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Stellar Synthesis of the Proton-Rich Heavy Elements

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The proton-rich heavy nuclei are produced by proton-capture and/or photo-nuclear reactions starting with the products of neutron capture.

The stellar conditions in which the proton-rich nuclei are produced must be such that the proton density $\rho_p=10\sim10^2 \text{gcm}^{-3}$, the neutron density $\rho_n=10^{-5}\sim10^{-4} \text{gcm}^{-3}$ and the other matter density $\rho\lesssim10^{-5} \text{gcm}^{-3}$ at the temperature $T=2.5\sim3\times10^9$ °K. It is, however, difficult to explain the abundance of all proton-rich nuclei for the equal temperature. It is possible if there exist the regions of different temperature. To form these nuclei the nuclear formation by the effective (p, γ) reaction occurs in the higher temperature region and the nuclear formation by the effective (γ, n) reaction in the lower temperature region.

§ 1. Introduction

Burbidge et al.[1] and Cameron[2] have explained the observed data by assuming that the heavy elements existing in the universe are formed through the neutron capture process. They divided its process into two types: The one is the slow neutron capture process (s-process) which occurs at a comparatively low temperature in the stellar interior, the other the rapid neutron capture process (r-process) which takes place during the initial stage of supernova explosion. The heavy nuclei being produced through these processes are subjected to beta-decays, so that they are located on the stability curve and on its right-hand side in the N-Z plane (see Fig. 1).

However, the proton-rich heavy nuclei which are located on the left-hand side of the stability curve cannot at all be produced by the neutron capture; hence it has been suggested that these nuclei are produced by proton capture and/or photo-nuclear reactions starting with the products of neutron capture processes. Burbidge et al.[1] explained qualitatively the data of Suess and Urey[3] assuming that these nuclei were initially produced by (p, γ) reactions operating on about 1% of the material which has already been synthesized through neutron capture processes. They showed that these reactions occur when the temperature $T=2\sim3\times10^9$ °K and the proton density $\rho_p\gtrsim10^2$ gcm^{-3}. It is impossible, however, to explain the relative abundances of all the proton-rich nuclei under the same stellar conditions and the nucleus $^{138}_{57}$La cannot be produced for a low temperature.

Moreover, for the temperatures above 2.5×10^9 degree the (γ, n) reaction time, $\tau_{\gamma n}$, is smaller than the (p, γ) reaction time, $\tau_{p\gamma}$, in the very heavy element

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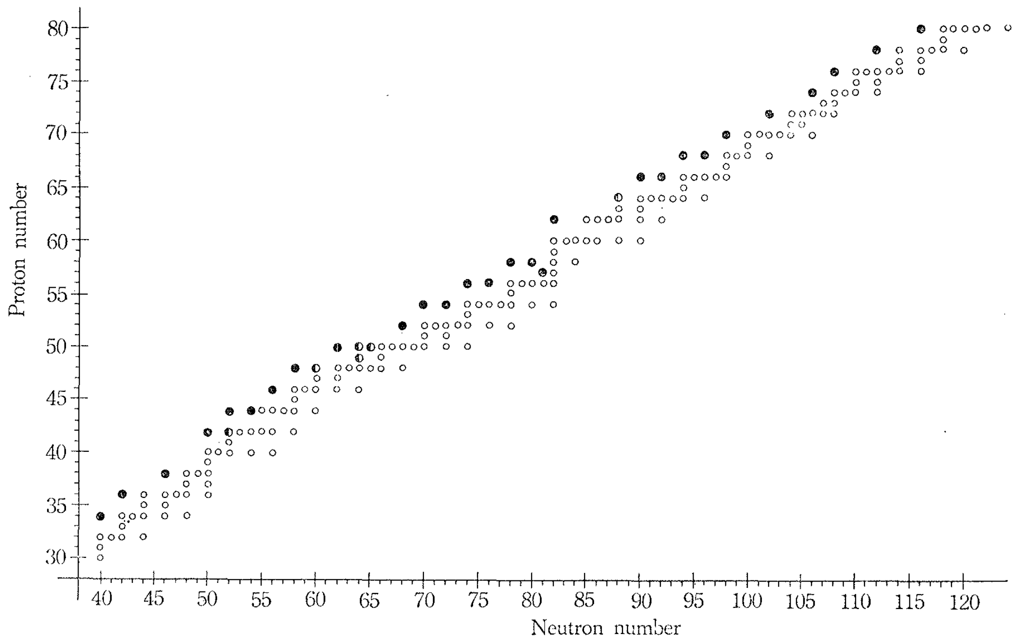
region; hence the formation of the proton-rich nuclei by the (p, γ) reaction would be almost impossible there.

In this paper we shall quantitatively deal with the process of forming the proton-rich heavy nuclei in the supernova envelope, and investigate whether the abundances of all proton-rich nuclei are explained under the same stellar conditions. As was suggested by Burbidge et al.[1] we assume the (p, γ) and (γ, n) reactions as the processes producing these nuclei. The comparison between the reaction times of two processes, (p, γ) and (γ, n), forming the proton-rich heavy nuclei from the seed nuclei which have already been synthesized tells us that $\tau_{\gamma n} < \tau_{p\gamma}$ at the temperatures of $2.5 \sim 3 \times 10^9$ °K in the region of elements having large mass number A and vice versa (see Fig. 5). However, at the temperature of 3×10^9 °K only the (p, γ) reaction of the reactions on the nuclei having the small mass number is responsible for the formation of the proton-rich nuclei, while at the temperature of 2.5×10^9 °K only the (γ, n) reaction of the reactions on the nuclei having the large mass number is effective. It is impossible to form the all proton-rich nuclei by both reactions at the equal temperature. Therefore, we consider the two regions having different temperatures. When the (p, γ) and (γ, n) reactions took place in two regions, the light elements of the proton-rich nuclei can be produced by (p, γ) reactions in the region of the higher temperature and the very heavy proton-rich nuclei can be produced by (γ, n) reactions in the region of lower temperature. The abundances of the proton-rich heavy nuclei are shown in Fig. 6.



● The proton-rich heavy nuclei.
◐ Of the proton-rich heavy nuclei, the nuclei having a possibility produced in weak loops of the s-process chain.
○ The nuclei produced by neutron capture processes.

Fig. 1. A chart of the proton-rich heavy nuclei.

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§ 2. Synthesis of the proton-rich heavy nuclei

(i) *Reaction rates*

Let us now consider the reaction rates for (p, γ), (γ, p), (γ, n) and (n, γ) reactions on the heavy nuclei. In general, for the (p, γ) reaction on a heavy nucleus (A, Z) a mean reaction time per one nucleus, $\tau_{p\gamma}$, is approximately given by using a statistical nonresonant formula as

$$1/\tau_{p\gamma}=\lambda_{p\gamma}=4.50\times 10^{9}\rho_p \frac{Z^{5/6}}{A^{1/6}T_9^{2/3}} \exp\left[1.26\{Z(A^{1/3}+1)\}^{1/2}-4.25(Z^2/T_9)^{1/3}\right], \tag{1}$$

where $\lambda_{p\gamma}$ is a rate of reaction per one nucleus per unit time, ρ_p the density of protons in gcm^{-3} and T_9 the temperature in units of 10^9 °K.

