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# Sterile Neutrinos in a Grand Unified Model

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Recent experimental results indicating the existence of neutrino oscillations may strongly suggest that at least one more light neutrino species is required in order to reconcile the existing data. In simple GUT frameworks, this fact seems not to preserve the parallelism between quarks and leptons. In this paper, we investigate an SO(10) grand unified model with a pair of extra generations in addition to the known three generations. Using the GUT relations, the obtained neutrino mass matrix naturally indicates that one of the  $SU(2)_L$  singlet (sterile) neutrinos is very light and has a large mixing with the muon neutrino. This can explain the atmospheric neutrino anomaly, and the existence of the hot dark matter neutrino is also indicated. The solar neutrino problem can be solved by considering the mixing with the muon neutrino consistently with quark mixing, namely, the Cabibbo angle.

# §1. Introduction

Accumulating data of several experiments, we are now convinced that neutrinos have non-vanishing masses and mixings. The observed solar neutrino deficits  $^{(1)-5)}$ compared to the standard solar model calculations<sup>6</sup>) can be explained in terms of matter induced resonant oscillation<sup>7</sup>) with the oscillation parameters  $\Delta m^2 \simeq$  $(0.4-1.1) \times 10^{-5} \text{ eV}^2$  and  $0.003 \lesssim \sin^2 2\theta_{ex} \lesssim 0.012$ .<sup>8),\*)</sup> The atmospheric neutrino anomaly  $^{(9)}$  - 13) also indicates the neutrino oscillation  $\nu_{\mu} \leftrightarrow \nu_{\tau,s}$  with  $\Delta m^2 \sim 10^{-(2-3)}$  $eV^2$  and  $0.8 \leq \sin^2 2\theta_{\mu x} \leq 1.^{14}$  Another indication of the existence of neutrino masses and mixings comes from astrophysics and cosmology. In particular, if the neutrino is to be considered as a natural candidate for the hot dark matter component, which is needed to explain the anisotropy of the cosmic microwave background radiation and so on, it is required neutrino masses to be a few eV.<sup>15)</sup> Within the known three neutrino frameworks, the only solution which can explain the above mentioned experimental results requires three almost degenerate mass eigenstates with masses  $\simeq O(eV)$ .<sup>16</sup> However, this requires fine-tuning or very hierarchical right-handed neutrino Majorana masses.<sup>17)</sup> Together with the large 2-3 mixing. this is apparently in contrast to the character of ordinary quark masses and mixings. Thus, the simultaneous explanation of the solar, the atmospheric, and the hot dark matter neutrino within the three generation scenario seems unnatural, in particular within GUT frameworks.<sup>18)</sup> In addition, accelerator and reactor experiments also constrain the allowed parameter regions. We comment on these matters below.

One of the natural ways to solve the problem is to introduce extra neutrinos which must be  $SU(2)_L \times U(1)_Y$  singlets (sterile) in view of the results of the LEP

<sup>\*)</sup> There is another solution with large mixing angle which is less preferable in view of the recent Superkamiokande reports on the day-night effect and the electron recoil energy spectrum.<sup>8)</sup>

data. Along this approach, many theoretical works have recently been made.<sup>19)</sup> However, if one assumes that the gauge unification or the left-right symmetry may be realized in nature, we should pursue an understanding of this neutrino spectrum from relations in some GUT framework.<sup>20)</sup> Then, the large mixing may be found to originate from the mixing with sterile neutrinos other than the ordinary three generations, since it is expected that the mixings between the ordinary neutrinos are small.

In this paper we present such a supersymmetric grand unified model based on the SO(10) gauge group in which an extra light neutrino is included and naturally has a large mixing with ordinary neutrinos. In this model, we add a pair of extra vector-like generations  $^{21)-25)}$  from which a sterile neutrino arises in addition to the ordinary three generations. The important feature of the model is that due to the existence of the extra generations (hereafter, we describe them as 4 and  $\overline{4}$  generations), all the gauge couplings become asymptotically non-free while preserving gauge coupling unification.<sup>22),23)</sup> This fact yields the strong convergence of Yukawa couplings to their infrared fixed points (IRFP),<sup>27)</sup> and with this property we can determine the texture of the quark and lepton mass matrices. In a previous paper,  $^{28)}$  we found that the texture is almost uniquely determined if we impose the condition that the masses of heavy up-type quarks (top and charm) are realized as their IRFP values. The most characteristic feature of this texture is that only the second generation strongly couples to the extra generations. This fact indicates that the muon neutrino may have a large mixing with the extra generations that is the origin of the atmospheric neutrino anomaly. Moreover, as we see below, using the GUT relations for Yukawa couplings, we can also fix the Majorana mass matrix of right-handed neutrinos. Then it is interesting to see how light neutrinos appear and how their mass matrix is predicted in this SO(10) model.

