity end of the AGB then, in accordance with the picture of Wood & Cahn (1977), several higher overtone modes could be excited simultaneously, giving rise to a seemingly chaotic light variability. Another point of view is that the irregular variability is indeed a purely stochastic phenomenon (Perdang 1985). The latter suggestion is somewhat off the main line of thought as we do not know of any other class of stars being destabilized in this way. Careful monitoring of irregular variables over long time spans should enable the discrimination between the two models, at least if, according to the multimode picture, the number of simultaneously excited modes is small.

Some variables on the AGB with nonstrictly repeating light variations have undergone long-term temporal analyses to identify chaotic signatures in their light curves. [For an introduction into the concepts and the language of chaotic dynamics and its applications in astrophysics see Buchler et al (1985).] Blacher & Perdang (1988) analyzed a number of Mira variables by applying a "variance-function" approach. Cannizzo et al (1990) investigated long series of observations of the Mira variables *o* Cet, R Leo, and V Boo to reconstruct the underlying dynamical attractor. Kolláth (1990) studied 150 years of data for the RV Tau star R Sct. Based on a shorter temporal sequence of observations, Buchler et al (1995) claimed the identification of a four-dimensional embedding space for the quasi-regular variability of R Sct. In this case the amplitude modulations could be understood from nonlinear interaction of only two simultaneously excited pulsation modes. The irregular light variation of RU Cam—a W Vir star whose regular pulsations disappeared in the recent past—was analyzed by Kolláth & Szeidl (1993). Except for R Sct, none of these investigations unveiled compelling evidence for a low-dimensional chaotic attractor. Despite all the efforts we have no evidence that deterministic chaos occurs frequently in irregularly pulsating stars. Perdang (1991, 1993) argued that stars with convective layers (AGB stars have extensive ones) might not become chaotic with low-dimensional attractors. In these convective envelopes a large number of unstable degrees of freedom are unlocked by convective motion. It is not necessarily clear that a few outstanding modes would dominate the dynamics and therefore reduce the dimension of the attractor. It must also be remembered that transitions to chaos through tangent bifurcation or cascades of period doublings as found in simulations (Buchler & Kovács 1987) or in simple model systems (Buchler & Goupil 1988, Tanaka & Takeuti 1988, Takeuti 1990) are based on hydrodynamics incorporating only radiative energy transport.

3.1.5 CEPHEID PULSATIONS After the onset of nondegenerate core He burning, the luminosity of stars more massive than about 2.2 $M_{\odot}$ drops, and they very closely evolve down the track on which they ascended the giant branch the first time. Stars with masses between about 2.2 and 3 $M_{\odot}$ settle at lower luminosity to burn most of their central helium before they ascend the giant branch a second time (AGB evolution). Stars with $M \gtrsim 3\ M_{\odot}$ perform blueward loops during that evolutionary phase. Below roughly 5 $M_{\odot}$ (in the framework of standard stellar evolution without semiconvection and overshooting) the blue loops are not sufficiently pronounced to let the star enter the instability strip. Higher-mass stars cross the strip two or more times as they perform one or more loops, and they appear as Cepheids. The review of Cox (1975) provides an exhaustive account of the early theoretical developments in the field of Cepheid pulsations. The proceedings of a conference dedicated to two centuries of observed Cepheid variability (Madore 1985) contain important contributions to the advancement of our understanding to the mid-1980s. Becker (1985) and

Chiosi (1990) provide instructive accounts of evolutionary aspects and of the persisting uncertainties in connection with Cepheid pulsations. In that regard, the masses cited above should be taken as rough guidelines only; the particular values are likely to change depending on the various physical assumptions and on input data required by stellar evolution codes. The existence and also the topology of the blueward loops of massive stars during the core He burning phase and the double-shell burning stage are known to depend sensitively on subtle issues in the numerical treatment of stellar interiors (Lauterborn et al 1971, Höppner et al 1978, Becker 1985).

Cepheids are well known for the mass-discrepancy problem, which persisted for several decades. Cox (1980) reviewed the topic and elaborated on the various methods of mass determination applicable to Cepheids. The essential point in the mass-discrepancy controversy was that any kind of mass estimates inferred from stellar pulsation theory turned out to be systematically lower than the predictions from stellar evolution theory. Presently, the Cepheid mass discrepancy can be considered as essentially reconciled. The improvements were achieved with the help of the new generation of opacity data (OPAL and OP). Extensive pulsation calculations by Moskalik et al (1992) showed that the period ratios are considerably reduced when employing the new Rosseland opacity tables (see Figure 1). The Z-bump in the Rosseland opacity near 10^5 K alters the stellar structure such that the low-order pulsation frequencies shift differentially and lower the period ratios considerably at a fixed stellar mass. The microlensing searches of recent years provided, as side results, extensive data collections on variable stars. The MACHO consortium presented data of beat Cepheids in the LMC (Alcock et al 1995); some of them appear to be pulsating simultaneously in the first and second overtone. Christensen-Dalsgaard & Petersen (1995) reconciled the observed period ratios rather well with linear adiabatic computations based on the new opacities and otherwise standard stellar physics. The masses of these double-mode Cepheids with simultaneously excited fundamental and first overtone modes are presently attributed to values derived from linear pulsation theory, which are close to evolutionary masses. Masses of around 4 $M_\odot$ are now considered to be an appropriate low-mass domain for double-mode Cepheids; they might require some modifications in a stellar evolution treatment or a careful tuning of the X:Y:Z ratios to force them into the instability strip. New stellar evolution tracks based on OPAL opacity data (Schaller et al 1992) show that the Z-bump tends to diminish the sizes of the blueward loops. A 5 $M_\odot$ star with $Y = 0.3$ and $Z = 0.02$ does not—in contrast to calculations with old opacity data—enter the instability strip anymore. A reduction of the heavy-element abundances, for example (Schaerer et al 1993), produces sufficiently elongated blue loops for 5 $M_{\odot}$ stars to enable them to appear as short-period Cepheids.

The occurrence and the shift with period of the location of a secondary maximum (or bump) in the light curve of Cepheids in the period range from 4 to 20 days is known as the *Hertzsprung progression.* The phase of the bump within the pulsational cycle changes monotonically as a function of period and hence as a function of mass. The observed bump location can be used to infer the mass of the pulsating star. Since the early days of nonlinear modeling of stellar pulsations (Christy 1968) it was clear that the mass deduced for a given bump phase did not agree with the standard mass-luminosity relation. Again, the results based on the latest opacity data, reported by Moskalik et al (1992), indicate that stellar-evolution masses are now in much better accord with the bump masses. The bump is attributed to the accidental 2:1 ratio of the period of the second overtone to that of the fundamental mode (Simon & Schmidt 1976). Stars with periods below about 20 days pass through this 2:1 resonance (Kovács & Buchler 1989). Because a period ratio is also involved in the mass determination of bump Cepheids, it is understandable that the modified opacities affected them in the same way as the beat Cepheid masses.

Despite providing a resolution to the Cepheid mass problem the new opacity data introduced new and unexpected complications. The period ratios admitted by the pulsation models, much like the ones for the RR Lyrae pulsators, appear to show a noticeable dependence on the particular contributions of heavy-element abundances to the Z abundance parameter. An easy and accurate mass determination for pulsators seems no longer possible without taking additional care in determining the chemical composition of the object.

The pulsational instability of Cepheids is well explained by linear pulsation theory; the nonlinear behavior, however, is not so comprehensively understood. Fernie (1990) analyzed observations and found that despite large amplitudes being predominantly associated with longer periods, large- and small-amplitude pulsators mix over essentially the whole extent of the instability strip. In particular, the case of α UMi is presently of interest. The amplitude of Polaris diminished exponentially during this century, and it was expected to stop pulsating or at least drop below the detection limit of some millimagnitudes by 1994. In contrast to this prediction, Krockenberger et al (1995) reported continuing pulsations at low but detectable amplitude. Fernie et al (1993) established that α UMi is not on the verge to cross the red edge of the instability strip. It has cooler neighbors in the instability strip that pulsate at significantly higher amplitudes. Other Cepheids are known that are unusual either in terms of their amplitude (γ Cyg, Butler 1992) or in terms of the modal behavior (HR 7308, Burki et al 1986). We currently lack any systematic studies addressing the long-term evolution of nonlinear limit cycles that would help us understand such stars. Direct numerical integration of the initial-value problem might be of little use. The amplitude-equation ansatz (Buchler & Goupil 1984) together with qualitative methods from dynamical-system theory could serve as starting points.

Short-period Cepheids (with $P \lesssim 7$ d) having low-amplitude sinusoidal light curves are classified as s-Cepheids. Those s-Cepheids with periods smaller than about 3 days are believed to be first overtone pulsators, mainly from their sinusoidal light curves (cf review by Simon 1990). This assignment is consistent with the P-L relation of LMC Cepheids obtained by the MACHO project (Alcock et al 1995), which shows that most of the single-mode pulsators with $P \lesssim 2.5$ d are first overtone pulsators. For some longer-period s-type Cepheids that still show symmetric light curves, the mode assignment remains unclear. We must remember that we do not understand the mechanisms shaping the light curve. Despite the analyses of simple model systems (Stellingwerf et al 1987) it is conceivable that the inspection of the light curve alone is insufficient to unambiguously identify the prevailing pulsation mode.

Simon & Lee (1981) introduced analyses combining phases and amplitude ratios of various terms of Fourier-decomposed light and velocity curves of pulsating stars. This allowed them to quantify geometrical properties of stars' temporal light and velocity variations. The method found broad application in quantitatively describing the properties of simulated and observed stellar pulsations. The ultimate hope was, and still is, that directly derivable numbers from observed pulsations let us infer stellar-physical quantities. Simon (1988) wrote a review, with many important references, addressing applications and the level of understanding of the content of the different Fourier components. Simulations of nonlinear pulsations serving as a basis for physically calibrating the Fourier components are few since the necessary numerical quality is difficult to achieve.

3.2 *Late Evolutionary Phases and Degenerate Stars*

Roughly speaking, the chemical evolution of low-mass stars ends with helium core burning. Such stars evolve up the AGB with recurrent He-shell flashes and leave it when the envelope mass drops below a critical level. The stars evolve rather rapidly at roughly constant luminosity to high temperatures. After the remaining nuclear shell source extinguishes, the stars settle on the cooling track of the white dwarfs. During this whole evolutionary phase a number of opportunities exist for these stars to become pulsationally unstable.

As mentioned previously, low-mass stars leaving the AGB might be observable as RV Tau stars. Another group of pulsating stars, low-mass F- and G-type supergiants (also known as UU Her stars) are considered to be post-AGB stars.