A last proton binding energy Q_p of a fixed N nucleus becomes progressively smaller as more and more protons are added, and the (γ, p) reactions become operative. A mean reaction time for the (γ, p) reaction being in a state of detailed balance,

$$1/\tau_{\gamma p}=\lambda_{\gamma p}=2.00\times 10^{10} \frac{T_9^{3/2}}{\rho_p \tau_{p\gamma}} \exp(-11.6 Q_p/T_9) \tag{2}$$

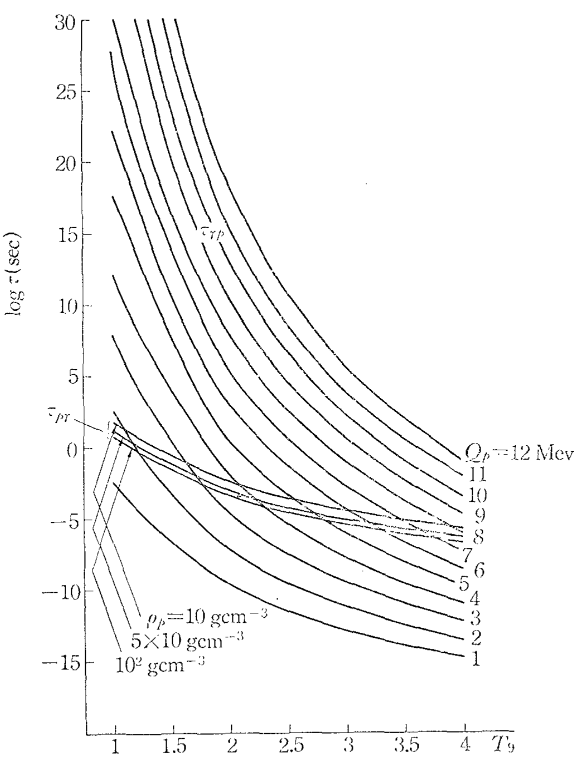

Fig. 2. (a) (p, γ) and (γ, p) reaction times for typical nuclei. $(Z, A)=(40, 90)$.

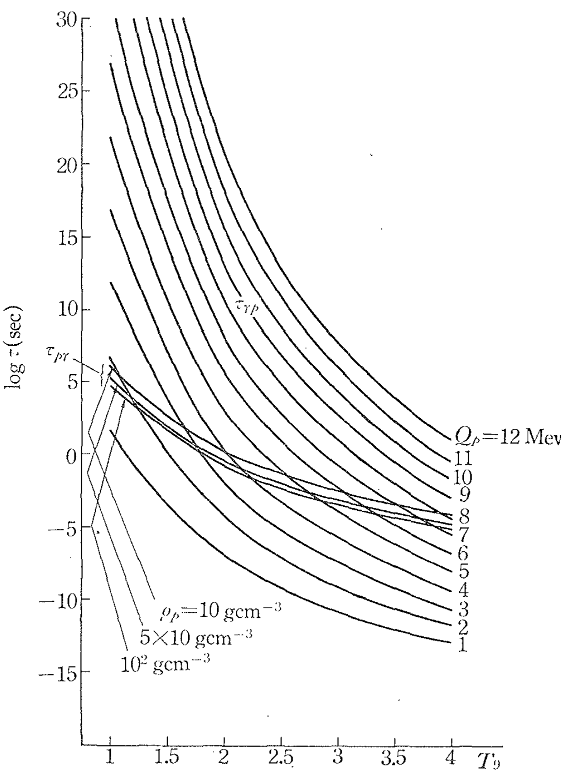

Fig. 2. (b) (p, γ) and (γ, p) reaction times for typical nuclei. $(Z, A)=(60, 142)$.

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or

$$\log \tau_{\gamma p} = -10.30 - 1.5 \log T_9 + 5.04 \frac{Q_p}{T_9} + \log \tau_{\gamma p} \rho_p, \tag{3}$$

with all logarithms to the base 10; $\lambda_{\gamma p}$ a reaction rate of the (γ, p) reaction and Q_p in units of Mev. For $Z=40$ and 60, the values of $\tau_{p\gamma}$ and $\tau_{\gamma p}$ are shown in Fig. 2 assuming $\rho_p = 10$, 5×10, $10^2\,\mathrm{gcm}^{-3}$.

On the other hand, a mean reaction time of the (γ, n) reaction, $\tau_{\gamma n}$, is also obtained from that of the (n, γ) reaction $\tau_{n\gamma}$. Write σ for the cross section of the (n, γ) reaction and let v and ρ_n be the velocity and density of neutrons responsible for its reaction, respectively, then

$$1/\tau_{n\gamma} = \lambda_{n\gamma} = 6.02\times10^{23} \rho_n \sigma v, \tag{4}$$

where $\lambda_{n\gamma}$ is a reaction rate per one nucleus per unit time. For the value σ, we use the cross sections evaluated[4] from the reactivity measurement for a neutron energy of 25 kev and for the unknown cross sections we interpolate or extrapolate their values from the known experimental ones considering the difference between even A and odd A nuclei for which a ratio (odd A)/(even A) $=1\sim4$ in the heavy element region. A mean reaction time of the (γ, n) reaction is also obtained from the equation for the equilibrium between (n, γ) and (γ, n) reactions.

$$1/\tau_{\gamma n} = \lambda_{\gamma n} = 2.00\times10^{10} \frac{T_9^{3/2}}{\rho_n \tau_{n\gamma}} \times \exp(-11.6 Q_n/T_9) \tag{5}$$

or

$$\log \tau_{\gamma n} = -10.30 - 1.5 \log T_9 + 5.04 \frac{Q_n}{T_9} + \log \tau_{n\gamma} \rho_n, \tag{6}$$

where $\lambda_{\gamma n}$ is a reaction rate and Q_n a last neutron binding energy in a nucleus (A, Z) in units of Mev. The relation between these reaction times $\tau_{n\gamma}$ and $\tau_{\gamma n}$ is shown in Fig. 3.

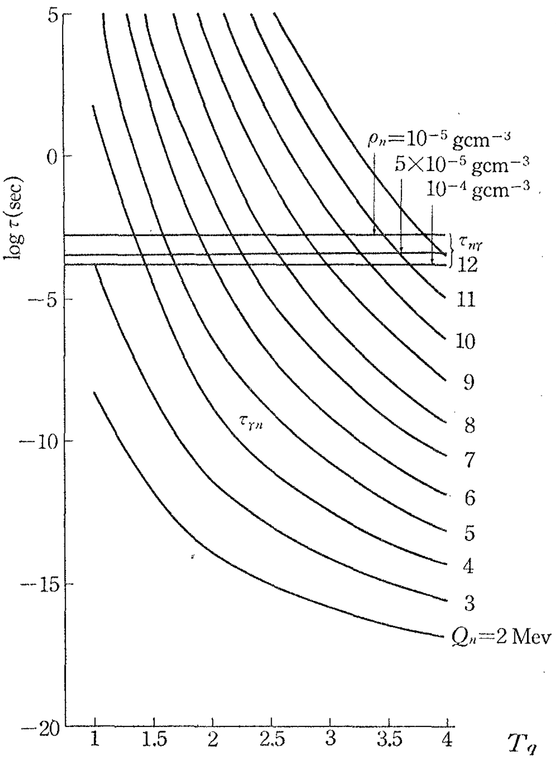

Fig. 3. (γ, n) and (n, γ) reaction times. (n, γ) reaction time is assumed to be constant for a given density ρ_n.

