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Stepped Spillway Flows and Air Entrainment

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Abstract

Stepped spillways have become a popular method for handling flood releases. The steps increase significantly the rate of energy dissipation taking place on the spillway face and reduce the size of the required downstream energy dissipation basin. The compatibility of stepped spillways with roller compacted concrete (RCC) and gabion construction techniques results in low additional cost for the spillway. This paper presents a review of recent developments for the design of stepped spillways, provides a discussion of the effects of air entrainment, and presents new calculation methods that take into account the effects of flow aeration on the flow characteristics and the rate of energy dissipation.

Key words : stepped spillway, air entrainment, dam, spillway, energy dissipation.

Résumé

Les évacuateurs de crues en marches d'escalier sont devenus une méthode courante pour décharger les crues. Les seuils des marches augmentent considérablement la dissipation d'énergie au long du déversoir, et, de ce fait, réduisent la taille du bassin de dissipation en aval de l'évacuateur de crues. La géométrie des déversoirs en marches d'escalier s'adapte très bien à des structures en gabions ou en béton compacté au rouleau (BCR), et n'entraîne qu'une augmentation raisonnable du coût de construction. Le présent article décrit de récents développements dans le dimensionnement de ce type de déversoir. Puis l'auteur discute les effets de l'entraînement d'air, et il présente une nouvelle méthode de calculs pour des écoulements aérés au dessus des évacuateurs de crues en marches d'escalier.

Mots clés : évacuateurs de crues en marches d'escalier, entraînement d'air, barrage, déversoirs, dissipation d'energie.

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Introduction

Presentation

Energy dissipation over dam spillways is usually achieved by : 1- a standard stilling basin downstream of the spillway where a hydraulic jump is created to dissipate a large amount of flow energy, 2- a high velocity water jet taking off from a flip bucket and impinging into a downstream plunge pool, or 3- the construction of steps on the spillway to assist in energy dissipation.

Water flowing over a rough or stepped face of a dam can dissipate a major proportion of its energy. The steps increase significantly the rate of energy dissipation taking place on the spillway face, and eliminate or reduce greatly the need for a large energy dissipator at the toe of the spillway. Stepped spillways have become a popular method for flood releases at roller compacted concrete (RCC) dams and gabions dams. The compatibility of the stepped spillway design with RCC construction techniques results in low additional cost for the spillway (FRIZELL and MEFFORD 1991). Gabions are used frequently in the construction of small dams because the construction of gabion structures is easy and cheap. Gabion-stepped spillways are the most common type of spillway used for gabion dams (STEPHENSON 1979a, DEGOUTTE et al. 1991).

So far, few analyses take into account the effects of air entrainment. In this paper the first part describes the flow characteristics and energy dissipation of non-aerated flows on a stepped spillway. In the second part, the effects of air entrainment on stepped spillway flows are discussed, and new formulations are proposed. The results are compared with recent experimental data (Table 1).

It must be noted that cavitation damage may occur on stepped spillways. But the risks of cavitation damage are reduced by the flow aeration. PETERKA (1953) and RUSSELL and SHEEHAN (1974) showed that 5 to 8% of air concentration next to the spillway bottom may prevent cavitation damage on concrete surfaces. Further the high rate of energy dissipation along stepped spillways reduces the flow momentum. The reduction of flow velocity and the resulting increase of flow depth reduce the risks of cavitation as the cavitation index increases.

Step geometry

A stepped channel consists of an open channel with a series of drops along the invert. The total fall is divided into a number of smaller falls. Various step geometries are used: horizontal step, inclined step and pooled step. The present paper presents results which are applicable to stepped spillways with horizontal steps. The geometry of a horizontal step is defined by its height h and horizontal length l as shown in figures 1B and 1C. The step height and length are related to the spillway slope α by :

$$[1] \qquad \tan\alpha = \frac{h}{l}$$

ESSERY and HORNER (1978) and PEYRAS et al. (1991, 1992) discussed experimental results obtained with inclined steps. VITTAL and POREY (1987) presented a cascade system of falls with pooled steps acting as intermediary energy dissipation basin.

Flow regimes above a stepped spillway

Two types of flow regime may occur above a stepped spillway : nappe flow and skimming flow (Fig. 1).

PEYRAS et al. (1991, 1992) indicated two types of nappe flow : 1- nappe flow with fully developed hydraulic jump (Fig. 1A) for low discharge and small flow depth, and 2- nappe flow with partially developed hydraulic jump (Fig. 1B). The flow from each step hits the step below as a falling jet, with the energy dissipation occurring by jet breakup in air, by jet mixing on the step and by the formation of a fully developed or partial hydraulic jump on the step (RAJARATNAM 1990).

In the skimming flow regime, the water flows down the stepped face as a coherent stream, skimming over the steps and cushioned by the recirculating fluid trapped between them (Fig. 1C). Along the upstream steps, the flow is smooth and no air entrainment occurs. Downstream the flow is characterized by a large amount of flow aeration and strong vortices at the step toes (Fig. 2). Most of the energy is dissipated by momentum transfer between the main stream and the recirculating fluid.

For small dams and weirs ELLIS (1989) and PEYRAS et al. (1991, 1992) suggested that higher energy dissipation might occur in the nappe flow regime than in the skimming flow regime. However nappe flow situations require relatively large steps, as detailed in the next paragraph. Such a geometry is not often practical but may apply to flat spillways, streams and stepped channels.

Un-Aerated Flow Characteristics

Nappe flow

Flow parameters

MOORE (1943) and RAND (1955) analyzed a single-step drop structure. Such a structure can be viewed as a single-step spillway. For a horizontal step, the flow conditions near the end of the step change from subcritical to critical at some section a short distance back from the edge. The flow depth at the brink of the step d_b is : $d_b = 0.715 * d_c$ where d_c is the critical flow depth (ROUSE 1936). Application of the momentum equation to the base of the overfall leads to (WHITE 1943) :

$$\text{[2]} \qquad \frac{d_1}{d_c} = \frac{2^{1/2}}{\frac{3}{2^{3/2}} + \sqrt{\frac{3}{2} + \frac{h}{d_c}}}$$

where d_1 is the flow depth at section 1 (Fig. 1A), and h the step height. The total head H_1 at section 1 can be expressed non-dimensionally as :

$$\text{[3]} \qquad \frac{H_1}{d_c} = \frac{d_1}{d_c} + \frac{1}{2} * \left(\frac{d_c}{d_1}\right)^2$$

The flow depth and total head at section 2 (Fig. 1A) are given by the classical hydraulic jump equations :

$$\text{[4]} \qquad \frac{d_2}{d_1} = \frac{1}{2} * \left(\sqrt{1 + 8*{Fr_1}^2} - 1\right)$$

[5] $$\frac{H_1 - H_2}{d_c} = \frac{(d_2 - d_1)^3}{4 * d_1 * d_2 * d_c}$$

where Fr_1 is the Froude number defined at section 1 : $Fr_1 = q_w/\sqrt{g*d_1^3}$. RAND (1955) assembled several sets of experimental data and developed the following correlations :

[6] $$\frac{d_1}{h} = 0.54 * \left(\frac{d_c}{h}\right)^{1.275}$$

[7] $$\frac{d_2}{h} = 1.66 * \left(\frac{d_c}{h}\right)^{0.81}$$

[8] $$\frac{d_p}{h} = \left(\frac{d_c}{h}\right)^{0.66}$$

where d_p is the height of water in the pool behind the overfalling jet (Fig. 1A).

Along a stepped spillway, critical flow conditions occur near to the end of each step, and equations [2] to [8] provide the main flow parameters for a nappe flow regime with fully developed hydraulic jump (Fig. 1A). PEYRAS et al. (1991, 1992) indicated that these equations can be applied also with reasonable accuracy to nappe flows with partially developed jump.

Energy dissipation

In a nappe flow situation with fully developed hydraulic jump, the head loss at any intermediary step equals the step height. The total head loss along the spillway ΔH equals the difference between the maximum head available H_{max} and the residual head at the bottom of the spillway H_1 (Eq. [3]) and can be written in dimensionless form :

[9] $$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{\frac{d_1}{d_c} + \frac{1}{2} * \left(\frac{d_c}{d_1}\right)^2}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right)$$

where H_{dam} is the dam height, and d_1 is given by equations [2] and [6] (Fig. 1A). The maximum head available and the dam height are related by : $H_{max} = H_{dam} + 1.5*d_c$. The residual energy is dissipated at the toe of the spillway by hydraulic jump in the dissipation basin. Combining equations [6] and [9] the total energy loss becomes :

[10] $$\frac{\Delta H}{H_{max}} = 1 - \left(\frac{0.54 * \left(\frac{d_c}{h}\right)^{0.275} + \frac{3.43}{2} * \left(\frac{d_c}{h}\right)^{-0.55}}{\frac{3}{2} + \frac{H_{dam}}{d_c}}\right)$$

On figure 3 the head loss (Eq. [10]) is plotted as a function of the critical flow depth and the number of steps, and compared with experimental data (MOORE 1943, RAND 1955, STEPHENSON 1979a). Figure 3 indicates that most of the flow energy is dissipated on the stepped spillway for large dams (i.e. large number of steps). Further, equation [10] shows a good agreement with the data for a single step structure.

Equations [9] and [10] were obtained for nappe flows with fully developed hydraulic jump. PEYRAS et al. (1991) performed experiments for nappe flows with fully and partially developed hydraulic jump. The rate of energy dissipation of nappe flows with partially developed hydraulic jump was within 10% of the values obtained for nappe flows with fully developed hydraulic jump for similar flow conditions. Therefore, it is believed that the equation [10] may be applied to most of the nappe flow situations with a reasonable accuracy.