Their semiregular low-amplitude variability has a timescale between 40 and 70 days. The high galactic latitudes and the strong infrared excesses in 89 Her and HD161796, typical members of the UU Her class, (Parthasarathy & Pottasch 1986, Likkel et al 1987) indicate that they are pre–planetary nebulae objects rather than population I, massive supergiants as some analyses suggested. Fernie & Sasselov (1989) studied the long-term behavior of UU Her stars and found their period and color changes to be one to two orders of magnitude smaller than what is expected if these variable stars had already left the AGB. Not only the evolutionary state but also the theoretical pulsation properties of their variability remain incompletely understood. Some of the variable F- and G-type supergiants are hotter than the blue edge of the classical instability strip. Pulsation calculations confirm the presence of pulsational instabilities at high temperatures for sufficiently large luminosity-to-mass ratios (Aikawa 1993, Gautschy 1993, Zalewski 1992, 1993). Seemingly, strange modes (cf GS95 Section 3.4) must be involved to explain the pulsational instabilities. These effects make a proper discussion of the pulsation physics of the strongly nonadiabatic envelopes of UU Her objects cumbersome. Nonlinear simulations of Aikawa (1993) and Zalewski (1993) indicated the presence of chaotic dynamics in certain $T_{\rm eff}$ and luminosity domains that could account for the observed irregular behavior. The light variations derived from nonlinear simulations were low (of the order of a few hundredths of a mag) and decrease towards high effective temperatures. This is not necessarily in agreement with observations. But again, the nonlinear modeling was restricted to purely radiative envelopes so that the relevance of these results still needs to be confirmed.

The peculiar variable star FG Sge, which was identified rather early as a post-AGB object [see Whitney (1978) for a historical account and references], is another example demonstrating that such stars can exhibit unusual pulsational behavior, at least in terms of the region of their pulsational instability. FG Sge is observed to have crossed the HR diagram from log $T_{\rm eff} \approx 4.7$ to 3.65 (van Genderen 1994) within about a century. Pulsations have been detected as far back as 1934 when the effective temperature was around 2×10^4 K (van Genderen & Gautschy 1995). These observed facts indicate a very broad instability domain. Application of the OPAL/OP data shows that the presence of the Z-bump strongly affects the pulsation modes and enhances their instability. Hence, the combination of the Z-bump, He II- and He I/H-ionization can drive pulsational instabilities essentially over the whole temperature range maximally coverable on the HR diagram by such objects if the L/M ratio is sufficiently high (Gautschy 1993, Zalewski 1993, van Genderen & Gautschy 1995).

When a post-AGB star reaches a surface temperature of around 3×10^4 K, it emits enough high-energy photons to efficiently photoionize the remaining circumstellar material and to let it appear as a planetary nebula. Observations of cool central stars show that some of them are variable (Méndez et al 1986, Bond & Ciardullo 1989, Hutton & Méndez 1993, Wlodarcyk & Zola 1990). Timescales are of the order of hours. The photometric amplitudes can reach a few hundredths of a magnitude. The variable cool central stars all show P Cygni-type line profiles, indicating strong stellar winds. Pulsation calculations (Gautschy 1993, Zalewski 1993) showed that cool central stars of planetary nebulae can be pulsationally unstable over a very broad effective-temperature range extending essentially from the AGB up to at least $\log T_{\rm eff} \approx 4.9$ depending on the heavy-element abundances and the L/M ratio. The trend indicates that the higher the L/M ratio ($\gtrsim 10^4\ L_\odot/M_\odot$) the more unstable are the stars. Therefore, we expect the low-mass branch (below about 0.60 $M_\odot$) of the central-star mass function to be pulsationally stable and more massive ones to be pulsationally unstable. Observationally, the available data are insufficient as yet to support or disprove such predictions. Additionally, we do not know how pulsational instabilities with large growth rates, as found in linear stability analyses, behave in the nonlinear regime. Since several radial modes were simultaneously excited over a broad temperature range, the final observable pulsational pattern in such stars could be rather complicated.

In the following we turn to the oscillations found in pre–white dwarfs and in the different families of white-dwarf stars. This has been a very active field of research in recent years, with particular emphasis on using white dwarf pulsations as seismological laboratories. Recent comprehensive review articles include those by Winget (1988a), who also provides some historical perspective, Winget (1988b), Kawaler & Hansen (1989), Kawaler (1990), and Brown & Gilliland (1994). For a review of the general physical properties of white dwarfs, see Koester & Chanmugam (1990) and references therein. Tassoul et al (1990) provide a wealth of information on evolutionary models of hydrogen- and helium-rich white dwarfs and their characteristics in pulsation analyses.

3.2.1 VARIABLE PG1159 STARS In the region of the HR diagram where the post-AGB tracks bend towards the white dwarfs' cooling sequences, at the knee (see Figure 1 of GS95), are the so-called PG1159 stars. These stars have very high effective temperatures ($7 \lesssim T_{\rm eff}/(10^4{\rm K}) \lesssim 17$), and they show spectroscopically strong deficiency of H but pronounced C and He features and the presence of O (see Werner 1992 and Dreizler et al 1995 for recent reviews). Due to the very high temperatures, determining the hydrogen content is extremely difficult; only relatively poor upper limits can be guessed, such as $\approx$ 10% by number for PG1159$-$035 (Werner 1995). Some of the PG1159 stars are known to be central stars of planetary nebulae.

A fraction of the PG1159 class shows photometric light variations with periods ranging from about 7 to about 30 min. Such oscillation modes are attributed to low-ℓ *g*-modes of high radial order. Variable PG1159 stars surrounded by planetary nebulae are sometimes referred to as variable planetary nebulae nuclei (PNNV), and those without signs of planetary nebulae as DOV or GW Vir stars. The pulsation periods of PNNVs are a factor of 2 or 3 longer than those of DOVs, indicating that the PNNVs have larger radii than the DOVs. From the evolutionary point of view it is plausible to assign the DOV stars to the vicinity or even to the early phases of the white dwarf cooling tracks. The PNNVs, in contrast, are still evolving towards higher temperatures at essentially constant luminosity and have not reached yet the knee. Bond et al (1993) list 9 PNNVs. Among them, RXJ2117.1+3412 can be regarded as a transition object between the PNNV and the DOV phase because the star has the shortest period among the PNNVs [ranging from about 11–22 min (Vauclair et al 1993)] and it has an extended low-surface-brightness planetary nebula. Long-term monitoring of the secular changes of periods of these hot pulsators should allow us to more precisely pin down their evolutionary status.

High-quality oscillation mode spectra were recently obtained from the WET consortium for the two DOV stars, GW Vir (PG1159−035) (Winget et al 1991) and PG2131+66 (Kawaler et al 1995). The results point towards $\ell = 1$ high-order *g*-modes being the dominant ones. From the periods and the period spacings, stellar masses of $\approx 0.6\ M_\odot$ were derived. From the departures from equidistant spacing between the periods, the depth of the composition transition (to a pure CO core) was estimated to be $\approx 3 \times 10^{-3}\ M_\odot$ for GW Vir (Kawaler & Bradley 1994) and PG2131 + 006 (Kawaler et al 1995). Furthermore, Winget et al (1991) obtained a rate of period change of $\dot{P} \approx -2.5 \times 10^{-11}$ for GW Vir, which corresponds to an evolutionary timescale of the order of 10^6 year.

The region on the HR diagram where variable PG1159 stars reside was determined by Werner et al (1995). However, constant and variable stars are intermixed in the instability region, indicating that luminosity and effective temperature are insufficient to characterize variable PG1159 stars. Pairs of spectroscopically identical stars are known in which one is variable and the other stable (Werner 1993).

The excitation mechanism of the PG1159 oscillations is thought to be partial ionization of the K-shell of C and/or O (Starrfield et al 1984, 1985). To obtain overstable pulsation modes, chemically homogeneous envelopes with sufficiently high C and O abundances must be invoked. The location and extent of the instability regions for stars evolving around the knee on the HR diagram depend, however, on the particular admixture of He in the CO-rich envelopes (Stanghellini et al 1991).

Vauclair (1990) proposed an alternative driving mechanism. His model focused on the levitation of chemical species in the strong radiation field of the PG1159 stars. Assuming equilibrium conditions, Vauclair (1990) found a strong nitrogen enhancement at a depth of the envelope where some of the g-modes achieve sufficiently high amplitudes and might be destabilized by the κ-mechanism by partial ionization of nitrogen. Unfortunately, however, the predicted surface composition contradicts the composition of GW Vir obtained by a detailed spectroscopic analysis by Werner et al (1991).

Kawaler (1988) investigated the effect of a H-burning shell on destabilizing H-rich nuclei of planetary nebulae. By accounting for temporal phase shifts between elemental abundances participating in the CNO cycle and the temperature perturbation, he found that pulsations with periods between 70 and 200 seconds were destabilized by the action of the ϵ-mechanism. For H-deficient central stars, Kawaler et al (1986) studied the effect of the ϵ-mechanism in a He-burning shell. Again, unstable g-modes were encountered. The low-ℓ g-modes had periods that are, however, about a factor of 3 to 4 shorter than those observed in DOV stars. Even under the favorable conditions for nuclear-driven instabilities as encountered in PNNV there is as yet no indication that nature permits such pulsations. Since the growth rates of modes excited by nuclear-burning shells are extremely small, the amplitudes of these modes may not grow sufficiently to be detected.

3.2.2 VARIABLE DB WHITE DWARFS (DBV) The eight presently known variable DB-type white dwarfs are all confined to a narrow effective-temperature range between about 21,500 and 24,000 K. The location of five of these variables on the HR diagram is shown in Figure 1 of GS95; the temperature determinations are those by Thejll et al (1991). A luminosity of $\log L/L_\odot = -1.3$ was assumed for all of them; only that of GD 358 is based on the estimate of Winget et al (1994). For DB stars, as for other white dwarf families, the quantification of physical parameters is still controversial. The effective temperatures obtained by Thejll et al (1991) are significantly lower than those by Liebert et al (1986) for example. The uncertainties in $T_{\rm eff}$, which exceed 1000 K, handicap the pulsational analyses of the driving mechanism by making it difficult to establish the borders of the instability strip.

The observed periods all fall into the interval between about 140 and 1000 seconds. Most DBV power spectra are complicated with many frequencies arranged in well discernible groups in frequency space. The power spectra are not always stable in time, although in part this may be due to undersampling of the data. Even in the WET multisite campaign this problem remained for PG1115 (Clemens et al 1993). Winget (1988b) considered the power spectrum of PG1351 to be the only resolved one; it is also the simplest one. Recently, a WET campaign on GD358 (Winget et al 1994) provided more than 180 significant peaks in its impressive power spectrum. From the observed triplet structures (indicating $\ell = 1$ modes) of the different radial orders, estimates of the amount of differential rotation were made. The period spacings between consecutive radial orders indicate a mass of 0.61 $M_\odot$. The different behavior of prograde and retrograde modes led to the postulation of a possible kG magnetic field in GD 358.

