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Research Article

Stiffness Calculation Method and Stiffness Characteristic Analysis of Bolted Connectors

Shi-kun Lu,[1,2] Deng-xin Hua,[1] Yan Li,[1] Fang-yuan Cui,[1] and Peng-yang Li[1]

[1]*Xi'an University of Technology, Xi'an 710048, China*
[2]*Laiwu Vocational and Technical College, Laiwu 271100, China*

Correspondence should be addressed to Yan Li; jyxy-ly@xaut.edu.cn

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At present, few scholars have studied the effect of surface roughness on assembly stiffness. The influence of the joint surface stiffness on the overall stiffness is neglected. In this paper, a new method for calculating the stiffness of bolted joints is presented. The effect of joint surface stiffness on the overall stiffness is considered. Firstly, the relationship between load and displacement between cylinder and cylinder (including the joint surface with certain roughness) is studied, and the stiffness characteristic expression of the joint surface is obtained; the results are compared with the traditional stiffness calculation theory, and then, the influence of bolt connection surface on bolt connection is studied and compared with the stiffness calculation results of traditional bolt connection. The results show that the theoretical model presented in this paper is more practical.

1. Introduction

Bolt connection is an important connection mode of mechanical connection. It is widely used in all kinds of mechanical connection and plays a very important role in the connection of mechanical equipment. Therefore, it is necessary to study the bolt connection. The stiffness of bolt connection also plays a very important role in the design of machine tools.

Many scholars have studied the stiffness characteristics of bolted connectors. Gomes et al. [1] studied the fracture behavior of bolted bolt preloading and surface treatment. Vilela et al. [2] performed a numerical simulation of the bolted connection. Some scholars [3] have established a theoretical calculation method for the stiffness of bolted connectors. Some scholars have studied the stiffness characteristics of bolted connectors by combining experiment with theory. Some scholars have also used the finite element method [4–8]. Pedersen analyzed the stiffness of bolted connectors and then obtained the formula for calculating the stiffness of bolted connectors by fitting.

Tocher [9] proposed a method for calculating the stiffness of bolted connectors. He believed that under the pressure of pretightening bolts, there was a conical force influence zone in the bolted connectors. In this case, the compressive stiffness coefficient k_{member} of the members could be calculated by the following formula:

$$k_{\text{member}} = \frac{E_{\text{member}}\pi}{4L_{\text{member-total}}} \cdot \left\{ \left(d_{\text{washer}} + \frac{L_{\text{member-total}}}{2} \tan\alpha \right)^2 - D_{\text{hole}}^2 \right\}. \quad (1)$$

The stiffness calculation method proposed by Tocher [9] is the stiffness calculation method of two members under the same condition. When two thicknesses are different, what is the calculation method of stiffness? In [8], Williams gives a method for calculating the stiffness of the joints under this condition. The formula for calculating the stiffness of member1 (upper member) under bolt pretension is as follows:

$$k_{\text{member1}} = \frac{\pi E_{\text{member1}} d_{\text{bolt}} \tan\alpha}{\ln\left[\left(2t_{\text{member1}}\tan\alpha + d_{\text{washer}} - d_{\text{bolt}}\right)\left(d_{\text{washer}} + d_{\text{bolt}}\right)/\left(2t_{\text{member1}}\tan\alpha + d_{\text{washer}} + d_{\text{bolt}}\right)\left(d_{\text{washer}} - d_{\text{bolt}}\right)\right]}. \tag{2}$$