(ii) (p, γ) *reaction*

In this subsection we shall formulate and solve the general problem of the nuclear formation in which the proton-rich heavy nuclei are produced by the (p, γ) or (γ, n) reaction on a fast time scale. As was described in

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§ 1, it is assumed that these processes occur at $2.5 \sim 3 \times 10^9$ degrees, and only about 1% of the heavy nuclei which have already been synthesized by the neutron capture process contribute to produce the proton-rich heavy nuclei.

Besides these processes, we must consider the reactions induced by α-particles, but their reactions occur much slower than those by protons, for example, a ratio of $\lambda_{\alpha\gamma}/\lambda_{p\gamma}$ being equal to $0.43 \cdot \rho_\alpha/\rho_p \cdot \exp\{41.5 - 10(633/T_9)^{1/3}\}$ for $(A, Z) = (120, 50)$, in which $\lambda_{\alpha\gamma}$ is a reaction rate of (α, γ) reaction and ρ_α the α-particle density in units of gcm^{-3}. Therefore, it is able to neglect the effect of the reactions induced by α-particles hereafter.

Next, it should be noted whether the number of protons which can be added to the heavy nuclei is limited by the positron decay times. In the region of the proton-rich heavy nuclei, a mean lifetime of β^+-decay τ_{β^+} is agreeable to the calculated values as positron emission takes place by a forbidden transition to the ground state of their nuclei. Hence for an unknown mean lifetime of β^+-decay we estimate it using the expression as

$$\tau_{\beta^+} \simeq 10^6/W_{\beta^+}^5 \ \mathrm{sec} \quad \text{(forbidden transition)},$$
$$W_{\beta^+} = M(A, Z) - M(A, Z-1) - 0.51, \tag{7}$$

where $M(A, Z)$ is the mass measured in units of Mev, for which we used the semi-empirical mass formula derived by Cameron.[5] Then the calculated value τ_{β^+} is of the order of 10 minutes at least for the nuclei of interest. Now the duration of a supernova explosion has been estimated to be $1 \sim 10^3$ sec, so we can neglect these β^+-decay processes in our rapid processes.

Thus, neglecting the α-induced reactions and β^+-decay reactions, we can write the time variation of abundances of nuclei in our process, in which is treated the (p, γ) reaction at first, as follows:

$$\frac{dn(0)}{dt} = \lambda_{p\gamma}(-1)\,n(-1) - \{\lambda_{p\gamma}(0) + \lambda_{\gamma p}(0)\}\,n(0) + \lambda_{\gamma p}(1)\,n(1),$$
$$\frac{dn(1)}{dt} = \lambda_{p\gamma}(0)\,n(0) - \{\lambda_{p\gamma}(1) + \lambda_{\gamma p}(1)\}\,n(1) + \lambda_{\gamma p}(2)\,n(2), \tag{8}$$

etc.,

which $n(i)$ is the relative abundance of a nucleus i, $\lambda_{p\gamma}(i)$ and $\lambda_{\gamma p}(i)$ are the reaction rates of the (p, γ) and (γ, p) reactions on a nucleus i, respectively. As protons are very abundant in comparison with the other nuclei, the reaction time which the seed nuclei capture protons is much shorter than the duration of supernova outburst. Hence we need not consider the (p, γ) reaction time on the initial several seed nuclei until the last proton binding energy in a nucleus decreases and the inverse reaction (γ, p) becomes effective. That is, when the proton capture process proceeds and the (γ, p) reaction rate becomes comparable with the (p, γ) reaction rate, a quasi-statistical equilibrium would

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be realized between (p, γ) and (γ, p) reactions for a short period. If the time during which the proton capture process takes place is long, the produced proton-rich nucleus is released from the system including such an equilibrium state.

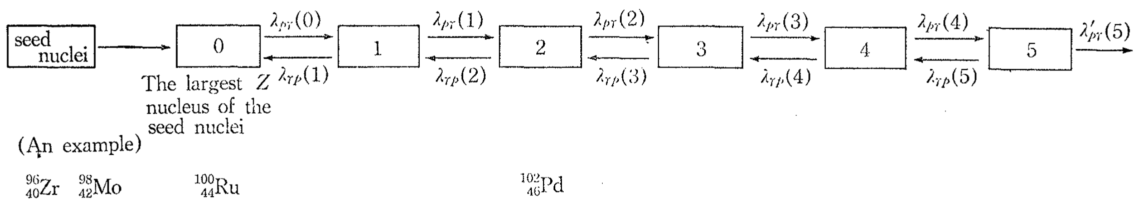

Fig. 4. Schematic diagram of the (p, γ) reactions.

When a nucleus reached one satisfying the condition such that $\lambda_{p\gamma}(0) \simeq \lambda_{\gamma p}(1)$, we call its nucleus a nucleus 0. We shall consider conveniently the system constitute of six nuclei. Then in most cases a nucleus 0 corresponds to an even-even nucleus which also is the largest Z nucleus of the seed nuclei having the same N (see Fig. 4). Thus we assume $\lambda_{\gamma p}(0)=0$ because of $\lambda_{p\gamma}(0) \gg \lambda_{\gamma p}(0)$ and put the initial conditions such that $n(0)=1$, $n(1)=n(2)=n(3)=\cdots=0$ at $t=0$. Then Eq. (8) are written in the following from a nucleus 0 to a nucleus 5 and their solutions can be obtained.

$$\begin{aligned}
\frac{dn(0)}{dt} &= -\lambda_{p\gamma}(0)\, n(0) + \lambda_{\gamma p}(1)\, n(1), \\
\frac{dn(1)}{dt} &= \lambda_{p\gamma}(0)\, n(0) - \{\lambda_{p\gamma}(1) + \lambda_{\gamma p}(1)\} n(1) + \lambda_{\gamma p}(2)\, n(2), \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\frac{dn(4)}{dt} &= \lambda_{p\gamma}(3)\, n(3) - \{\lambda_{p\gamma}(4) + \lambda_{\gamma p}(4)\} n(4) + \lambda_{\gamma p}(5)\, n(5), \\
\frac{dn(5)}{dt} &= \lambda_{p\gamma}(4)\, n(4) - \{\lambda'_{p\gamma}(5) + \lambda_{\gamma p}(5)\} n(5),
\end{aligned} \tag{9}$$

where the reaction rate of a nucleus 5, $\lambda'_{p\gamma}(5)$, is expressed as the released effect from the system in a quasi-equilibrium from a nucleus 0 to 5 and is approximately given as

$$\lambda'_{p\gamma}(5) = \frac{\lambda_{p\gamma}(6)}{\lambda_{p\gamma}(6) + \lambda_{\gamma p}(6)} \cdot \lambda_{p\gamma}(5). \tag{10}$$

The value $\lambda_{p\gamma}$ decreases gradually as the mass number increases, for which a ratio $\lambda_{p\gamma}(0)/\lambda_{p\gamma}(5) \simeq 4 \sim 5$ through each process, while the value $\lambda_{\gamma p}$ increases rapidly with the form $\exp(-11.6\, Q_p/T_9)$. Also, for the same N nuclei $\lambda_{\gamma p}(Z \text{ odd}) \gg \lambda_{\gamma p}$ (Z even) from the odd-even effect of a last proton binding energy in nucleus. When we assume $T_9=3$ and $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$ for the

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reason that it will be discussed later, $\lambda_{\gamma p}(5)$, $\lambda_{\gamma p}(3)$ and $\lambda_{\gamma p}(1)$ are all larger by the order two or more than the other λ, and it tells us $\lambda_{\gamma p}(5) \gg \lambda_{\gamma p}(3) \gg \lambda_{\gamma p}(1)$.