Conditions for nappe flow regime

A number of dams have been built in South Africa with stepped spillways. From this experience STEPHENSON (1991) suggested that the most suitable conditions for nappe flow situations are :

[11a] $\tan\alpha = \frac{h}{l} < 0.20$ and

[11b] $\frac{d_c}{h} < \frac{1}{3}$

Skimming flow

In the skimming flow regime, the external edges of the steps form a pseudo-bottom over which the flows pass. Beneath this, horizontal axis vortices develop, filling the zone between the main flow and the step. These vortices are maintained through the transmission of shear stress from the fluid flowing past the edges of the steps (Fig. 1C). In addition small-scale vorticity will be generated continuously at the corner of the steps.

Onset of skimming flow

For horizontal steps, the onset of skimming flow is a function of the discharge (i.e. critical depth), and the step height and length. Experimental data obtained by ESSERY and HORNER (1978) and PEYRAS et al. (1991) showed that the onset flow conditions may be estimated as (Fig. 4) :

[12] $\left(\frac{d_c}{h}\right)_{onset} = 1.01 - 0.37 * \frac{h}{l}$

and skimming flows occur for : $d_c/h > (d_c/h)_{onset}$.

It must be noted that the data of PEYRAS et al. (1991) were obtained on a gabion stepped spillway model. The infiltration through the gabion is likely to affect the flow conditions, and may explain smaller values of $(d_c/h)_{onset}$ than for SORENSEN's data.

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Uniform flow conditions

Assuming a long stepped spillway and that the uniform flow conditions are reached before the end of the spillway, the uniform flow depth can be deduced from the momentum equation :

[13] $$\tau_o * P_w = \rho_w * g * A * \sin\alpha$$

where P_w is the wetted perimeter, ρ_w the water density, g the gravity constant, A the channel cross-section and τ_o the average shear stress between the skimming flow and the recirculating fluid underneath. The average bottom shear stress τ_o is defined as for an open channel flow (HENDERSON 1966, STREETER and WYLIE 1981) :

[14] $$\tau_o = \frac{f}{8} * \rho_w * V_o$$

where f is the Darcy coefficient (or friction factor) and V_o the uniform non-aerated flow velocity. For a wide channel, the uniform flow parameters V_o and d_o are deduced from the continuity and momentum equations, and can be written in dimensionless form :

[15] $$\frac{V_o}{V_c} = \sqrt[3]{\frac{8 * \sin\alpha}{f}}$$

[16] $$\frac{d_o}{d_c} = \sqrt[3]{\frac{f}{8 * \sin\alpha}}.$$

where V_c is the critical velocity. It must be emphasized that these results were obtained for non-aerated flows. The effects of air entrainment on the flow properties are discussed in the next paragraph.

MORRIS (1955) and KNIGHT and MACDONALD (1979) analysed 'quasi-smooth flows' (i.e. skimming flows) over large roughness elements of rectangular cross-section. Their results indicated that the classical flow resistance calculations must be modified to take into account the shape of the roughness element. The shear stress τ_o, indicated in Fig. 1C and used in equation [13], represents the turbulent shear stress between the main stream and the recirculating fluid trapped between the steps of the spillway. For a stepped spillway, the steps form the dominant surface roughness (SORENSEN 1985). If the roughness height k_s is estimated as the depth of a step normal to the flow (i.e. $k_s = h * \cos\alpha$), the dimensions of the step are defined completely by k_s and the spillway slope. Dimensional analysis suggests that the friction factor is a function of a Reynolds number, the roughness height k_s and the spillway slope:

[17] $$f = f_1\left(\text{Re} ; \frac{k_s}{D_H} ; \alpha\right)$$

where Re is the Reynolds number defined as : $\text{Re} = \rho_w * \frac{V_o * D_H}{\mu_w}$, D_H the hydraulic diameter : $D_H = 4*A/P_w$, and μ_w the dynamic viscosity of water. If the uniform flow conditions are known, the Darcy coefficient can be deduced from the momentum equation [13] :

[18] $$f = \frac{8 * g * \sin\alpha * d_o^2}{q_w^2} * \frac{D_H}{4}$$

where q_w is the discharge per unit width. SORENSEN (1985) and DIEZ-CASCON et al. (1991) measured flow depths at the bottom of long stepped spillway models. Their data were re-analysed using equation [18] and neglecting the aeration of the flow. The results are presented in figure 5, with the friction factor plotted as a function of the relative roughness. Figure 5 indicates friction factors in the range of 0.6 to 3.5, with an average value of 1.30. Such large values of the friction factor imply smaller flow velocity and greater flow depth than on smooth spillway, and enhance the energy dissipation.

HARTUNG and SCHEUERLEIN (1970) studied open channel flows on rockfill dams, with great natural roughness and steep slopes (Table 1). For slopes in the range 6 to 34 degrees, and in absence of air entrainment, their results are presented as :

[19] $$\frac{1}{\sqrt{f}} = -3.2 * \log_{10}\left((1.7 + 8.1 * \sin\alpha) * \frac{k_s}{D_H}\right)$$

For a slope of 30 degrees and $k_s/D_H = 0.1$, equation [19] provides a value of the friction factor : f = 1.7 of similar order of magnitude as the results obtained on stepped spillways (Fig. 5).

Energy dissipation

In skimming flow, most of the energy is dissipated in the maintenance of stable depression vortices. If uniform flow conditions are reached at the downstream end of the spillway, the total head loss is :

[20] $$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_o}{d_c} * \cos\alpha + \frac{E}{2} * \left(\frac{d_c}{d_o}\right)^2}{\frac{H_{dam}}{d_c} + \frac{3}{2}}$$

where E is the kinetic energy correction coefficient. Using equation [16] the head loss may be rewritten in terms of the friction factor, the spillway slope, the critical depth and the dam height :

[21] $$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8 * \sin\alpha}\right)^{1/3} * \cos\alpha + \frac{E}{2} * \left(\frac{f}{8 * \sin\alpha}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}}$$

Equation [21] was computed for a slope (α = 52 degrees) close to the geometry used by SORENSEN (1985) and DIEZ-CASCON et al. (1991), and using two values of the friction factor : f = 0.03 and f = 1.30, that represent average flow resistance on smooth spillways and stepped spillways. The results are plotted on figure 6 for an uniform velocity distribution (i.e. E = 1), and compared with the data of SORENSEN (1985), DIEZ-CASCON et al. (1991) and STEPHENSON (1991). Figure 6 indicates a good agreement between the experimental data and the equation [21] computed with a friction factor f = 1.30 and α = 52 degrees. Further, a comparison between the energy dissipation on smooth and stepped spillway shows a larger energy dissipation occurring on stepped spillways.

For a high dam, the residual energy term is small and the equation [21] is similar to the expression obtained by STEPHENSON (1991) :

$$\frac{\Delta H}{H_{max}} = 1 - \left(\left(\frac{f}{8 * \sin\alpha} \right)^{1/3} * \cos\alpha + \frac{E}{2} * \left(\frac{f}{8 * \sin\alpha} \right)^{-2/3} \right) * \frac{d_c}{H_{dam}} \qquad [22]$$

Equation [22] shows that the energy loss ratio increases with the height of the dam. For high dams, it becomes more appropriate to talk of residual head H_{res} than total head loss :

$$\frac{H_{res}}{d_c} = \left(\frac{f}{8 * \sin\alpha} \right)^{1/3} * \cos\alpha + \frac{E}{2} * \left(\frac{f}{8 * \sin\alpha} \right)^{-2/3} \qquad [23]$$

Equations [21] and [23] suggest that the total energy dissipation above the spillway and the residual energy at the bottom of the spillway are functions of the friction factor, spillway slope, discharge (i.e. critical depth) and dam height. These calculations (Eq. [21] and [23]) depend critically upon the estimation of the friction factor. Figure 5 shows a large scatter of friction factor values observed on model. Further, it will be subsequently shown that the friction factor is affected significantly by the rate of aeration. Therefore the equations [21] and [23] must be used with caution.

<u>Notes on gabion stepped spillways</u>

Gabions are extensively used for earth retaining structures and for hydraulic structures (e.g. weirs, channel linings). Their advantages are : 1- their stability, 2- their low cost and 3- that they are flexible and porous (STEPHENSON 1979a and 1979b).

The design of gabion structures is limited by the stability of the gabions. This imposes limitations on the flow rate and flow velocity that can be accommodated. When gabions are laid parallel to the slope, there are risks of sliding or overturning failure. Stability problems may occur for discharges larger than 1 m^2/s (PEYRAS et al. 1991, 1992). Gabion stepped spillways are more stable. STEPHENSON (1991) suggested that stability problems will occur for flow velocities higher than approximatively 4 m/s. PEYRAS et al. (1991, 1992) indicated that gabion stepped spillways are appropriate for discharges per unit width up to 3 m^2/s (i.e. d_c = 0.97 m). For discharges larger than 1.5 m^2/s however, gabion wires must be reinforced or the steps must be protected by concrete caps. Inclined gabion-stepped spillways can also be used. Larger energy dissipation is observed, but their construction requires greater care.

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Effects of Air Entrainment on Stepped Spillway Flows

Introduction

The flow aeration, also called self-aeration[1], was initially studied because of the effects of entrained air on the thickness of the flow. Air entrainment increases the bulk flow depth, and this is used as a design parameter for the height of spillway sidewalls (FALVEY 1980). Also, the presence of air within the boundary layer reduces the shear stress between the flow layers and hence the shear force. The resulting drag reduction reduces the energy dissipation above the spillway and hence its efficiency. Further, the presence of air within high-velocity flows may prevent or reduce the damage caused by cavitation (MAY 1987). Recently, air entrainment on spillways and chutes has been recognized also for its contribution to the air-water transfer of atmospheric gases such as oxygen and nitrogen (WILHELMS and GULLIVER 1989). This process must be taken into account for the re-oxygenation of polluted streams and rivers, and also to explain the high fish mortality downstream of large hydraulic structures.