The pulsational driving of the DB variability is attributed to partial second He ionization. Driving of g-modes seems to occur, and to lead to results consistent with observations, for helium-rich surface layers ranging from 10^{-8} to 10^{-2} M_* (Bradley & Winget 1994). Helium-layer masses below about 10^{-6} M_* (Pelletier et al 1986) are, however, expected to lead to mixing that would transform DB white dwarfs into stars with carbon-enhanced surface layers. The important point is that the mass of the He-rich layers does not seem to be crucial for discriminating between stability and instability. Its thickness is pivotal, however, in determining the size of the mode trapping cycles and hence in the selection of eventually observable modes (Bradley et al 1993). Results from multisite studies of GD 358 and PG1115 indicate rather low-mass He blankets of $\approx 10^{-6} M_*$ and $\lesssim 10^{-4}$ M_*, respectively, on these two DBV stars (Clemens et al 1993, Winget et al 1994). In theoretical modeling the convective efficiency plays an important role in fixing the exact location of the edges of the instability strip. The blue boundaries are usually adjusted to concur with the hottest known DBV stars by adapting the convective efficiency in the model envelopes. With the presently adopted dialects of the mixing-length formalism for convection, unusually efficient convection zones are called for to reach agreement with observational data (see, for example, Bradley & Winget 1994 for a discussion).

3.2.3 VARIABLE DA WHITE DWARFS (DAV) At still lower effective temperatures along the white-dwarf cooling sequences are the variable hydrogen-rich white dwarf (DAV or ZZ Cet) stars. Spectroscopy of the atmospheres of DA white dwarfs shows a black-body continuum with superposed H-absorption lines; helium and metals are essentially absent (Koester & Chanmugam 1990). It is believed that most, if not all DA white dwarfs, will oscillate when they enter the appropriate temperature interval. There are some indications that some stable DA stars exist inside the ZZ Cet instability domain (Kepler & Nelan 1993). The exact temperature range of the instability domain is a matter of debate. As mentioned before, also for the DAV stars, spectroscopic determination of physical parameters is complex. According to the latest studies (Bergeron et al 1995) the blue edge of the ZZ Ceti variables is believed to lie between 12,500 K and 13,700 K. Their paper contains a comprehensive discussion of the involved difficulties.

The low-amplitude ($\lesssim 0.2$ mag) photometric variability of the ZZ Ceti stars is usually multiperiodic with periods ranging from about 100 to more than 1000 s. Some of the closely spaced periods are considered to be multipletsplittings due to rotation. Others must be sets of excited modes belonging to different radial orders. The power spectra of DAV stars have considerably fewer features than are encountered in DBV and DOV stars. Hence, they are less well suited for detailed seismological analyses. Due to the relatively low number of simultaneously excited oscillation modes, Clemens (1993) tried to identify similarities between the single ZZ Ceti stars to find a typical mean oscillation spectrum for a whole sample. One important conclusion from this approach was the inference of thick, i.e. $\approx 10^{-4}\ M_*$, hydrogen surface layers, a conclusion derived by assigning $\ell = 1$ to the dominant modes. This identification is supported, at least in G117$-$B15A, by multicolor observations by Robinson et al (1995). For some time the large-amplitude DAV stars were considered to be promising candidates to study nonlinear mode coupling and mode switching through the observed changes in their power spectra on short timescales. Kepler (1984) and O'Donoghue (1986) challenged the mode-switching interpretation by attributing the variable power to undersampling of the data. Theoretical modeling and discussions of ZZ Ceti power spectra were recently attempted by Brickhill (1992a,b).

If the mass of the surface hydrogen layer is large enough ($\gtrsim 10^{-14}\ M_*$), partial hydrogen ionization can in principle excite low-degree *g*-mode oscillations of DAV stars (Dolez & Vauclair 1981, Winget et al 1982). The major source of uncertainty in theoretically destabilizing DA white dwarfs is the coincidence of the driving region with the bottom of the convection zone, which is caused by the ionization of hydrogen. In the equilibrium models for DAV stars, below about 12,000 K, only a small fraction of the total energy passing through the convection zone is transported by photon diffusion. Hence, convection is dominating locally. Despite the convective turn-over timescale being shorter than the observed oscillation periods of ZZ Ceti stars, the nonradial stability analyses are usually carried out neglecting the perturbation of the convective flux.

It is generally agreed on that the theoretical location of the blue edge for the ZZ Ceti oscillations depends sensitively on the parameterization of convective efficiency in the equilibrium stellar model (e.g. Cox et al 1987, Bradley & Winget 1994). Only unusually extended convection zones (compared with other stellar applications) seem to lead to blue edges that agree with present observational data. Indeed, two-dimensional convection simulations on white dwarfs by Ludwig et al (1994) resulted in much shallower convection zones than those postulated on pulsational grounds. The thicknesses were, though, compatible with usual MLT convection zones and a mixing length of about 1.5 pressure scale-heights. The figures in Ludwig et al (1994) show that a constant mixing-length approach cannot adequately describe the stratification of the surface regions. Although a reliable time-dependent convection theory is necessary to exactly solve the convection-oscillation coupling, convection in ZZ Ceti stars seems to be tractable because the convective turn-over timescale is much shorter than the relevant oscillation periods. Brickhill (1990) discussed the effect of turbulent pressure and turbulent stress on the oscillatory motion. He concluded that the horizontal motion of oscillation should be nearly independent of depth in the convection zones. Applying this result to the stability analysis for ZZ Ceti stars, Brickhill (1991) showed that the convective perturbation tends to drive oscillations and that it is the most important agent for exciting *g*-modes in ZZ Ceti stars rather than the κ-mechanism. Brickhill's theory can be checked by comparing it with the observed range of effective temperatures of ZZ Ceti stars and with the observed periods. In reaching conclusions care is advised as the $T_{\rm eff}$ calibrations for DAV stars by spectroscopic analyses are still uncertain by the order of 500 K or more (Koester & Allard 1993, Bergeron et al 1995).

The question of the thickness of the superficial H-rich layers on DAV stars and their influence on the instability was extensively discussed in the 1980s. In contrast to long prevailing results admitting only thin hydrogen layers ($M_{\rm H}/M_* \lesssim 10^{-8}$, Winget et al 1982), Bradley & Winget (1994), and Fontaine et al (1994) joined—for different reasons—the conclusion of Cox et al (1987) that nonradial *g*-mode instabilities of DA white dwarfs can occur for hydrogen layers at least as massive as $10^{-4}\ M_*$. Instabilities of *g*-modes are presently considered rather insensitive to the hydrogen mass floating on the surface. For GD 165 (Bergeron et al 1993), observations pointed indeed to a thick hydrogen layer, exceeding possibly $10^{-4}\ M_*$, depending on the assumption about the spherical degree of the observed oscillation modes. Based on mean power spectra properties, Clemens (1993) also favored thick hydrogen layers. The thickness of the hydrogen layer controls the trapping properties, and hence it is believed to be responsible for selecting the eventually observable modes. An extensive study of the trapping properties in stratified DAV stars was undertaken by Brassard et al (1992).

The often very stable oscillation modes of white-dwarf variables are considered to be accurate clocks for measuring their evolutionary timescales. Attempts are being made to deduce cooling rates of these degenerate pulsators from the secular variation of the periods that were monitored over more than ten years (O'Donoghue & Warner 1987, Kepler et al 1990, Kepler 1993). Upper bounds determined for $\dot{P}/P$ as yet do not contradict the standard theory of cooling white dwarfs. For the DOV star PG1159−035, the observed negative $\dot{P}$ has at first sight the wrong sign if it is assumed to be located on the cooling sequence (Winget et al 1991). Detailed modeling of DOV stars suggested, though, that the way a particular mode is trapped influences the sign of its $\dot{P}$; oscillation modes confined in the outermost layers of stars having already reached their cooling track can well exhibit period decreases (Kawaler & Bradley 1994).

Radial mode instabilities in white dwarfs were studied for both DAV stars [see Cox (1974) for early references and e.g. Cox et al (1980), Saio et al (1983)] and for DBV stars (Kawaler 1993). The theoretical studies revealed instabilities for both classes of white dwarfs. Observationally, no signature of radial modes, with expected periods below a few seconds (Robinson 1984, Kawaler et al 1994), has been detected so far. It is unclear at present if the lack of observable radial modes points to theoretical inadequacies or to very low amplitudes that are below the present detection threshold.

4. PULSATIONS IN EVOLVED VERY MASSIVE STARS

Clear evidence was collected during the Geneva-photometry monitoring campaigns that stars of luminosity class I show low-level photometric variability in essentially all spectral types between O and K (Grenon 1993). In the following, we address some aspects of massive-star stability theory that might provide explanations in terms of pulsational instabilities. In particular, we consider stars more massive than about 30 $M_\odot$ as they evolve off the zero-age main sequence. When such stars return towards the main-sequence region after their red-supergiant phase they may have—due to significant mass loss—a considerably higher L/M ratio than when they left the ZAMS. Strong winds and hence large mass-loss rates are also considered essential for massive stars to evolve into Wolf-Rayet stars. A self-consistent physical picture for the evolution towards the Wolf-Rayet stage has not yet emerged, however. A number of difficult fluid-dynamical problems, such as semiconvection, convective overshooting, mass loss, and rotation with accompanying instabilities influence the evolution of massive stars crucially, and most of them defy a satisfactory treatment at present. For recent numerical studies concerning the evolution of massive stars we refer to e.g. Maeder & Meynet (1987), Schaller et al (1992), or Langer et al (1994) and the literature cited therein.

4.1 *Luminous Blue Variables (LBVs)*

By LBVs we designate all those variable massive stars with luminosities exceeding some 10^4 $L_\odot$ having envelopes with considerable amounts of hydrogen and which are located to the red of the ZAMS on the HR diagram. Hence, we discuss within the same framework all variable stars occupying the uppermost part of Figure 1 of GS95; the data for these variables were adopted from van Genderen

(1989). Our definition is somewhat wider than what is typically encountered in the literature (see e.g. Humphreys 1989, Humphreys & Davidson 1994). In our picture of LBV stars, the α Cyg-like supergiants with luminosities around 10^4 $L_\odot$ and masses of the order of 10 $M_\odot$ (Lucy 1976) as well as intermediate variable supergiants are also included. The very luminous variables discovered in nearby galaxies (in particular in M31 and M33) were called Hubble-Sandage variables in the past (Viotti 1992). Based on the characteristics of their variability there are, however, good reasons to assume that they can be attributed to what is called LBV in the Galaxy and in the Magellanic clouds.