Taking account of the above relations among the reaction rates we can solve Eq. (9),

$$n(i) = A_i e^{-\delta_0 t} + \sum_{k=1}^{5} B_i e^{-\delta_k t} \quad (i=0\sim 5), \tag{11}$$

where δ_0, δ_1, δ_2, δ_3, δ_4 and δ_5 are the approximate solutions of the following equation:

$$\begin{aligned} x^6 &+ \lambda_{\gamma p}(5)\, x^5 + \lambda_{\gamma p}(5)\, \lambda_{\gamma p}(3)\, x^4 + \lambda_{\gamma p}(5)\, \lambda_{\gamma p}(3)\, \{\lambda_{\gamma p}(1) \\ &+ \lambda_{\gamma p}(4)\}\, x^3 + \lambda_{\gamma p}(5)\, \lambda_{\gamma p}(4)\, \lambda_{\gamma p}(3)\, \lambda_{\gamma p}(1)\, x^2 + \lambda_{\gamma p}(5) \\ &\times \lambda_{\gamma p}(4)\, \lambda_{\gamma p}(3)\, \{\lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1) + \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) + \lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2)\}\, x \\ &+ \lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)\, \lambda_{p\gamma}(2)\, \lambda_{p\gamma}(3)\, \lambda_{p\gamma}(4)\, \lambda'_{p\gamma}(5) = 0. \end{aligned} \tag{12}$$

It may be allowed to neglect the higher terms in comparison with the first term of (11) considering $\delta_0 \ll \delta_1, \delta_2, \cdots \delta_5$. Then $n(i)$ have the following expression,

$$\begin{aligned} & n(i) \cong A_i e^{-\delta_0 t}, \\ & \delta_0 \cong \frac{\lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)\, \lambda_{p\gamma}(2)\, \lambda_{p\gamma}(3)\, \lambda_{p\gamma}(4)\, \lambda'_{p\gamma}(5)}{\lambda_{\gamma p}(3)\, \lambda_{\gamma p}(4)\, \lambda_{\gamma p}(5)\, \{\lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1) + \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) + \lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2)\}}, \\ & A_0 : A_1 : A_2 \cong \lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2) : \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) : \lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1), \end{aligned} \tag{13}$$

where A_i, etc., are the coefficients expressing the distribution of nuclei for $t \ll 1/\delta_0$ (i.e. $e^{-\delta_0 t} \simeq 1$), and A_3, A_4, A_5 are smaller by order two or more than A_0, A_2.

From (9) $\sim$ (13) we can evaluate the relative abundances of the proton-rich nuclei to their seed nuclei. Under the above conditions $T_9=3$, $\rho_p = 10 \sim 10^2$ gcm^{-3} the proton-rich nucleus corresponds to a nucleus 2 with the exception that $^{113}_{49}$In and $^{138}_{57}$La correspond to a nuclei 1 and $^{94}_{42}$Mo to a nucleus 0, in which $^{139}_{57}$La (only a seed nucleus of $^{138}_{57}$La) is exceptionally an odd-even nucleus. Thus $n(0)$, $n(1)$ and $n(2)$ are given by

$$n(0) \cong A_0 e^{-\delta_0 t} \cong \frac{\lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2)}{\lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)}\, e^{-\delta_0 t},$$

$$n(1) \cong A_1 e^{-\delta_0 t} \cong \frac{\lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2)}{\lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)}\, e^{-\delta_0 t}$$

and

$$n(2) \cong A_2 e^{-\delta_0 t} \cong \frac{\lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)}{\lambda_{\gamma p}(1)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{\gamma p}(2) + \lambda_{p\gamma}(0)\, \lambda_{p\gamma}(1)}\, e^{-\delta_0 t}. \tag{14}$$

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Since the nucleus ${}^{138}_{57}$La is a double shielded nucleus, which is not so proton-rich, it is unable to produce it by the β-decay processes considered as the freezing reaction in the cooling stage from the unstable nucleus, which is the product of the neutron capture process. Even if we also assume that ${}^{138}_{57}$La is the product of the proton-capture reaction from ${}^{137}_{56}$Ba, this nucleus cannot be produced at $T_9<3$ because $\lambda_{\gamma p}({}^{139}_{58}\mathrm{Ce}\rightarrow{}^{138}_{57}\mathrm{La})\ll\lambda_{p\gamma}({}^{138}_{57}\mathrm{La}\rightarrow{}^{139}_{58}\mathrm{Ce})$, ${}^{138}_{57}$La being the Z odd-N odd nucleus. To produce ${}^{138}_{57}$La through the proton-capture process the temperature needs to be at least $3\times10^9\,°\mathrm{K}$. The astrophysical condition of $T_9>3$ would be implausible in the supernova envelope containing the excessive abundance of hydrogen. We therefore take $T_9=3$ for $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$ to produce ${}^{138}_{57}$La. When the temperature rises to about $3\times10^9\,°\mathrm{K}$, one must notice that the (γ, n) reaction time becomes shorter than the (p, γ) reaction time in the very heavy element region as indicated in Fig. 5. Namely, the (γ, n) reaction on their elements first occur and the seed nuclei for the (p, γ) process are almost brought off. Hence the formation of the proton-rich nuclei by the (p, γ) reaction is impossible there. Moreover in the envelope the neutron density is so low ($10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$) that the (γ, n) reaction rate is much larger than the (n, γ) reaction rate in the very heavy element region for this temperature. These facts tell us that the formation of the proton-rich nuclei in the very heavy element region is impossible in either process at $T_9=3$.