In nappe flows, air entrainment occurs near the impact of the falling jet with the horizontal step and at the hydraulic jump. Large de-aeration occurs also downstream of the impact of the falling jet and downstream of the jump. Altogether, the net flow aeration is small, and it is believed that the effect of air entrainment on nappe flows can be neglected. This section of the paper presents an analysis of the effect of air entrainment on skimming flows. Although the flow conditions are different between a smooth and a stepped spillway, it is believed that the mechanisms of air entrainment are similar. Once the flow becomes fully developed, the stepped spillway behaves in the same way as a smooth one.

Mechanisms of air entrainment

Air entrainment is caused by turbulent velocities acting at the air-water free surface. Through this interface, air is continuously trapped and released. Air entrainment occurs when the turbulent kinetic energy is large enough to overcome both surface tension and gravity effects. The turbulent velocity normal to the free surface v' must overcome the surface tension pressure (ERVINE and FALVEY 1987) and be greater than the bubble rise velocity component for the bubbles to be carried away. These conditions are :

[24] $$v' > \sqrt{\frac{8 * \sigma}{\rho_w * d_{ab}}}$$ and

[25] $$v' > u_r * \cos\alpha$$

where σ is the surface tension, d_{ab} the air bubble diameter, and u_r the bubble rise velocity. For bubble sizes in the range 1 to 100 mm, calculations using the equations [24] and [25] suggest that air entrainment occurs for turbulent velocities v' greater than 0.1 to 0.3 m/s (CHANSON 1992). The flow conditions above a stepped spillway are characterized by a high degree of turbulence, and both velocity conditions are satisfied. As a consequence, large quantities of air are entrained along a stepped spillway.

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[1]Natural aeration occurring at the free surface of high velocity flows is referred as free surface aeration or self-aeration.

For stepped spillway flows (Fig. 2), the entraining region follows a region where the flow over the spillway is smooth and glassy. Next to the boundary, however, turbulence is generated and the boundary layer grows until the outer edge of the boundary layer reaches the free surface. When the outer edge of the boundary layer reaches the free surface, the turbulence may initiate natural free surface aeration. The location of the start of air entrainment is called the point of inception. Downstream of the point of inception, a layer containing a mixture of both air and water extends gradually through the fluid. Far downstream, the flow becomes uniform, and for a given discharge, the flow depth and the air concentration and velocity distributions do not vary along the chute. This region is defined as the uniform equilibrium flow region.

Definitions

The local air concentration C is defined as the volume of air per unit volume. The characteristic water flow depth d is defined as :

[26] $$d = \int_0^{Y_{90}} (1 - C) * dy$$

where y is measured perpendicular to the spillway surface and Y_{90} is the depth where the local air concentration is 90%. The depth averaged mean air concentration C_{mean} is defined as:

[27] $$(1 - C_{mean}) * Y_{90} = d$$

The average water velocity U_w is defined as :

[28] $$U_w = \frac{q_w}{d}$$

Point of inception

On smooth spillways, the position of the point of inception is primarily a function of the discharge and the spillway roughness. KELLER and RASTOGI (1977) suggested :

[29a] $$\frac{L_I}{k_s} = f_2\left(F_* \; ; \sin\alpha\right)$$

[29b] $$\frac{d_I}{k_s} = f_3\left(F_* \; ; \sin\alpha\right)$$

where L_I is the distance from the start of growth of the boundary layer to the point of inception, d_I is the depth at the point of inception measured normal to the free surface, and F* is a Froude number defined in terms of the roughness height : $F_* = q_w / \sqrt{g * \sin\alpha * k_s^3}$. For smooth concrete spillways, WOOD et al. (1983) estimated equation [29] as :

[30] $$\frac{L_I}{k_s} = 13.6 * (\sin\alpha)^{0.0796} * (F_*)^{0.713}$$

[31] $$\frac{d_I}{k_s} = \frac{0.223}{(\sin\alpha)^{0.04}} * (F_*)^{0.643}$$

On stepped spillways, the position of the start of air entrainment is a function of the discharge, spillway roughness, step geometry and spillway geometry. SORENSEN (1985) recorded the position of the start of air entrainment and the flow depth at the nearest measurement station. His results are presented as L_I/k_s and d_I/k_s versus the Froude number F_* as shown on figure 7. SORENSEN's (1985) results are also compared with equations [30] and [31], where the roughness k_s was estimated as the depth of a step normal to the free surface (i.e. $k_s = h * \cos\alpha$). Figure 7 shows that the equation [30] overestimates the location of the point of inception by approximatively 40%. This result indicates that the growth of the boundary layer is enhanced by the geometry of the steps.

Uniform flow region

Average air concentration

On stepped spillways, a large quantity of air is entrained along the channel, and the amount of air entrained is usually defined in terms of the average air concentration[2].

The analysis of self-aerated flow measurements on smooth spillways (STRAUB and ANDERSON 1958, AIVAZYAN 1986, Table 2) showed that the average air concentration for uniform flow conditions C_e is independent of the upstream geometry and flow conditions (i.e. discharge, flow depth, roughness) and is a function of the slope only (WOOD 1983, CHANSON 1992). Figure 8 shows the average air concentration C_e as a function of the slope α for STRAUB and ANDERSON's (1958) data obtained on a model and field data presented by AIVAZYAN (1986). For slopes flatter than 50 degrees, the average air concentration may be estimated as :

[32] $$C_e = 0.9 * \sin\alpha$$

Using the data of HARTUNG and SCHEUERLEIN (1970) obtained with great natural roughness and steep slopes (Table 1), KNAUSS (1979) indicated that the quantity of air entrained was estimated as :

[33] $$C_e = 1.44 * \sin\alpha - 0.08$$

This result is of similar form as the equation [32]. Figure 8 compares the equations [32] and [33] with experimental data. These results show a comparable rate of air entrainment for both smooth and rough flows. On stepped spillways, the uniform air concentration is expected to be similar to the results obtained on smooth spillway, where the mean air concentration is a function of the slope only (Table 3).

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[2]The quantity of air entrained within the flow is related to the mean air concentration by : $q_{air}/q_w = C_{mean}/(1 - C_{mean})$

Friction factor

For uniform aerated flows the momentum equation yields :

$$[34] \quad f_e = \frac{8 * g * \sin\alpha * d_e^2}{q_w^2} * \left(\frac{D_H}{4}\right)$$

where f_e is the friction factor for the uniform air-water mixture and d_e is the water flow depth (i.e. Eq. [26]) in uniform equilibrium flow. If f is the friction factor of non-aerated flow, dimensional analysis suggests that the ratio f_e/f is a function of the average air concentration, the Reynolds number and the relative roughness :

$$[35] \quad \frac{f_e}{f} = f_4\left(C_e ; Re ; \frac{k_s}{D_H}\right)$$

In uniform self-aerated flows for a smooth channel, the data of JEVDJEVICH and LEVIN (1953), STRAUB and ANDERSON (1958) and AIVAZYAN (1986) were analyzed using the equation [34]. The results are presented in figure 9, where the ratio f_e/f is plotted as a function of the average air concentration, f being calculated using the Colebrook-White formula. For these data, the effect of the Reynolds number and relative roughness on the ratio f_e/f, is small, and the equation [35] is estimated as :

$$[36] \quad \frac{f_e}{f} = 0.5 * \left(1 + \tanh\left(0.70 * \frac{0.490 - C_e}{C_e * (1 - C_e)}\right)\right)$$

where : tanh(x) = (exp(x) - exp(-x))/(exp(x) + exp(-x)). A general trend is that, for a given non-aerated friction factor, the friction factor for aerated flow f_e decreases when the average air concentration increases.

Further, HARTUNG and SCHEUERLEIN (1970) studied open channel flows on rockfill dams. The extremely rough bottom induced a highly turbulent flow with air entrainment. In presence of air entrainment, their results are presented as :

$$[37] \quad \frac{f_e}{f} = \frac{1}{(1 - 3.2 * \sqrt{f} * \log_{10}(1 - C_e))^2}$$

where C_e is estimated from the equation [33] and f is the non-aerated friction factor estimated by equation [19]. Their results show also a reduction in the ratio f_e/f with an increase in air concentration (Fig. 9). Also in fully rough turbulent flows, equation [37] suggests that the ratio f_e/f decreases with increasing roughness.

Figure 9 indicates a similar trend on both smooth and extremely rough channels ; that is, a substantial drag reduction when the air concentration increases above 10 to 20% (i.e. $\alpha > 10$ degrees). It is believed that the same mechanisms of drag reduction apply also to stepped spillways, and that the equations [36] and [37] can be used to provide a first estimate of the relative friction factor of aerated flows on stepped spillways.

Uniform flow parameters on stepped spillways

In uniform self-aerated flows, the flow parameters and energy dissipation can be deduced from the chute geometry (i.e. slope, roughness, width) and from the discharge. For any slope α, the average air concentration for uniform flow C_e can be obtained from figure 8. If the value of the friction factor for non-aerated flows, f, is available, figure 9 can be used to provide the friction factor for an aerated flow f_e as a function of the mean air concentration C_e. The characteristic depth d_e may be deduced from the equation [34]. For a wide channel (i.e. $D_H \sim 4*d_e$), the equation [34] yields :

[38] $$d_e = \left(\frac{q_w^2 * f_e}{8 * g * \sin\alpha}\right)^{1/3}$$

Knowledge of the equilibrium air concentration C_e, friction factor f_e and flow depth d_e provides the characteristic depth Y_{90} (Table 3) : $Y_{90} = d_e/(1 - C_e)$. The depth Y_{90} takes into account the bulk of the flow, and may be used as a design parameter for the height of sidewalls.