## §2. Quark and charged lepton mass matrix

Before going into the neutrino masses, we first summarize the ingredients of the previously obtained results which we need to analyze the neutrino mass matrix. As stressed above, in asymptotically non-free models, the IRFP behavior can determine the fate of the low-energy quark Yukawa couplings almost uniquely; all the quark Yukawa couplings with appreciable strength grow to be of order 1. Thus, in the present model, the dominant elements in the quark mass matrices are of the order of either the electroweak scale or the invariant mass scale at which the extra generations are decoupled (it is expected to be on the order of  $\text{TeV}^{24)-26}$ ). Another characteristic feature is the down to charged lepton mass ratio strongly enhanced by the strong gauge couplings. It requires that the down and charged lepton sectors, in particular bottom and tau, couple to Higgs fields of the  $\overline{126}$  representation of SO(10), which induces the ratio 1 : 3 for Yukawa couplings at the GUT scale. Combined with the enormous QCD enhancement factor of about  $5 \sim 6$  (in contrast to  $\sim 3$  in the MSSM), it can correctly reproduce the low-energy experimental value of the bottom-tau ratio,  $\sim 1.7$ . Note that the right-handed neutrino Majorana masses come from the standard gauge singlet component of  $\overline{126}$ -Higgs and therefore may be

proportional to the down and charged lepton sectors.

Since realistic texture should yield typical hierarchical structures, we can first fix the leading part of mass matrices (hereafter for simplicity, w and M are used symbolically to represent electroweak scale masses and invariant masses of the pair of extra generations, respectively). Among the 5 × 5 Dirac mass matrices, it is easily seen that the matrix elements relevant to the first generation can be neglected because of the hierarchy structures. Thus, we shall express the mass matrices in  $4 \times 4$  forms hereafter. The forms of the dominant elements in the quark and charged lepton mass textures at the GUT scale turn out to be as follows:<sup>28)</sup>

$$m_{u} = \begin{pmatrix} 2 & 3 & 4 & \bar{4} \\ & & & \\ 3 \\ 4 \\ \bar{4} \end{pmatrix} \begin{pmatrix} & & w \\ & & & \\ w & & & \\ & & & M \\ & & & M \end{pmatrix}, \quad m_{d} = \begin{pmatrix} 2 & 3 & 4 & \bar{4} \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & &$$

The above texture has the following characteristic properties: (i) The charm quark mass as well as the top quark mass is determined from their IRFP values. The charmto-top mass ratio is suppressed by the factor  $w^2/M^2$ , which comes from the existence of the heavy extra generations. (ii) It is interesting that the 2-4 (4-2) elements reach their IRFPs at a low energy whose value is of order 1. This indicates that the second generation is strongly coupled to the extra generations. (iii) The charged lepton masses are reproduced quite successfully by assuming that the relevant Higgs fields belong to the  $\overline{126}$  representation of SO(10), as noted above. (iv) The  $\epsilon$  parameter in the 3-3 elements is needed to reproduce the correct bottom-to-strange (or tau-to-mu) mass ratio, and its value is predicted to be  $\sim 0.2$ . Within this approximation, taking the parameters as  $M_{\rm GUT} \sim 5 \times 10^{16}$  GeV,  $\alpha_{\rm GUT} \sim 0.3$  and  $\tan \beta \sim 20$ , for example, we obtain the low-energy predictions at the  $M_Z$  scale,  $m_t \sim$  180,  $m_c \sim$  1.0,  $m_b \sim$ 3.1,  $m_s \sim 0.08, m_\tau \sim 1.75$  and  $m_\mu \sim 0.10$  (in GeV). These are in good agreement with the experimental data.<sup>29)</sup> The full mass matrices including quark mixing angles can be obtained by introducing hierarchically very small (less than order  $\epsilon^3$ ) Yukawa couplings. After all, we can obtain a reasonable  $5 \times 5$  GUT-scale texture which explains the experimental values of the CKM mixing angle. It should be stressed that the above mentioned texture is found to be actually the only possibility left in view of the IRFP structure.

