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STELLAR WEAK INTERACTION RATES FOR INTERMEDIATE-MASS NUCLEI. IV. INTERPOLATION PROCEDURES FOR RAPIDLY VARYING LEPTON CAPTURE RATES USING EFFECTIVE log (*ft*)–VALUES

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ABSTRACT

Simple expressions for continuum electron and positron capture phase space factors and the associated neutrino energy loss integrals are presented in terms of standard Fermi integrals. Continuous approximations to the relevant Fermi integrals and their first derivatives are made. These allow the computation of effective log (*ft*)–values, at each temperature and density point, for the continuum lepton capture rates considered in the earlier papers in this series. Since the effective log (*ft*)–values have most of the rapid temperature and density dependence associated with the phase space integrals removed, interpolation in temperature and density to obtain stellar rates is greatly facilitated in speed and accuracy. Computer simulations of stellar evolution will be able to implement more accurately our calculations of the stellar nuclear weak interaction rates of intermediate-mass nuclei. Generalization of the Fermi integral expressions for the lepton continuum capture phase space factors are given for astrophysical environments where there exists an equilibrium distribution of electron-type neutrinos. These allow rough estimates of the effect of neutrino blocking on our tabulated rates and estimates of total neutrino capture rates.

Subject headings: neutrinos — nuclear reactions

I. INTRODUCTION

The calculation of the stellar weak interaction rates of intermediate-mass nuclei was considered in the previous papers in this series: Fuller, Fowler, and Newman (1980, 1982*a*, *b*; hereafter F^2N I, II, and III, respectively). The stellar electron and positron decay rates, and the stellar continuum electron and positron capture rates of these nuclei, as well as the neutrino energy loss rates associated with these processes, were tabulated as functions of density and temperature. Tables of these rates were given on an abbreviated temperature and density grid in F^2N III, and tables of rates on a detailed temperature and density grid were provided on magnetic tape for use in computer calculations of stellar evolution and nucleosynthesis.

The stellar weak interaction rates of nuclei are, in general, very sensitive functions of temperature and density (see F^2N I and II). Temperature dependence of the rates arises through thermal excitation of parent excited states and through the lepton distribution functions inherent in the integrands of the decay and continuum capture phase space factors.

For electron and positron decay most of the temperature dependence is due to thermal population of parent excited states at all but the lowest temperatures and highest densities. In general, only a few transitions will contribute to these decay rates, and hence the variation of the rates with temperature is usually not so large that rates cannot be accurately interpolated in temperature and density with our grid. Density dependence of these decay rates is minimal. In the case of electron emission decay, however, there may be considerable density dependence because of Pauli blocking of final state electrons when the density is high and the temperature is low. In practice this does not present much of a problem for interpolation since the electron emission decay rate is usually very small in these conditions.

The temperature and density dependence of continuum electron and positron capture can be quite different. In addition to temperature sensitivity introduced through thermal population of parent excited states, there is considerable temperature and density sensitivity introduced into the capture rates through the lepton distribution functions in the integrands of the continuum capture phase space factors (see eqs. [3a], [3b], and [4a], F^2N I). This sensitivity of the capture rates means that interpolations in temperature and density on our standard grid to obtain a rate can be difficult, requiring a high-order interpolation routine and a relatively large amount of computer time to interpolate an accurate rate.

A typical transition for which electron capture rates are needed in stellar evolution calculations is $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn} - 4.2064$ MeV, where we give the nuclear Q_n-value for the transition. The sensitivity problem can be illustrated with this transition as an example, considered at some typical presupernova conditions. At a temperature $T_9 \equiv T/10^9$ K $= 1.0$ or $kT = 0.0862$ MeV and a density of $\rho Y_e = 10^8$ g cm^{-3} (where the number of electrons per baryon Y_e is defined by eq. [19a] below), the log base 10 of the continuum electron capture rate is $\log \lambda_{ec} = \log \lambda_{Z+1} = -13.265$ according to the F^2N standard rate table tape. Equation (2a), which follows, defines $\log \lambda_{Z+1}$. At a neighboring grid point of the same temperature and $\rho Y_e = 10^9$ g cm^{-3}, the log base 10 of the capute rate is $\log \lambda_{ec} = -2.628$; in other words, the rate is nearly 11 orders of magnitude faster than at the neighboring grid point. This increase in rate is due almost entirely to the lepton distribution functions, which change dramatically as the electron total Fermi energy changes from $W_F = 2.436$ MeV below the capture threshold at the first grid point to $W_F = 5.176$ MeV above the capture threshold at the second. The interpolation problem can be greatly eased by defining a simple continuum capture phase space integral, based on the parent ground-state to daughter ground-state transition Q-value, and dividing this into the F^2N tabulated rates at each temperature and density grid point to obtain a table of effective ft-values which are relatively temperature and density insensitive. This procedure requires a formulation of the capture phase space factors which is simple enough to use many times in the inner loop of stellar evolution or nucleosynthetic computer programs. Such a formulation in terms of standard Fermi integrals is presented in the next two sections, along with approximations for the requisite Fermi integrals which are continuous and have continuous first derivatives at zero argument. Section IV explains the calculation of effective log (ft)–values from the formulae deveoped in §§ II and III. Section IV also presents example tables of effective log (ft)–values for continuum electron and positron capture on the standard F^2N temperature and density grid, and outlines the procedure for reconstructing a stellar rate from an interpolated effective log (ft)–value. Average neutrino (antineutrino) emission energies are calculated from the total neutrino (antineutrino) energy loss rates and the associated electron capture–positron emission (positron capture–electron emission) rates. These average neutrino (antineutrino) energies are slowly varying functions of temperature and density and are tabulated on the standard F^2N grid. Finally, § V uses the generalized formulae of § II to outline a procedure for estimating the effects of neutrino phase space blocking of the capture rates, and a method to obtain total neutrino (antineutrino) capture rates.

II. LEPTON CONTINUUM CAPTURE PHASE SPACE INTEGRALS

In accordance with the approach stated in the introduction we now drop all reference to individual states of the parent and daughter nuclei and concentrate on the lepton capture reactions:

$$^{A}(Z+1) + e^- \rightleftarrows {}^{A}Z + \nu + Q_n \,, \tag{1a}$$

$$^{A}(Z-1) + e^+ \rightleftarrows {}^{A}Z + \bar{\nu} + Q_n \,, \tag{1b}$$

where the Q_n represent the *nuclear* mass-energy differences of the ground states of the parent (left-hand side) and daughter (right-hand side) nuclei. We will be interested in neutrino (antineutrino) continuum capture, the reverse of reactions (1a), (1b), as well as electron (positron) continuum capture, not necessarily under conditions of beta equilibrium. Appropriate modification in the notation of equation (I.1) (hereafter an "I" designates equations from F^2N I, "II" from F^2N II, etc.) and equations (A19) and (A19′) of Fowler and Hoyle (1964) then lead to

$$\lambda_{Z\pm 1} = \ln 2 \, \frac{I_e}{\langle ft \rangle_e} \tag{2a}$$

for electron or positron continuum capture, with a modified phase space factor defined by

$$I_e \equiv f_e / \langle G_e \rangle \tag{3}$$

and

$$\lambda_Z = \ln 2 \, \frac{I_\nu}{\langle ft \rangle_\nu} \tag{2b}$$

for neutrino or antineutrino continuum capture, with $I_\nu \equiv f_\nu / \langle G_e \rangle$, where e represents either e^- or e^+ as appropriate and ν represents either ν or $\bar{\nu}$.

In equations (2a) and (2b) the t are half-lives and the f_e are the phase space factors defined in equation (I.3b), while the f_ν are given by interchanging $S_\mp$ (S_e in our notation) and S_ν in that equation. In applying equation (I.3b) we have removed $G(\pm Z, w) = G_e$ (in our notation) from under the integral sign and replaced it by an average value $\langle G_e \rangle$ in front of the integral sign. Note the $\langle G_e \rangle$ appears in both directions of reactions (1a) and (1b). This procedure is valid in fair approximation except for low-energy positron capture where the rates are negligible in any case. At the risk of being pedantic we note that the Z in $G(\pm Z, w)$ must be replaced by $(Z+1)$ for equation (1a) and by $(Z-1)$ for equation (1b).

In equation (2a) the quantity $\langle ft \rangle_e$ represents an effective value for $ft/\langle G_e \rangle$ for the left-to-right reactions (1a) or (1b). Similarly in equation (2b) the quantity $\langle ft \rangle_\nu$ represents an effective value for $ft/\langle G_e \rangle$ for the right-to-left reactions (1a) and (1b). One may be tempted to invoke detailed balance at this point to connect $\langle ft \rangle_e$ and $\langle ft \rangle_\nu$. This is not valid since by our original premise we have essentially approximated complex nuclear systems comprised of many excited states and Gamow-Teller resonances by just the ground states in evaluating I_ν and I_e. The connection between $\langle ft \rangle_e$ and $\langle ft \rangle_\nu$ will become more transparent in the discussions of beta equilibrium in the last section. Detailed balance does relate I_ν and I_e as shown in equation (5d) below. This is an important generalization of the nondegenerate expression for detailed balance to the case for initial and final reactants of arbitrary degeneracy.

As stated in the Introduction the effective $\langle ft \rangle_e$-values are to be determined from equation (2a) using the weak interaction rates previously calculated (F^2N I–III) for electron and positron capture. In addition, some average modified phase space factor I_e should be used in equation (2a), but that would defeat the purpose of our approach which attempts to ease calculational difficulties in the problem. What we have done is to use modified phase space factors calculated for parent ground-state to daughter ground-state transitions and solved for $\langle ft \rangle_e$. The user must do likewise, except to reverse the procedure in calculating λ from our tables of $\langle ft \rangle_e$ and in using procedures for calculating I factors now to be described.

Using equations (I.3b) and (I.4a), and with some change in notation, it is possible to write

$$I_e = \left(\frac{kT}{m_e c^2}\right)^5 \int_{\eta_e^L}^{\infty} \eta_e^2 (\eta_e + \zeta_n)^2 [1 + \exp(\eta_e - \eta_e^{\mathrm{F}})]^{-1} \{1 - [1 + \exp(\eta_e + \zeta_n - \eta_\nu^{\mathrm{F}})]^{-1}\} d\eta_e \,, \tag{4a}$$

and, with obvious generalization in which η_ν in I_ν is replaced by $\eta_\nu = \eta_e + \zeta_n$, one has

$$I_\nu = \left(\frac{kT}{m_e c^2}\right)^5 \int_{\eta_e^L}^{\infty} \eta_e^2(\eta_e + \zeta_n)^2[1 + \exp(\eta_e + \zeta_n - \eta_\nu^{\mathrm{F}})]^{-1}\{1 - [1 + \exp(\eta_e - \eta_e^{\mathrm{F}})]^{-1}\}d\eta_e\,. \tag{4b}$$

In these equations all symbols under the integral sign are energies in kT units: $\eta_e = W_e/kT$, with W_e = total $e^{\pm}$-energy (rest mass energy plus kinetic); $\eta_\nu = W_\nu/kT$, $\eta_e^{\mathrm{F}} = W_e^{\mathrm{F}}/kT$, and $\eta_\nu^{\mathrm{F}} = W_\nu^{\mathrm{F}}/kT$, where the superscript "F" designates *total* chemical potentials, which in most of our applications reduce to total Fermi energies (kinetic plus rest mass energy). The quantity η_e^L is defined by $\eta_e^L = W_e^L/kT$, where W_e^L is the appropriate lower limit of integration given by $W_e^L = m_e c^2$ when $Q_n + m_e c^2 > 0$ and by $W_e^L = |Q_n|$ when $Q_n + m_e c^2 \leq 0$. Thus for the no threshold case $\eta_e^L = m_e c^2/kT$, while for the threshold case $\eta_e^L = |\zeta_n|$, where $\zeta_n = Q_n/kT$ and we have used $W_\nu = W_e + Q_n$. Note, however, that η_e^{F} and η_ν^{F} represent four independent quantities for e^-, e^+, ν, and $\bar{\nu}$ to be determined from the appropriate number densities in the standard way from the ambient conditions for temperature and density. A useful reference is Bethe, Applegate, and Brown (1980).