In order to produce the proton-rich nuclei by the (γ, n) reaction in the very heavy element region, we must lower the temperature for the densities $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$ and $\rho_n=10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$, these densities being the suitable values in the envelope of a supernova. As shown in Eqs. (4) and (5), the value of $\lambda_{n\gamma}$ is almost independent of temperature, while $\lambda_{\gamma n}$ changes sharply with the temperature. Therefore, if we adopt $T_9=2.5$ to increase the value of $\lambda_{\gamma n}$ for the same neutron density, the quasi-equilibrium state between (γ, n) and (n, γ) reactions realizes for a short period and the proton-rich nuclei can be produced through the (γ, n) process on the seed nuclei in the very heavy element region. In the light element region, at the temperature of $T_9=2.5$ the (p, γ) reaction time is shorter than the (γ, n) reaction time, so that the (p, γ) reaction is favorable as in the case of $T_9=3$ (see Fig. 5). In this case, however, the proton-rich nuclei cannot be produced by the (p, γ) reaction because of $\lambda_{p\gamma}>\lambda_{\gamma p}$ for $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$. For $T_9=2.5$, therefore, the proton-rich nuclei in the light element region cannot be made in any case.

Here we assume that there exist two regions of $T_9=2.5$ and three for the densities of $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$ and $\rho_n=10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$. Then the light nuclei of the proton-rich nuclei would be produced by the (p, γ) reactions in the region of $T_9=3$, and the very heavy nuclei of the proton-rich nuclei would be produced by the (γ, n) reactions in the region of $T_9=2.5$.

The relation between the proton-rich heavy nuclei and their expected formation processes is shown in Table I.

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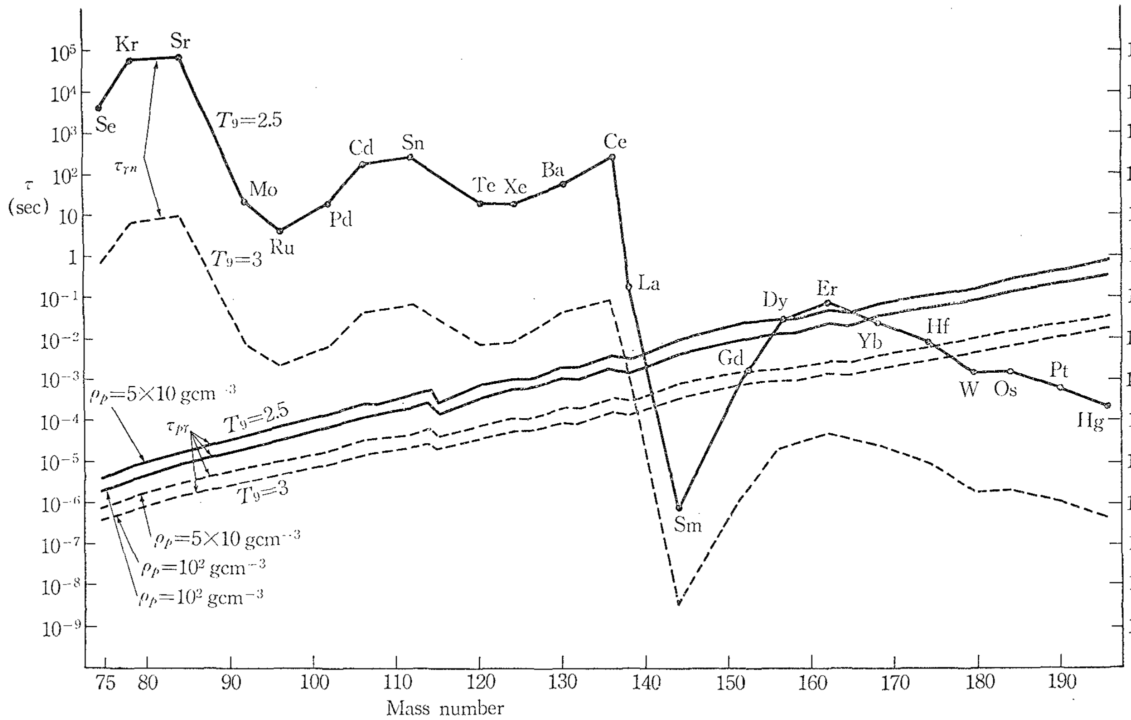

Fig. 5. The reaction times of (p, γ) and (γ, n) reactions for the production of proton-rich heavy nuclei.

(iii) (γ, n) *reaction*

The general formula forming the proton-rich nuclei through the (γ, n) process is treated with the same consideration as in the (p, γ) process. For $\lambda_{p\gamma}$ and $\lambda_{\gamma p}$ in Eqs. (8) and (9), we may substitute $\lambda_{\gamma n}$ and $\lambda_{n\gamma}$, respectively. Since $\lambda_{\gamma n}(0) \leq \lambda_{n\gamma}(0)$ for $\rho_n = 10^{-5} \sim 10^{-4}\,\text{gcm}^{-3}$, $\lambda_{n\gamma}(0)$ being not negligible in comparison with $\lambda_{\gamma n}(0)$, $\lambda_{\gamma n}(0)$ is replaced by $\lambda_{\gamma n}(0) + \lambda_{n\gamma}(0)$. Then the following equations are obtained:

$$\frac{dn(0)}{dt} = -\{\lambda_{\gamma n}(0) + \lambda_{n\gamma}(0)\}\,n(0) + \lambda_{n\gamma}(1)\,n(1),$$

$$\frac{dn(1)}{dt} = \lambda_{\gamma n}(0)\,n(0) - \{\lambda_{\gamma n}(1) + \lambda_{n\gamma}(1)\}\,n(1) + \lambda_{\gamma n}(2)\,n(2),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (15)$$

$$\frac{dn(4)}{dt} = \lambda_{\gamma n}(3)\,n(3) - \{\lambda_{\gamma n}(4) + \lambda_{n\gamma}(4)\}\,n(4) + \lambda_{n\gamma}(5)\,n(5),$$

$$\frac{dn(5)}{dt} = \lambda_{\gamma n}(4)\,n(4) - \{\lambda'_{\gamma n}(5) + \lambda_{n\gamma}(5)\}\,n(5),$$

$$\lambda'_{\gamma n}(5) = \frac{\lambda_{\gamma n}(6)}{\lambda_{\gamma n}(6) + \lambda_{n\gamma}(6)}\,\lambda_{\gamma n}(5). \qquad (16)$$

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To explain the formation of the proton-rich nuclei through the quasi-equilibrium state between the (γ, n) and (n, γ) processes during the initial phases of the supernova explosion, we require the conditions such that $\rho_n=10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$ for $T_9=2.5$ as discussed before. Taking here the value $\rho_n=10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$, $\lambda_{\gamma n}(1)$ and $\lambda_{\gamma n}(3)$ are larger by order one or more than the other all reaction rates $\lambda\,(\lambda_{\gamma n}(1)>\lambda_{\gamma n}(3))$, and $\lambda_{n\gamma}$'s values are almost constant $(\lambda_{n\gamma}(N\ \text{odd})/\lambda_{n\gamma}(N\ \text{even})=2\sim4)$ for fixed Z nuclei. Similarly to the case of (p, γ) reaction, we neglect the time until the seed nuclei capture the γ-rays and reach the nucleus 0, and take the initial conditions that $n(0)=1$, $n(1)=n(2)=\cdots=n(5)=0$ at $t=0$, assuming the system from the nucleus 0 to the nucleus 5. The nucleus 2 generally corresponds the proton-rich heavy nucleus except ${}^{138}_{57}\mathrm{La}$ which corresponds to the nuclei 1. In the case that the proton-rich heavy nuclei have two isotopes, these nuclei correspond to the nuclei 0 and 2. Since the dependence of Q_n on the mass number is not so large as that of Q_p, the value $\lambda_{\gamma n}$ does not sharply increase with the mass number for fixed Z nuclei. The total number of seed nuclei will be distributed among comparatively many species of nuclei for each process, and A_i express the distribution coefficients of the produced nuclei for a short period.