The calculation formulas of the upper part stiffness k_{member21} of the member2 (lower member) with larger thickness and the lower part k_{member22} of the member2 (lower member) with larger thickness can be, respectively, expressed as

$$\begin{aligned} k_{\text{member21}} &= \frac{\pi E_{\text{member2}} d_{\text{bolt}} \tan\alpha}{\ln\left\{\begin{array}{l}\left[\left(t_{\text{member2}} - t_{\text{member1}}\right)\tan\alpha + 2t_{\text{member1}}\tan\alpha + d_{\text{washer}} - d_{\text{bolt}}\right]\left(2t_{\text{member1}}\tan\alpha + d_{\text{washer}} + d_{\text{bolt}}\right)/ \\ \left[\left(t_{\text{member2}} - t_{\text{member1}}\right)\tan\alpha + 2t_{\text{member1}}\tan\alpha + d_{\text{washer}} + d_{\text{bolt}}\right]\left(2t_{\text{member1}}\tan\alpha + d_{\text{washer}} - d_{\text{bolt}}\right)\end{array}\right\}}, \\ k_{\text{member22}} &= \frac{\pi E_{\text{member2}} d_{\text{bolt}} \tan\alpha}{\ln\left\{\begin{array}{l}\left[\left(t_{\text{member2}} + t_{\text{member1}}\right)\tan\alpha + d_{\text{washer}} - d_{\text{bolt}}\right]\left(d_{\text{washer}} + d_{\text{bolt}}\right)/ \\ \left[\left(t_{\text{member2}} + t_{\text{member1}}\right)\tan\alpha + d_{\text{washer}} + d_{\text{bolt}}\right]\left(d_{\text{washer}} - d_{\text{bolt}}\right)\end{array}\right\}}. \end{aligned} \tag{3}$$

According to their relationship, the total stiffness of bolted connections can be expressed as

$$k_{\text{member}} = \frac{k_{\text{member1}} k_{\text{member21}} k_{\text{member22}}}{k_{\text{member21}} k_{\text{member22}} + k_{\text{member1}} k_{\text{member22}} + k_{\text{member1}} k_{\text{member21}}}. \tag{4}$$

Shigley and Mischke [10] have also studied the calculation formula of bolting stiffness as follows:

$$\left\{\begin{array}{l} k_{\text{member}} = \dfrac{\pi E_{\text{member}} d_{\text{bolt}} \tan\alpha}{2\ln\left[\left(d_{\text{washer}} + L_{\text{member-total}}\tan\alpha - d_{\text{bolt}}\right)\left(d_{\text{washer}} + d_{\text{bolt}}\right)/\left(d_{\text{washer}} + L_{\text{member-total}}\tan\alpha + d_{\text{bolt}}\right)\left(d_{\text{washer}} - d_{\text{bolt}}\right)\right]}, \\ k_{\text{member}} = \dfrac{0.577\pi E_{\text{member}} d_{\text{washer}}}{2\ln\left[5\left(0.577L_{\text{member-total}} + 0.5d_{\text{bolt}}\right)/\left(0.577L_{\text{member-total}} + 2.5d_{\text{bolt}}\right)\right]}, \quad \alpha = 30^{\circ},\ d_{\text{washer}} = 1.5d_{\text{washer}}. \end{array}\right. \tag{5}$$

In addition, more and more scholars [11, 12] have studied related aspects, such as Juvinall and Marshek [13], and proposed a simpler stiffness calculation method for estimating the clamping area. This stiffness formula is obtained when the gap between bolt and connector is very small.

The German Industrial Association (VDI) [14] provides a formula for calculating the stiffness of members. The formulas for calculating the stiffness of members in two different cases are given, which is a revision of the stiffness theory of Shigley and Mischke members.

In addition to the establishment of theoretical models, many scholars have carried out finite element analysis on the stiffness of fasteners and fitted the stiffness calculation formula.

Sethuraman and Kumar [15] used the finite element method to analyze the bolts in two different ways and fitted the stiffness calculation formula.

Wileman et al. [16] found the stiffness of bolt members is analyzed by the finite element method and the stiffness calculation formula is fitted based on the assumption that the diameter of washer type screw is 1.5 times. In addition, Zhang and Poirier [17] have carried out finite element analysis and fitted the stiffness calculation formula.

All the above studies on the stiffness of bolted joints do not consider the effect of surface roughness on the stiffness.