The LBVs belong to the most luminous stars that exist and hence are of considerable interest for extragalactic applications. Light and radial velocity vary on timescales from weeks to possibly centuries. The long-term variations, with brightness changes of several magnitudes, are usually referred to as outbursts, which may or may not repeat (see references in van Genderen 1989 or Wolf 1992). Interestingly, the light variability is essentially a reflection of variable bolometric correction only. The bolometric luminosity remains roughly constant during an outburst so that the stars move horizontally across the HR diagram (see Figure 7 in Wolf 1992). Short-term variability, which is also observed, occurring on the timescale of weeks to months and having small amplitudes, shows recurring patterns that might indicate a pulsational origin (cf de Jager 1980, Sterken 1989). The observational data base is, despite considerable efforts from the observers' side, rather meager for conclusive analyses. Probably the best temporal coverage exists for HD 160529 (Sterken et al 1991). For this star, which is not unlike other LBVs, a characteristic timescale of 57 days was derived. This number is compatible with periods of radial modes for a star with the parameters suggested by Sterken et al (1991). It is presently unknown if the short-term variations are also occurring at constant bolometric luminosity. Short-term, small-scale variability is not considered to be an outstanding characteristic behavior for LBV stars as understood by Humphreys (1989) since this is also found for lower luminosity supergiants that do not show eruptions, such as e.g. R 71 or S Dor. For the discussion of the pulsation hypothesis as the origin of the short-term variability such a distinction is possibly not vital.

Recently, Glatzel & Kiriakidis (1993b) and Kiriakidis et al (1993) reported on extensive linear, radial stability analyses of massive stellar-evolution models. They followed stars between 20 and 200 $M_\odot$ through the hydrogen-burning phase. A sketch of the situation is shown in the upper left part of Figure 3. The first study (Glatzel & Kiriakidis 1993b), which was based on the old Los Alamos opacity data, showed radial instabilities when the stars evolved away from the ZAMS. An impressive enhancement of the strength of instability was

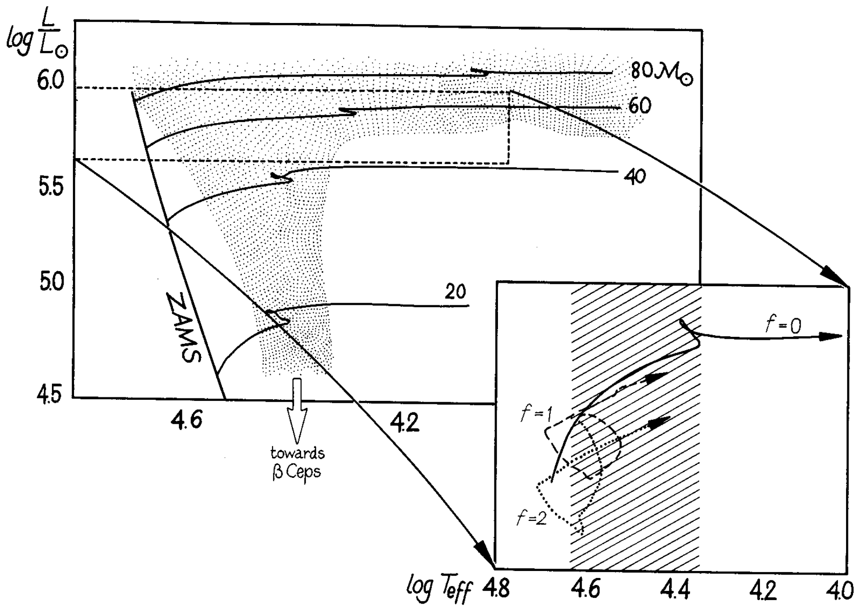

Figure 3 Luminous part of the HR diagram. Evolutionary paths of massive-star evolution according to Kiriakidis et al (1993). The dotted region outlines the extent of the radial instabilities encountered. The inset at the lower right, adapted from Langer et al (1994), shows the influence of mass loss on the evolutionary paths. The mass-loss rates were scaled with the growth rates of the most unstable mode from linear stability computations assuming different efficiencies f over the hatched interval.

later achieved with the use of OPAL opacity data (Kiriakidis et al 1993). The instability domain is sketched by the dotted area in Figure 3.

The extent of the instability regions depends on the amount of heavy elements. Even for low Z $(= 0.004)$, pulsational instability was encountered at masses above 65 $M_{\odot}$ in regions of the HR diagram that are compatible with observed LBVs. For higher heavy-element abundances, the instability region extended to higher $T_{\rm eff}$ as well as to lower luminosities and seemed to confine itself to the S-bend phase of evolution and eventually towards the instability domain of the β Cepheids (see Figure 3).

The radial-mode instabilities in massive stars can achieve growth rates that are several magnitudes larger than those found in classical pulsators. This led to the suspicion that these pulsations become violent enough to induce mass loss. Preliminary nonlinear simulations (Kiriakidis 1992) resulted in very rapidly growing pulsations indeed. They appear to be a mixture of the different unstable modes found in the linear analyses. The eventual nonlinear motion led to considerable mass loss; stable limit cycles were not often found. Whether the pulsations occurring in the stellar models of Kiriakidis et al (1993) can be causally connected with the eruptive phases of the LBVs remains to be seen. At least part of the short-term behavior can be attributed to pulsational instabilities.

Again, as in other high L/M stars such as helium stars, the unstable modes in the massive stars of Kiriakidis et al (1993) were found to be strange modes. They must be attributed to the strong influence of the He II ionization zone and the Z-bump on the acoustic cavity of the stellar envelopes, giving rise to a rich unstable oscillation spectrum.

In a first attempt, Langer et al (1994) used results from the linear pulsation calculations of Kiriakidis et al (1993) to parameterize the mass loss in their stellar evolution calculations. The mass-loss rate within the hatched area in the lower-right inset of Figure 3 was assumed to be proportional to the growth rate of the most unstable pulsation mode. The influence of different scaling factors f of this mass-loss rate on the evolutionary tracks is displayed. Langer et al (1994) performed such calculations in their attempt to devise an evolutionary scenario to interweave consistently the different spectroscopic subgroups of very massive stars.

Model envelopes considered appropriate for α Cyg-like objects showed that the essential prerequisite for pulsational instabilities to develop is a sufficiently high L/M ratio (Gautschy 1992). Hence, the short-term variability of low-luminosity (compared with typical LBVs), intermediate-type supergiants might be understandable in basically the same framework as the LBVs. The quantitative results of Gautschy (1992) are probably outdated, due to the old Los Alamos data used to construct the envelopes. The existence of the Z-bump in the new opacity data reduces the L/M ratio for pulsational instabilities to occur in very luminous stars.

4.2 *Wolf-Rayet Stars*

Wolf-Rayet (WR) stars—either as type WN or WC—occupy the area on the HR diagram between $4.5 \lesssim \log L/L_{\odot} \lesssim 6$ and at $\log T_{\rm eff} \gtrsim 4.6$ (Langer et al 1994). The WR stars, whose masses are estimated to be higher than about 4 $M_{\odot}$, probably originate from stars more than 40 $M_{\odot}$ on the main sequence after experiencing heavy mass loss during their early evolution. Photometrically, as well as spectroscopically, some WR stars show variability timescales of several hours (e.g. Vreux 1986, Koenigsberger & Auer 1987, Gosset et al 1989, van Genderen et al 1990). Clear periodicities have not been established, and the physical origin of the variability is not known. The large mass-loss rates associated with WR stars and the estimates that the momentum in the wind frequently exceeds the momentum contained in the radiation field led rather early to the conjecture that pulsations could be involved. Only recently, Lucy & Abbott

(1993) showed that multiple scattering in expanding WR envelopes can, under suitable conditions, transfer sufficient momentum into the wind for it to achieve magnitudes comparable with observational estimates. Besides pulsations of the WR envelopes, hydrodynamic instabilities in the dense winds are considered to be the source of light and radial-velocity variations. Matthews & Beech (1987) argued that pulsations might not be observable at all spectroscopically due to the long geometrical paths of photons through the extensive expanding atmospheres, which would smear out variations on shor timescales.

For many years unstable pulsation modes were sought, in particular ones driven by the ϵ-mechanism. The rather compact structure of WR stars assures considerable relative amplitudes of the pulsational displacement in the innermost regions of the stars, facilitating the efficiency of nuclear burning on driving pulsational instabilities (Noels & Gabriel 1981, Maeder 1985, Cox & Cahn 1988). In sufficiently massive stars the radial fundamental mode is destabilized by the ϵ-mechanism during He core burning. All periods of unstable modes remained below one hour. The very low growth rates (of the order of the evolutionary timescale of the stars) could hardly be responsible for strong pulsation-driven mass loss. To investigate the longer-period domain the stability of g-modes, which can be trapped in shell-burning regions, was studied. These modes appeared particularly appropriate for explaining the WN phases during massive-star evolution. The quasi-adiabatic analyses of Noels & Scuflaire (1986) and Scuflaire & Noels (1986) revealed very brief evolutionary phases during which the g_1-mode at low and intermediate spherical degree ($\ell < 10$) became weakly unstable. The resulting periods at fractions of a day compared, though, rather favorably with the observed timescales. Cox & Cahn (1988), in their fully nonadiabatic analysis, could not verify the Noels-Scuflaire results. The origin of the discrepancy remains unresolved.

In view of the rather dramatic instabilities found recently by Glatzel et al (1993) the ϵ-driven instability loses much of its appeal. Glatzel and collaborators performed stability analyses on homogeneous helium main-sequence star models. Their models were chosen to be appropriate approximations for WC-type WR stars. Besides the well-known instability of the radial fundamental mode—setting in above about 15 $M_\odot$—due to ϵ-driving, they encountered strongly unstable modes crossing the regular acoustic mode spectrum. The growth times of these unstable modes eventually reached values of only a few dynamical timescales. Such violently unstable modes can be imagined as promising candidates to at least *initiate* mass loss. Presently available are only linear analyses on static background models. It would be highly desirable to follow the nonlinear evolution of these unstable modes, of which several can occur simultaneously. It is not yet clear how a possible onset of mass loss affects the pulsational instability, i.e. if the oscillation modes are stabilized and how strongly, due to the presence of a velocity field. In any case, from the point of view of relevant instabilities the situation in the WR domain is comparable to the one of LBV stars: The strange modes that were missed in earlier studies were identified as the dominating pulsation modes.

5. THE FUTURE

On the observational side, the CCD-photometry experiment of Gilliland & Brown (1992) proved that many smaller university observatories located in mediocre climatic environments could be revitalized to perform profitable variable star work if the data obtained are analyzed with appropriate care. Either space-borne experiments or collaborative campaigns around the globe to monitor variable stars and provide very long time series of observations will allow detailed analysis of multiperiodic oscillators. In particular, such approaches will enable the detection of closely spaced frequencies in the power spectra and suppress the cumbersome sidelobe-effects due to monitoring gaps.

The increasing spectral resolution and stability of spectrographs are going to provide radial-velocity data of stars with the attempted resolution below 1 m sec^{-1} in the near future. Such accuracy will allow solar-type oscillations to be discovered in distant stars. The monitoring of equivalent-width variations (Kjeldsen et al 1995) might turn out to be a competitive alternative to searching for solar-type oscillations in stars requiring off-the-shelf spectrographs only.