$$\cdots A_3 : A_2 : A_1 : A_0 : A_{-1}\cdots$$

$$=\cdots\frac{\lambda_{\gamma n}(2)}{\lambda_{n\gamma}(3)}:1:\frac{\lambda_{n\gamma}(2)}{\lambda_{\gamma n}(1)}:\frac{\lambda_{n\gamma}(2)\,\lambda_{n\gamma}(1)}{\lambda_{\gamma n}(1)\,\lambda_{\gamma n}(0)}:\frac{\lambda_{n\gamma}(2)\,\lambda_{n\gamma}(1)\,\lambda_{n\gamma}(0)}{\lambda_{\gamma n}(1)\,\lambda_{\gamma n}(0)\,\lambda_{\gamma n}(-1)}\cdots \tag{17}$$

where A_2 is normalized to 1, and the values A_4, $A_5\cdots$ are smaller by order two or more than that of A_2.

Considering the above relations, the relative abundances of the objective nuclei are obtained as follows.

$$\begin{aligned} n(2) &\simeq A_2 e^{-\delta_0' t}, \\ n(1) &\simeq A_1 e^{-\delta_0' t}, \\ n(0) &\simeq A_0 e^{-\delta_0' t}, \end{aligned} \tag{18}$$

$$\delta_0' \simeq \frac{\{\lambda_{\gamma n}(0)+\lambda_{n\gamma}(0)\}\,\lambda_{\gamma n}(1)\,\lambda_{\gamma n}(2)\,\lambda_{\gamma n}(3)\,\lambda_{\gamma n}(4)\,\lambda'_{\gamma n}(5)}{\lambda_{n\gamma}(3)\,\lambda_{n\gamma}(4)\,\lambda'_{n\gamma}(5)\,\{(\lambda_{\gamma n}(0)+\lambda_{n\gamma}(0))\,(\lambda_{\gamma n}(1)+\lambda_{n\gamma}(2))+\lambda_{n\gamma}(1)\,\lambda_{n\gamma}(2)\}}.$$

Using (14) and (18), the relative abundances of the proton-rich nuclei to the seed nuclei can be calculated for two processes, (p, γ) and (γ, n), the former is responsible for the large A nuclei at $T_9=3$, $\rho_p=10\sim10^2\,\mathrm{gcm}^{-3}$, and the latter for the small A nuclei at $T_9=2.5$, $\rho_n=10^{-5}\sim10^{-4}\,\mathrm{gcm}^{-3}$. In Table I we show the relative abundances of all proton-rich nuclei to their seed nuclei, their ratios being about 1% or less. In either case, we assume that one percent of the seed nuclei contribute to our processes, namely the abundance of the nucleus 0 at $t=0$, $n(0)$ is one percent of the abundance of all seed nuclei.

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Then the abundances of the proton-rich heavy nuclei can be obtained as a function of time.

Table I. The relative abundances of proton-rich nuclei to the seed nuclei.[3)] $^{113}_{49}$In is assumed to be produced by (p, γ) and β^+-decay processes.

Proton-rich nuclei	Seed nuclei in (p, γ) reactions		Proton-rich nuclei	Seed nuclei in (p, γ) reactions	
$^{74}_{34}$Se	$^{72}_{32}$Ge $^{71}_{31}$Ga $^{70}_{30}$Zn	0.030	$^{120}_{52}$Te	$^{118}_{50}$Sn $^{116}_{48}$Cd	0.011
$^{78}_{36}$Kr	$^{76}_{34}$Se $^{75}_{33}$As $^{74}_{32}$Ge	0.0061	$^{124}_{54}$Xe	$^{122}_{52}$Te $^{121}_{51}$Sb $^{120}_{50}$Sn	0.0055
$^{84}_{38}$Sr	$^{82}_{36}$Kr $^{81}_{35}$Br $^{80}_{34}$Se	0.0023	$^{126}_{54}$Xe	$^{124}_{52}$Te $^{123}_{51}$Sb $^{122}_{50}$Sn	0.0091
$^{92}_{42}$Mo	$^{90}_{40}$Zr $^{89}_{39}$Y $^{88}_{38}$Sr $^{87}_{37}$Rb $^{86}_{36}$Kr	0.0058	$^{130}_{56}$Ba	$^{128}_{54}$Xe $^{127}_{53}$I $^{126}_{52}$Te $^{124}_{50}$Sn	0.0020
$^{94}_{42}$Mo	$^{93}_{41}$Nb $^{92}_{40}$Zr	0.021	$^{132}_{56}$Ba	$^{130}_{54}$Xe $^{128}_{52}$Te	0.0022
$^{96}_{44}$Ru	$^{93}_{41}$Nb $^{92}_{40}$Zr	0.0082	$^{136}_{58}$Ce	$^{134}_{56}$Ba $^{133}_{55}$Cs $^{132}_{55}$Xe $^{130}_{52}$Te	0.0014
$^{98}_{44}$Ru	$^{96}_{42}$Mo $^{94}_{40}$Zr	0.0034	$^{138}_{58}$Ce	$^{136}_{56}$Ba $^{134}_{54}$Xe	0.0080
$^{102}_{46}$Pd	$^{100}_{44}$Ru $^{98}_{42}$Mo $^{96}_{40}$Zr	0.0024	$^{138}_{57}$La	$^{137}_{56}$Ba	0.0044
$^{106}_{48}$Cd	$^{104}_{46}$Pd $^{103}_{45}$Rh $^{102}_{44}$Ru $^{100}_{42}$Mo	0.011	$^{144}_{62}$Sm	$^{142}_{60}$Nd $^{141}_{59}$Pr $^{140}_{58}$Ce $^{139}_{57}$La $^{138}_{56}$Ba $^{136}_{54}$Xe	0.0014
$^{108}_{48}$Cd	$^{107}_{47}$Ag $^{106}_{46}$Pd $^{108}_{44}$Ru	0.013	$^{152}_{64}$Gd	$^{151}_{63}$Eu $^{150}_{62}$Sm $^{148}_{60}$Nd	0.0062
$^{112}_{50}$Sn	$^{110}_{48}$Cd $^{109}_{47}$Ag $^{108}_{46}$Pd	0.032	$^{156}_{66}$Dy	$^{154}_{64}$Gd $^{153}_{63}$Eu $^{152}_{62}$Sm $^{150}_{60}$Nd	0.00079
$^{113}_{49}$In	$^{111}_{48}$Cd	0.041	$^{158}_{66}$Dy	$^{156}_{64}$Gd $^{154}_{62}$Sm	0.0017
$^{114}_{50}$Sn	$^{112}_{48}$Cd $^{110}_{46}$Pd	0.030	$^{162}_{68}$Er	$^{160}_{66}$Dy $^{159}_{65}$Tb $^{158}_{64}$Gd	0.0011
$^{115}_{50}$Sn	$^{113}_{48}$Cd	0.042	$^{164}_{68}$Er	$^{162}_{66}$Dy $^{160}_{64}$Gd	0.016