They all assume that the surface is smooth. In reality, the surface of machined parts has a certain roughness, which cannot be smooth. Therefore, the characteristics of the joint surface will certainly have an impact on the stiffness of bolted joints. At present, there are few related studies and the effect of joint stiffness on the overall stiffness has been neglected. Therefore, it is necessary to study. In this paper, a new method for calculating the stiffness of bolted joints is presented and the effect of joint stiffness on the overall stiffness is considered. The results of the proposed method are compared with those of the traditional method. Research in this paper shows that the theoretical model presented in this paper is more practical. The structure of this article is shown in Figure 1.

2. Stiffness Test of Joint Surface

2.1. Stiffness Testing Theory of Joint Surface. Because the machined surface is not smooth and consists of a series of uneven peaks and valleys (as shown in Figure 2), forming a certain roughness, the contact of the two rough surfaces forms a joint surface. Many scholars have studied the stiffness of the joint surface: some studies are based on the fractal theory [17–22] from a micro perspective and some scholars have modeled the stiffness of the joint surface based on experiments. The stiffness characteristic formula is obtained by test [23–26]. The experimental results show that there is a relationship between the normal pressure of the joint surface and the comprehensive deformation of the joint microconvex body as follows:

$$\lambda_{\text{joint}} = c_{\text{normal}} p_{\text{normal}}^{m_{\text{normal}}}. \tag{6}$$

Formula (6) is shifted, and the derivative is obtained. The relationship between the comprehensive deformation and stiffness of the microconvex body on the joint surface is obtained as follows:

$$k_{\text{joint}} = \frac{1}{m_{\text{normal}}} c_{\text{normal}}^{-(1/m_{\text{normal}})} \lambda_{\text{joint}}^{(1-m_{\text{normal}})/m_{\text{normal}}} \cdot s_{\text{joint}}. \tag{7}$$

The relationship between contact stress and stiffness of joint surface can be obtained by substituting formula (6) with formula (7):

$$k_{\text{joint}} = \frac{1}{c_{\text{normal}} m_{\text{normal}}} p_{\text{normal}}^{1-m_{\text{normal}}} \cdot s_{\text{joint}}. \tag{8}$$

Given the interface composed of cylinder1 and cylinder2 as shown in Figure 3, the area of the interface is S_{cylinder}, the height of cylinder1 is $H_{\text{cylinder1}}$, the elastic modulus is $E_{\text{cylinder1}}$, the height of cylinder2 is $H_{\text{cylinder2}}$, and the elastic modulus is $E_{\text{cylinder2}}$. Under the action of load F_{normal}, the total deformation can be expressed as follows:

$$\lambda_{\text{total}} = \frac{c_{\text{normal}} \left(F/A_{\text{cylinder}}\right)^{m} A_{\text{cylinder}} E_{\text{cylinder1}} E_{\text{cylinder2}} + F\left(H_{\text{cylinder1}} E_{\text{cylinder2}} + H_{\text{cylinder2}} E_{\text{cylinder1}}\right)}{A_{\text{cylinder}} E_{\text{cylinder1}} E_{\text{cylinder2}}}. \tag{9}$$

If the material of two contact cylinders is the same, that is, $E_{\text{cylinder1}} = E_{\text{cylinder21}} = E_{\text{cylinder}}$, formula (9) can be reduced to

$$\lambda_{\text{total}} = \frac{c_{\text{normal}} \left(F/A_{\text{cylinder}}\right)^{m} A_{\text{cylinder}} E_{\text{cylinder}} + F\left(H_{\text{cylinder1}} + H_{\text{cylinder2}}\right)}{A_{\text{cylinder}} E_{\text{cylinder}}}. \tag{10}$$

2.2. Experimental Test of Joint Surface. This paper designs an experiment as shown in Figure 4. The experimental device consists of a load-bearing frame (fixed frame), a bolt pretightening device, a load transmission mechanism, a displacement sensor, a load sensor, a displacement display, a load display, and a data transmission wire.