It will be clear that the integrals in equations (4a) and (4b) involve Fermi integrals, and by elementary algebraic manipulation we have been able to show

$$I_e = \left(\frac{kT}{m_e c^2}\right)^5 [1 - \exp(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^{\mathrm{F}})]^{-1} I\,, \tag{5a}$$

$$I_\nu = \left(\frac{kT}{m_e c^2}\right)^5 [\exp(\eta_e^{\mathrm{F}} + \zeta_n - \eta_\nu^{\mathrm{F}}) - 1]^{-1} I\,, \tag{5b}$$

$$(I_e - I_\nu) = \left(\frac{kT}{m_e c^2}\right)^5 I\,, \tag{5c}$$

and

$$I_\nu/I_e = \exp(\eta_\nu^{\mathrm{F}} - \eta_e^{\mathrm{F}} - \zeta_n) \rightarrow \exp(\zeta_n) \text{ as } \eta_\nu^{\mathrm{F}} \rightarrow \eta_e^{\mathrm{F}}\,, \tag{5d}$$

where the integral I is given below.

The limit for I_ν/I_e given in equation (5d) as η_ν^{F} approaches η_e^{F} is not usually applicable to situations discussed in this paper. However, it shows that the nondegenerate result for $\eta_\nu^{\mathrm{F}} = \eta_e^{\mathrm{F}} = 0$ is more general than might be anticipated. It is important to note, especially when ζ_n is negative, that $I_\nu = I_e$ for $\eta_\nu^{\mathrm{F}} = \eta_e^{\mathrm{F}} + \zeta_n$. Doing the integral for I in equation (5), we obtain

$$\begin{aligned} I &\equiv \int_{\eta_e^L}^{\infty} \eta_e^2(\zeta_n + \eta_e)^2 \left(\frac{1}{1 + e^{\eta_e - \eta_e^{\mathrm{F}}}} - \frac{1}{1 + e^{\eta_e + \zeta_n - \eta_\nu^{\mathrm{F}}}}\right) d\eta_e \\ &= F_4(\eta_e^{\mathrm{F}} - \eta_e^L) - F_4(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L) + (2\zeta_n + 4\eta_e^L)[F_3(\eta_e^{\mathrm{F}} - \eta_e^L) - F_3(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L)] + [6(\eta_e^L)^2 + 6\eta_e^L\zeta_n + \zeta_n^2] \\ &\quad \times [F_2(\eta_e^{\mathrm{F}} - \eta_e^L) - F_2(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L)] + [4(\eta_e^L)^3 + 6(\eta_e^L)^2\zeta_n + 2\eta_e^L(\zeta_n)^2][F_1(\eta_e^{\mathrm{F}} - \eta_e^L) - F_1(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L)] \\ &\quad + [(\eta_e^L)^4 + 2\zeta_n(\eta_e^L)^3 + (\eta_e^L)^2(\zeta_n)^2][F_0(\eta_e^{\mathrm{F}} - \eta_e^L) - F_0(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L)]\,, \end{aligned} \tag{5e}$$

where we have used the standard definition of relativistic Fermi integrals of order k:

$$F_k(\eta) \equiv \int_0^{\infty} \frac{x^k}{1 + \exp(x - \eta)}\, dx\,. \tag{5f}$$

For conditions in which neutrinos are trapped and thermalized and in near beta equilibrium, the increasing value of η_ν^{F} approaches $\eta_e^{\mathrm{F}} + \zeta_n$, and the expressions for I_e and I_ν in equations (5a), (5b) are undefined. Using l'Hopital's rule and the well-known property of relativistic Fermi integrals, $\partial F_x(\eta)/\partial\eta = kF_{k-1}(\eta)$, we have been able to show that as $\eta_\nu^{\mathrm{F}} \rightarrow \eta_e^{\mathrm{F}} + \zeta_n$,

$$\begin{aligned} I_\nu \rightarrow I_e \rightarrow \left(\frac{kT}{m_e c^2}\right)^5 &\{4F_3(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L) + 3(2\zeta_n + 4\eta_e^L)F_2(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L) \\ &+ 2[6(\eta_e^L)^2 + 6\eta_e^L\zeta_n + \zeta_n^2]F_1(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L) + [4(\eta_e^L)^3 + 6(\eta_e^L)^2\zeta_n + 2\eta_e^L(\zeta_n)^2]F_0(\eta_\nu^{\mathrm{F}} - \zeta_n - \eta_e^L)\}\,. \end{aligned} \tag{5g}$$

The procedure for converting our tables of $\langle ft \rangle_e$ into stellar rates in now clear. The user must have on hand values for W_e^{F} and W_ν^{F}. These are commonly available in the equation-of-state subroutines in the stellar evolution computer programs in which our rates are usually employed. If these total Fermi energies are not readily available and standard approximations are used to compute them from ambient lepton densities and temperatures, then we caution that our tables of $\langle ft \rangle_e$ were computed using accurate values of W_e^{F} from numerical integration, not an approximation. Hence, introduction of an approximate value of W_e^{F} in the reverse procedure may result in an inaccurate and unfaithful calculation of the capture rate from our tables of $\langle ft \rangle_e$. We have used

$$W_\nu^{\mathrm{F}} = \eta_\nu^{\mathrm{F}} kT = -\infty$$

in our calculation of $\langle ft \rangle_e$ from our original rate tables in F^{2}N I, II, and III. Note the important difference in I_ν/I_e from equation (5d) for free streaming neutrinos, $\eta_\nu^{\mathrm{F}} = -\infty$, and for the marginally degenerate case, $\eta_\nu^{\mathrm{F}} \approx 0$.

Over much of the temperature and density conditions encountered during stellar evolution, neutrinos are not thermalized, and freely stream out of the star with interaction mean free paths large compared to relevant stellar dimensions. In this case neutrino phase space is empty, the neutrino occupation probability per state is zero, and $\eta_\nu^{\rm F} = -\infty$. In this case only I_e is relevant, and it reduces to

$$I_e = \left(\frac{kT}{m_e c^2}\right)^5 \{F_4(\tilde{\eta}_e) + (4\eta_e^L + 2\zeta_n)F_3(\tilde{\eta}_e) + [6(\eta_e^L)^2 + 6\eta_e^L\zeta_n + (\zeta_n)^2]F_2(\tilde{\eta}_e) + [4(\eta_e^L)^3 + 6(\eta_e^L)^2\zeta_n + 2\eta_e^L(\zeta_n)^2]F_1(\tilde{\eta}_e) + [(\eta_e^L)^4 + 2(\eta_e^L)^3\zeta_n + (\eta_e^L)^2(\zeta_n)^2]F_0(\tilde{\eta}_e)\}\,, \tag{6a}$$

where $\tilde{\eta}_e \equiv \eta_e^{\rm F} - \eta_e^L$. For the threshold case, $\eta_e^L = |\zeta_n|$, $\zeta_n < -m_e c^2/kT$, $Q_n < -m_e c^2$, one has

$$I_e = \left(\frac{kT}{m_e c^2}\right)^5 [F_4(\tilde{\eta}_e) + 2|\zeta_n|F_3(\tilde{\eta}_e) + \zeta_n^2 F_2(\tilde{\eta}_e)]\,, \tag{6b}$$

where $\tilde{\eta}_e = (W_e^{\rm F} - |Q_n|)/kT$. For the no threshold case, $W_e^L = m_e c^2$, and $\tilde{\eta}_e = U_e^{\rm F}/kT$ is just the plasma degeneracy parameter, where we define $U_e^{\rm F} \equiv W_e^{\rm F} - m_e c^2$. For the threshold case, $W_e^L = |Q_n|$, and $\tilde{\eta}_e$ represents an effective plasma degeneracy parameter for electron (positron) capture; where a threshold exists, the effective degeneracy parameter is less than that for the plasma, reflecting the smaller number of electrons (positrons) in the Fermi-Dirac distributions with energy sufficient to overcome the energy barrier.

Equally as important as electron and positron capture rates are the associated rates of neutrino energy emission. The rate of neutrino energy emission in units of $m_e c^2 s^{-1}$ is defined

$$\pi_\nu = \ln 2\,\frac{J_e^\nu}{\langle ft\rangle_e}\,, \tag{7a}$$

where, as before, we factor an average value of G_e out of the neutrino emission phase space factors (eq. [I.6]) to obtain $J_e^\nu = f_e^\nu/\langle G_e\rangle$. In analogy with equation (4a),

$$J_e^\nu = \left(\frac{kT}{m_e c^2}\right)^6 \int_{\eta_e^L}^\infty \eta_e^2(\zeta_n + \eta_e)^3[1 + \exp(\eta_e - \eta_e^{\rm F})]^{-1}\{1 - [1 + \exp(\eta_e + \zeta_n - \eta_\nu^{\rm F})]^{-1}\}d\eta_e\,, \tag{7b}$$

and

$$J_e^\nu = \left(\frac{kT}{m_e c^2}\right)^6 \frac{1}{1 - \exp(\eta_\nu^{\rm F} - \zeta_n - \eta_e^{\rm F})}\,J^\nu\,, \tag{7c}$$

with

$$\begin{aligned} J^\nu = {} & \int_{\eta_e^L}^\infty \eta_e^2(\zeta_n + \eta_e)^3 \left\{\frac{1}{1 + e^{\eta_e - \eta_e^{\rm F}}} - \frac{1}{1 + e^{\eta_e + \zeta_n - \eta_\nu^{\rm F}}}\right\}d\eta_e \\ = {} & F_5(\eta_e^{\rm F} - \eta_e^L) - F_5(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n) + (5\eta_e^L + 3\zeta_n)[F_4(\eta_e^{\rm F} - \eta_e^L) - F_4(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n)] \\ & + [10(\eta_e^L)^2 + 12\eta_e^L\zeta_n + 3(\zeta_n)^2][F_3(\eta_e^{\rm F} - \eta_e^L) - F_3(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n)] \\ & + [10(\eta_e^L)^3 + 18(\eta_e^L)^2\zeta_n + 9\eta_e^L\zeta_n^2 + \zeta_n^3][F_2(\eta_e^{\rm F} - \eta_e^L) - F_2(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n)] \\ & + [5(\eta_e^L)^4 + 12(\eta_e^L)^3\zeta_n + 9(\eta_e^L)^2\zeta_n^2 + 2\eta_e^L\zeta_n^3][F_1(\eta_e^{\rm F} - \eta_e^L) - F_1(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n)] \\ & + [(\eta_e^L)^5 + 3(\eta_e^L)^4\zeta_n + 3(\eta_e^L)^3\zeta_n^2 + (\eta_e^L)^2\zeta_n^3][F_0(\eta_e^{\rm F} - \eta_e^L) - F_0(\eta_\nu^{\rm F} - \eta_e^L - \zeta_n)]\,. \end{aligned} \tag{7d}$$

The limit as $\eta_\nu^{\rm F} \to \eta_e^{\rm F} + \zeta_n$ follows in obvious fashion as in equation (5g).

In direct analogy with the calculation of stellar lepton capture rates, the rates of stellar neutrino energy emission can be computed by interpolating the appropriate value of $\langle ft\rangle_e$ and evaluating equations (7a)–(7d) with appropriate values of $W_e^{\rm F}$ and $W_\nu^{\rm F}$. We caution readers that where neutrinos are partially trapped and thermalized, so that the full machinery of equation (7d) is used, the neutrino energy emission rate is not equivalent to the energy taken away by neutrinos, the neutrino energy loss rate. In that case the energy loss rate from a particular zone of a star must be computed from a detailed treatment of neutrino transport. In most applications of equations (7a)–(7d) the free streaming neutrino limit, $\eta_\nu^{\rm F} = -\infty$, will be employed, with obvious generalization of equations (6a) and (6b).

We provide an alternative and somewhat more physically illuminating method for computing the neutrino energy emission rate. Just as $\langle ft\rangle_e$ is tabulated at each of our temperature and density points, we also tabulate the average neutrino emission energy

$$\langle\epsilon_\nu\rangle \equiv \frac{\pi_\nu}{\lambda_{Z+1}} \quad \text{and} \quad \langle\epsilon_{\bar\nu}\rangle \equiv \frac{\pi_{\bar\nu}}{\lambda_{Z-1}}\,, \tag{8}$$

expressed in MeV. Since the energy emission rate closely tracks the rate of electron or positron capture, the value of $\langle\epsilon_\nu\rangle$ or $\langle\epsilon_{\bar\nu}\rangle$ is slowly varying with temperature and density, facilitating accurate interpolation. The interpolated value of $\langle\epsilon_\nu\rangle(\langle\epsilon_{\bar\nu}\rangle)$ is multiplied by the appropriate electron (positron) capture rate (computed from an interpolated $\langle ft\rangle_e$ and eqs. [2a] and [5a] plus [5d]) to obtain the neutrino (antineutrino) energy emission rate.