Proton-rich nuclei	Seed nuclei in (γ, n) reactions		Proton-rich nuclei	Seed nuclei in (γ, n) reactions	
$^{196}_{80}$Hg	$^{198,199,200,201,202,204}_{80}$Hg	0.0016	$^{158}_{66}$Dy	$^{160,161,162,163,164}_{66}$Dy	0.00090
$^{190}_{78}$Pt	$^{192,194,195,196,198}_{78}$Pt	0.00062	$^{156}_{66}$Dy	$^{160,161,162,163,164}_{66}$Dy	0.00052
$^{184}_{76}$Os	$^{186,187,188,189,190,192}_{76}$Os	0.00018	$^{152}_{64}$Gd	$^{154,155,156,157,158,160}_{64}$Gd	0.0020
$^{180}_{74}$W	$^{182,183,184,186}_{74}$W	0.0012	$^{144}_{62}$Sm	$^{147,148,149,150,151,154}_{62}$Sm	0.016
$^{174}_{72}$Hf	$^{176,177,178,179,180}_{72}$Hf	0.0018	$^{138}_{57}$La	$^{139}_{57}$La	0.0009
$^{168}_{70}$Yb	$^{170,171,172,173,174,176}_{70}$Yb	0.0014	$^{138}_{58}$Ce	$^{140,142}_{58}$Ce	0.0025
$^{164}_{68}$Er	$^{166,167,168,170}_{68}$Er	0.015	$^{136}_{58}$Ce	$^{140,142}_{58}$Ce	0.0020
$^{162}_{68}$Er	$^{166,167,168,170}_{68}$Er	0.0010			

§ 3. Results and discussions

As shown in § 2, we divide the build-up process of the proton-rich heavy

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nuclei into two processes and evaluate their abundances. The time giving rise to these processes, t, may be of the same order of magnitude as or shorter than the duration of the supernova explosion ($1\sim10^3$ sec). Therefore we obtain the abundances of the objective nuclei for $1\lesssim t\lesssim10^3$ sec. Our calculated results are summarized in Fig. 6 and are shown in Table II. Since these processes are considered to occur in the supernova envelope, one may assume that the density of other elements is very low ($\rho\lesssim10^{-5}$).

Table II. Summary of our results.

Process	Temperature (in 10^9 °K)	Density (gcm^{-3})	Time scale (sec)	Our results (see Fig. 6)	Remarks
		$\rho_p=10$	$1\lesssim t\lesssim10^2$	Abund. of ^{124}Xe, ^{138}La and ^{144}Sm are too low $\frac{^{94}\text{Mo}}{^{96}\text{Ru}}=\frac{1}{25}$: small. (observed value : 2.7)	
$(p,\ \gamma)$	3	$\rho_p=5\times10$	$1\lesssim t\lesssim10^3$	Abund. of ^{74}Se, ^{84}Sr, and ^{98}Ru decrease sharply with t. At $t\gtrsim10$ these nuclei are very low-abundant. ^{138}La and ^{144}Sm are low-abundant.	Se, Sr and Ru can also be produced by p-capture process from ^{58}Ni.[6]
		$\rho_p=10^2$	$1\lesssim t\lesssim10^2$	$\frac{^{94}\text{Mo}}{^{96}\text{Ru}}=10^{-2}$: very small. ^{74}Se, ^{84}Sr and ^{138}La are hardly produced. Abund. of ^{138}Ce and ^{98}Ru decrease sharply with t.	In addition to synthesis by the p-capture, ^{94}Mo can also be made in weak loops of the s-process chain.[1]
		$\rho_n=10^{-5}$	$1\lesssim t\lesssim10^2$	^{196}Hg and ^{190}Pt are hardly produced. Abund. of ^{184}Os and ^{180}W decrease sharply with t.	In any case, ^{136}Ce, ^{138}Ce, La, Sm and Gd are hardly produced by $(\gamma,\ n)$ process. ^{164}Er, ^{162}Er, ^{158}Dy and ^{156}Dy also be produced by $(p,\ \gamma)$ process.
$(\gamma,\ n)$	2.5	$\rho_n=5\times10^{-5}$	$1\lesssim t\lesssim10^2$	$\frac{^{164}\text{Er}}{^{162}\text{Er}}, \frac{^{158}\text{Dy}}{^{156}\text{Dy}}$: large. Abund. of ^{196}Hg decreases sharply with t.	
		$\rho_n=10^{-4}$	$1\lesssim t\lesssim10^2$	Abund. of Dy is very low. ^{164}Er/^{162}Er$\simeq300$. (observed : 9/5) ^{158}Dy/^{156}Dy$\simeq600$. (observed : 10) Abund. of ^{196}Hg decreases sharply with t.	

When $T_9=2.5$ or 3, $\rho_p=10\sim10^2$ gcm^{-3} and $\rho_n=10^{-5}\sim10^{-4}$ gcm^{-3}, there are the proton-rich nuclei having the possibilities produced by both reactions. These nuclei correspond to the ones such that the two reaction rates produced them are almost equal to each other ($\lambda_{p\gamma}\simeq\lambda_{\gamma n}$).

Considering the ambiguity of the values of the reaction rates, we take $^{136}_{58}$Ce,

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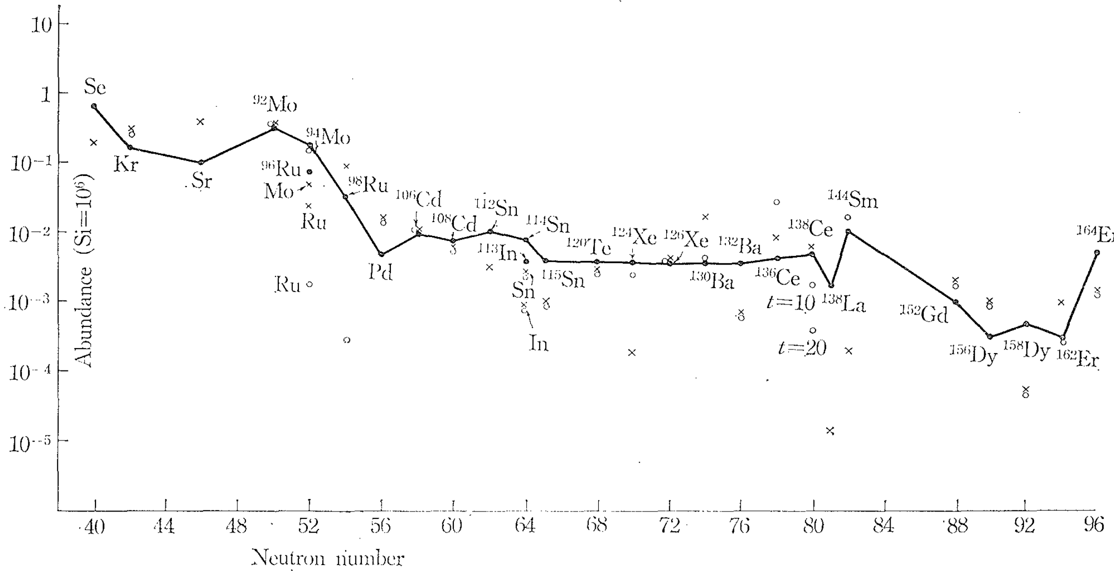
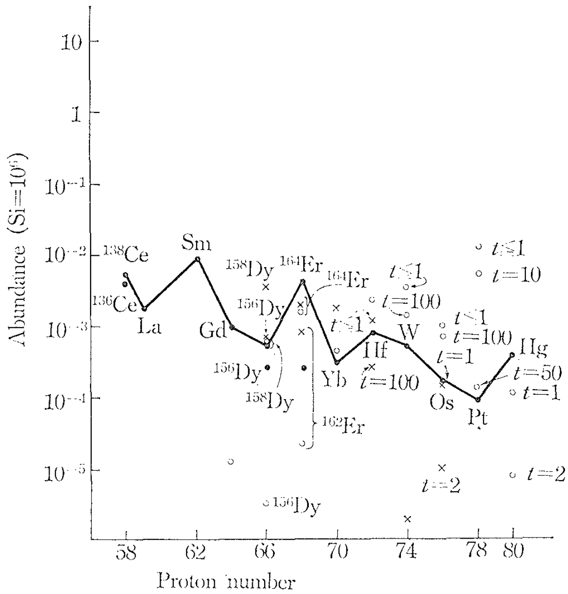