#### §3. Neutrino mass and mixing

Let us proceed to the neutrino masses,  $m_{\nu}^{D}$  (Dirac) and  $m_{\nu}^{R}$  (right-handed Majorana). Once we fix the texture of the quark and charged lepton, the SO(10) gauge

symmetry can relate the neutrino mass texture to the quark mass textures. This time we have one more scale of the right-handed neutrino Majorana mass  $M_R$  in addition to M and w, among which a hierarchy exists:  $w < M \ll M_R$ .

Now, let us consider the mixing of the first generation, which is responsible for the solar neutrino problem. In the quark sector, it is known that the 1-2 mixing, that is, the Cabibbo angle is properly reproduced from the down-quark part only:  $\sin \theta_C \simeq$  $(m_d/m_s)^{1/2} \sim 0.22.30$  According to the GUT relation between quark and lepton Dirac mass matrices, the corresponding lepton 1-2 mixing angle is  $(m_e/m_\mu)^{1/2} \sim$ 0.07, which is disfavored more than at a  $2\sigma$  level for the MSW small angle solution.<sup>31)</sup> However, the lepton mixing consists of two parts, the charged lepton and neutrino parts. Since the GUT relations lead to a small mixing in the charged lepton sector, the large mixing angle  $(\sin\theta \sim 1/\sqrt{2})$  of the second generation required by the recent Superkamiokande report should come from the neutrino side in the present model. Then the lepton 1-2 mixing is predicted such that  $\sin \theta_{e\mu} \sim (m_e/m_{\mu})^{1/2} \times$  $1/\sqrt{2} \sim 0.05$ , which is now well within the desired range for the solar neutrino problem. After all, we do not have to consider the mixing of the first generation neutrino with the other generations, if only the second generation neutrino mixes strongly with the other generations except the first one. 32 It is noted that from the Superkamiokande atmospheric neutrino data (the zenith angle distribution of the e-like and  $\mu$ -like events data) and the recent results of the CHOOZ long-baseline oscillation experiment, <sup>33)</sup> large angle  $\nu_e \leftrightarrow \nu_\mu$  oscillation is found to be disfavored for the solution to the atmospheric neutrino anomaly.<sup>14)</sup> Hence the above described mechanism seems to work naturally and to be a likely scenario in GUT models. In the following, therefore, we can consider the  $4 \times 4$  neutrino mass matrices. From the quark texture  $(2 \cdot 1)$ , we can obtain the following texture for neutrinos:

Here we have used the GUT relation  $m_{\nu}^{D} = m_{u}$  and the fact that  $m_{\nu}^{R}$  comes from the 126-Higgs fields, namely,  $m_{\nu}^{R} \propto m_{d} (m_{e})$ . The above neutrino texture indicates the following: (i) One extra (sterile) neutrino in the  $\bar{4}$  generation is left to be almost massless and may couple strongly to the second generation (muon) neutrino. (ii) The third generation right-handed Majorana mass is slightly smaller than the others. This yields a heavier left-handed tau neutrino which could be the hot dark matter component. In the above texture we have assumed that the up-type quarks as well as neutrinos couple to the 10-Higgs, and, in particular, that the  $\bar{4}$ - $\bar{4}$  elements do not come from 126-Higgs (not 126). This may be easily realized when one introduces relevant Higgs multiplets with a flavor U(1) (gauge) symmetry (see the Appendix). However, it is interesting that almost all parts of the above texture can be fixed from the characteristic IR property of this model without such symmetry arguments.

As seen from the textures (2.1), (2.2) and (3.1), the third generation is almost

decoupled and can be neglected in the following analyses. In the remaining part, the masses of two of the six neutrinos (the second and fourth right-handed neutrinos) are on the order of the intermediate scale  $M_R$ . In this way the neutrino texture is reduced to a  $4 \times 4$  matrix with light elements. Then, the problem is whether the mixing angle between light neutrinos can become very large. After integrating out the heavy right-handed neutrinos of the second and fourth generations, we obtain the following mass matrix in the basis of  $(\nu_{2_2}, \nu_{\bar{4}_1}, \nu_{4_2}, \nu_{\bar{4}_2})$  (the second subscripts represent the transformation properties under the  $SU(2)_L$ ),

$$\begin{pmatrix} 2\alpha m & \alpha m' & m \\ \alpha m' & m' & w \\ m & m' & -m & M \\ & & & M \end{pmatrix},$$
(3.2)