The model of displacement sensor is SPN-S4V, and the repetition accuracy of the displacement sensor used is 0.1 μm. The sensing accuracy of the weighing sensor is 1/1000, its transmitting accuracy is 1/1000, and its range is 0–300 kg.

The principle of joint surface deformation measurement is as follows: applying torque to pretightening bolt and transforming torque into load. Load is transferred from transmission mechanism to tested parts and joint surface. Load makes joint surface deformed. Force sensor measures the applied load, and displacement sensor measures the deformation of joint surface.

The material of the experimental workpiece is steel 45 (Figure 5). The joint surface is machined by milling of the NC machining center. When machined the workpiece surface, the spindle speed of the NC machining center is $s = 800$ r/min, the feed speed of the NC machining center is $f = 480$ mm/min, the diameter of the milling cutter head is 120 mm, and the arc radius of the cutting cutter is 1 mm. It is known that the surface roughness of the workpiece

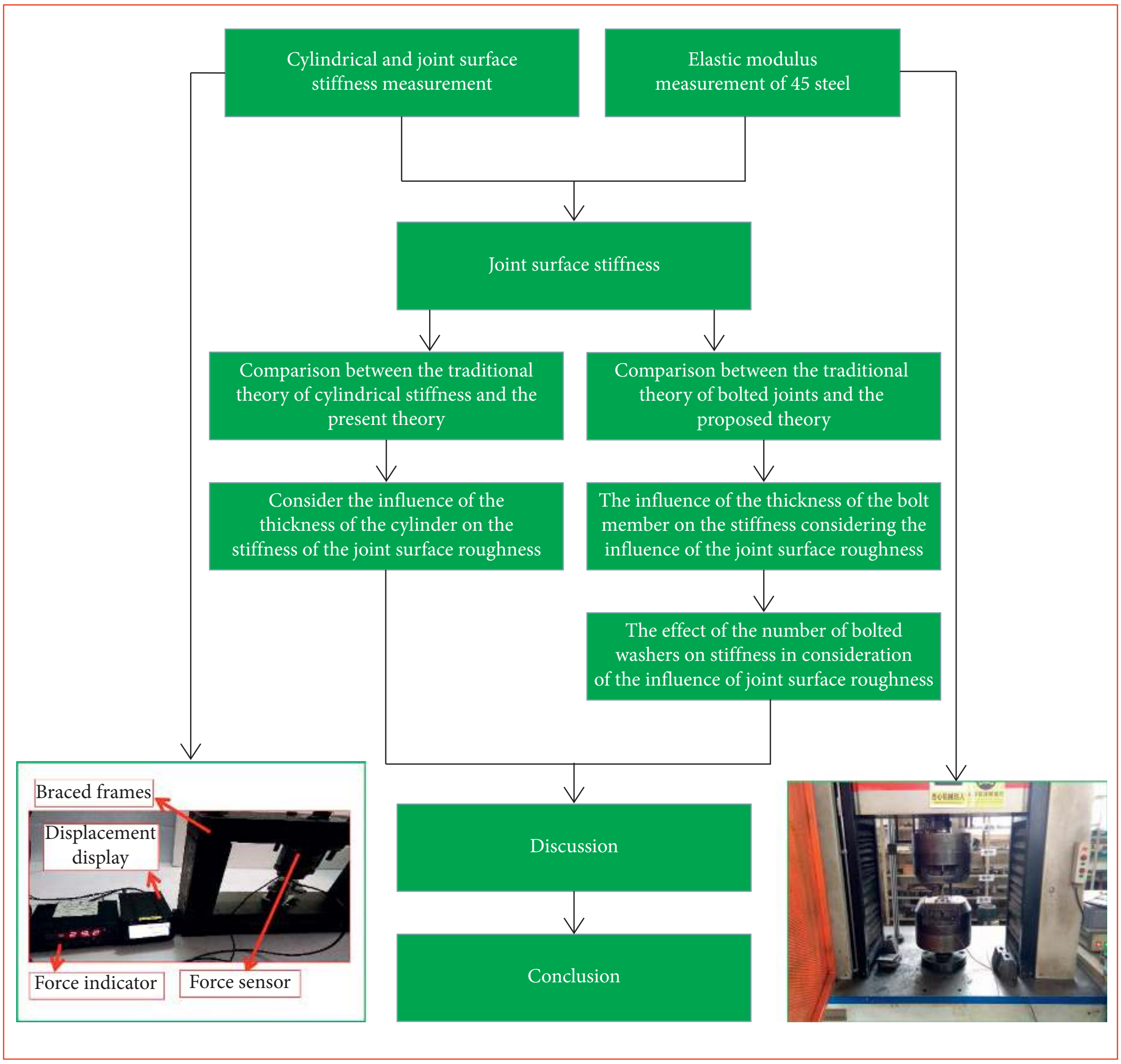

Figure 1: Content diagram of articles.

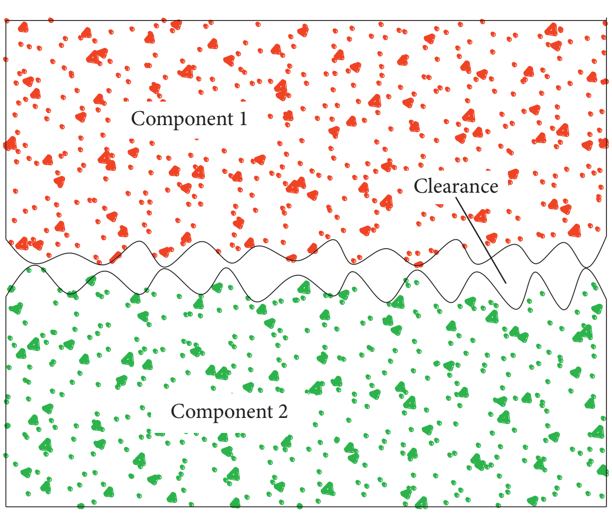

Figure 2: Enlarged view of rough surface interface.

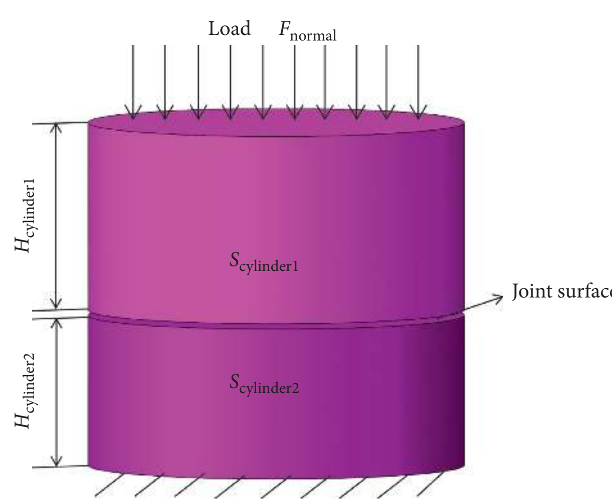

Figure 3: Rough cylindrical joint surface.