Large homogeneous observational data sets of survey projects (such as the microlensing projects MACHO, EROS, OGLE) provide an important basis for statistical studies with pulsating stars. First results from different projects are available (Cook et al 1995, Beaulieu et al 1995, Udalski et al 1994). The number densities of pulsating variables at their different locations throughout the HR diagram will help to test and improve, when deduced from statistically meaningful samples, our understanding of the underlying stellar evolution.

The points mentioned above contribute to establishing stellar pulsations as a reliable tool to study a variety of aspects of the internal constitution of stars. Pulsation theory is now entering the era of a detail-rich quantitative theory. Despite its glorious history several important and exciting topics within stellar pulsation theory still need to be developed or even correctly formulated. Pulsation-convection interaction, hydromagnetic waves, and pulsation-rotation coupling are examples of basic fluid-dynamical processes that still need much effort before their effects on stellar structure and pulsation are understood quantitatively.

Nonlinear pulsation simulations including time-dependent convection and detailed radiation transport need further development. In particular, in cases when mass loss is expected to set in, reliable nonlinear solutions are required to get even a glimpse of the final state of the pulsating system. The long-term nonlinear behavior of pulsating stars is far from clear. And results from decade-long monitoring efforts prove that even "simple" pulsating stars do not necessary have very stable limit cycles. As for nonradial pulsations, no numerical methods are known to exist that would allow, with sufficient spatial resolution, unstable modes to be followed into their nonlinear regimes.

However, phenomena will certainly be discovered that will provide new insights into these issues.

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CORRIGENDUM TO PART I

Equation (18) should read

$$-\sigma^2\xi_r - \frac{1}{r^4\rho}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Gamma_1 pr^4\frac{\mathrm{d}\xi_r}{\mathrm{d}r}\right) - \frac{1}{\rho r}\left\{\frac{\mathrm{d}}{\mathrm{d}r}[(3\Gamma_1 - 4)p]\right\}\xi_r = 0.$$

Literature Cited

The following abbreviations of conference proceedings are used in the text and in the reference list:

Bo90: *Confrontation between Stellar Pulsation and Evolution, ASP Conf. Ser.* Vol. 11, ed. C Cacciari, G Clementini. San Francisco: Astron. Soc. Pac. (1990)

Buda88: *Multimode Stellar Pulsations,* ed. G Kovács, L Szabados, B Szeidl. Budapest: Kultura (1988)

CT95: *Astrophysical Applications of Stellar Pulsation, ASP Conf. Ser.* Vol. 82, IAU Coll. 155, ed. RS Stobie, PA Withelock. San Francisco: Astron. Soc. Pac. (1995)

GONG92: *Seismic Investigation of the Sun and the Stars, ASP Conf. Ser.* Vol. 42, ed. TM Brown. San Francisco: Astron. Soc. Pac. (1993)

GONG94: *Helio- and Asteroseismology, ASP Conf. Ser.* Vol. 76, ed. RK Ulrich, EJ Rhodes Jr, W Däppen. San Francisco: Astron. Soc. Pac. (1995)

ITS92: *Inside the Stars, ASP Conf. Ser.* Vol. 40, IAU Coll 137, ed. WW Weiss, A Baglin. San Francisco: Astron. Soc. Pac. (1993)

PSSS: *Progress of Seismology of the Sun and Stars,* ed. Y Osaki, H Shibahashi, *Lecture Notes Phys.* 367. New York: Springer-Verlag (1990)

Vic92: *New Perspectives on Stellar Pulsation and Pulsating Variable Stars,* ed. JM Nemec, JM Matthews. Cambridge: Cambridge Univ.

Press (1993)
Wd88: *White Dwarfs,* IAU Coll. 114, ed. G Wegner, *Lecture Notes Phys.* 328. New York: Springer-Verlag (1989)
Wd92: *White Dwarfs: Advances in Observation and Theory,* ed. MA Barstow, NATO ASI Ser. C, Vol. 403. Dordrecht: Kluwer (1993)

*****

Aerts C, De Pauw MD, Waelkens C. 1992. *Astron. Astrophys.* 266:294–306
Aerts C, Waelkens C. 1993. *Astron. Astrophys.* 273:135–46
Africano J. 1977. *Inf. Bull. Variable Stars* 1301
Aikawa T. 1987. *Astrophys. Space Sci.* 139:81–93
Aikawa T. 1993. *MNRAS* 262:893–900
Alcock C, Allsman RA, Axelrod TS, Bennett DP, Cook KH, et al. 1995. *Astron. J.* 109:1653–62
Ando H. 1986. *Astron. Astrophys.* 163:97–104
Andreasen GK. 1988. *Astron. Astrophys.* 201:72–79
Andreasen GK, Hejlesen PM, Petersen JO. 1983. *Astron. Astrophys.* 121:241–49
Baade D. 1982. *Astron. Astrophys.* 105:65–75
Baade D. 1984. *Astron. Astrophys.* 135:101–6
Baade D. 1987. In *Physics of Be Stars,* ed. A Slettebak, TP Snow, pp. 361–83. Cambridge: Cambridge Univ. Press
Baade D. 1989. *Astron. Astrophys.* 222:200–4
Babcock HW 1958. *Ap. J. Suppl.* 3:141–210
Baglin A. 1972. *Astron. Astrophys.* 19:45–50
Baglin A. 1976. In *Multiple Periodic Variable Stars,* IAU Coll. 29, ASSL Vol. 29, pp. 223–46, ed. WS Fitch. Dordrecht: Reidel
Baglin A, Breger M, Chevalier C, Hauck B, le Contel J-M, et al. 1973. *Astron. Astrophys.* 23:221–40
Bailyn CD. 1995. *Rev. Astron. Astrophys.* 33:133–62
Balmforth NJ, Gough DO, Merryfield WJ. 1990. In *From Miras to Planetary Nebulae: Which Path for Stellar Evolution?* ed. MO Mennessier, A Omont, pp. 85–87. Gif-sur-Yvette: Editions Frontières
Balona LA. 1985. *MNRAS* 217:L17–21
Balona LA. 1986a. *MNRAS* 219:111–29
Balona LA. 1986b. *MNRAS* 220:647–56
Balona LA. 1987. *MNRAS* 224:41–52
Balona LA. 1990a. In PSSS, pp. 443–48
Balona LA. 1990b. *MNRAS* 245:92–100
Balona LA. 1992. *MNRAS* 256:425–36
Balona LA. 1993. *MNRAS* 260:795–802
Balona LA. 1994. *MNRAS* 267:1060–70
Balona LA, Engelbrecht CA. 1981. *MNRAS* 212:889–97
Balona LA, Hearnshaw JB, Koen C, Collier A, Machi I, et al. 1994b. *MNRAS* 267:103–10
Balona LA, Koen C. 1994. *MNRAS* 267:1071–80
Balona LA, Krisciunas K, Cousins AWJ. 1994a. *MNRAS* 270:905–13
Balona LA, Rozowsky J. 1991. *MNRAS* 251:L66–68
Balona LA, Sterken C, Manfroid J. 1991. *MNRAS* 252:93–101
Beaulieu JP, Grison P, Tobin W, Pritchard JD, Ferlet R, et al. 1995. *Astron. Astrophys.* 303:137–54
Becker SA. 1985. In *Cepheids: Theory and Observations,* ed. BF Madore, pp. 104–25. Cambridge: Cambridge Univ. Press
Belmonte JA, Roca Cortés T, Vidal I, Schmider FX, Michel E, et al. 1993. In ITS92, pp. 739–41
Bergeron P, Fontaine G, Brassard P, Lamontagne R, Wesemael F, et al. 1993. *Astron. J.* 106:1987–99
Bergeron P, Wesemael F, Lamontagne R, Fontaine G, Saffer RA, Allard NF. 1995. *Ap. J.* 449:258–79
Berthomieu G, Gonczi G, Graff Ph, Provost J, Rocca A. 1978. *Astron. Astrophys.* 70:597–606
Bessell MS, Brett JM, Scholz M, Wood PR. 1989. *Astron. Astrophys.* 213:209–25
Blacher S, Perdang J. 1988. In Buda88, pp. 283–99
Bond HE, Ciardullo R. 1989. In Wd88, pp. 473–76
Bond HE, Ciardullo R, Kawaler SD. 1993. *Acta Astron.* 43:425–30
Bono G, Caputo F, Stellingwerf RF. 1994. *Ap. J.* 423:294–304
Bono G, Stellingwerf RF. 1993. In Vic92, pp. 275–276
Bowen GH. 1988. *Ap. J.* 329:299–317
Bradley PA, Winget DE. 1994. *Ap. J.* 421:236–44
Bradley PA, Winget DE, Wood MA. 1993. *Ap. J.* 406:661–73
Brassard P, Fontaine G, Wesemael F, Hansen CJ. 1992. *Ap. J. Suppl.* 80:369–401
Breger M. 1979. *Publ. Astron. Soc. Pac.* 91:5–26
Breger M, Handler G, Nather RE, Winget DE, Kleinman SJ, et al. 1995. *Astron. Astrophys.* 297:473–82
Brickhill AJ. 1990. *MNRAS* 246:510–17
Brickhill AJ. 1991. *MNRAS* 251:673–80
Brickhill AJ. 1992a. *MNRAS* 259:519–28
Brickhill AJ. 1992b. *MNRAS* 259:529–35
Brown TM, Christensen-Dalsgaard J, Weibel-Mihalas B, Gilliland RL. 1994. *Ap. J.* 427:1013–34
Brown TM, Gilliland RL. 1994. *Annu. Rev. Astron. Astrophys.* 32:37–82
Buchler JR, Goupil M-J. 1984. *Ap. J.* 279:394–400
Buchler JR, Goupil M-J. 1988. *Astron. Astro-*