where *m* and *m'* are masses induced by the seesaw mechanism<sup>34)</sup>  $(m \sim \frac{w^2}{M_R}, m' \sim \frac{wM}{M_R})$  and are much smaller than *w* and *M*. Therefore we are left with two very light neutrinos with masses  $\sim O(m, m')$ , which come mainly from  $\nu_{2_2}$  and  $\nu_{\bar{4}_1}$ . In the above matrix, *M* is an invariant mass of the extra lepton doublets. Its value is estimated as  $M \gtrsim 200$  GeV if one takes account of the constraints for the extra vector-like quark masses ( $\gtrsim 1$  TeV) from the FCNC<sup>25)</sup> and *S*, *T* and *U* parameters,<sup>26)</sup> and the relative QCD enhancement factor ( $\sim 5$ ) between quarks and leptons in this model. There also appear non-zero matrix elements with a factor  $\alpha$  which come from the induced neutrino Dirac mass elements via the one-loop renormalization group. This  $\alpha$ , representing the ratio of the induced to tree-level Dirac masses, is almost independent of the input parameters (tan  $\beta$ ,  $\alpha_{\rm GUT}$ , etc.) and its typical value is  $|\alpha| \sim 0.1$ . By diagonalizing the mass matrix (3·2), the mixing angle between the light neutrinos ( $\nu_{2_2}, \nu_{\overline{4}_1}$ ) becomes

$$\tan 2\theta = \frac{2m'\alpha\cos\phi - 2m\sin\phi}{m'\sin 2\phi + m(2\alpha + \sin^2\phi)}, \qquad (3.3)$$
$$\tan\phi \equiv \frac{w}{M}.$$

Since m/m',  $\tan \phi \sim w/M \ll 1$ , we have

$$\tan 2\theta \sim \frac{\alpha}{\sin \phi} \,. \tag{3.4}$$

By taking typical values of  $\alpha$  and w, the mixing angle becomes

$$\sin^2 2\theta \sim \frac{1}{1 + \left(\frac{350}{M \,(\text{GeV})}\right)^2 \cos^2 \beta},\tag{3.5}$$

with  $\tan \beta$ , the ratio of the vacuum expectation values of two doublet Higgses. From this, for  $\tan \beta \gtrsim 3$ , we can naturally obtain a large mixing angle for a suitable parameter range ( $M \gtrsim 200$  GeV) (Fig. 1).

To be more exact, some of the blank elements in the matrix (3.2) are radiatively induced as well if the invariant masses come from Yukawa couplings to a singlet field.<sup>28)</sup> Then the light neutrino mass matrix becomes

$$\begin{pmatrix} 2\alpha m & \alpha m' & m & \gamma M\\ \alpha m' & \alpha' m'' & m' & w\\ m & m' & -m & M\\ \gamma M & w & M \end{pmatrix}, \qquad (3.6)$$



Fig. 1. The mixing angle between the second and anti-fourth generations.

where m'' represents the seesaw induced mass  $(m'' \sim \frac{M^2}{M_R})$ , and  $\alpha'$  and  $\gamma$  are the relative ratios of the renormalization group induced mass parameters to the tree level ones. They are again almost independent of the input values. The typical values are  $|\alpha'| \sim 0.01$  and  $|\gamma| \sim 0.1$ . This texture (3.6) is just a realization of the recently proposed singular seesaw matrix, <sup>35)</sup> and two of the above four neutrinos remain very light. An analytic expression for the mixing angle of the remaining two neutrinos is

$$\tan 2\theta = 2\left(-m''\alpha'\cos\phi\cos\phi' + m'(\sin\phi\sin\phi' + \alpha\cos\phi'\cos2\phi) - m\cos\phi(\sin\phi' - 2\alpha\sin\phi\cos\phi')\right) / \left(m''\alpha'(\sin^2\phi + \cos^2\phi\cos^2\phi') + m'\left(\cos\phi\sin2\phi' - \alpha\sin2\phi(1 + \cos^2\phi')\right) + m\left(\sin^2\phi' + \sin\phi\sin2\phi' + 2\alpha(\cos^2\phi - \sin^2\phi\cos^2\phi')\right)\right), \quad (3.7)$$
$$\tan\phi = \frac{\gamma M}{1 + 1}, \quad \tan\phi' = \frac{\gamma}{1 + 1}. \quad (3.8)$$

$$an \phi = \frac{\gamma M}{w}, \quad \tan \phi' = \frac{\gamma}{\sin \phi}.$$
(3.8)