Note that $\pi_\nu(\pi_{\bar\nu})$ is the total neutrino (antineutrino) energy emission rate from both electron capture and β^+ decay (positron capture and β^- decay), while $\langle\epsilon_\nu\rangle(\langle\epsilon_{\bar\nu}\rangle)$ in equation (8) above involves only the capture processes. At fairly low temperature and density, $\pi_\nu(\pi_{\bar\nu})$ will be dominated by positron emission (electron emission), and $\langle\epsilon_\nu\rangle(\langle\epsilon_{\bar\nu}\rangle)$ will not reflect the average lepton capture neutrino emission energy. However, multiplying $\langle \epsilon_\nu \rangle (\langle \epsilon_{\bar{\nu}} \rangle)$ by the electron (positron) capture rate will still result in the correct total neutrino (antineutrino) energy emission rate. Where lepton capture rates dominate, $\langle \epsilon_\nu \rangle (\langle \epsilon_{\bar{\nu}} \rangle)$ is indeed the average capture $\nu(\bar{\nu})$ energy.

Evaluation of equations (5b), (5e), (6), and (7d) for the purpose of computing a stellar lepton capture rate or neutrino energy emission rate from an interpolated value of $\langle ft \rangle_e$ now requires calculations of the relativistic Fermi integrals for appropriate values of η_e^F, η_ν^F, η_e^L, and ζ_n.

III. EXPRESSIONS FOR RELATIVISTIC FERMI INTEGRALS

There exist standard expansions for the relativistic Fermi integrals $F_k(\eta)$ defined in equation (5f). For $\eta < 0$,

$$F_k(\eta) = (k!)e^\eta \sum_{i=0}^{\infty} (-e^\eta)^i/(i+1)^{k+1} \approx (k!)e^\eta(1 - e^\eta/2^{k+1} + \cdots) \sim (k!)e^\eta , \tag{9a}$$

(see Chiu 1968), and for $\eta > 1$ the Sommerfeld expansion gives

$$F_k(\eta) = \frac{\eta^{k+1}}{k+1}\left(\sum_{i=0}^{\infty} a_{2i}\eta^{-2i}\right) , \tag{9b}$$

where

$$a_0 = 1 , \quad a_2 = \frac{\pi^2}{6} k(k+1) , \quad a_4 = \frac{7\pi^4}{360}(k-2)(k-1)(k)(k+1) ,$$

$$a_6 = \frac{31\pi^2}{15120}(k-4)(k-3)(k-2)(k-1)(k)(k+1) ,$$

and so on. For the computational procedure outlined in this paper we must find approximate expressions for the $F_k(\eta)$ which are continuous and convergent across $\eta = 0$, and which have continuous derivatives in η across $\eta = 0$. In addition, these approximate expressions must be algebraically simple, since they may be evaluated many times in the inner loop of stellar evolution computer codes.

Note that the Sommerfeld expansion of $F_k(\eta)$ for $\eta > 1$ is known to be slowly convergent for η near unity, and not convergent at all for $0 \leq \eta \leq 1$. By contrast, the nondegenerate expansion, equation (9a), is very rapidly convergent, even near $\eta = 0$. The procedure we follow here is to exploit the rapid convergence of equation (9a) for $\eta < 0$ by using recursion relations for the sum or difference of $F_k(\eta)$ and $F_k(-\eta)$ to define an approximation for $\eta \geq 0$.

We follow Bludman and van Riper (1978) and note that the differential recursion relation $dF_k(\eta)/d\eta = kF_{k-1}(\eta)$ can be written

$$F_k(\eta) = F_k(0) + k\int_0^\eta F_{k-1}(\eta')d\eta' ,$$

$$F_k(-\eta) = F_k(0) + k\int_0^{-\eta} F_{k-1}(\eta')d\eta' . \tag{10a}$$

Since $F_0(\eta)$ can be integrated exactly to give $F_0(\eta) = \ln(1 + e^\eta) = \eta + \ln(1 + e^{-\eta})$, we have

$$F_0(\eta) - F_0(-\eta) = \eta , \tag{10b}$$

and, using equation (10a), we find

$$F_1(\eta) + F_1(-\eta) = 2F_1(0) + \tfrac{1}{2}\eta^2 . \tag{10c}$$

Noting that $F_k(0) = (1 - 2^{-k})(k!)\zeta(k+1) = \pi^2/12$ $(k = 1)$, $3(1.202)/2$ $(k = 2)$, $7\pi^4/120$ $(k = 3)$, $45(1.0369)/2$ $(k = 4)$, and $= \pi^6/645$ $(k = 5)$, we can continue to integrate equation (10c) to obtain the recursion relations for $k = 1, 2, 3$ (Bludman and van Riper 1978) and for $k = 4, 5$:

$$F_1(\eta) + F_1(-\eta) = \frac{\pi^2}{6} + \frac{1}{2}\eta^2 , \tag{11a}$$

$$F_2(\eta) - F_2(-\eta) = \frac{\pi^3}{3}\eta + \frac{1}{3}\eta^3 , \tag{11b}$$

$$F_3(\eta) + F_3(-\eta) = \frac{7\pi^4}{60} + \frac{\pi^2}{2}\eta^2 + \frac{1}{4}\eta^4 , \tag{11c}$$

$$F_4(\eta) - F_4(-\eta) = \frac{28\pi^4}{60}\eta + \frac{2}{3}\pi^2\eta^3 + \frac{\eta^5}{5} , \tag{11d}$$

$$F_5(\eta) + F_5(-\eta) = \frac{31\pi^6}{126} + \frac{7\pi^2}{6}\eta^2 + \frac{5}{6}\pi^2\eta^4 + \frac{\eta^6}{6} . \tag{11e}$$

For $\eta \le 0$ we adopt the leading term in equation (9a) for our approximation:

$$F_k(\eta) \approx F_k{}^{<}(\eta) = k!\,e^{\eta} \qquad \eta \le 0 \,. \tag{12}$$

In the $\eta > 0$ case we use the recursion relations (11a)–(11e) to define an approximation for $F_k(\eta)$ in terms of $F_k{}^{<}(\eta)$, equation (12). For example, if $\eta > 0$ and $k = 1$, then

$$F_1(\eta) = \frac{\pi^2}{6} + \frac{1}{2}\eta^2 - F_1(-\eta) \approx \frac{\pi^2}{6} + \frac{1}{2}\eta^2 - e^{-\eta} \equiv F_1{}^{>}(\eta) \qquad \eta > 0 \,, \tag{13a}$$

whereas for $\eta \le 0$,

$$F_1(\eta) \approx e^{\eta} \equiv F_1{}^{<}(\eta) \qquad \eta \le 0 \,. \tag{13b}$$

We require that these expressions and their derivatives be continuous at $\eta = 0$. Note that while the derivatives are continuous,

$$\left.\frac{dF_1{}^{>}}{d\eta}\right|_{\eta=0} = 1 = \left.\frac{dF_1{}^{<}}{d\eta}\right|_{\eta=0} \,,$$

the functions themselves are not,

$$F_1{}^{>}(0) = \frac{\pi^2}{6} - 1 \quad \text{and} \quad F_1{}^{<}(0) = 1 \,.$$

If we approximate the $\pi^2/6(\approx 1.645)$ by 2.0 in equation (13a), we find $F_1{}^{>}(0) = F_1{}^{<}(0)$, ensuring continuity in both the function and its derivative at $\eta = 0$. For the relativistic Fermi integrals of rank 2 we adopt for $\eta \le 0$

$$F_2(\eta) \approx F_2{}^{<}(\eta) = 2e^{\eta} \qquad \eta \le 0 \,, \tag{14a}$$

so that for $\eta > 0$ equation (11b) implies

$$F_2(\eta) \approx F_2{}^{>}(\eta) = \frac{\pi^2}{3}\eta + \frac{1}{3}\eta^3 + 2e^{-\eta} \qquad \eta > 0 \,. \tag{14b}$$

Note that these expressions for $F_2(\eta)$ are continuous at $\eta = 0$, but their first derivatives are not. By approximating the coefficient of the linear term in η in equation (14b) $\pi^2/3$ (≈ 3.289) by 4.0, both equations (14a) and (14b) and their derivatives are continuous across $\eta = 0$. We have continued this procedure for $k = 3, 4, 5$ to obtain the following approximations for $F_k(\eta)$:

$$F_0(\eta) = \ln(1 + e^{\eta}) \approx \begin{cases} e^{\eta} & \text{for } \eta \ll 0 \\ \eta & \text{for } \eta \gg 0 \end{cases} \,, \tag{15a}$$

$$F_1(\eta) \approx \begin{cases} e^{\eta} \\ \dfrac{\eta^2}{2} + 2 - e^{-\eta} \end{cases} \,, \tag{15b}$$

$$F_2(\eta) \approx \begin{cases} 2e^{\eta} \\ \dfrac{\eta^3}{3} + 4\eta + 2e^{-\eta} \end{cases} \,, \tag{15c}$$

$$F_3(\eta) \approx \begin{cases} 6e^{\eta} \\ \dfrac{\eta^4}{4} + \dfrac{\pi^2}{2}\eta^2 + 12 - 6e^{-\eta} \end{cases} \,, \tag{15d}$$

$$F_4(\eta) \approx \begin{cases} 24e^{\eta} \\ \dfrac{\eta^5}{5} + \dfrac{2}{3}\pi^2\eta^3 + 48\eta + 24e^{-\eta} \end{cases} \,, \tag{15e}$$

$$F_5(\eta) \approx \begin{cases} 120e^{\eta} \\ \dfrac{\eta^6}{6} + \dfrac{5}{6}\pi^2\eta^4 + \dfrac{7\pi^2}{6}\eta^2 + 240 - 120e^{-\eta} \end{cases} \,, \tag{15f}$$

where equation (15a) is exact and in equations (15b)–(15f) the upper expressions are for $\eta \le 0$ and the lower expressions are for $\eta \ge 0$. These are the expressions for $F_k(\eta)$ we have used in equations (5a) plus (5e) and (2a) to tabulate $\langle ft \rangle_e$ from our previous rate calculations. As a consequence, the user must employ these expressions for $F_k(\eta)$ in a reconstruction of $\lambda_{Z\pm1}$ from $\langle ft \rangle_e$ in order to avoid inaccuracies.

The approximations (15b)–(15f) for $F_1(\eta)$ through $F_5(\eta)$ are asymptotically exact for $|\eta| \gg 0$. The largest error between the exact F_k

and the equation (15) approximation is for $0 \leq \eta \leq 1$. In the approximation for $F_1(\eta)$, the largest error occurs near $\eta = 0.4$ where the approximation is $\sim 24\%$ larger than the exact F_1. The approximation for F_2 has its largest error ($\sim 17\%$ high) near $\eta = 1$. The approximation for F_3 has its largest error ($\sim 6\%$) near $\eta = 0$, while that for F_4 has its largest error ($\sim 4\%$) near $\eta = 0.8$. The expressions for F_5 differs from the exact value by no more than a few percent near $\eta = 0$. In all cases equation (15b)–(15f) approximations accurately track the temperature and density dependence of the exact Fermi integrals over many orders of magnitude. Equation (15d) for $F_4(\eta)$, for instance, gives $F_4(-10.00) \approx 1.090 \times 10^{-3}$ ("exact" numerical integration differs by less than 0.02%); $F_4(-2) \approx 3.248$ (exact $F_4 = 3.235$); $F_4(+0.4) \approx 35.71$ (exact $F_4 \approx 34.38$); $F_4(+10.00) \approx 2.706 \times 10^4$ (exact $F_4 \approx 2.703 \times 10^4$).