phys. 190:137–47
Buchler JR, Kovács G. 1986. *Ap. J.* 308:661–68
Buchler JR, Kovács G. 1987. *Ap. J. Lett.* 320:L57–62
Buchler JR, Perdang JM, Spiegel EA. 1985. *Chaos in Astrophysics*, NATO ASI Ser. C, Vol. 161. Dordrecht: Reidel
Buchler JR, Serre T, Kolláth Z, Mattei Z. 1995. *Phys. Rev. Lett.* 73:842–45
Buonanno R, Corsi CE, Fusi Pecci F. 1989. *Astron. Astrophys.* 216:80–108
Burki G, Schmidt EG, Arellano Ferro A, Fernie JD, Sasselov D, et al. 1986. *Astron. Astrophys.* 168:139–46
Buta RJ, Smith MA. 1979. *Ap. J.* 232:213–35
Butler RP. 1992. *Ap. J. Lett.* 394:L25–27
Campos AJ, Smith MA. 1980. *Ap. J.* 238:250–65
Cannizzo JK, Goodings DA, Mattei JA. 1990. *Ap. J.* 357:235–42
Caputo F, Castellani V, Tornambé A. 1989. *Astron. Astrophys.* 222:121–24
Chiosi C. 1990. In Bo90, pp. 158–92
Christensen-Dalsgaard J. 1993. In ITS92, pp. 483–96
Christensen-Dalsgaard J, Frandsen S. 1983. *Sol. Phys.* 82:469–86
Christensen-Dalsgaard J, Petersen JO. 1995. *Astron. Astrophys.* 299:L17–20
Christy RF. 1968. *Q. J. R. Astron. Soc.* 9:13–39
Clemens JC. 1993. *Baltic Astron.* 2:407–34
Clemens JC, Barstow MA, Nather RE, Winget DE, Bradley PA, et al. 1993. In Wd92, pp. 515–21
Clement CM, Kinman TD, Suntzeff NB. 1991. *Ap. J.* 372:273–80
Cook KH, Alcock C, Allsman RA, Axelrod TS, Freeman KC, Peterson BA, et al. 1995. In CT95, pp. 221–31
Cousens A. 1983. *MNRAS* 203:1171–82
Cox AN. 1980. *Annu. Rev. Astron. Astrophys.* 18:15–41
Cox AN. 1983. In *Astrophysical Processes in Upper Main Sequence Stars*, Saas-Fee Course No.13, pp. 82–100
Cox AN, Cahn JH. 1988. *Ap. J.* 326:804–12
Cox AN, Hodson SW, Starrfield SG. 1980. In *Nonradial and Nonlinear Stellar Pulsation,* ed. HA Hill, WA Dziembowski, *Lecture Notes Phys.* 125:458–66. New York: Springer-Verlag
Cox AN, King DS, Hodson SW. 1979. *Ap. J.* 231:798–807
Cox AN, McNamara BJ, Ryan W. 1984. *Ap. J.* 284:250–56
Cox AN, Starrfield SG, Kidman RB, Pesnell WD. 1987. *Ap. J.* 317:303–24
Cox JP. 1974. *Rep. Prog. Phys.* 37:563–698
Cox JP. 1975. *Mem. Soc. R. Sci. Liège*, Coll. 8, 6^{e} Ser. 8:129–59
Cox JP. 1984. *Ap. J.* 280:220–27
Cugier H. 1993. *Acta Astron.* 43:27–38
Cugier H, Boratyn DA. 1992. *Acta Astron.* 42:191–209
Cugier H, Dziembowski WA, Pamyatnykh AA. 1994. *Astron. Astrophys.* 291:143–54
de Jager C. 1980. *The Brightest Stars*, *Geophys. Astrophys. Monogr.,* Vol. 19. Dordrecht: Reidel
de Jager C, Nieuwenhuijzen H. 1991. *Instabilities in Evolved Super- and Hypergiants*. Amsterdam: R. Netherlands Acad. Arts Sci.
Deming D, Mumma MJ, Espenak F, Jennings DE, Kostink T, et al. 1989. *Ap. J.* 343:456–67
Deupree RG. 1977a. *Ap. J.* 211:509–26
Deupree RG. 1977b. *Ap. J.* 214:502–9
Deupree RG, Hodson SW. 1977. *Ap. J.* 218:654–58
Diethelm R. 1990. *Astron. Astrophys.* 239:186–92
Diethelm R. 1996. *Astron. Astrophys.* 307:803–6
Dolez N, Gough DO. 1982. In *Pulsations in Classical and Cataclysmic Variable Stars,* ed. JP Cox, CJ Hansen, pp. 248–56. Boulder: JILA
Dolez N, Vauclair G. 1981. *Astron. Astrophys.* 102:375–85
Dorman B. 1992a. *Ap. J. Suppl.* 80:701–24
Dorman B. 1992b. *Ap. J. Suppl.* 81:221–50
Dreizler S, Werner K, Heber U. 1995. In *White Dwarfs, Proc. 9th European Workshop on White Dwarfs,* ed. D Koester, K Werner, *Lecture Notes Phys.* 443:160–70. New York: Springer-Verlag
Dukes RJ Jr. 1974. *Ap. J.* 192:81–91
Dziembowski WA. 1977a. *Acta Astron.* 27:95–126
Dziembowski WA. 1977b. *Acta Astron.* 27:203–11
Dziembowski WA. 1982. *Acta Astron.* 32:147–71
Dziembowski WA, Goode PR. 1985. *Ap. J. Lett.* 296:L27–30
Dziembowski WA, Goode PR. 1986. In *Seismology of the Sun and Distant Stars,* ed. DO Gough, NATO ASI C, Vol. 169, pp. 441–51. Dordrecht: Reidel
Dziembowski WA, Królikowska M. 1985. *Acta Astron.* 35:5–28
Dziembowski WA, Królikowska M. 1990. *Acta Astron.* 40:19–26
Dziembowski WA, Królikowska M, Kosovichev A. 1988. *Acta Astron.* 38:61–75
Dziembowski WA, Moskalik P, Pamyatnykh AA. 1993. *MNRAS* 265:588–600
Dziembowski WA, Pamyatnykh AA. 1991. *Astron. Astrophys.* 248:L11–14
Dziembowski WA, Pamyatnykh AA. 1993. *MN-*

RAS 262:204–12
Eggen OJ. 1979. *Ap. J. Suppl.* 41:413–34
Fadeyev YuA, Fokin AB. 1985. *Astrophys. Space Sci.* 111:355–74
Feast MW, Glass IS, Whitelock PA, Catchpole RM. 1989. *MNRAS* 241:375–92
Fernie JD. 1990. *Ap. J.* 354:295–301
Fernie JD. 1994. *MNRAS* 271:L19–20
Fernie JD, Kamper KW, Seager S. 1993. *Ap. J.* 416:820–24
Fernie JD, Sasselov DD. 1989. *Publ. Astron. Soc. Pac.* 101:513–15
Feuchtinger MU, Dorfi EA, Höfner S. 1993. *Astron. Astrophys.* 273:513–23
Feuchtinger MU, Dorfi EA. 1994. *Astron. Astrophys.* 291:209–25
Fontaine G, Brassard P, Wesemael F, Tassoul M. 1994. *Ap. J. Lett.* 428:L61–64
Fox MW, Wood PR. 1982. *Ap. J.* 259:198–212
Gabriel M. 1969. In *Low-Luminosity Stars,* ed. SS Kumar, pp. 267–77. New York: Gordon & Breach
Gabriel M, Noels A, Scuflaire R, Mathys G. 1985. *Astron. Astrophys.* 143:206–8
Gautschy A. 1987. *Vistas Astron.* 30:197–241
Gautschy A. 1992. *MNRAS* 259:82–88
Gautschy A. 1993. *MNRAS* 265:340–46
Gautschy A, Saio H. 1993. *MNRAS* 262:213–19
Gautschy A, Saio H. 1995. *Annu. Rev. Astron. Astrophys.* 33:75–113 (GS95)
Gehmeyr M. 1992a. *Ap. J.* 399:265–71
Gehmeyr M. 1992b. *Ap. J.* 399:272–83
Gehmeyr M. 1993. *Ap. J.* 412:341–50
Gies D. 1994. In *Pulsation, Rotation and Mass Loss in Early-Type Stars, IAU Symp. 162,* ed. LA Balona, H Herichs, JM Le Contel, pp. 89–99. Dordrecht: Kluwer
Gilliland RL, Brown TM. 1992. *Astron. J.* 103:1945–54
Gilliland RL, Brown TM, Duncan DK, Suntzeff NB, Lockwood GW, et al. 1991. *Astron. J.* 101:541–61
Gingold RA. 1976. *Ap. J.* 204:116–30
Gingold RA. 1985. *Mem. Soc. Astron. Ital.* 56:169–91
Glatzel W, Kiriakidis M. 1993a. *MNRAS* 262:85–92
Glatzel W, Kiriakidis M. 1993b. *MNRAS* 263:375–84
Glatzel W, Kiriakidis M, Fricke KJ. 1993. *MNRAS* 262:L7–11
Gloria KA. 1990. *Publ. Astron. Soc. Pac.* 102:338–43
Gosset E, Vreux J-M, Manfroid J, Sterken C, Walker EN, Haefner R. 1989. *MNRAS* 238:97–113
Gough DO, Toomre J. 1991. *Annu. Rev. Astron. Astrophys.* 29:627–85
Grenon M. 1993. In ITS92, pp. 693–707
Guzik JA, Cox AN. 1993. *Astrophys. Space Sci.* 210:307–9
Haniff CA, Gehz AM, Gorham PW, Kulkarni SR, Matthews K, Neugebauer G. 1992. *Astron. J.* 103:1662–67
Heller CH, Kawaler SD. 1988. *Ap. J. Lett.* 329:L43–46
Höfner S, Feuchtinger M, Dorfi EA. 1995. *Astron. Astrophys.* 297:815–27
Höppner W, Kähler H, Roth ML, Weigert A. 1978. *Astron. Astrophys.* 63:391–99
Hughes SMG. 1993. In Vic92, pp. 192–200
Humphreys RM 1989. In *Physics of Luminous Blue Variables*, ed. K Davidson, AFJ Moffat, HJGLM. Lamers, ASSL Vol. 157, pp. 3–14. Dordrecht: Kluwer
Humphreys RM, Davidson K. 1994. *Publ. Astron. Soc. Pac.* 106:1025–51
Hutton RG, Méndez RH. 1993. *Astron. Astrophys.* 267:L8–10
Jerzykiewicz M, Wenzel W. 1977. *Acta Astron.* 27:35–50
Jones TJ, Bryja CO, Gehrz RD, Harrison TE, Johnson JJ, et al. 1990. *Ap. J. Suppl.* 74:785–817
Jura M. 1986. *Ap. J.* 309:732–36
Jura M, Kleinmann SG. 1992. *Ap. J. Suppl.* 83:329–49
Kambe E, Ando H, Hirata R. 1990. *Publ. Astron. Soc. Jpn.* 42:687–710
Kambe E, Ando H, Hirata R. 1993a. *Astron. Astrophys.* 273:435–50
Kambe E, Ando H, Hirata R, Walker GAH, Kennelly EJ, Matthews JM. 1993b. *Publ. Astron. Soc. Pac.* 105:1222–31
Kambe E, Osaki Y. 1988. *Publ. Astron. Soc. Jpn.* 40:313–29
Kawaler SD. 1988. *Ap. J.* 334:220–28
Kawaler SD. 1990. In Bo90, pp. 494–512
Kawaler SD. 1993. *Ap. J.* 404:294–304
Kawaler SD, Bond HE, Sherbert LA, Watson TK. 1994. *Astron. J.* 107:298–305
Kawaler SD, Bradley PA. 1994. *Ap. J.* 427:415–28
Kawaler SD, Hansen CJ. 1989. In Wd88, pp. 97–108
Kawaler SD, O'Brien MS, Clemens JC, Nather RE, Winget DE, et al. 1995. *Ap. J.* 450:350–63
Kawaler SD, Winget DE, Hansen CJ, Iben I Jr. 1986. *Ap. J. Lett.* 306:L41–44
Kenelly EJ, Walker GAH, Merryfield WJ. 1992. *Ap. J. Lett.* 400:L71–74
Kepler SO. 1984. *Ap. J.* 278:754–60
Kepler SO. 1993. *Baltic Astron.* 2:444
Kepler SO, Nelan EP. 1993. *Astron. J.* 105:608–13
Kepler SO, Vauclair G, Dolez N, Chevreton M, Barstow MA, et al. 1990. *Ap. J.* 357:204–7
Kiriakidis M. 1992. *Stabilität und Pulsationen*