Energy dissipation

If the flow is uniform at the downstream end of the spillway, the energy dissipation along the stepped spillway with aerated flow ΔH_e is :

[39] $$\frac{\Delta H_e}{H_{max}} = 1 - \frac{\left(\frac{f_e}{8 * \sin\alpha}\right)^{1/3} * \cos\alpha + \frac{E}{2} * \left(\frac{f_e}{8 * \sin\alpha}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}}$$

Equation [39] differs from the equation [21] by using the friction factor for aerated flow. Equation [39] was computed for a slope α = 52 degrees and two values of non-aerated friction factor : f = 0.03 and 1.30. In figure 6, the results are compared with non-aerated flow calculations (Eq. [21]), and the data of SORENSEN (1985), DIEZ-CASCON et al. (1991) and STEPHENSON (1991), neglecting the effects of air entrainment. It must be emphasized that the measurements of SORENSEN (1985), DIEZ-CASCON et al. (1991) and STEPHENSON (1991) took into account the flow bulking due to air entrainment, and that the measured flow depths were not the uniform flow depth d_e. Therefore the friction factor values and energy dissipation computed from their data (Eq. [18] and [20]) are overestimated and differ from equations [34] and [39].

Figure 6 shows that the rate of energy dissipation on a smooth spillways is affected much more by air entrainment than on stepped spillways. As the mean air concentration increases with the slope and the friction factor decreases with the mean air concentration, the effects of air entrainment are more significant on steep slopes. On stepped spillways, air entrainment seems to have little effects on the energy dissipation. But it is more appropriate to consider the residual energy. For aerated flows the residual energy at the bottom of the spillway $(H_{res})_e$ is :

[40] $$\frac{(H_{res})_e}{d_c} = \left(\frac{f_e}{8 * \sin\alpha}\right)^{1/3} * \cos\alpha + \frac{E}{2} * \left(\frac{f_e}{8 * \sin\alpha}\right)^{-2/3}$$

The relative increase in residual energy due to flow aeration is :

$$[41] \quad \frac{(H_{res})_e}{H_{res}} = \left(\frac{f_e}{f}\right)^{1/3} * \left(\frac{1 + 4 * \frac{E}{f} * \tan\alpha * \left(\frac{f}{f_e}\right)}{1 + 4 * \frac{E}{f} * \tan\alpha} \right)$$

where H_{res} is obtained from equation [23] and f_e/f is estimated from equations [36] and [37]. The aeration of the flow decreases the friction factor and increases the kinetic energy of the flow. As a result, the residual energy increases with the air concentration. Equation [41] is plotted as a function of the mean air concentration for a smooth spillway (i.e. f = 0.03) and a stepped spillway (i.e. f = 1.30) on figure 10. Figure 10 shows that the residual energy is affected by the flow aeration for mean air concentrations larger than 40%. Figure 8 and table 3 indicate that a mean air concentration of 40% is obtained for a slope : α = 30 degrees. Hence these results suggest that the effects of air entrainment on the residual energy cannot be neglected for slopes larger than 30 degrees, for both smooth and stepped spillway types. Although figure 6 suggests that the total energy dissipation is only slightly overestimated, figure 10 shows that the residual energy is strongly underestimated if the effect of air entrainment is neglected.

Discussion

Table 4 presents a comparison between the energy dissipation in nappe flows (Eq. [10]) and skimming flows (Eq. [21] and [39]) for various slopes and dam heights.

The results indicate that nappe flow situations do not always provide the maximum energy dissipation. If the spillway is long enough (i.e. if uniform flow conditions are obtained), and for identical flow conditions and dam height, the maximum energy dissipation along the spillway is obtained for a skimming flow regime above a relatively flat stepped spillway. On steep spillways, the flow aeration reduces the flow resistance and hence the rate of energy dissipation.

For short stepped spillways, equations [21] and [39] are incorrect as uniform flow conditions do not occur. On such cases, it is believed that nappe flow situations provide the maximum energy dissipation as suggested by ELLIS (1989) and PEYRAS et al. (1992).

Gradually varied flow region

On smooth spillways, a simple analysis of the continuity equation for air, and the energy equation provides two simultaneous differential equations in terms of the average air concentration and the flow depth in the gradually varied flow region (CHANSON 1992). These equations can be solved with explicit numerical methods to determine the air entrainment which will occur on chutes and spillways. However, predictions of self-aeration depend upon the estimation of the air bubble rise velocity, the entrainment velocity and the friction factor. At the present time, no such experimental data is available for prototype stepped spillways. Although the air entrainment mechanisms are comparable to those observed on smooth spillways, the high level of turbulence is likely to modify the estimations of the rise velocity and entrainment velocity. Additional measurements on stepped spillways are required also to estimate the flow resistance.

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Conclusion

Two types of flow regime exist above a stepped spillway : nappe flow and skimming flow. For flat slopes or low discharges, the water proceeds in a series of plunges from one step to the next in what is called nappe flow. For steep slopes and large discharges, the water flows as a coherent stream over large recirculating vortices trapped between the steps and the main stream in a skimming flow regime. Equation [12] provides an estimate of the onset of skimming flow.

In both nappe flow and skimming flow regimes, the design of a stepped spillway is a very efficient method to dissipate a large part of the flow energy along the spillway, and up to 99% of the total head available can be dissipated (Table 4).

Flow conditions above a stepped spillway are affected by the air entrainment. A comparison between aerated skimming flows and self-aerated flows above smooth and extremely rough channels showed that the mean air concentration tends to an uniform air concentration C_e as function of the slope only (Eq. [32] and [33]) Further, the presence of air reduces the friction factor in uniform aerated flows for slopes steeper than 10 degrees. The observed drag reduction reduces the total energy dissipation above the spillway (Fig. 5), and hence the efficiency of a stepped spillway for slopes steeper than 30 degrees (Fig. 10).

At the present time, there is a lack of information on the estimate of the non-aerated friction factor and on the flow properties in the gradually varied flow region (Fig. 2). Measurements of air concentration and velocity in aerated flows above stepped spillways are also required.

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List of Symbols

A	cross-sectional area (m^2);
C	air concentration defined as the volume of air per unit volume;
C_e	equilibrium depth averaged air concentration for uniform flow;
C_{mean}	depth averaged air concentration defined as : $(1 - Y_{90}) * C_{mean} = d$;
D_H	hydraulic diameter (m) defined as : $D_H = 4 * \frac{A}{P_w}$;
d	1- flow depth measured normal to the channel slope at the edge of a step;
	2- characteristic depth (m) defined as : $d = \int_{y=0}^{y=Y_{90}} (1 - C) * dy$;
d_I	flow depth at the inception point (m);
d_{ab}	air bubble diameter (m);
d_b	flow depth at the brink of a step (m);
d_c	critical flow depth (m);

d_e	uniform aerated flow depth (m);
d_o	uniform non-aerated flow depth (m);
d_p	flow depth in the pool beneath the nappe (m);
E	kinetic energy correction coefficient (Coriolis coefficient);
Fr	Froude number defined as : $Fr = \frac{q_w}{\sqrt{g * d^3}}$;
F_*	Froude number defined as : $F_* = \frac{q_w}{\sqrt{g * \sin\alpha * k_s^3}}$;
f	friction factor of non-aerated flow;
f_e	friction factor for aerated flow;
g	gravity constant (m/s^2);
H	total head (m);
H_{dam}	dam height (m);
H_{max}	maximum head available (m) : $H_{max} = H_{dam} + \frac{3}{2} * d_c$;
H_{res}	residual head at the bottom of the spillway (m);
h	height of steps (m);
k_s	1- equivalent uniform sand roughness (m),
	2- step dimension normal to the flow : $k_s = h * \cos\alpha$;
L	distance along the spillway (m);
L_I	distance from the start of growth of boundary layer to the point of inception (m);
l	horizontal length of steps (m);
P_w	wetted perimeter (m);
Q	discharge (m^3/s);
q	discharge per unit width (m^2/s);
Re	Reynolds number defined as : $Re = \rho_w * \frac{U_w * D_H}{\mu_w}$;
U_w	flow velocity (m/s) : $U_w = q_w/d$;
u_r	rise bubble velocity (m/s);
V	velocity (m/s);
V_c	critical velocity (m/s);
V_o	uniform non-aerated flow velocity (m/s);
v'	root mean square of lateral component of turbulent velocity (m/s);
W	channel width (m);
Y_{90}	characteristic depth (m) where the air concentration is 90%;

y	distance from the bottom measured perpendicular to the spillway surface (m);
α	spillway slope;
ΔH	head loss (m);
μ	dynamic viscosity ($N.s/m^2$);
ρ	density (kg/m^3);
σ	surface tension between air and water (N/m);
τ_o	average bottom shear stress (Pa).

Subscript

air	air flow;
c	critical flow conditions;
e	equilibrium uniform aerated flow;
o	uniform non-aerated flow;
w	water flow.