IV. EFFECTIVE *ft*-VALUES

The weak interaction transition rates of a nucleus in the stellar interior are, in general, sensitive functions of the lepton distribution functions and the nuclear level structure. The reader is referred to the earlier papers in this series (F^2N I, II, III) for an in depth discussion of these points. The electron and positron continuum capture rates are extremely sensitive functions of temperature and density. It is difficult to interpolate in the F^2N III rate tables and tapes to find, for instance, electron capture rates at high density to a level of accuracy commensurate with that of the nuclear data which went into the calculations of the rate tables. We have presented above a method to remove most of the temperature and density variation of the lepton capture rates and to yield tables of slowly varying effective $\log \langle ft \rangle_e$–values, in which it is considerably easier to interpolate accurately. As outlined above, the similarly rapid temperature and density dependence of the neutrino energy emission rate tables is removed by tabulating average neutrino (antineutrino) energy per decay, $\langle \epsilon_\nu \rangle (\langle \epsilon_{\bar{\nu}} \rangle)$, instead of energy emission rate.

Where they are important, the electron and positron emission rates of the nuclei considered here are relatively slowly varying, so that rates may be accurately interpolated in temperature and density. Consider the electron decay rate of the transition ^{56}Mn $(e^- \bar{\nu})^{56}$Fe + 4.2064 MeV. The log base 10 of this rate at $\rho = 10^6$ g cm^{-3} and $T_9 = 0.5$ is $\log \lambda_{\beta^-} = -2.498$, while for the same temperature and a density of $\rho = 10^8$ g cm^{-3}, the value is $\log \lambda_{\beta^-} = -3.206$. There is quite a slow variation in density until the electron Fermi energy becomes of the order of the electron emission Q-value for this decay, $Q_n \approx 4.2064$ MeV, at which point most of the lower energy transitions in ^{56}Mn$(e^- \bar{\nu})^{56}$Fe will be Pauli blocked. At $\rho = 10^9$ g cm^{-3} the electron total Fermi energy is $W_e^{\rm F} = 5.176$ MeV $> Q_n$ and $\log \lambda_{\beta^-} = -10.766$, a relatively large decrease from the rate at $\rho = 10^8$ g cm^{-3}. Note, however, that the rate is very small here and relatively unimportant compared to the reverse rate of continuum electron capture $\log \lambda_{e^-} = -2.628$. Whenever Pauli blocking of electron emission at high density occurs and it is difficult to interpolate in our tables, the rates are very small, and the inverse capture rates are dominant.

Effective $\log \langle ft \rangle_e$ values can be defined for continuum electron and positron capture as outlined above using equation (2a) to write

$$\log \langle ft \rangle_e \equiv \log(\ln 2) + \log I_e - \log \lambda_{Z \pm 1} + \log \alpha \,, \tag{16}$$

where $\lambda_{Z \pm 1}$ is taken from our original rate tables and I_e is evaluated as explained in the last two sections. In the calculation of $\langle ft \rangle_e$ from the F^2N rate tables the quenching factor, α, has been set equal to 1. Table 1 presents values of $\log \langle ft \rangle_e$ for continuum electron and positron capture for some selected example nuclei on an abbreviated grid of temperature and density. Table 1 also contains the logarithms of the average neutrino and antineutrino energies $\langle \epsilon_\nu \rangle$ and $\langle \epsilon_{\bar{\nu}} \rangle$ for each decay pair. These are designated by $\log \langle \nu \rangle$ and $\log \langle \bar{\nu} \rangle$, respectively. Similar information is available for the full range of nuclei and temperatures and densities treated in F^2N I, II, III in computer readable form upon request.

Where dashes appear in Table 1 in lieu of numerical entries the logarithm of the corresponding rates is less than -99.999. For these cases we have not computed values for $\log \langle ft \rangle_{e^-}$, $\log \langle ft \rangle_{e^+}$, $\log \langle \nu \rangle$, or $\log \langle \bar{\nu} \rangle$. Estimates for these extremely low rates can be obtained by reasonable extrapolation of adjacent entries in the tables.

It is well known that only roughly one-half of the shell model Gamow-Teller strength is seen in high energy (p, n) experiments (see Bloom and Goodman 1982). This quenching factor is, as yet, not well determined, but we have included α in equation (16) as a simple means of taking account of quenching. At relatively lower temperatures and densities ($kT < 1$ MeV, $\rho Y_e \lesssim 10^9$), where our rate calculations are dominated by experimentally determined information, $\alpha = 1$ should be employed. At higher temperatures and densities where the Gamow-Teller resonances dominate, the user is encouraged to select a value of α according to his personal prejudice, remembering that α is identical to 1 for free nucleons. However, it should be kept in mind that the F^2N II calculation of the resonance energies places the strength slightly higher than more sophisticated shell model calculation (Bloom and Fuller 1984), implying that the shell model strength is already slightly quenched. All measurements of Gamow-Teller strength to date employ (p, n) reactions and thus measure $T^> \to T^<$ transitions, whereas most of the electron capture transitions treated in F^2N are $T^< \to T^>$ transitions. We refer the reader to Ajzenberg-Selove *et al.* (1984) for an experimental discussion of quenching in this direction.

In the standard physics of nuclear β-decay the *ft*–value is an inverse measure of the matrix element for a particular transition between individual states of the parent and daughter nuclei. Equations (I.2a) and (I.2b) give the relation between the *ft*-value and the Fermi and Gamow-Teller matrix elements. The effective $\langle ft \rangle_e$ values defined here are inverse measures of the *total* matrix element strength contributing to the *total* transition rate between parent and daughter, representing the contributions of transitions involving many nuclear states of both nuclei.

Thus many transitions may contribute in such a weak decay; for example, electron capture on thermally populated parent states going to highly excited states of the daughter. These transitions each have, in general, a different Q-value, yet the phase space factor I_e used in equation (16) employs only the ground-state to ground-state Q-value. In a sense $\langle ft \rangle_e$ is then only an accurate measure of the total matrix element strength available to the extent that the important transitions have Q-values similar to the ground-state to ground-state Q-value.

TABLE 1

CONTINUUM ELECTRON AND POSITRON CAPTURE DATA

T_9 \ $\log \rho/\mu_e$	$p \longrightarrow n$: 6	7	8	9	10	11	$n \longrightarrow p$: 6	7	8	9	10	11
	$\log \beta^+$						$\log \beta^-$					
0.4	-	-	-	-	-	-	-3.137	-4.997	-20.216	-54.694	-	-
1	-	-	-	-	-	-	-3.105	-4.303	-10.400	-24.209	-54.136	-
1.5	-	-	-	-	-	-	-3.077	-3.962	-8.026	-17.258	-37.226	-80.287
2	-	-	-	-	-	-	-3.054	-3.734	-6.779	-13.722	-28.711	-61.020
3	-	-	-	-	-	-	-3.029	-3.457	-5.468	-10.138	-20.157	-41.704
5	-	-	-	-	-	-	-3.043	-3.215	-4.345	-7.217	-13.266	-26.215
10	-	-	-	-	-	-	-3.112	-3.139	-3.469	-4.926	-8.041	-14.560
30	-	-	-	-	-	-	-3.204	-3.205	-3.219	-3.375	-4.384	-6.695
100	-	-	-	-	-	-	-3.247	-3.247	-3.247	-3.251	-3.294	-3.737
	$\log \langle ft \rangle_{e^-}$						$\log \langle ft \rangle_{e^+}$					
0.4	3.051	3.058	3.037	3.028	3.026	3.025	3.528	3.527	3.528	3.526	-	-
1	3.050	3.062	3.038	3.028	3.025	3.025	3.324	3.327	3.328	3.331	3.335	-
1.5	3.046	3.062	3.040	3.028	3.026	3.025	3.243	3.243	3.243	3.247	3.251	3.258
2	3.043	3.055	3.042	3.028	3.025	3.025	3.191	3.193	3.192	3.194	3.197	3.202
3	3.040	3.048	3.045	3.028	3.026	3.025	3.131	3.131	3.130	3.131	3.133	3.137
5	3.034	3.040	3.052	3.028	3.026	3.025	3.087	3.080	3.077	3.078	3.079	3.081
10	3.034	3.034	3.051	3.033	3.026	3.025	3.061	3.059	3.049	3.046	3.046	3.048
30	3.035	3.035	3.036	3.044	3.034	3.026	3.051	3.051	3.050	3.044	3.037	3.037
100	3.037	3.037	3.037	3.038	3.039	3.042	3.048	3.048	3.048	3.048	3.046	3.038
	$\log \langle \nu \rangle$						$\log \langle \bar{\nu} \rangle$					
0.4	-0.963	-0.960	-0.036	0.500	0.908	1.273	-0.399	-0.924	-1.013	-1.014	-	-
1	-0.537	-0.530	-0.015	0.502	0.908	1.273	-0.369	-0.619	-0.671	-0.671	-0.670	-
1.5	-0.343	-0.335	0.015	0.506	0.909	1.274	-0.350	-0.506	-0.554	-0.554	-0.554	-0.554
2	-0.203	-0.196	0.052	0.509	0.909	1.273	-0.317	-0.440	-0.487	-0.487	-0.487	-0.488
3	-0.004	0.002	0.136	0.521	0.912	1.274	0.096	-0.217	-0.305	-0.306	-0.307	-0.306
5	0.247	0.250	0.301	0.558	0.919	1.275	0.485	0.447	0.381	0.375	0.375	0.375
10	0.584	0.585	0.594	0.686	0.948	1.282	0.719	0.717	0.712	0.710	0.710	0.710
30	1.096	1.096	1.096	1.101	1.151	1.339	1.142	1.142	1.141	1.138	1.135	1.135
100	1.634	1.634	1.634	1.634	1.634	1.650	1.647	1.647	1.647	1.647	1.646	1.643

T_9 \ $\log \rho/\mu_e$	$^{48}Ca \longrightarrow {}^{48}K$: 6	7	8	9	10	11	$^{48}K \longrightarrow {}^{48}Ca$: 6	7	8	9	10	11
	$\log \beta^+$						$\log \beta^-$					
0.4	-	-	-	-	-	-	-0.878	-0.899	-1.050	-2.104	-38.373	-
1	-89.496	-89.496	-89.496	-89.496	-89.496	-89.496	-0.630	-0.646	-0.760	-1.541	-18.730	-83.322
1.5	-59.098	-59.098	-59.098	-59.098	-59.098	-59.098	-0.504	-0.518	-0.620	-1.339	-14.057	-57.118
2	-43.876	-43.876	-43.876	-43.876	-43.876	-43.876	-0.442	-0.455	-0.553	-1.245	-11.435	-43.743
3	-28.639	-28.639	-28.639	-28.639	-28.639	-28.639	-0.387	-0.399	-0.491	-1.150	-8.503	-30.050
5	-16.440	-16.438	-16.437	-16.437	-16.437	-16.437	-0.351	-0.360	-0.442	-1.045	-5.820	-18.770
10	-7.413	-7.409	-7.394	-7.389	-7.389	-7.389	-0.341	-0.344	-0.393	-0.831	-3.383	-9.902
30	-1.489	-1.489	-1.487	-1.475	-1.460	-1.459	-0.411	-0.412	-0.418	-0.497	-1.236	-3.493
100	1.839	1.839	1.840	1.841	1.858	1.927	-0.524	-0.524	-0.524	-0.527	-0.563	-0.958
	$\log \langle ft \rangle_{e^-}$						$\log \langle ft \rangle_{e^+}$					
0.4	-	-	-	4.556	32.122	53.841	6.387	6.386	6.387	6.385	-	-
1	4.267	4.265	4.270	4.269	14.757	24.851	5.879	5.882	5.883	5.886	5.890	-
1.5	4.091	4.092	4.096	4.092	10.945	18.241	5.671	5.671	5.671	5.675	5.678	5.685
2	3.976	3.975	3.980	3.980	8.900	14.725	5.548	5.549	5.548	5.550	5.553	5.558
3	3.840	3.839	3.840	3.846	6.820	11.023	5.409	5.409	5.409	5.410	5.412	5.416
5	3.708	3.709	3.709	3.712	5.207	7.929	5.287	5.281	5.280	5.280	5.281	5.283
10	3.651	3.651	3.655	3.677	4.177	5.571	5.165	5.161	5.149	5.145	5.146	5.147
30	3.724	3.724	3.725	3.742	3.916	4.452	4.546	4.546	4.545	4.542	4.537	4.537
100	3.201	3.201	3.201	3.201	3.206	3.256	3.169	3.169	3.169	3.169	3.168	3.166
	$\log \langle \nu \rangle$						$\log \langle \bar{\nu} \rangle$					
0.4	-	-	-	-0.961	0.278	1.088	0.481	0.469	0.404	0.294	-0.989	-
1	-0.576	-0.576	-0.575	-0.573	0.286	1.089	0.529	0.520	0.470	0.284	-0.596	-0.596
1.5	-0.395	-0.395	-0.394	-0.394	0.325	1.096	0.545	0.536	0.489	0.285	-0.425	-0.425
2	-0.265	-0.265	-0.265	-0.263	0.358	1.102	0.550	0.542	0.497	0.290	-0.305	-0.306
3	-0.081	-0.081	-0.081	-0.078	0.402	1.108	0.555	0.548	0.505	0.303	-0.140	-0.141
5	0.158	0.158	0.158	0.160	0.480	1.117	0.559	0.553	0.516	0.340	0.057	0.058
10	0.532	0.533	0.530	0.511	0.654	1.138	0.608	0.599	0.549	0.429	0.284	0.285
30	1.201	1.200	1.201	1.210	1.253	1.381	1.105	1.105	1.104	1.094	1.050	1.039
100	1.725	1.725	1.725	1.725	1.727	1.746	1.566	1.566	1.566	1.565	1.565	1.564