We now consider the numerical estimations. Since the third generation neutrino is identified with the hot dark matter component and it is almost decoupled from the other generations, the intermediate scale  $M_R$  is mainly determined from the eigenvalue  $m_3$ . We find that the desired tau neutrino mass is obtained if we take  $M_R$  as  $10^{12}$  GeV  $\lesssim M_R \lesssim 10^{13}$  GeV (Fig. 2). Then, for the solar and atmospheric neutrino anomalies, the value  $\Delta m^2$  and the mixing angles depend on the other parameters and are especially sensitive to  $\tan \beta$  and M, as indicated above. In Figs. 3-5, we display acceptable solutions as an example, and typical values of the masses and mixing angles are

$$\Delta m_{12}^2 \simeq 1.0 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{e\mu} \simeq 0.012, \quad (3.9)$$

$$\Delta m_{2\bar{4}}^2 \simeq 1.1 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\mu s} \simeq 0.82, \qquad (3.10)$$

$$m_3 \simeq a \text{ few eV},$$
 (3.11)



Fig. 2. The  $M_R$  dependence of the eigenvalue  $m_3$  (mass of the hot dark matter neutrino).



Fig. 4. The predicted value of  $\Delta m^2$  for the atmospheric neutrino anomaly.



Fig. 3. The predicted value of  $\Delta m^2$  for the solar neutrino anomaly.



Fig. 5. The predicted value of  $\sin^2 2\theta$  for the atmospheric neutrino anomaly.

for  $M_R \sim 4 \times 10^{12}$  GeV,  $\tan \beta \sim 30$  and  $M \sim 250$  GeV. These are in good agreement with the experimental observations of the solar, atmospheric and hot dark matter neutrinos.

A few comments are in order concerning other experimental results. In this model, the sterile neutrino has a large mixing with the muon neutrino, and this solves the atmospheric neutrino anomaly. In this scheme, the positive LSND results of the  $\nu_e \leftrightarrow \nu_\mu$  oscillation<sup>36)</sup> can be reconciled at a  $3\sigma$  level only<sup>37)</sup> with indirect oscillation<sup>38)</sup> through the tau neutrino, or it can be certainly explained by the sterile neutrino with a heavier mass. However, at this time the zenith angle dependence of the atmospheric neutrino data is not explained due to a large value of  $\Delta m^2$ . On the other hand, the recent results of the KARMEN experiment <sup>39)</sup> seem to exclude almost the entire allowed parameter region of the LSND, so it may not be necessary to take the LSND results seriously in this paper. The discrimination between two oscillation scenarios,  $\nu_{\mu} \leftrightarrow \nu_{\tau}$  and  $\nu_{\mu} \leftrightarrow \nu_s$ , for the solution to the atmospheric neutrino anomaly will be made by ongoing and forthcoming experiments observing various quantities.<sup>40)</sup> The recent Superkamiokande reports indicate that the observed suppression of the NC-induced  $\pi^0$  events is consistent with  $\nu_{\mu} \leftrightarrow \nu_{\tau}$  oscillation, but they have not excluded  $\nu_{\mu} \leftrightarrow \nu_s$  oscillation as yet. The cosmological and astrophysical implications of the existence of the fourth light neutrino should also be addressed. In particular, the big-bang nucleosynthesis scenario may severely constrain the effective number of light neutrino species to be less than four and then exclude the large mixing angle between active and sterile neutrinos.<sup>41)</sup> However, according to recent estimations,<sup>42)</sup> more than four light neutrinos are acceptable, and there is no constraint on the mixing angles. Even if the constraint is revalued and the allowed number turns out to be less than four, there is an interesting and simple mechanism which has recently been proposed.<sup>43)</sup> In order to avoid the constraints, a large lepton asymmetry ( $\geq 10^{-5}$ ) is required for which a small mixing between the active (tau) and sterile neutrinos is needed. This can be easily realized in the present model.

### §4. Summary and discussion

In summary, we have investigated a supersymmetric SO(10) model with a pair of extra vector-like generations. In this model, the textures are almost uniquely determined by the IRFP structures due to the asymptotic non-freedom of gauge couplings and the GUT relations between quark and lepton. We have particularly examined the neutrino sector and found the following: (i) By assuming that the  $\overline{4}$ generation couples to 10-Higgs, one of the extra  $SU(2)_L$  singlet neutrinos becomes very light, and has a very large mixing with the muon neutrino. This large mixing can explain the atmospheric neutrino anomaly. (ii) The texture requires that the third generation right-handed neutrino is slightly lighter than the others, resulting in the heavier left-handed tau neutrino to become a hot dark matter candidate. (iii) The solar neutrino problem can be explained by the mixing with the muon neutrino consistently with the mixing angle expected from the GUT relation with the Cabibbo angle.