von massereichen Sternen. PhD thesis. Univ. Göttingen
Kiriakidis M, El Eid MF, Glatzel W. 1992. *MNRAS* 255:L1–5
Kiriakidis M, Fricke KJ, Glatzel W. 1993. *MNRAS* 264:50–62
Kjeldsen H, Bedding TR, Viskum M, Frandsen S. 1995. AJ 109:1313–19
Koenigsberger G, Auer LH. 1987. *Publ. Astron. Soc. Pac*. 99:1080–83
Koester D, Allard N. 1993. In Wd92, pp. 237–43
Koester D, Chanmugam G. 1990. *Rep. Prog. Phys*. 53:837–915
Kolláth Z. 1990. *MNRAS* 247:377–86
Kolláth Z, Szeidl B. 1993. *Astron. Astrophys.* 277:62–68
Kovács G, Buchler JR. 1988a. *Ap. J.* 324:1026–41
Kovács G, Buchler JR. 1988b. *Ap. J.* 334:971–94
Kovács G, Buchler JR. 1989. *Ap. J.* 346:898–905
Kovács G, Buchler JR. 1993. *Ap. J.* 404:765–72
Kovács G, Buchler JR, Marom A. 1991. *Astron. Astrophys.* 252:L27–30
Kovács G, Buchler JR, Marom A, Iglesias CA, Rogers FJ 1992. *Astron. Astrophys.* 259:L46–48
Kovács G, Shlosman I, Buchler JR. 1986. *Ap. J.* 307:593–608
Krisciunas K, Aspin C, Geballe TR, Akazawa H, Claver CF, et al. 1993. *MNRAS* 263:781–88
Krisciunas K, Handler G. 1995. *Inf. Bull. Variable Stars* 4195
Krockenberger M, Noyes RW, Sasselov DD. 1995. *Bull. Astron. Soc. Am.* 26:1366
Kurtz DW. 982. *MNRAS* 200:807–59
Kurtz DW. 1988. In Buda88, pp. 95–106
Kurtz DW. 1989. *MNRAS* 238:1077–84
Kurtz DW. 1990. *Annu. Rev. Astron. Astrophys.* 28:607–55
Kurtz DW. 1995. In GONG94, pp. 606–617
Kurtz DW, Shibahashi H. 1986. *MNRAS* 223:557–79
Kurtz DW, Marang F. 1995. *MNRAS* 276:191–98
Kurtz DW, Martinez P, van Wyk F, Marang F, Roberts G. 1994. *MNRAS* 268:641–53
Kurtz DW, Garrison RF, Koen C, Hofmann GF, Viranna NB. 1995. *MNRAS* 276:199–205
Lampens P. 1987. *Astron. Astrophys.* 172:173–78
Langer N, Hamann W-R, Lennon M, Najarro F, Pauldrach AWA, Puls J. 1994. *Astron. Astrophys.* 290:819–33
Lauterborn D, Refsdal S, Weigert A. 1971. *Astron. Astrophys.* 10:97–117
Ledoux P. 1941. *Ap. J.* 94:537–48
Lee U. 1985a. *Publ. Astron. Soc. Jpn.* 37:261–77
Lee U. 1985b. *Publ. Astron. Soc. Jpn.* 37:279–91
Lee U. 1988. *MNRAS* 232:711–24
Lee U, Baraffe I. 1995. *Astron. Astrophys.* 301:419–32
Lee U, Jeffery CS, Saio H. 1992. *MNRAS* 254:185–91
Lee U, Saio H. 1986. *MNRAS* 221:365–76
Lee U, Saio H. 1990a. *Ap. J.* 349:570–79
Lee U, Saio H. 1990b. *Ap. J. Lett.* 359:L29–32
Lee U, Saio H. 1990c. *Ap. J.* 360:590–603
Lee U, Saio H. 1993. *MNRAS* 261:415–24
Lee Y-W, Demarque P, Zinn R. 1990. *Ap. J.* 350:155–72
Lee Y-W, Demarque P, Zinn R. 1994. *Ap. J.* 423:248–65
Li Y, Stix M. 1994. *Astron. Astrophys.* 286:815–23
Liebert J, Wesemael F, Hansen CJ, Fontaine G, Shipman HL, et al. 1986. *Ap. J.* 309:241–52
Likkel L, Omont A, Morris M, Forveille T. 1987. *Astron. Astrophys.* 173:L11–14
Lomb NR. 1978. *MNRAS* 185:325–33
Lucy LB, Abbott DC. 1993. *Ap. J.* 405:738–46
Ludwig H-G, Jordan S, Steffen M. 1994. *Astron. Astrophys.* 284:105–17
Madore BF. 1985. *Cepheids: Theory and Observations.* Cambridge: Cambridge Univ. Press
Maeder A. 1985. *Astron. Astrophys.* 147:300–8
Maeder A, Meynet G. 1987. *Astron. Astrophys.* 182:243–63
Magalhães JA, Weir AL, Conrath BJ, Gierasch PJ, Leroy SS. 1990. *Icarus* 88:39–72
Martinez P, Kurtz DW. 1990. *MNRAS* 242:636–52
Mateo M. 1993. In *Blue Stragglers,* ed. RE Saffer, *ASP Conf. Ser.* Vol. 53, pp. 74–96
Mathias P, Aerts C, Pauw MD, Gillet D, Waelkens C. 1994. *Astron. Astrophys.* 283:813–26
Mathys G. 1985. *Astron. Astrophys.* 151:315–21
Matthews JM. 1988. *MNRAS* 235:L7–11
Matthews JM. 1990a. *Astron. Astrophys.* 229:452–56
Matthews JM. 1990b. In PSSS, pp. 385–91
Matthews JM. 1991. *Publ. Astron. Soc. Pac.* 103:5–19
Matthews JM. 1993. In GONG92, pp. 303–16
Matthews JM, Beech M. 1987. *Ap. J. Lett.* 313:L25–29
Méndez RH, Fortez JC, López RH. 1986. *Rev. Mex. Astron. Astrof.* 13:119–29
Michaud G. 1970. *Ap. J.* 160:641–58
Michaud G. 1980. *Astron. J.* 85:589–98
Miller RH. 1991. *J. Comput. Phys.* 93:469–76
Moffett TJ. 1989. In *The Use of Pulsating Stars*

in Fundamental Problems of Astronomy, IAU Coll. 111, ed. EG Schmidt, pp. 191–204. Cambridge: Cambridge Univ. Press
Moskalik P. 1985. *Acta Astron.* 35:229–54
Moskalik P. 1995. In CT95, pp. 44–55
Moskalik P, Buchler JR. 1990. *Ap. J.* 355:590–601
Moskalik P, Buchler JR., Marom A. 1992. *Ap. J.* 385:685–93
Moskalik P, Dziembowski WA. 1992. *Astron. Astrophys.* 256:L5–8
Mosser B, Mékarnia D., Maillard JP, Gay J, Gautier D, Delache P. 1993. *Astron. Astrophys.* 267:604–22
Nemec JM, Mateo M. 1990. In Bo90, pp. 64–85
Nemec JM, Nemec AFL, Lutz TE. 1994. *Astron. J.* 108:222–46
Noels A, Gabriel M. 1981. *Astron. Astrophys.* 101:215–22
Noels A, Scuflaire R. 1986. *Astron. Astrophys.* 161:125–29
North P, Paltani S. 1994. *Astron. Astrophys.* 288:155–64
O'Donoghue D. 1986. *MNRAS* 220:L19–22
O'Donoghue D, Warner B. 1987. *MNRAS* 228:949–55
Osaki Y. 1971. *Publ. Astron. Soc. Jpn.* 23:485–502
Osaki Y. 1977. *Publ. Astron. Soc. Jpn.* 29:235–48
Osaki Y. 1986a. In *Seismology of the Sun and the Distant Stars*, ed. DO Gough, pp. 453–63. Dordrecht: Reidel
Osaki Y. 1986b. *Publ. Astron. Soc. Pac.* 98:30–32
Ostlie DA, Cox AN. 1986. *Ap. J.* 311:864–72
Parthasarathy M, Pottasch SR. 1986. *Astron. Astrophys.* 154:L16–19
Pelletier G, Fontaine G, Wesemael F, Michaud G, Wegner G. 1986. *Ap. J.* 307:242–52
Percy JR, Lane MJ. 1977. *Astron. J.* 82:353–59
Perdang J. 1985. *Physicalia* 7:239–303
Perdang J. 1991. In *ESO Workshop on Rapid Variability of OB-Stars: Nature and Diagnostic Value*, ed. D Baade, pp. 349–61. Garching: ESO
Perdang J. 1993. In *Cellular Automata: Prospects in Astrophysical Applications*, ed. J Perdang, A Lejeune, pp. 342–68. Singapore: World Scientific
Petersen JO. 1976. In *Multiple Periodic Variable Stars, IAU Coll. 29*, ed. WS Fitch, pp. 195–222. Dordrecht: Reidel
Pijpers FP, Habing HJ. 1989. *Astron. Astrophys.* 215:334–46
Radick RR, Lockwood GW, Thomson DT, Warnock III A, Hartmann LW, et al. 1983. *Publ. Astron. Soc. Pac.* 95:621–34
Robinson EL. 1984. *Astron. J.* 89:1732–34
Robinson EL, Mailloux TM, Zhang E, Koester D, Stiening RF, et al. 1995. *Ap. J.* 438:908–16
Romanov YuS, Udovichenko SN, Frolov MS. 1987. *Sov. Astron. Lett.* 13:29–31
Rood R. 1990. In Bo90, pp. 11–21
Saio H, Winget DE, Robinson EL. 1983. *Ap. J.* 265:982–95
Sandage A. 1982a. *Ap. J.* 252:553–73
Sandage A. 1982b. *Ap. J.* 252:574–81
Sandage A. 1990a. *Ap. J.* 350:603–30
Sandage A. 1990b. *Ap. J.* 350:631–44
Sandage A. 1993a. *Astron. J.* 106:687–702
Sandage A. 1993b. *Astron. J.* 106:703–18
Sandage A. 1993c. *Astron. J.* 106:719–25
Sandage A, Diethelm R, Tammann GA. 1994. *Astron. Astrophys.* 283:111–20
Sandage A, Katem B, Sandage M. 1981. *Ap. J. Suppl.* 46:41–74
Schaerer D, Meynet D, Maeder A, Schaller G. 1993. *Astron. Astrophys Suppl.* 98:523–27
Schaller G, Schaerer D, Meynet G, Maeder A. 1992. *Astron. Astrophys Suppl.* 96:269–331
Scuflaire R, Noels A. 1986. *Astron. Astrophys.* 169:185–88
Shibahashi H. 1983. *Ap. J.* 275:L5–9
Shibahashi H. 1991. In *Challenges to Theories of the Structure of Moderate-Mass Stars*, ed. D Gough, J Toomre, Lecture Notes Phys. 388:393–410. New York: Springer-Verlag
Shibahashi H, Saio H. 1985. *Publ. Astron. Soc. Jpn.* 37:245–59
Shibahashi H, Takata M. 1993. *Publ. Astron. Soc. Pac.* 45:617–41
Shobbrook RR, Herbison-Evans D, Johnston ID, Lomb NR. 1969. *MNRAS* 145:131–40
Shobbrook RR, Lomb NR, Herbison-Evans D. 1972. *MNRAS* 156:165–80
Simon NR. 1988. In *Pulsation and Mass Loss in Stars*, ed. R Stalio, LA Willson, pp. 27–50. ASSL. Vol. 148, Dordrecht: Kluwer
Simon NR. 1989. *Ap. J. Lett.* 343:L17–20
Simon NR. 1990. In Bo90, pp. 193–208
Simon NR, Cox AN, Hodson SW. 1980. *Ap. J.* 237:550–57
Simon NR, Lee AS. 1981. *Ap. J.* 248:291–97
Simon NR, Schmidt EG. 1976. *Ap. J.* 205: 162–64
Slettebak A. 1988. *Publ. Astron. Soc. Pac.* 100:770–84
Smak J. 1970. *Acta Astron.* 20:75–91
Smith MA. 1977. *Ap. J.* 215:574–83
Smith MA. 1980a. In *Nonradial and Nonlinear Stellar Pulsation*, ed. HA Hill, WA Dziembowski, *Lecture Notes Phys.* 125:60–75 New York: Springer-Verlag
Smith MA. 1980b. *Ap. J.* 240:149–60
Smith MA. 1983. *Ap. J.* 265:338–53
Smith MA. 1985. *Ap. J.* 297:206–23
Smith MA. 1989. *Ap. J. Suppl.* 71:357–86
Smith MA, Fitch WS, Africano JL, Goodrich