TABLE 1—*Continued*

$^{48}Sc \rightarrow {}^{48}Ca$

T_9 \ $\log \rho/\mu_e$	6	7	8	9	10	11
			$\log \beta^+$			
0.4	-36.749	-36.749	-36.749	-36.749	-36.749	-36.749
1	-17.810	-17.810	-17.810	-17.810	-17.810	-17.810
1.5	-13.655	-13.655	-13.655	-13.655	-13.655	-13.655
2	-11.600	-11.600	-11.600	-11.600	-11.600	-11.600
3	-9.576	-9.573	-9.573	-9.573	-9.573	-9.573
5	-7.530	-7.520	-7.517	-7.517	-7.517	-7.517
10	-4.374	-4.370	-4.356	-4.352	-4.352	-4.352
30	-0.171	-0.171	-0.168	-0.147	-0.124	-0.122
100	1.816	1.816	1.816	1.819	1.843	1.947
			$\log \langle ft \rangle_{e^-}$			
0.4	32.406	35.081	35.686	11.767	7.116	3.801
1	15.156	16.280	17.163	9.284	6.075	3.795
1.5	11.607	12.277	13.023	8.431	5.842	3.791
2	9.937	10.369	10.978	7.929	5.724	3.789
3	8.393	8.596	8.950	7.311	5.591	3.784
5	7.193	7.232	7.284	6.557	5.414	3.777
10	5.799	5.801	5.807	5.642	5.007	3.756
30	3.748	3.748	3.749	3.760	3.751	3.546
100	2.555	2.555	2.556	2.555	2.559	2.572
			$\log \langle \nu \rangle$			
0.4	0.296	0.426	0.148	-0.837	0.674	1.000
1	0.307	0.432	0.536	-0.296	0.676	1.002
1.5	0.321	0.441	0.542	-0.150	0.680	1.005
2	0.340	0.452	0.539	-0.070	0.682	1.007
3	0.388	0.476	0.518	0.073	0.688	1.010
5	0.471	0.484	0.465	0.308	0.694	1.016
10	0.642	0.646	0.652	0.624	0.719	1.037
30	1.185	1.185	1.187	1.196	1.212	1.257
100	1.689	1.689	1.689	1.689	1.690	1.707

$^{48}Ca \rightarrow {}^{48}Sc$

T_9	6	7	8	9	10	11
			$\log \beta^-$			
0.4	-49.175	-49.243	-49.806	-62.019	-	-
1	-20.199	-20.266	-20.820	-26.570	-53.820	-
1.5	-13.731	-13.795	-14.318	-18.399	-35.715	-78.542
2	-10.436	-10.493	-10.959	-14.108	-26.462	-58.683
3	-7.005	-7.048	-7.423	-9.778	-17.210	-38.644
5	-4.112	-4.136	-4.423	-5.959	-9.474	-22.344
10	-1.493	-1.496	-1.557	-1.860	-3.364	-9.787
30	0.993	0.993	0.989	0.942	0.419	-1.705
100	1.645	1.645	1.645	1.642	1.611	1.253
			$\log \langle ft \rangle_{e^+}$			
0.4	21.813	21.811	21.812	21.811	-	-
1	11.913	11.916	11.917	11.919	11.924	-
1.5	9.569	9.569	9.570	9.574	9.577	9.584
2	8.370	8.370	8.370	8.372	8.374	8.380
3	7.132	7.135	7.134	7.135	7.137	7.141
5	6.041	6.041	6.042	6.042	6.043	6.045
10	4.641	4.638	4.628	4.625	4.625	4.626
30	3.528	3.528	3.527	3.520	3.514	3.514
100	2.718	2.718	2.718	2.717	2.717	2.712
			$\log \langle \bar{\nu} \rangle$			
0.4	-0.969	-0.970	0.027	-0.988	-	-
1	-0.541	0.059	0.039	-0.589	-0.448	-
1.5	-0.201	0.222	0.077	-0.419	-0.330	-0.421
2	0.075	0.262	0.136	-0.311	-0.252	-0.300
3	0.263	0.310	0.213	-0.148	-0.084	-0.131
5	0.362	0.351	0.273	0.090	0.108	0.075
10	0.689	0.685	0.657	0.638	0.365	0.332
30	1.090	1.090	1.085	1.028	0.811	0.738
100	1.603	1.603	1.603	1.603	1.602	1.599

$^{56}Fe \rightarrow {}^{56}Mn$

T_9 \ $\log \rho/\mu_e$	6	7	8	9	10	11
			$\log \beta^+$			
0.4	-	-	-	-	-	-
1	-55.626	-55.626	-55.626	-55.626	-55.626	-55.626
1.5	-36.301	-36.301	-36.301	-36.301	-36.301	-36.301
2	-26.613	-26.613	-26.613	-26.613	-26.613	-26.613
3	-16.930	-16.930	-16.930	-16.930	-16.930	-16.930
5	-9.272	-9.271	-9.270	-9.270	-9.270	-9.270
10	-3.845	-3.842	-3.833	-3.830	-3.829	-3.829
30	0.200	0.200	0.202	0.215	0.229	0.230
100	2.633	2.633	2.633	2.635	2.653	2.728
			$\log \langle ft \rangle_{e^-}$			
0.4	4.124	4.216	4.468	4.851	3.221	2.554
1	3.722	3.794	4.050	4.844	3.219	2.554
1.5	3.495	3.557	3.832	4.759	3.215	2.554
2	3.338	3.382	3.648	4.538	3.207	2.553
3	3.146	3.166	3.363	4.122	3.179	2.550
5	3.020	3.027	3.115	3.673	3.110	2.541
10	3.057	3.058	3.072	3.227	2.963	2.528
30	2.591	2.591	2.592	2.597	2.592	2.474
100	2.145	2.145	2.145	2.146	2.148	2.164
			$\log \langle \nu \rangle$			
0.4	-0.964	-0.967	-0.977	-0.170	0.413	1.109
1	-0.538	-0.536	-0.559	-0.136	0.414	1.109
1.5	-0.346	-0.338	-0.353	-0.053	0.420	1.109
2	-0.208	-0.197	-0.192	0.049	0.430	1.110
3	-0.012	-0.004	0.027	0.171	0.462	1.112
5	0.238	0.241	0.271	0.383	0.533	1.116
10	0.569	0.569	0.574	0.620	0.688	1.131
30	1.082	1.082	1.083	1.084	1.094	1.243
100	1.668	1.668	1.669	1.668	1.670	1.683

$^{56}Mn \rightarrow {}^{56}Fe$

T_9	6	7	8	9	10	11
			$\log \beta^-$			
0.4	-2.782	-2.881	-3.570	-19.398	-91.370	-
1	-2.498	-2.589	-3.206	-10.766	-38.047	-
1.5	-2.420	-2.509	-3.103	-8.616	-26.033	-69.093
2	-2.375	-2.460	-3.032	-7.484	-19.936	-52.243
3	-2.308	-2.381	-2.899	-5.884	-13.693	-35.240
5	-2.160	-2.198	-2.533	-3.621	-8.370	-21.339
10	-0.860	-0.864	-0.898	-1.268	-3.747	-10.299
30	0.981	0.981	0.975	0.903	0.199	-2.055
100	1.628	1.628	1.628	1.625	1.590	1.201
			$\log \langle ft \rangle_{e^+}$			
0.4	6.308	6.307	6.308	6.306	-	-
1	5.689	5.692	5.693	5.696	5.700	-
1.5	5.465	5.464	5.465	5.468	5.472	5.479
2	5.320	5.322	5.320	5.323	5.326	5.331
3	5.132	5.134	5.132	5.133	5.135	5.139
5	4.923	4.919	4.917	4.918	4.920	4.921
10	4.609	4.606	4.596	4.594	4.594	4.596
30	3.524	3.524	3.523	3.513	3.502	3.502
100	2.730	2.730	2.729	2.729	2.727	2.720
			$\log \langle \bar{\nu} \rangle$			
0.4	0.206	0.163	-0.103	-0.914	-0.990	-
1	0.223	0.185	-0.022	-0.560	-0.597	-
1.5	0.226	0.193	0.004	-0.412	-0.426	-0.426
2	0.229	0.199	0.028	-0.287	-0.306	-0.306
3	0.237	0.213	0.081	0.117	-0.141	-0.141
5	0.343	0.312	0.295	0.350	0.052	0.057
10	0.634	0.626	0.584	0.462	0.278	0.290
30	1.171	1.170	1.166	1.118	0.951	0.919
100	1.635	1.635	1.635	1.635	1.634	1.632

TABLE 1—*Continued*

$^{56}Co \rightarrow {}^{56}Fe$

log β^{+}

T_9 \ log ρ/μ_e	6	7	8	9	10	11
0.4	-4.877	-4.877	-4.877	-4.877	-4.877	-4.877
1	-3.727	-3.727	-3.727	-3.727	-3.727	-3.727
1.5	-3.495	-3.495	-3.495	-3.495	-3.495	-3.495
2	-3.376	-3.376	-3.376	-3.376	-3.376	-3.376
3	-3.205	-3.204	-3.204	-3.204	-3.204	-3.204
5	-2.837	-2.830	-2.828	-2.828	-2.828	-2.828
10	-1.691	-1.688	-1.677	-1.673	-1.673	-1.673
30	0.918	0.919	0.920	0.933	0.948	0.949
100	2.693	2.693	2.693	2.695	2.715	2.797

log $\langle ft \rangle_{e^-}$

T_9	6	7	8	9	10	11
0.4	6.467	6.320	4.062	3.139	2.694	2.468
1	5.378	5.242	4.007	3.130	2.691	2.465
1.5	5.130	4.961	3.958	3.123	2.689	2.464
2	4.950	4.775	3.907	3.117	2.686	2.463
3	4.575	4.451	3.818	3.103	2.683	2.460
5	3.964	3.951	3.621	3.072	2.676	2.458
10	3.257	3.258	3.223	2.961	2.657	2.454
30	2.583	2.584	2.585	2.586	2.519	2.410
100	2.114	2.114	2.114	2.115	2.117	2.118

log $\langle \nu \rangle$

T_9	6	7	8	9	10	11
0.4	0.292	0.387	-0.034	0.497	0.905	1.272
1	0.311	0.419	0.065	0.503	0.906	1.272
1.5	0.313	0.395	0.125	0.510	0.908	1.272
2	0.312	0.366	0.169	0.518	0.909	1.273
3	0.303	0.316	0.243	0.532	0.912	1.273
5	0.358	0.359	0.371	0.572	0.920	1.275
10	0.612	0.613	0.621	0.701	0.951	1.282
30	1.124	1.124	1.125	1.131	1.175	1.351
100	1.680	1.680	1.680	1.680	1.682	1.697