Table 1 - Experiments on stepped spillways and rockfill bottom channels

Reference	Slope degree	h (1) m	q_w (1) m^2/s	Re	k_s/D_H (2)	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(6)
STEPPED SPILLWAYS						
MOORE (1943)	--	0.152 & 0.457	0.017 to 0.186			Drop structure. One step. W = 0.279 m.
RAND (1955)	--	0.198	0.25E-3 to 4.1E-3			Drop structure. One step. W = 0.5 m.
ESSERY and HORNER (1978)	11.3 to 40.1	0.03 to 0.05				Stepped spillway (30 steps).
STEPHENSON (1979a)	18.4 to 45.0	0.10 and 0.15				Gabion stepped spillway (1, 2, 3 & 4 steps).
SORENSEN (1985)	52.0	0.061	0.005 to 0.235			Stepped spillway model (scale 1/10). 7 steps.
SORENSEN (1985)	52.0	0.024	0.006 to 0.111	6.5E+4 to 3.3E+5	0.092 to 0.199	Stepped spillway model (scale 1/25). 59 steps.
DIEZ-CASCON et al. (1991)	53.1	0.06	0.025 to 0.200	1.6E+5 to 6.5E+5	0.125 to 0.323	Stepped spillway model (scale 1/10). H_{dam} = 3.8 m. W = 0.8 m.
PEYRAS et al. (1991, 1992)	18.4 to 45.0	0.20	0.045 to 0.268			Gabion stepped spillway model (scale 1/5). 5 steps.
STEPHENSON (1991)	54.5					Stepped spillway : Kennedy's vale model.
ROCKFILL BOTTOM CHANNELS						
HARTUNG and SCHEUERLEIN (1970)	6.0 to 34.0			8.5E+4 to 2E+6	0.02 to 0.2	Flow on rockfill bottom channel.

Note : (1) model dimensions

(2) on stepped spillway : $k_s = h * \cos\alpha$

Table 2 - Flow conditions for STRAUB and ANDERSON's (1958), AIVAZYAN's (1986) and JEVDJEVICH and LEVIN's (1953) data

Experiment (1)	Slope (degrees) (2)	k_s (mm) (3)	k_s/D_H (4)	Re (5)	C_e (6)	Comments (7)
St Anthony Falls ([1])	7.5 to 75	0.71	3E-3 to 1.6E-2	4.7E+5 / 2E+6	0.15 to 0.73	Spillway model (W = 0.457 m). Artificial roughness.
AIVAZYAN ([2])	14 to 31	0.1 to 10	5E-4 to 0.04	1.7E+5 / 2.8E+7	0.21 to 0.54	Prototype and large spillway model (W = 0.25 to 6 m). Planed boards, wood, rough concrete and rough stone masonry.
Mostarsko Blato ([3])	---	10 to 20	0.015 to 0.035	8.3E+4 / 3E+7	0.58 to 0.66	Prototype. Wide channel (W = 5.75 m). Stone lining.

Note : ([1]) STRAUB and ANDERSON (1958)

([2]) AIVAZYAN (1986)

([3]) JEVDJEVICH and LEVIN (1953)

Table 3 - Average air concentration in uniform self-aerated flows

Slope degrees (1)	C_e ([1]) (2)	Y_{90}/d_e ([1]) (3)	f_e/f ([2]) (4)
7.5	0.1608	1.192	0.968
15.0	0.2411	1.318	0.871
22.5	0.3100	1.449	0.765
30.0	0.4104	1.696	0.614
37.5	0.5693	2.322	0.389
45.0	0.6222	2.647	0.313
60.0	0.6799	3.124	0.228
75.0	0.7209	3.583	0.168
0.0	0.000	1.000	1.000

Note : ([1]) Data from STRAUB and ANDERSON (1958)

([2]) Computed from the equation [36]

Table 4 - Energy dissipation on stepped spillways

H_{dam}/d_c	ΔH / H_{max}							
	Nappe Flow (1)		Skimming Flow (2)					
	d_c/h=0.1	d_c/h=0.6	α = 30 deg.		α = 45 deg.		α = 60 deg.	
			Unaerated	Aerated	Unaerated	Aerated	Unaerated	Aerated
	Eq. [10]	Eq. [10]	Eq. [21]	Eq. [39]	Eq. [21]	Eq. [39]	Eq. [21]	Eq. [39]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	0.446	0.762	0.856	0.828	0.846	0.723	0.842	0.629
20	0.704	0.873	0.923	0.908	0.918	0.852	0.916	0.802
30	0.798	0.913	0.948	0.937	0.944	0.899	0.942	0.865
40	0.846	0.934	0.960	0.952	0.957	0.823	0.956	0.897
50	0.876	0.947	0.968	0.962	0.966	0.938	0.965	0.917
60	0.896	0.955	0.973	0.968	0.971	0.948	0.971	0.931
70	0.911	0.962	0.977	0.972	0.975	0.955	0.975	0.940
80	0.922	0.966	0.980	0.976	0.978	0.961	0.978	0.948
90	0.930	0.980	0.982	0.978	0.981	0.965	0.980	0.953
100	0.937	0.973	0.984	0.981	0.983	0.969	0.982	0.958
120	0.948	0.977	0.986	0.984	0.985	0.974	0.985	0.965
150	0.958	0.982	0.989	0.987	0.988	0.979	0.988	0.972
200	0.968	0.986	0.992	0.990	0.991	0.984	0.991	0.979

Notes : (1) The number of steps equals : $H_{dam}/h = H_{dam}/d_c * d_c/h$.

(2) Calculations made assuming f = 1.30.

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Fig. 1 - Flow regimes above a stepped spillways

Fig. 1A - Nappe flow with fully developed hydraulic jump

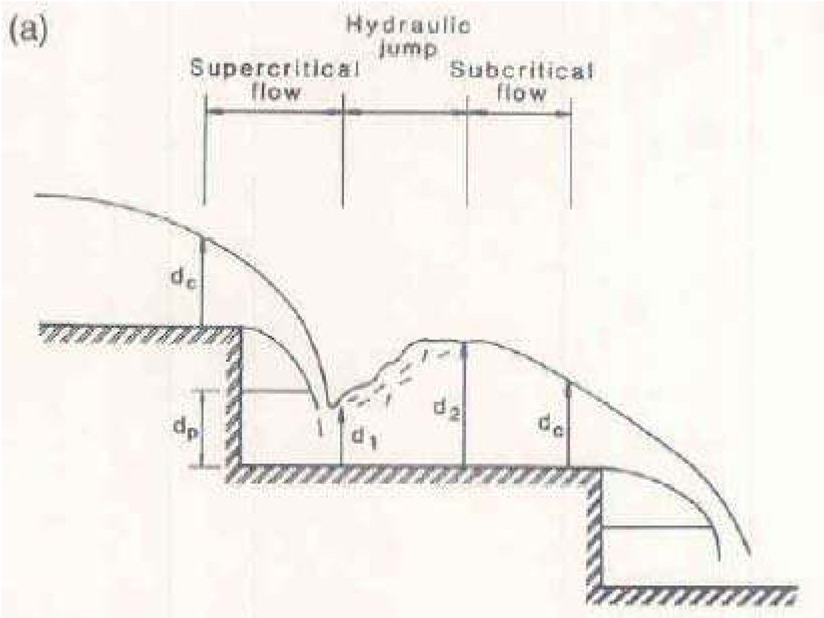

Fig. 1B - Nappe flow with partially developed hydraulic jump

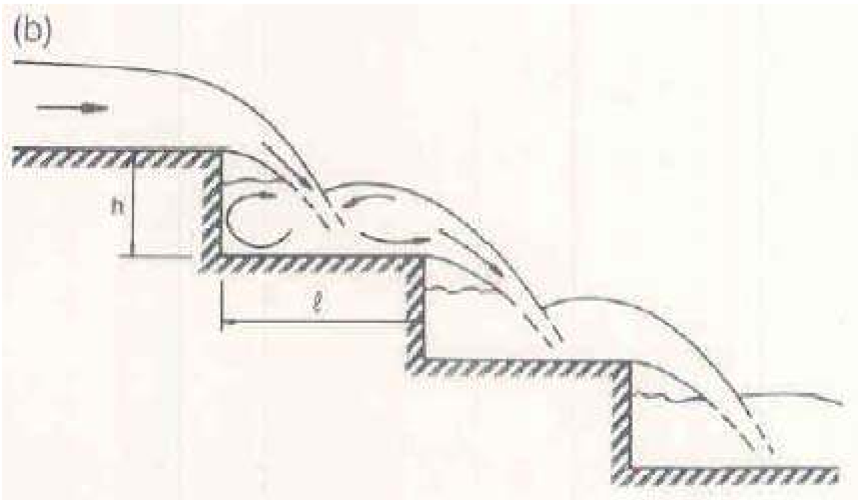

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Fig. 1C - Skimming flow above a stepped spillway