◎ Abundances of proton-rich nuclei
× $\rho_p=10$ g cm^{-3}, $1\lesssim t$ (sec)$\lesssim 10^2$ (calculated abund.)
○ $\rho_p=10^2$ g cm^{-3}, $1\lesssim t$ (sec)$\lesssim 10^2$ (calculated abund.)
Se, Sr, La are hardly produced

Fig. 6 (a). The abundances of proton-rich heavy nuclei[3] and our results. The calculated abundances of proton-rich nuclei produced by (p, γ) reactions.



◎ Abundances of proton-rich nuclei
× $\rho_n=10^{-5}$gcm, $1\lesssim t$(sec)$\lesssim 10^2$ (calculated abund.)
○ $\rho_n=5\times10^{-5}$gcm^{-3}, $1\lesssim t$ (sec) $\lesssim 10^2$ (calculated abund.)

Fig. 6 (b). The abundances of proton-rich heavy nuclei[3] and our results. The calculated abundances of proton-rich nuclei produced by (γ, n) reactions.

$^{138}_{58}$Ce, $^{138}_{57}$La, $^{144}_{62}$Sm, $^{152}_{64}$Gd, $^{156}_{66}$Dy, $^{158}_{66}$Dy, $^{162}_{68}$Er and $^{164}_{68}$Er as the products through both processes, and evaluate their abundances.

In the case of the (p, γ) reaction, the abundances of the proton-rich nuclei are mostly decided by the magnitude of A_i in (14), and their abundances are almost constant for time t. However, the abundances of $^{74}_{34}$Se, $^{84}_{38}$Sr and $^{98}_{44}$Ru sharply decrease with t at large proton density ($\rho_p\gtrsim 5\times10$ gcm^{-3}), and become low-abundant. In these nuclei, there is a probability that they may be formed through the successive proton-capture processes[6] from $^{58}_{28}$Ni for $T_9=0.5\sim2$ and $\rho_p=10^{-5}\sim10^2$ gcm^{-3} in the supernova envelope. As $^{113}_{49}$In is the Z odd-N even nucleus, the last proton binding energy of $^{114}_{50}$Sn is larger than

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that of $^{113}_{49}$In ($\lambda_{p\gamma}(^{113}_{49}\mathrm{In}) \gg \lambda_{\gamma p}(^{114}_{50}\mathrm{Sn})$) ; hence $^{113}_{49}$In cannot be produced by the (p, γ) reactions. For this nucleus, it seems plausible to consider that the unstable nucleus $^{113}_{50}$Sn is produced by the (p, γ) reaction on $^{111}_{48}$Cd which has already been synthesized by neutron-capture and after the explosion of the supernova $^{113}_{50}$Sn is subjected to β^+-decay to produce $^{113}_{49}$In. At $T_9=3$ and $\rho_p=10\,\mathrm{gcm}^{-3}$ $^{138}_{57}$La is made from $^{137}_{56}$Ba, but its abundance is lower than the observed one by about order two.

In the case of the (γ, n) reaction, the abundances of the very heavy proton-rich nuclei sharply decrease with time t. At $\rho_n > 10^{-5}\,\mathrm{gcm}^{-3}$ the abundance ratios among two isotopes ($^{164}_{68}$Er/$^{162}_{68}$Er and $^{158}_{66}$Dy/$^{156}_{66}$Dy) built by this process are in disagreement with the observed data. For $^{136}_{58}$Ce, $^{138}_{58}$Ce, $^{138}_{57}$La and $^{152}_{64}$Gd, it seems probable to consider that their nuclei cannot be produced by the (γ, n) reactions but can be produced only by the (p, γ) reactions. However, we have a number of uncertainties for the value of the reaction rates. For a last nucleon binding energy, there is some difference between the experimental values by Everling et al.[7] and the values using the semi-empirical mass formula by Cameron.[5] If the error between these values is 1 Mev, the difference of the two (γ, n) reaction rates using their values is of order two or three at $T_9=2.5\sim3$. Especially, the calculated abundances of the nuclei for which the time dependence is very large are not so reliable, these nuclei being $^{74}_{34}$Se, $^{84}_{38}$Sr, $^{98}_{44}$Ru, $^{138}_{58}$Ce, $^{138}_{57}$La, $^{184}_{76}$Os, $^{190}_{78}$Pt and $^{196}_{80}$Hg.

Finally we shall consider the foregoing astrophysical conditions in which the synthesis of the proton-rich heavy elements takes place. In general the temperature continuously changes in any region. However, the effective temperature by which the (p, γ) or (γ, n) reaction takes place is sharply determined, so that $T_9\simeq3$ for the (p, γ) reaction and $T_9\simeq2.5$ for the (γ, n) reaction. When we conveniently take the two independent regions of different temperatures in which the formation of the objective nuclei by the (p, γ) reaction is effective in the region of $T_9=3$ and the formation by the (γ, n) reaction is effective in the region of $T_9=2.5$, it would not give us so different results from true ones in our case. Therefore we assume that about one percent of abundances of the heavy-nuclei which have already been synthesized by neutron-capture are mixed into these two regions and give rise to our process. It will not be allowed that the temperature changes with time and the two effective reactions successively occur at different temperatures. Here, basing on the shock-theory of supernova explosion,[8] the mass fraction to the total mass of supernova in the stellar layer being heated to $T_9=3$ is $0.01\sim0.1$; meanwhile the ratio of the total mass in Galaxy to the ejected mass from the supernova is about 0.1.[1] Therefore it is considered, for example, that the heavy-nuclei existing in this layer would give rise to the effective (p, γ) reaction at $T_9=3$.

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