$^{56}Fe \rightarrow {}^{56}Co$

log β^{-}

T_9 \ log ρ/μ_e	6	7	8	9	10	11
0.4	-68.988	-70.693	-85.785	-	-	-
1	-28.226	-29.392	-35.464	-49.273	-79.199	-
1.5	-19.112	-19.999	-24.043	-33.275	-53.244	-96.305
2	-14.537	-15.219	-18.261	-25.204	-40.193	-72.502
3	-9.969	-10.405	-12.417	-17.087	-27.105	-48.653
5	-6.404	-6.579	-7.713	-10.586	-16.635	-29.584
10	-3.994	-4.022	-4.354	-5.812	-8.928	-15.447
30	-2.758	-2.760	-2.773	-2.930	-3.940	-6.251
100	-2.390	-2.390	-2.391	-2.395	-2.438	-2.881

log $\langle ft \rangle_{e^+}$

T_9	6	7	8	9	10	11
0.4	6.240	6.239	6.240	-	-	-
1	4.994	4.996	4.998	4.999	5.003	-
1.5	4.672	4.671	4.672	4.675	4.678	4.686
2	4.468	4.469	4.468	4.471	4.473	4.479
3	4.138	4.138	4.137	4.138	4.140	4.144
5	3.529	3.522	3.520	3.520	3.521	3.524
10	2.982	2.979	2.970	2.965	2.964	2.966
30	2.805	2.805	2.804	2.798	2.791	2.790
100	2.558	2.558	2.558	2.557	2.556	2.551

log $\langle \bar{\nu} \rangle$

T_9	6	7	8	9	10	11
0.4	-0.876	-0.881	-1.010	-	-	-
1	-0.353	-0.594	-0.661	-0.660	-0.661	-
1.5	-0.317	-0.479	-0.533	-0.532	-0.532	-0.532
2	-0.284	-0.408	-0.459	-0.459	-0.459	-0.459
3	-0.100	-0.283	-0.348	-0.348	-0.349	-0.348
5	0.375	0.286	0.156	0.147	0.147	0.147
10	0.704	0.703	0.695	0.689	0.690	0.689
30	1.132	1.132	1.132	1.129	1.125	1.125
100	1.611	1.611	1.611	1.611	1.610	1.608

$^{56}Ni \rightarrow {}^{56}Co$

log β^{+}

T_9 \ log ρ/μ_e	6	7	8	9	10	11
0.4	-35.400	-35.400	-35.400	-35.400	-35.400	-35.400
1	-14.981	-14.981	-14.981	-14.981	-14.981	-14.981
1.5	-10.443	-10.443	-10.443	-10.443	-10.443	-10.443
2	-8.168	-8.168	-8.168	-8.168	-8.168	-8.168
3	-5.844	-5.843	-5.843	-5.843	-5.843	-5.843
5	-3.702	-3.698	-3.696	-3.696	-3.696	-3.696
10	-1.111	-1.109	-1.099	-1.096	-1.096	-1.096
30	1.457	1.457	1.459	1.469	1.481	1.482
100	3.202	3.202	3.202	3.204	3.222	3.295

log $\langle ft \rangle_{e^-}$

T_9	6	7	8	9	10	11
0.4	5.343	4.988	4.483	3.011	2.524	2.331
1	5.284	4.970	4.426	3.009	2.523	2.332
1.5	5.195	4.943	4.351	3.006	2.523	2.332
2	5.102	4.898	4.254	3.000	2.523	2.331
3	4.812	4.691	4.035	2.988	2.522	2.331
5	4.010	3.995	3.634	2.947	2.518	2.331
10	3.011	3.012	2.985	2.790	2.496	2.326
30	2.316	2.316	2.317	2.322	2.292	2.244
100	1.896	1.895	1.896	1.896	1.899	1.909

log $\langle \nu \rangle$

T_9	6	7	8	9	10	11
0.4	-0.239	-0.019	0.196	0.402	0.871	1.258
1	-0.172	0.005	0.169	0.406	0.871	1.258
1.5	-0.094	0.037	0.143	0.410	0.872	1.259
2	-0.022	0.071	0.124	0.416	0.872	1.258
3	0.092	0.120	0.132	0.433	0.874	1.259
5	0.278	0.271	0.270	0.483	0.882	1.260
10	0.627	0.629	0.632	0.680	0.924	1.269
30	1.151	1.151	1.153	1.160	1.199	1.358
100	1.700	1.700	1.700	1.700	1.702	1.717

$^{56}Co \rightarrow {}^{56}Ni$

log β^{-}

T_9 \ log ρ/μ_e	6	7	8	9	10	11
0.4	-38.647	-44.535	-56.526	-90.974	-	-
1	-19.645	-21.007	-24.352	-38.418	-68.344	-
1.5	-14.363	-14.721	-17.212	-26.546	-46.514	-89.576
2	-11.262	-11.484	-13.554	-20.534	-35.523	-67.832
3	-8.167	-8.343	-9.815	-14.459	-24.477	-46.025
5	-5.784	-5.876	-6.672	-9.549	-15.599	-28.548
10	-4.051	-4.071	-4.328	-5.712	-8.828	-15.346
30	-2.325	-2.326	-2.339	-2.484	-3.472	-5.780
100	-1.543	-1.543	-1.544	-1.548	-1.589	-2.027

log $\langle ft \rangle_{e^+}$

T_9	6	7	8	9	10	11
0.4	12.937	12.936	12.937	-	-	-
1	8.581	8.584	8.585	8.587	8.591	-
1.5	7.684	7.684	7.684	7.689	7.692	7.699
2	7.258	7.259	7.258	7.259	7.262	7.268
3	6.789	6.788	6.788	6.787	6.790	6.794
5	5.997	5.987	5.985	5.985	5.986	5.989
10	5.071	5.068	5.057	5.053	5.054	5.055
30	3.837	3.837	3.836	3.830	3.823	3.822
100	2.937	2.936	2.937	2.936	2.935	2.931

log $\langle \bar{\nu} \rangle$

T_9	6	7	8	9	10	11
0.4	-0.879	-1.033	-1.000	-0.999	-	-
1	-0.555	-0.178	-0.694	-0.624	-0.625	-
1.5	-0.031	-0.128	-0.506	-0.470	-0.470	-0.469
2	0.002	-0.085	-0.389	-0.369	-0.369	-0.369
3	0.025	-0.050	-0.250	-0.245	-0.245	-0.245
5	0.212	0.077	-0.094	-0.105	-0.105	-0.105
10	0.698	0.690	0.624	0.560	0.558	0.559
30	1.137	1.137	1.136	1.133	1.131	1.131
100	1.599	1.599	1.599	1.599	1.598	1.597

TABLE 1—*Continued*

	^{60}Zn → ^{60}Cu						^{60}Cu → ^{60}Zn					
T_9 \ log ρ/μ_e	6	7	8	9	10	11	6	7	8	9	10	11
	log β^+						log β^-					
0.4	-2.290	-2.290	-2.290	-2.290	-2.290	-2.290	-63.448	-67.402	-82.962	-	-	-
1	-2.290	-2.290	-2.290	-2.290	-2.290	-2.290	-27.859	-29.727	-35.953	-48.929	-77.490	-
1.5	-2.284	-2.284	-2.284	-2.284	-2.284	-2.284	-19.971	-21.232	-25.407	-33.086	-52.282	-95.343
2	-2.251	-2.251	-2.251	-2.251	-2.251	-2.251	-16.047	-16.972	-20.120	-25.130	-39.598	-71.907
3	-2.085	-2.084	-2.084	-2.084	-2.084	-2.084	-12.143	-12.696	-14.754	-17.103	-26.802	-48.349
5	-1.768	-1.763	-1.761	-1.761	-1.761	-1.761	-8.802	-8.923	-9.326	-10.466	-16.303	-29.252
10	-0.552	-0.550	-0.543	-0.540	-0.540	-0.540	-4.290	-4.296	-4.372	-5.042	-7.962	-14.480
30	1.840	1.841	1.843	1.858	1.875	1.876	-0.783	-0.784	-0.792	-0.888	-1.713	-3.994
100	2.846	2.846	2.847	2.849	2.871	2.964	0.510	0.510	0.510	0.506	0.469	0.060
	log $\langle ft \rangle_{e^-}$						log $\langle ft \rangle_{e^+}$					
0.4	4.020	3.973	3.747	3.311	2.817	2.398	5.379	5.378	5.379	-	-	-
1	4.032	3.971	3.742	3.310	2.817	2.397	5.486	5.489	5.490	5.493	5.497	-
1.5	4.038	3.967	3.734	3.309	2.817	2.397	5.579	5.578	5.579	5.582	5.585	5.593
2	4.019	3.955	3.722	3.306	2.815	2.397	5.650	5.651	5.651	5.652	5.655	5.661
3	3.926	3.898	3.674	3.296	2.812	2.397	5.696	5.698	5.697	5.697	5.699	5.704
5	3.740	3.758	3.575	3.275	2.802	2.395	5.549	5.546	5.545	5.546	5.548	5.550
10	3.314	3.317	3.306	3.130	2.756	2.390	5.022	5.021	5.014	5.012	5.012	5.014
30	2.349	2.349	2.351	2.356	2.322	2.228	3.605	3.605	3.604	3.600	3.595	3.595
100	1.795	1.794	1.795	1.794	1.797	1.801	2.697	2.697	2.697	2.698	2.696	2.691
	log $\langle \nu \rangle$						log $\langle \bar{\nu} \rangle$					
0.4	0.348	0.550	0.547	0.634	0.856	1.217	-0.954	-1.038	-1.012	-	-	-
1	0.349	0.551	0.546	0.635	0.856	1.216	-0.540	-0.662	-0.664	-0.303	-0.601	-
1.5	0.352	0.550	0.546	0.637	0.857	1.217	-0.409	-0.514	-0.524	-0.207	-0.432	-0.432
2	0.358	0.551	0.548	0.641	0.857	1.217	-0.286	-0.392	-0.409	-0.144	-0.314	-0.315
3	0.388	0.550	0.559	0.652	0.861	1.218	-0.055	-0.135	-0.119	-0.038	-0.154	-0.154
5	0.505	0.579	0.599	0.684	0.870	1.219	0.273	0.271	0.330	0.114	0.035	0.035
10	0.743	0.749	0.779	0.807	0.914	1.230	0.613	0.605	0.528	0.330	0.263	0.263
30	1.220	1.220	1.222	1.238	1.269	1.373	1.088	1.088	1.088	1.081	1.060	1.056
100	1.689	1.690	1.689	1.690	1.690	1.706	1.604	1.604	1.604	1.604	1.604	1.602

The case of electron capture on ^{56}Fe is illustrative of these points. The ground-state to ground-state Q-value for $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ is $Q_n = -4.2064$, and is therefore a threshold transition. In F^{2}N II, the $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ transition was calculated to have a total Gamow-Teller sum rule of log $(ft) = 2.58$ and a Gamow-Teller resonance excitation energy estimated to be 3.777 MeV in ^{56}Mn, implying a resonance Q-value of $Q_R = -4.2064 - 3.7770 = -7.9834$ MeV. At low temperature and high density, where $W_e^{\mathrm{F}} < |Q_R|$, we expect the electron capture transition rate to be dominated by discrete transitions. At densities for which $W_e^{\mathrm{F}} > |Q_R|$ we expect the ^{56}Fe (ground state)→ ^{56}Mn (resonance state) to be the dominant transition. Consequently, $\langle ft \rangle_e$ should reflect the average values of ft for discrete transitions for $W_e^{\mathrm{F}} < |Q_R|$ and should approximate the resonance ft-values for $W_e^{\mathrm{F}} > |Q_R|$.

An examination of $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ in Table 1 shows that at a density of $\rho Y_e = 10^6$ g cm^{-3} and a temperature of $T_9 = 1.0$ ($W_e^{\mathrm{F}} \approx 0.672$ MeV), log $\langle ft \rangle_e = 3.726$. Note that here $W_e^{\mathrm{F}} < |Q_n|$, so that most of the electron capture proceeds on excited states of ^{56}Fe which have smaller values of $|Q_n|$ than does the ground-state transition. Electron capture transitions from highly excited states would have larger phase factors than the value I_e chosen for the calculation of $\langle ft \rangle_e$. For a given electron capture *rate* in which these other transitions contribute, $\langle ft \rangle_e$ must be smaller than the average physical discrete transition ft-values [log $(ft) \approx 5.0$] in order to compensate for the unrealistically small value of I_e.