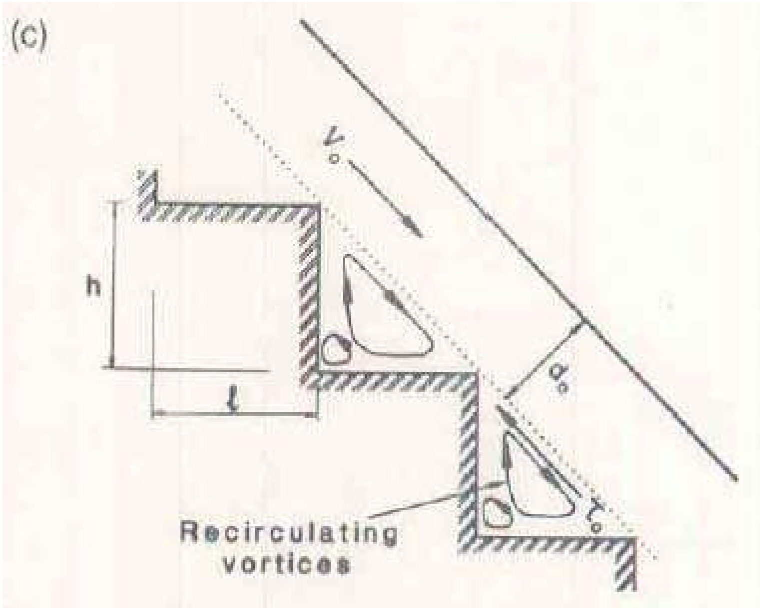

Fig. 2 - Flow regions above a long stepped spillway

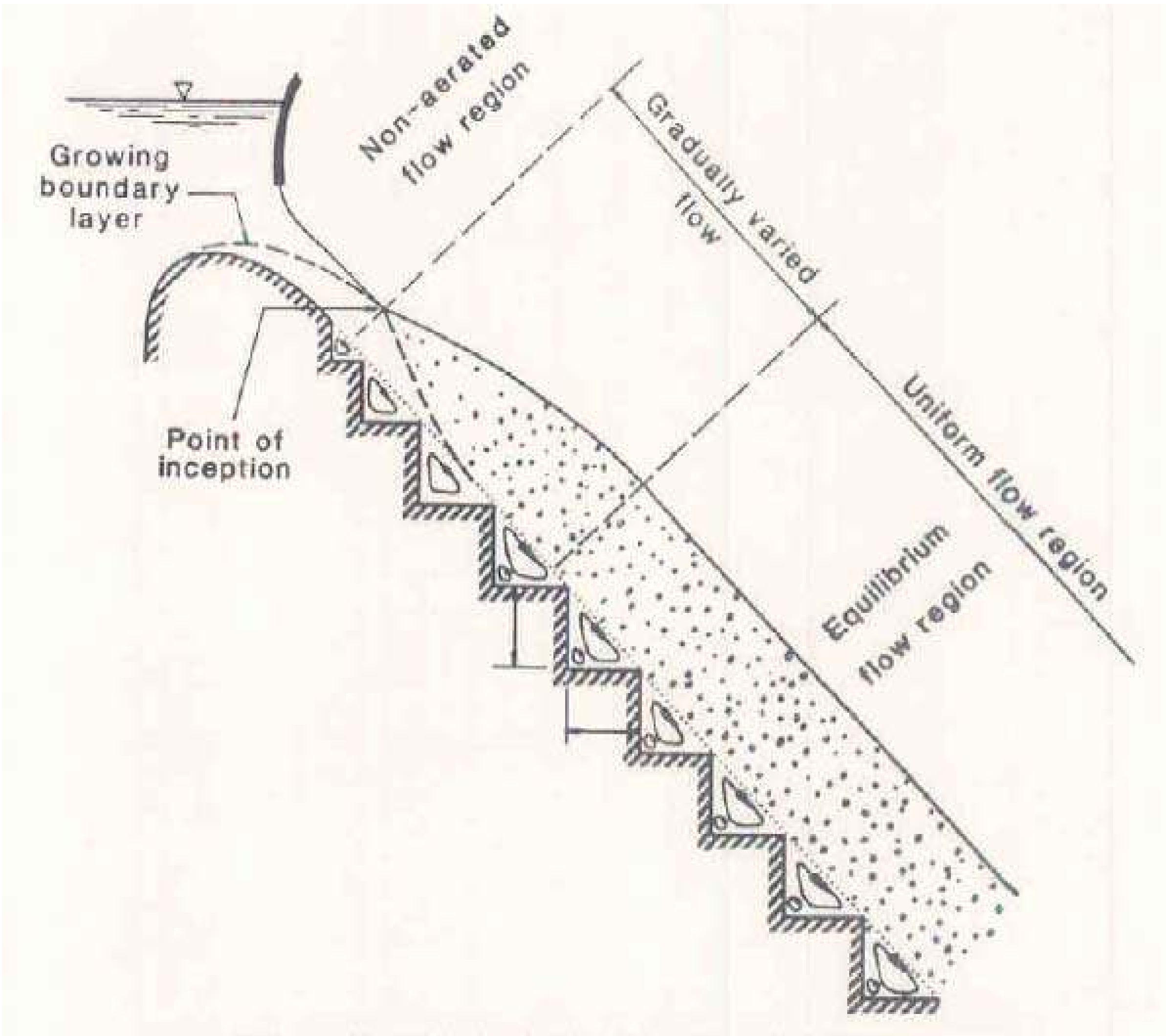

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Fig. 3 - Energy dissipation in nappe flow regime as a function of the number of steps : comparison between the equation [10] and the data of MOORE (1943), RAND (1955) and STEPHENSON (1979a)

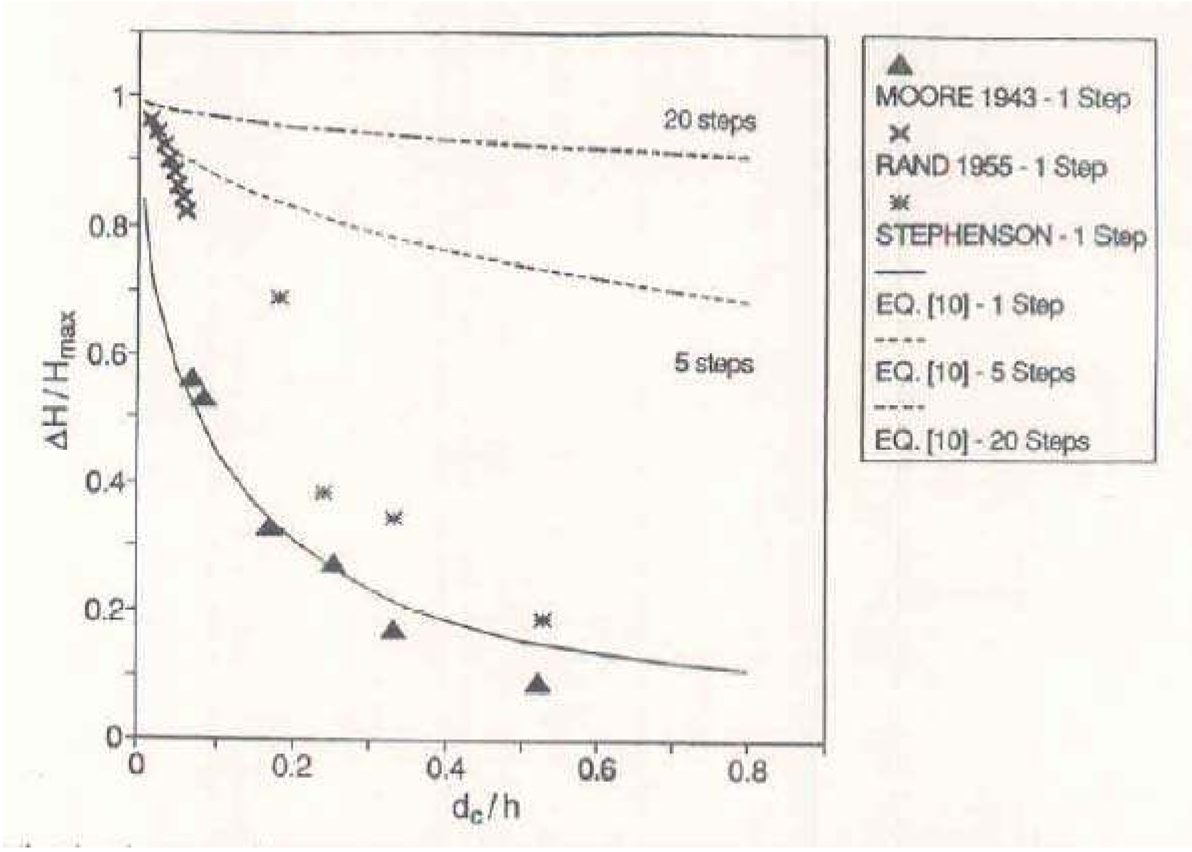


Fig. 4 - Onset of Skimming flow - ESSERY and HORNER (1978) - PEYRAS et al. (1991).