Noting that the supersymmetry breaking scale is of the same order as the invariant masses of the extra generations, we may discover the extra fermions when supersymmetry is found. Moreover, using muon colliders,<sup>44)</sup> the extra generations may be explored easily, since in the present model the second generation strongly couples to the extra generations. It is interesting that the extra generations appear themselves via the second generation in the neutrino sector. We would like also to stress that neutrinos are more appropriate subjects to be investigated in seeking the extra generations, and we hope that the sterile neutrino scenario will be confirmed by a new generation of experiments.

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## Appendix

The texture zeros can arise due to symmetries in the underlying string or GUT theory. In this appendix, we give an example which reproduces the textures adopted in this paper. Although there may be many possibilities that realize the desired texture and among them there might exist simpler choices, it would be instructive to see how the desired patterns of the texture come about from such kind of flavor symmetry.

Let us consider the case in which the matter and Higgs fields have additional flavor U(1) charges. We consider the Higgs multiplets of the SO(10) representation  $\Phi_{1,2}(210)$ ,  $\Delta_{1,2}(126)$ ,  $\bar{\Delta}_{1,2}(\overline{126})$ ,  $H_{1,2,3,4}(10)$ , and  $\theta(1)$  as well as the matter superfields  $\Psi_{1,2,3,4}(16)$  and  $\bar{\Psi}_{\bar{4}}(\overline{16})$ . Their charges under the U(1) symmetry are given in Table I. Then, the gauge and flavor invariant superpotential becomes

$$W = (H_1 + \bar{\Delta}_1)\Psi_2\Psi_4 + H_2\Psi_3\Psi_3 + \bar{\Delta}_1\theta\Psi_3\Psi_3 + \bar{\Delta}_2\Psi_4\Psi_4 + H_3\bar{\Psi}_{\bar{4}}\bar{\Psi}_{\bar{4}} + H_1\bar{\Delta}_1\Phi_2 + H_3\Delta_1\Phi_1 + W_m + W_G.$$
(A·1)

The term  $W_m$  contains the relevant mass terms of the above Higgs fields by some of which the U(1) flavor symmetry may be softly broken. Suppose that the SO(10)gauge symmetry is broken down to the standard gauge group by  $W_G$  for an appropriate choice of Higgs couplings (probably, including more Higgs multiplets (45-, 54-Higgs) in addition to the above ones). The vacuum expectation values of singlet components in the  $\Phi$  can break not only the SO(10) but also D-parity.<sup>45)</sup> This parity breaking is favored for several phenomenological reasons<sup>46)</sup> and in particular it can suppress direct left-handed neutrino Majorana mass terms<sup>47)</sup> which we do not consider in this paper. As is easily seen, since all the desired Yukawa couplings are contained in the above superpotential, we need to introduce terms in  $W_m$  (and  $W_G$ ) so that one linear combination of the doublet Higgses may remain light.<sup>48)</sup> This can be easily done by the choice of the softly broken mass terms in  $W_m$ , for example,

$$W_m = m_1 H_1 H_4 + m_2 H_2 H_4 + m_3 \Delta_1 \overline{\Delta}_2 + m_4 \Delta_2 \overline{\Delta}_1 + m_5 \Delta_2 \Delta_2. \quad (A.2)$$

With these terms, together with the other ones in W, a pair of linear combinations of  $H_1, H_2$  (for up-type doublet Higgs) and  $H_3, \bar{\Delta}_1, \bar{\Delta}_2$  (for down-type doublet Higgs) remain light in the low-energy region and give mass terms to the matter superfields, provided that the phenomenologically favored breaking chain<sup>49</sup> is assumed.

$\overline{\Psi_2}$	$\Psi_3$	$\Psi_4$	$ar{arPsi}_{ar{4}}$	$H_1$	$H_2$	$H_3$	$H_4$	$\Delta_1$	$\Delta_2$	$ar{\Delta}_1$	$\bar{\Delta}_2$	$\Phi_1$	$\Phi_2$	θ
3	1	0	4	-3	-2	-8	1	-1	-6	-3	0	9	6	1

Table I. U(1) quantum number assignments.

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