BD, Halbedel W, et al. 1984. *Ap. J.* 282:226–35
Smith MA, McCall ML. 1978. *Ap. J.* 223:221–33
Smith MA, Polidan RS. 1993. *Ap. J.* 408:323–36
Stanghellini L, Cox AN, Starrfield S. 1991. *Ap. J.* 383:766–78
Starrfield S, Cox AN, Kidman RB, Pesnell WD. 1984. *Ap. J.* 281:800–10
Starrfield S, Cox AN, Kidman RB, Pesnell WD. 1985. *Ap. J. Lett.* 293:L23–27
Stellingwerf RF. 1974. *Ap. J.* 192:139–44
Stellingwerf RF. 1975. *Ap. J.* 195:441–66
Stellingwerf RF. 1982. *Ap. J.* 262:330–38
Stellingwerf RF. 1984a. *Ap. J.* 277:322–26
Stellingwerf RF. 1984b. *Ap. J.* 277:327–32
Stellingwerf RF. 1984c. *Ap. J.* 284:712–18
Stellingwerf RF, Bono G. 1993. In Vic92, pp. 252–60
Stellingwerf RF, Gautschy A, Dickens RJ. 1987. *Ap. J. Lett.* 313:L75–79
Sterken C. 1989. In *Physics of Luminous Blue Variables,* ed. K Davidson, AFJ Moffat, HJGLM Lamers, ASSL Vol. 157, pp. 59–66. Dordrecht: Kluwer
Sterken C, Gosset E, Jüttner A, Stahl O, Wolf B, Axer M. 1991. *Astron. Astrophys.* 247:383–92
Sterken C, Jerzykiewicz M. 1993. *Space Sci. Rev.* 62:95–171
Stothers RB. 1992. *Ap. J.* 392:706–9
Stryker LL. 1993. *Publ. Astron. Soc. Pac.* 105:1081–100
Szeidl B. 1988. In Buda88, pp. 45–65
Takata M, Shibahashi H. 1995a. *Publ. Astron. Soc. Jpn.* 47:219–31
Takata M, Shibahashi H. 1995b. Preprint
Takeuti M. 1990. In *The Numerical Modelling of Nonlinear Stellar Pulsations,* ed. JR Buchler, NATO ASI Series C, Vol 302, pp. 121–41. Dordrecht: Reidel
Takeuti M, Petersen JO. 1983. *Astron. Astrophys.* 117:352–56
Tanaka Y, Takeuti M. 1988. *Astrophys. Space Sci.* 148:229–37
Tassoul M. 1980. *Ap. J. Suppl.* 43:469–90
Tassoul M, Fontaine G, Winget DE. 1990. *Ap. J. Suppl.* 72:335–86
Thejll P, Vennes S, Shipman HL. 1991. *Ap. J.* 370:355–369
Tuchman Y. 1991. *Ap. J.* 383:779–83
Tuchman Y, Lèbre A, Mennessier MO, Yarri A. 1993. *Astron. Astrophys.* 271:501–7
Tuthill PG, Haniff CA, Baldwin JE, Feast MW. 1994. *MNRAS* 266:745–51
Udalski A, Kubiak M, Szymański M, Kałużny J, Mateo M, Krzemiński W. 1994. *Acta Astron.* 44:317–86
Unno W, Osaki Y, Ando H, Saio H, Shibahashi H, 1989. *Nonradial Oscillations of Stars.* Tokyo: Univ. Tokyo Press. 2nd ed.
van Genderen AM. 1989. *Astron. Astrophys.* 208:135–40
van Genderen AM. 1994. *Astron. Astrophys.* 284:465–76
van Genderen AM, Gautschy A. 1995. *Astron. Astrophys.* 294:453–68
van Genderen AM, van der Hucht KA., Larsen I. 1990. *Astron. Astrophys.* 229:123–32
Vassiliadis E, Wood PR. 1993. *Ap. J.* 413:641–57
Vauclair G. 1990. In PSSS, pp. 437–442
Vauclair G, Belmonte JA, Pfeiffer B, Chevreton M, Dolez N, et al. 1993. *Astron. Astrophys.* 267:L35–38
Vauclair S, Dolez N. 1990. In PSSS, pp. 399–403
Vauclair S, Dolez N, Gough DO. 1991. *Astron. Astrophys.* 252:618–24
Viotti R. 1992. In *Variable Star Research: An International Perspective,* ed. JR Percy, JA Mattei, C Sterken, pp. 194–204. Cambridge: Cambridge Univ. Press
Vogt SS, Penrod GD. 1983. *Ap. J.* 275:661–82
Vreux J-M. 1986. *Publ. Astron. Soc. Pac.* 97: 274–279
Waelkens C. 1987. In *Stellar Pulsation,* ed. AN Cox, WM Sparks, SG Starrfield. In *Lecture Notes Phys.* 274:75–78. New York: Springer-Verlag
Waelkens C. 1991. *Astron. Astrophys.* 246:453–68
Waelkens C, Rufener F. 1985. *Astron. Astrophys.* 152:6–14
Waelkens C, Van den Abeele K, Van Winckel H. 1991. *Astron. Astrophys.* 251:69–74
Walker GAH, Yang S, Fahlman GG. 1979. *Ap. J.* 233:199–204
Wallerstein G, Cox AN. 1984. *Publ. Astron. Soc. Pac.* 96:677–91
Watson RD. 1988. *Astrophys. Space Sci.* 140:255–90
Wehlau A, Bohlender D. 1982. *Astron. J.* 87:780–91
Werner K. 1992. In *The Atmospheres of Early-Type Stars,* ed. U Heber, CS Jeffery, *Lecture Notes Phys.* 401:273–87. New York: Springer-Verlag
Werner K. 1993. In Wd92, pp. 67–75
Werner K. 1995. *Astron. Astrophys.* In press
Werner K, Heber U, Hunger K. 1991. *Astron. Astrophys.* 244:437–61
Werner K, Rauch T, Dreizler S, Heber U. 1995. In CT95, pp. 96–97
Whitelock PA. 1990. In Bo90, pp. 365–378
Whitelock PA, Menzies J, Feast M, Marang F, Carter B, et al. 1994. *MNRAS* 267:711–42
Whitney CA. 1978. *Ap. J.* 220:245–50

Winget DE. 1988a. In *Advances in Helio- and Asteroseismology, IAU Symp. 123,* ed. J Christensen-Dalsgaard, S Frandsen, pp. 305–24. Dordrecht: Reidel
Winget DE. 1988b. In Buda88, pp. 181–97
Winget DE, Nather RE, Clemens JC, Provencal JL, Kleinman S, et al. 1991. *Ap. J.* 378:326–46
Winget DE, Nather RE, Clemens JC, Provencal JL, Kleinman S, et al. 1994. *Ap. J.* 430:839–49
Winget DE, Van Horn HM, Tassoul M, Hansen CJ, Fontaine G, Carroll BW. 1982. *Ap. J. Lett.* 252:L65–68
Wlodarczyk K, Zola S. 1990. In Bo90, pp. 586–88
Wolf B. 1992. In *Reviews in Modern Astronomy,* ed. G Klare, 5:1–15. Berlin: Springer-Verlag
Wolff SC. 1983. In *The A-type Stars, NASA SP-463,* pp. 93-111. Washington DC: NASA
Wood PR. 1979. *Ap. J.* 227:220–31
Wood PR. 1990a. In *From Miras to Planetary Nebulae: Which Path for Stellar Evolution?* ed. MO Mennessier, A Omont, pp. 67–83. Gif-sur-Yvette:Editions Frontières
Wood PR. 1990b. In Bo90, pp. 355–64
Wood PR. 1995. In CT95, pp. 127–38
Wood PR, Cahn JH. 1977. *Ap. J.* 211:499–508
Worrell JK. 1987. In *Stellar Pulsation,* ed. AN Cox, WM Sparks, SG Starrfield, *Lecture Notes Phys.* 274:289–92. New York: Springer-Verlag
Xiong D-R. 1981. *Acta Astron. Sinica* 22:350–56
Yang S, Walker GAH, Hill GM, Harmanec P. 1990. *Ap. J. Suppl.* 74:595–608
Yee HC, Sweby PK, Griffiths DF. 1991. *J. Comput. Phys.* 97:249–310
Zalewski J. 1992. *Publ. Astron. Soc. Jpn.* 44:27–43
Zalewski J. 1993. *Acta Astron.* 43:431–40
Ziebarth K. 1970. *Ap. J.* 162:947–62