At higher density, and higher electron Fermi energy, the situation is different. For $5.29 \times 10^8 \lesssim \rho Y_e (\mathrm{g\ cm^{-3}}) \lesssim 3.68 \times 10^9$, then $|Q_n| \lesssim W_e^{\mathrm{F}} \lesssim |Q_R|$ for $^5\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$. In this range of density the dominant electron capture transitions will be from the ground state to discrete daughter states below ~3.8 MeV excitation, and we expect $\langle ft \rangle_e$ to reflect the average discrete state transition ft-value, log $(ft) \approx 5.0$. Indeed, Table 1 gives log $\langle ft \rangle_e = 4.844$ for $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ at $T_9 = 1.0$ and $\rho Y_e = 10^9$ g cm^{-3}, corresponding to $W_e^{\mathrm{F}} = 5.176$ MeV. For $T_9 = 1.0$ and $\rho Y_e = 10^{11}$ g cm^{-3}, $W_e^{\mathrm{F}} = 23.93$ MeV $> |Q_R|$, so we expect $\langle ft \rangle_e$ to be roughly the resonance ft-value. Table 1 gives log $\langle ft \rangle_e \approx 2.554$, slightly smaller than the resonance value of log $ft \approx 2.58$. At a temperature of $T_9 = 1.0$ and a density of $\rho Y_e = 10^{10}$ g cm^{-3} the total electron Fermi energy is $W_e^{\mathrm{F}} = 11.114$ MeV $> |Q_R|$, so we expect the Gamow-Teller resonance to dominate the transition. However, since W_e^{F} is not too much larger than $|Q_R|$, the difference between $|Q_R|$ and $|Q_n|$ is not negligible in the phase space factor. Since $|Q_n|$ is used for computing I_e and $\langle ft \rangle_e$, we expect $\langle ft \rangle_e$ to be larger than the resonance ft-value; Table 1 for $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ at this grid point gives log $\langle ft \rangle_e \approx 3.219$.

At a fixed low density $\langle ft \rangle_e$ decreases as the temperature rises. This is due to thermal population of the parent states, particularly the Gamow-Teller resonance state (which then can serve as an electron capture target state, allowing a transition to the daughter ground state). In the case of $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$, log $\langle ft \rangle_e$ decreases monotonically from 3.726 at $T_9 = 1.0$ and $\rho Y_e = 10^6$ g cm^{-3} to 2.591 at $T_9 = 30.0$ and the same density.

In the introduction the problem of interpolation was presented with an example rate for the electron capture transition $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$. The effective ft-value procedure eliminates the large variation in rate between neighboring grid points, with a much smaller variation in $\langle ft \rangle_e$. For instance, as pointed out in the introduction the electron capture transition $^{56}\mathrm{Fe}(e^-, \nu)^{56}\mathrm{Mn}$ has a rate which changes by nearly 11 orders of magnitude between $T_9 = 1.0$ and $\rho Y_e = 10^8$ g cm^{-3} and $T_9 = 1.0$ and $\rho Y_e = 10^9$ g cm^{-3}. By contrast, log $\langle ft \rangle_e$ changes from 4.052 at the first grid point to 4.844 at the latter grid point: a change of a little more than a factor of 6 in $\langle ft \rangle_e$.

V. NEUTRINO BLOCKING OF ELECTRON CAPTURE, NEUTRINO CAPTURE, AND THE APPROACH TO BETA EQUILIBRIUM

The effective ft-values, $\langle ft \rangle_e$, have been calculated from the F^{2}N tables of capture rates, $\lambda_{Z\pm 1}$, using equation (16). In so doing, I_e was calculated employing $\eta_e{}^{\mathrm{F}}$ from the F^{2}N tabulation and setting $\eta_\nu{}^{\mathrm{F}} = \eta_{\bar{\nu}}{}^{\mathrm{F}} = -\infty$. In other words, the effective $\langle ft \rangle_e$ were computed in the free-streaming neutrino limit in which the occupation probability per neutrino phase space state is zero. Since $\langle ft \rangle_e$ is really a measure of the nuclear transition matrix element and is relatively insensitive to changes in lepton phase space distributions as outlined above, the capture rate in the presence of a nonvanishing thermal distribution of neutrinos can be calculated by employing the ambient value of $\eta_\nu{}^{\mathrm{F}}$ and $\eta_{\bar{\nu}}{}^{\mathrm{F}}$ in the calculation of I_e (eq. [5]) in equation (16) and solving for $\lambda_{Z\pm 1}$. This gives the effect of neutrino (antineutrino) blocking on the continuum electron (positron) capture rate. In what follows we restrict our discussion to neutrino blocking, since the application to antineutrino blocking follows quite simply. In this section we let subscript e refer to processes involving electrons, and subscript e^+ to processes involving positions.

Most stellar collapse computer calculations will compute a neutrino chemical potential once neutrino trapping has set in ($\rho Y_e > 10^{11}$ g cm^{-3}). The distribution of neutrinos is very nearly Fermi-Dirac in character. Holes in the Fermi-Dirac neutrino distribution are quickly filled by weak processes (Arnett 1982). If a rough estimate for the neutrino chemical potential $W_\nu{}^{\mathrm{F}}$ exists, then equations (5) and (16) and the table of effective ft-values allow an estimate of the neutrino-blocked lepton capture rate.

Where a thermal distribution of neutrinos exist, ν-capture can be important. The computational machinery for calculating the neutrino capture rate has been presented above: use equation (2b) with I_ν being calculated from equations (5b) and (5e). This procedure requires a knowledge of $\langle ft \rangle_\nu$, which is not related in any obvious manner to $\langle ft \rangle_e$; however, there is a simple way to compute directly the neutrino (antineutrino) capture rate from the electron (positron) capture rate using detailed balance.

Consider the rate of electron capture, $\lambda_e{}^{ij}$ from the ith state of the parent nucleus X to the jth state of the daughter nucleus Y,

$$e^- + X^i \Leftrightarrow Y^j + \nu + Q_{ij}\,, \tag{17a}$$

with

$$Q_{ij} = E_i - E_j + Q_{00}\,, \tag{17b}$$

where E_i and E_j are the excitation energies of parent state i and daughter state j, respectively, so that $\zeta_{ij} = Q_{ij}/kT$, and $\zeta_n = Q_{00}/kT$. The rate between parent state i and daughter state j is just

$$\lambda_e{}^{ij} = \ln 2\, \frac{I_e{}^{ij}}{(ft)_{ij}}\,, \tag{17c}$$

where $(ft)_{ij}$ is the appropriate ft-value between these states and $I_e{}^{ij}$ is evaluated from equation (5) with neutrino blocking. The total electron capture rate is given by a sum over daughter states and a sum over parent states, weighting each by a Boltzman factor

$$P_i = (2J_i + 1) \exp(-E_i/kT)/G_X\,, \tag{17d}$$

where J_i is the spin of state i and G_X is the nuclear partition function of parent X, so that

$$\lambda_e = \sum_i P_i \sum_j \lambda_e{}^{ij}\,. \tag{17e}$$

In like manner we can proceed to calculate the reverse neutrino capture rate in equation (17a) and sum over states as

$$\lambda_\nu = \sum_j P_j \sum_i \lambda_\nu{}^{ji}\,, \tag{18a}$$

where, following the above notation,

$$\lambda_\nu{}^{ji} = \ln 2\, \frac{I_\nu{}^{ji}}{(ft)_{ji}}\,, \tag{18b}$$

and now

$$P_j = (2J_j + 1) \exp(-E_j/kT)/G_Y\,. \tag{18c}$$

Whereas we could not immediately use detailed balance to relate $\langle ft \rangle_\nu$ to $\langle ft \rangle_e$ because these quantities involve averaging over states, we can invoke detailed balance between individual parent and daughter states. In particular, for states i and j we have

$$(ft)_{ji} = \left(\frac{2J_j + 1}{2J_i + 1}\right)(ft)_{ij}\,. \tag{18d}$$

Note that from equation (5d) we have

$$I_\nu{}^{ji} = I_e{}^{ij} \exp(\eta_\nu{}^{\mathrm{F}} - \eta_e{}^{\mathrm{F}} - \zeta_{ij})\,. \tag{18e}$$

Equations (18d) and (18e) can be used in equation (18a) to obtain

$$\lambda_\nu = \frac{\exp(\eta_\nu{}^{\mathrm{F}} - \eta_e{}^{\mathrm{F}} - \zeta_n)}{G_Y} \sum_i (2J_i + 1) \exp\left(\frac{E_i}{kT}\right) \sum_j \frac{\ln 2 I_e{}^{ij}}{(ft)_{ij}}\,, \tag{18f}$$

where the first sum is clearly G_X, and we obtain

$$\lambda_\nu = \frac{G_X}{G_Y} \exp\,(\eta_\nu{}^{\rm F} - \eta_e{}^{\rm F} - \zeta_n)\lambda_e\,. \tag{18g}$$

We emphasize that $\zeta_n = Q_{00}/kT$.

The generalization to positrons and antineutrinos is obvious. If we know the total electron (positron) capture rate in the presence of neutrino (antineutrino) blocking, then we can obtain the total reverse neutrino (antineutrino) capture rate in the presence of electron (positron) blocking using equation (18g). The values of the partition functions G_X and G_Y can be computed from simple fitting formulae given by Woosley *et al.* (1978) for the nuclei considered by $\mathrm{F^2N}$ I, II, III.

The tables of effective log (ft)–values allow calculation of continuous electron (positron) capture rates and, through equation (18g), the reverse neutrino (antineutrino) capture rates for the free nucleons and all of the nuclei considered in $\mathrm{F^2N}$ I, II, III. Note that the use of the detailed balance relation (18g) *does not* require either beta equilibrium or nuclear statistical equilibrium. It only requires that the nuclear levels be thermally populated according to the Boltzmann distributions (17d) and (18c), a situation satisfied under almost all circumstances of astrophysical interest (see Ward and Fowler 1979). With both the forward, lepton capture rates and the reverse, neutrino capture rates in hand, it is logical to ask what the total neutronization (or protonization) rates are.

The number of electrons per baryon Y_e is defined as

$$Y_e = \frac{n_{em}}{\rho N_{\rm A}}\,, \tag{19a}$$

where $N_{\rm A}$ is Avogadro's number and n_{em} is the number of matter electrons. We hereby denote by Y_- the total number of electrons, n_e, per baryon, including pair electrons. Similarly, we take Y_+ to be total number of positrons per baryon, so that in general

$$Y_e = Y_- - Y_+\,, \tag{19b}$$

Consider now the reactions

$$e^- + p \Leftrightarrow n + \nu\,. \tag{20a}$$

The rate of change of the total number of electrons due to these reactions, $(\partial n_e/\partial t)^f$, is

$$\left(\frac{\partial n_e}{\partial t}\right)^f = -\rho N_{\rm A} X_p \lambda_e{}^{fp} + \rho N_{\rm A} X_n \lambda_\nu{}^{fn}\,, \tag{20b}$$

where $\lambda_e{}^{fp}$ is the electron capture rate on free protons, $\lambda_\nu{}^{fn}$ is the neutrino capture rate on free neutrons, f denotes free nucleons, and where X_p and X_n are the free proton and neutron mass fractions, respectively. We denote the total neutronization rate by $\dot{Y}_- = dY_-/dt$ and the contribution to the neutronization rate from electron capture on free protons plus neutrino capture on free neutrons as $(\dot{Y}_-{}^f) = (\partial n_e/\partial t)^f$. We then have

$$\dot{Y}_-{}^{fn} = X_n \lambda_\nu{}^{fn} - X_p \lambda_e{}^{fp}\,. \tag{20c}$$

We can generalize equation (20c) to the case of electron capture on a heavy parent nucleus and neutrino capture on a daughter nucleus:

$$\dot{Y}_-{}^h = \frac{X_D{}^h}{A}\lambda_\nu{}^h - \frac{X_P{}^h}{A}\lambda_e{}^h\,, \tag{20d}$$

where $\lambda_e{}^h$ is the electron capture rate on the parent nucleus (mass fraction $X_P{}^h$) in the presence of neutrino blocking, $\lambda_\nu{}^h$ is the rate of neutrino capture on the daughter nucleus (mass fraction $X_D{}^h$) in the presence of electron blocking, and A is the atomic mass of the parent or daughter nucleus.