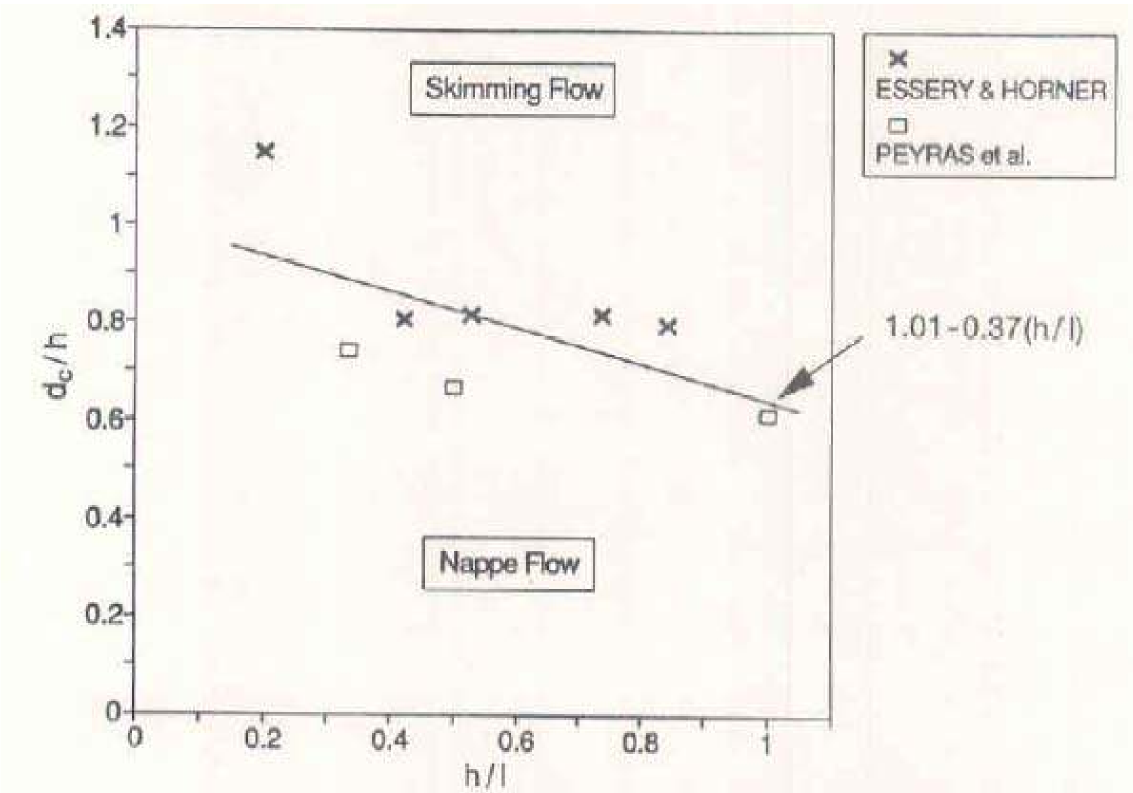
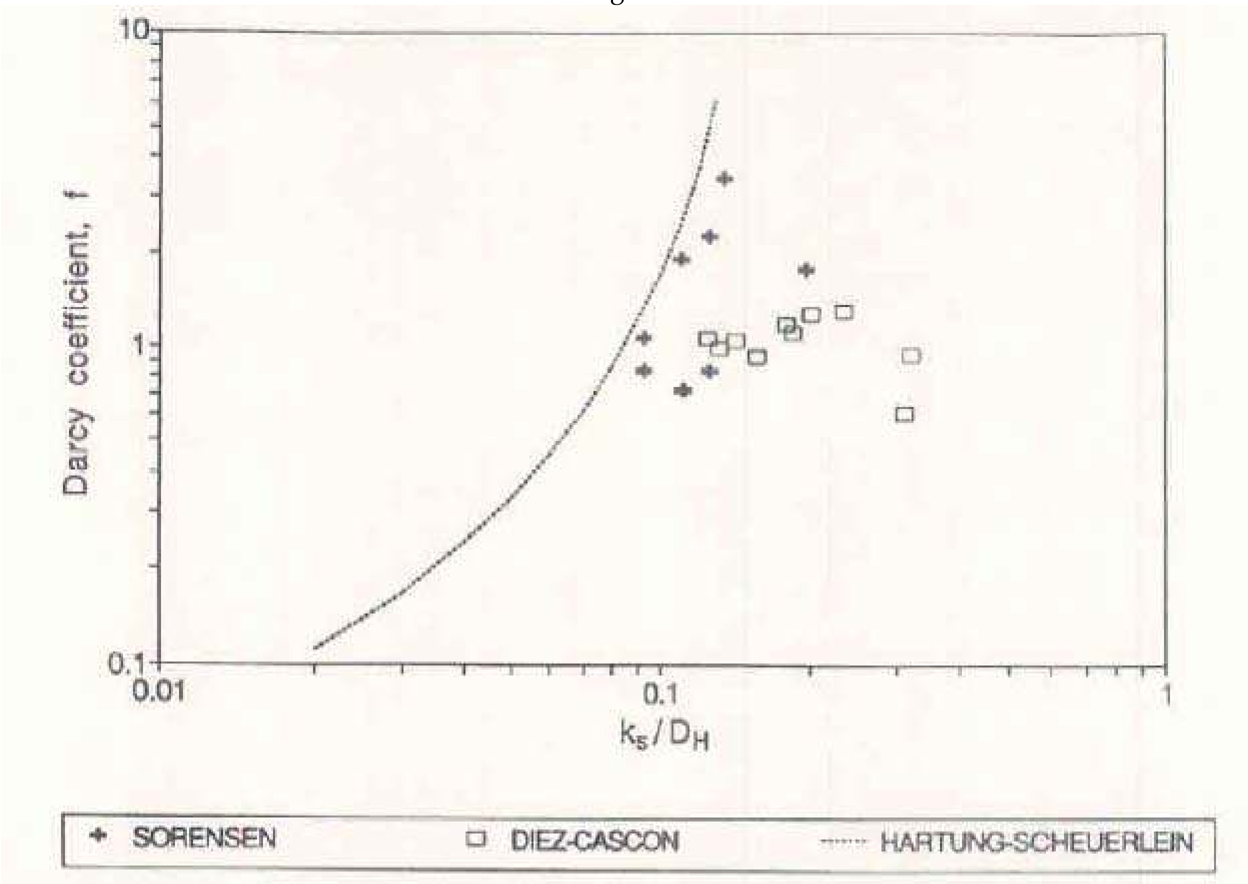


Fig. 5 - Non-aerated friction factor on stepped spillways

SORENSEN (1985) - DIEZ-CASCON et al. (1991) - HARTUNG and SCHEUERLEIN (1970) for a slope α = 30 degrees



CHANSON, H. (1993). "Stepped Spillway Flows and Air Entrainment." *Can. Jl of Civil Eng.*, Vol. 20, No. 3, June, pp. 422-435 (ISSN 0315-1468).

Fig. 6 - Energy dissipation in skimming flow regime : comparison between the equations [21] and [39], and the data of SORENSEN (1985), DIEZ-CASCON et al. (1991) and STEPHENSON (1991)

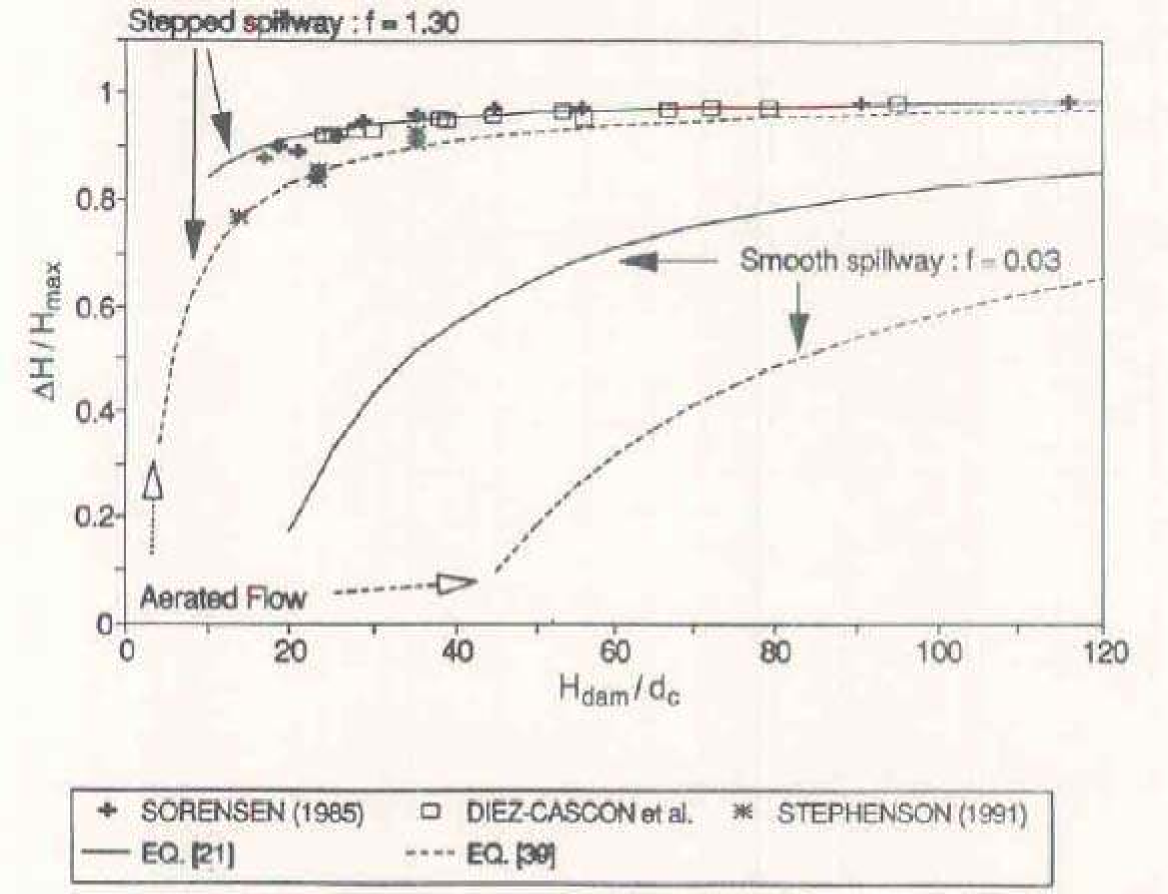
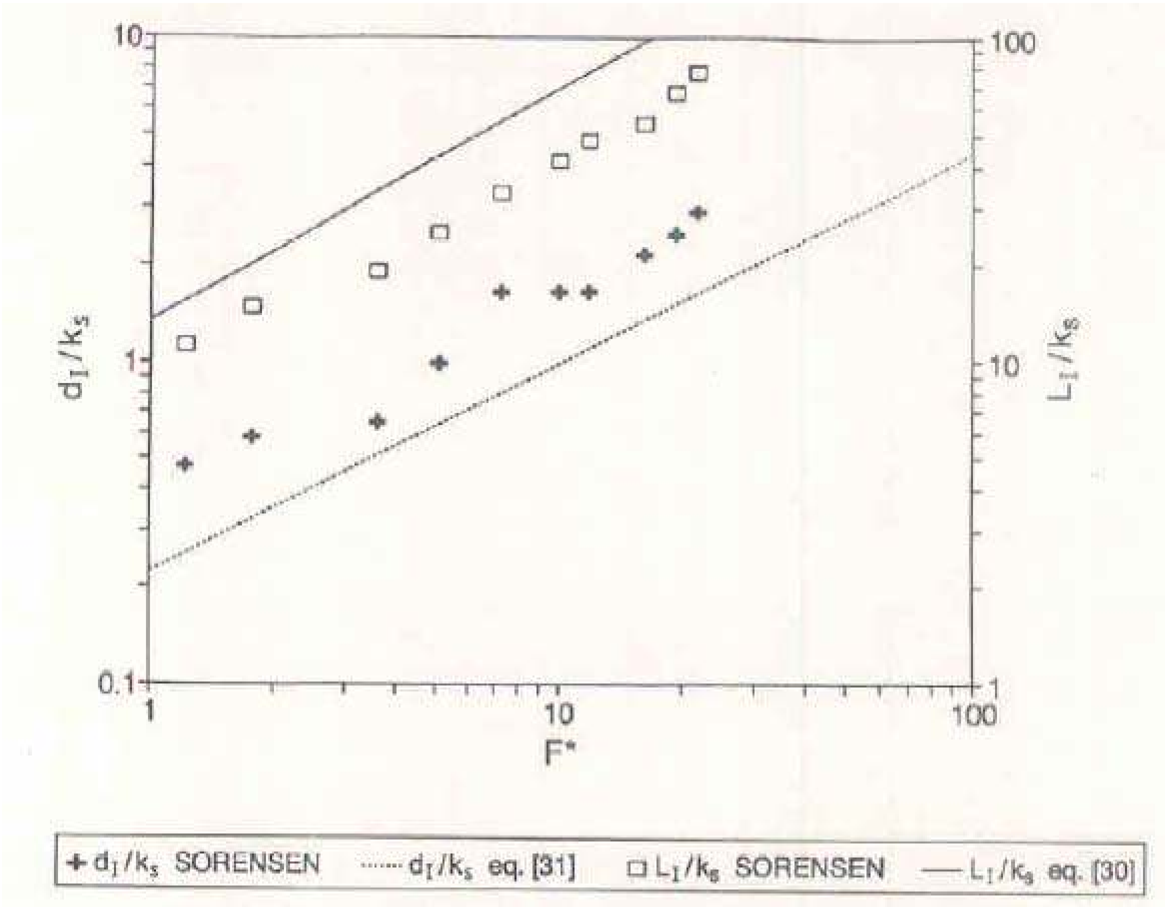