In beta equilibrium the chemical potentials of the interacting leptons are related in a special way to the chemical potentials of the neutron and proton. This relationship can be seen in the stoichiometry of reaction (20a). We define

$$\Delta \equiv (W_e^{\rm F} - W_\nu^{\rm F}) - (W_n^{\rm C} - W_p^{\rm C}) = (W_e^{\rm F} - W_\nu^{\rm F}) - (U_n{}^C - U_p{}^C + M_n - M_p)\,, \tag{21a}$$

where the W's are the total chemical potentials as defined above (Fermi energies for leptons), $U_n{}^C$ and $U_p{}^C$ are the neutron and proton *kinetic* chemical potentials, respectively, and M_n and M_p are the neutron and proton masses, respectively. Note that our $U_n{}^C$ and $U_p{}^C$ correspond to the Bethe *et al.* (1979) μ_n and μ_p, respectively. We can divide by kT to obtain

$$\delta \equiv \frac{\Delta}{kT} = (\eta_e{}^{\rm F} - \eta_\nu{}^{\rm F}) - \left(\mu_n{}^C - \mu_p{}^C + \frac{\delta M}{kT}\right), \tag{21b}$$

where our $\mu_n{}^C \equiv U_n{}^C/kT$, $\mu_p{}^C \equiv U_p{}^C/kT$, and $\delta M \equiv M_n - M_p \approx 1.293$ MeV.

In beta equilibrium we must have $\Delta = 0$, or $\delta = 0$. The same condition on the lepton and nucleon chemical potentials in beta equilibrium results from the reactions

$$e^+ + n \Leftrightarrow p + \bar{\nu} \tag{22}$$

when the equilibrium conditions $W_{e^+}{}^{\rm F} = -W_{e^-}{}^{\rm F}$ and $W_{\bar{\nu}}^{\rm F} = -W_\nu^{\rm F}$ are used. The same equilibrium conditions hold for processes similar to equations (20a) and (22) but proceeding on heavy nuclei. It is useful to cast total neutronization rates in terms of Δ.

From equation (18g) we can relate λ_ν for neutrino capture on free neutrons to λ_e for electron capture on free protons. Substituting in equation (20c) we obtain for the neutronization rate due to free nucleons,

$$\dot{Y}_-^{\,f} = \left[X_n \exp\left(\eta_\nu{}^{\rm F} - \eta_e{}^{\rm F} + \frac{\delta M}{kT}\right) - X_p\right]\lambda_e{}^{fp}\,, \tag{23a}$$

where $(\zeta_n)^f = -\delta M$ and where we note that the ratio of the partition functions in equation (18g) is unity for the free nucleons. Using the definition of δ in equation (21a), we obtain

$$\dot{Y}_-^{\,f} = [X_n \exp(-\delta - \delta\mu^C) - X_p]\,\lambda_e{}^{fp}\,, \tag{23b}$$

where $\delta\mu^C \equiv \mu_n{}^C - \mu_p{}^C = \hat{\mu}/kT$ and $\hat{\mu}$ is the Bethe *et al.* (1979) difference in kinetic chemical potential between neutrons and protons. Over the typical range of temperatures and densities of interest in the stellar collapse problem the free neutrons and protons are nondegenerate, so that

$$X_p \approx X_n \exp(-\delta\mu^C)\,. \tag{23c}$$

Equation (23b) then becomes

$$\dot{Y}_-^{\,f} \approx X_p \lambda_e{}^{fp}[\exp(-\delta) - 1]\,. \tag{23d}$$

In the beta equilibrium limit $\delta \to 0$, so that $\dot{Y}_-^{\,f} \to 0$. The analogous protonization rate for reaction (22) follows in a similar manner:

$$\dot{Y}_+^{\,f} = \lambda_{e^+}{}^{fn}[X_p \exp(\delta\mu^C + \delta) - X_n] \tag{24a}$$

$$\approx X_n \lambda_{e^+}{}^{fn}[\exp(\delta) - 1]\,, \tag{24b}$$

and has the same zero limit as beta equilibrium is approached.

We can generalize the above neutronization and protonization rates for the case of heavy nuclei. Starting with equation (20d) for the neutronization rate, we define the parent nucleus to be X and the daughter Y (as in eq. [17a]), with mass fractions X_X and X_Y, respectively, and use equation (18g) to obtain

$$\dot{Y}_-^{\,h} = \left[\frac{X_Y}{A}\frac{G_X}{G_Y}\exp(\eta_\nu{}^{\rm F} - \eta_e{}^{\rm F} - \zeta_n) - \frac{X_X}{A}\right]\lambda_{e^-}{}^{h}\,. \tag{25a}$$

We can now employ nuclear statistical equilibrium (NSE) to relate the abundances of species X and Y. Following Burbidge *et al.* (1957),

$$\frac{X_Y}{X_X} = \frac{X(N+1, Z-1)}{X(N, Z)} \approx \frac{G_Y}{G_X}\frac{X_n}{X_p}\exp\left(\frac{\Delta Q}{kT}\right), \tag{25b}$$

where ΔQ is the difference between the nuclear Q-values for Y and X. The nuclear Q-value is defined as

$$Q(N, Z) = ZM_p + NM_n - M(Z, N)\,, \tag{25c}$$

where $M(Z, N)$ is the nuclear mass of a nucleus with charge Z and atomic number $A = Z + N$, so that

$$\Delta Q = Q(N+1, Z-1) - Q(N, Z) = Q_n + \delta M\,.$$

We can show then that on substitution of the NSE relation (25b) into equation (25a), one obtains

$$\dot{Y}_-^{\,h} \approx \frac{X_X}{A}\,\lambda_e{}^{h}[\exp(-\delta) - 1]\,, \tag{25d}$$

and, in similar fashion,

$$\dot{Y}_+^{\,h} \approx \frac{X_Y}{A}\,\lambda_{e^+}{}^{h}[\exp(\delta) - 1]\,. \tag{25e}$$

Each expression has the appropriate zero limit as beta equilibrium ($\delta = 0$) is approached.

The total neutronization rate is given by the sum of the contributions from free nucleons and heavy nuclei,

$$\dot{Y}_- = \dot{Y}_-^{\,f} + \sum_h \dot{Y}_-^{\,h}\,. \tag{26}$$

If the user of Table 1 knows $\eta_e{}^{\rm F}$, $\eta_\nu{}^{\rm F}$, $U_n{}^C$, and $U_p{}^C$, then the capture rate on each nucleus can be calculated as outlined in the previous section, δ can be computed, and the contribution to the neutronization rate can be computed for each species using equations (23d) and (25d).

Many current efforts to compute models of stellar collapse and explosion cannot implement our rate tables, either because the computer codes are not large enough to internalize our tables or because the special conditions considered run beyond the limits of our grid of temperatures, densities, or nuclear masses. We encourage the full use of our tables and the fitting procedures considered above wherever possible, because this insures the most accurate weak interaction rates. Where this is simply not possible, usually in the high temperatures and high densities of stellar collapse, we now provide typical average values of Q_n and $\log\langle ft\rangle_e$ which will allow a good estimate of lepton capture rates to be made by employing the appropriate phase space factors (I_e, I_ν) presented in this paper.

Where nuclear statistical equilibrium obtains and where the values of the nuclear chemical potentials are known, then a good approximation for the average electron capture Q_n for the mean nucleus is

$$Q_n \approx -(U_n{}^C - U_p{}^C + M_n - M_p)\,, \tag{27}$$

while for neutrino capture Q_n is the negative of equation (27). See definitions after equation (21a).

Where the mean nucleus is unblocked (neutron number less than 40 and proton number greater than 20), the lepton captures will proceed principally through the GT-resonances which will be roughly at an excitation of 3.0 MeV above the daughter ground state. We suggest taking an effective *ft*-value typical of ^{56}Fe, where $\log \langle ft \rangle_e \approx 3.2$ for $W_e^F < |Q_R| \approx |Q_n| + 3.0$ MeV, and $\log \langle ft \rangle_e \approx 2.6$ for $W_e^F > |Q_R|$. For a typical blocked nucleus we take an effective *ft*-value based on the simple temperature-dependent unblocking model of Fuller (1982). A blocked mean nucleus will have a Q_n given by equations (27) but may have a Q_R different from the unblocked case. In applying our fitting formulae we recommend using Q_n from equation (27) for the unblocked cases and $Q_n - 5$ MeV in place of Q_n for the blocked cases. Recall that usually $Q_n < 0$ for electron capture and $Q_n > 0$ for neutrino capture. This information is summarized as

$$\log \langle ft \rangle_e \approx \begin{cases} \Rightarrow 3.2 \quad W_e^{\rm F} < |Q_R| & \text{unblocked: use } Q_n \text{ in } I_e \text{ or } I_\nu\,; \\ \Rightarrow 2.6 \quad W_e^{\rm F} > |Q_R| & \\ \Rightarrow 2.6 + 25.9/T_9 & \text{blocked: use } Q_n - 5 \text{ MeV in } I_e \text{ or } I_\nu\,. \end{cases} \tag{28}$$

The expressions in equation (28) follow on taking $|M_{\rm GT}|^2 = 10$ for the total amount of possible Gamow-Teller strength and noting that the strength available for lepton capture depends on thermal unblocking. Since it costs $\sim$5.13 MeV to pull a neutron out of a filled $1f_{5/2}$ orbit and place it in the *gd*-shell, the available strength will be roughly $|M_{\rm GT}|^2 \approx 10 \exp(-5.13/kT) \approx 10 \exp(-59.53/T_9) \approx \text{dex}\,(1 - 25.9/T_9)$ at temperature T_9.

The $\log \langle ft \rangle_e$ given in equation (28) is consistent with the thermal unblocking result of Fuller (1982). Recently Cooperstein and Wambach (1985) have shown that there may be more thermal unblocking near the blocking point, so that the coefficient of $(T_9)^{-1}$ in the blocked case of equation (28) will be smaller. However, some of the values for $\log \langle ft \rangle_e$ given in equation (28) have not been corrected for Gamow-Teller quenching which could increase the numerical constant 2.6 to as much as 2.9. The numerical constant 3.2 is based on experimental data and should not be changed.

In many circumstances of stellar evolution, collapse, and explosion, the lepton capture rates on free neutrons and protons will dominate those on nuclei. This is particularly true in the high entropy environments of supernova shock passage (see Brown, Bethe, and Baym 1982). Our fitting formulae allow accurate calculation of the lepton weak rates for free nucleons within a few percent of our actual numerical calculation. We recommend employing $\log \langle ft \rangle_e = 3.035$ for all processes involving free nucleons.

Frequently the rate of change of entropy corresponding to a given lepton capture rate is a desired quantity. If the entropy per baryon, S, is given in units of Boltzmann's constant then for a given species i,

$$\frac{\partial}{\partial t}\left(\frac{S}{k}\right) = \left[\frac{(\bar{\epsilon}_\nu)_i - \Delta}{kT}\right] \dot{Y}_-{}^i\,, \tag{29}$$

where k is Boltzmann's constant, $\dot{Y}_-{}^i$ is the neutronization rate due to species i, and $(\bar{\epsilon}_\nu)^i$ is the energy of the *escaped* neutrino. Where neutrinos are trapped and thermalized, $(\bar{\epsilon}_\nu)_i = 0$ for all species i, and with $\delta = \Delta/kT$ one has

$$\frac{\dot{S}}{k} = -\delta(\dot{Y}_-{}^i) \tag{30}$$

which holds for each species i independently, where $\dot{Y}_-{}^i$ is the neutronization rate corresponding to electron capture on i and $\dot{S}/k$ is the entropy change due to that process. Equation (30) also holds for the total neutronization rate, $\dot{Y}_-$ in equation (26), where then $\dot{S}/k$ is the total entropy change rate.

VI. CONCLUSION

We have presented here effective *ft*-values for continuum electron (positron) capture and average neutrino (antineutrino) emission energies. These quantities are relatively slowly varying compared with the bare capture and neutrino loss rates, facilitating rapid and accurate computer interpolation. The lepton capture rates can be reconstructed easily with the approximate phase space factor expressions presented above. These expressions contain most of the rapid temperature and density variation associated with the capture rates. We have also presented a means of calculating the effect of neutrino (antineutrino) blocking on the electron (positron) capture rates. A simple formula based on detailed balance enables the computation of a neutrino (antineutrino) capture rate once the electron (positron) capture rate has been calculated from our tables. The approach to beta equilibrium is briefly considered. Tables of effective *ft*-values and average neutrino energies in computer readable form on magnetic tape can be obtained by writing to Michael J. Newman.

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