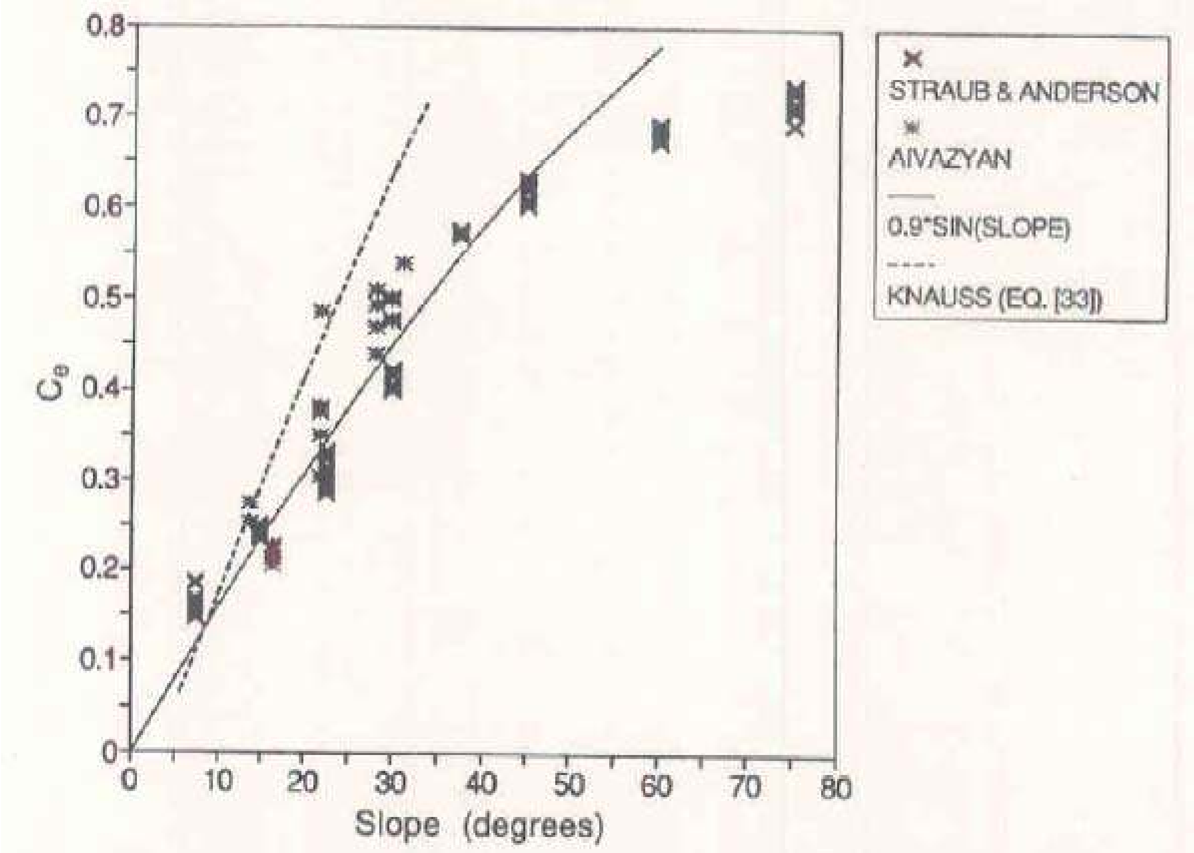
Fig. 7 - Characteristics of the point of inception - SORENSEN (1985)



CHANSON, H. (1993). "Stepped Spillway Flows and Air Entrainment." *Can. Jl of Civil Eng.*, Vol. 20, No. 3, June, pp. 422-435 (ISSN 0315-1468).

Fig. 8 - Uniform equilibrium air concentration C_e as a function of the spillway slope

STRAUB and ANDERSON (1958) - AIVAZYAN (1986) - KNAUSS (1979)



CHANSON, H. (1993). "Stepped Spillway Flows and Air Entrainment." *Can. Jl of Civil Eng.*, Vol. 20, No. 3, June, pp. 422-435 (ISSN 0315-1468).

Fig. 9 - Relative friction factor f_e/f as a function of the uniform air concentration C_e on smooth spillways : STRAUB and ANDERSON (1958), JEVDJEVICH and LEVIN (1953) and AIVAZYAN (1986), and rockfill channels : HARTUNG and SCHEUERLEIN (1970)

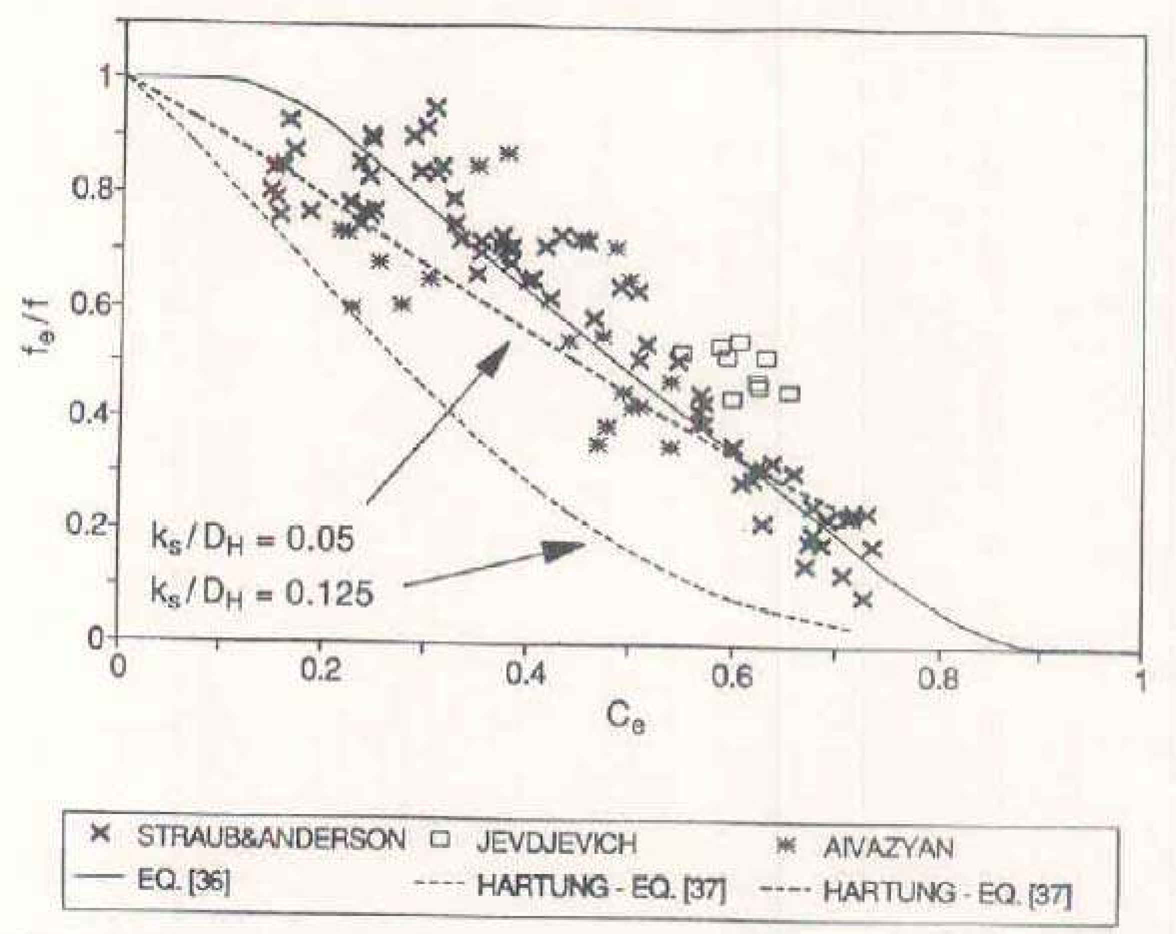


Fig. 10 - Effects of air entrainment and spillway slope on the residual energy - Eq. [41]