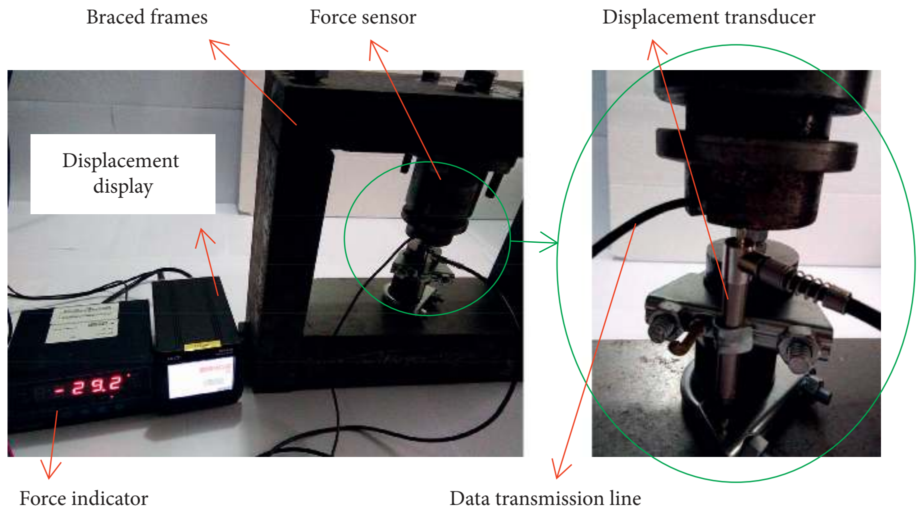

Figure 4: Experiment of joint stiffness.

Figure 5: Tested piece.

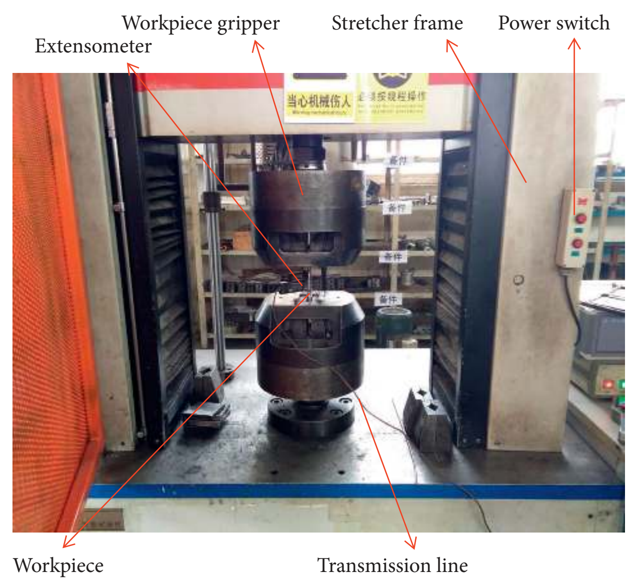

Figure 6: Tensile test of 45 steel.

machined is between 4.6 microns measured by using a white light interferometer. When measuring, the joint surface is cleaned, impurities and oil are removed, and the contact between the joint surface is dry friction contact.

2.3. Measurement of Elastic Modulus of Materials. Since the elastic modulus of steel varies within a certain range, for a specific steel, if it is desired to accurately know its elastic modulus, it can only be obtained by a test method.

Elastic modulus test standard parts are made of the same batch of materials as the deformation test above, and then, the tensile test is carried out on the precision tensile testing machine (Figure 6) to determine the elastic modulus of materials, and the precision extensometer is used in the testing process. The material was pulled apart during the test (Figure 7).

Figure 7: Deformation of standard test parts after tension.

The model of the material tensile testing machine used is DNS600, and its maximum tensile load can reach 60 tons. Material fixture is a hydraulic fixture. The initial clamping load is 1000 N. With the continuation of the stretching process, the clamping load of the hydraulic fixture increases rapidly and the final clamping load is far greater than 1000 N. Therefore, when the material is broken (Figure 7), the clamping load is far greater than 1000 N. Hence, we can see that the material is clamped into a flat shape (Figure 7).

In the course of measurement, the model of extensometer used is CBY1 50-5, its accuracy can reach 1/10,000, and the measurement error is 1/100.

The material of the test standard is 45 steel, and the chemical composition of C45 steel is as follows: 0.45% C, 0.04% S, 0.25% Si, max. 0.3% Cr, max. 0.3% Ni, 0.65% Mn, max. 0.3% Cu, and 0.04% P [27].

The elastic modulus of the material can be calculated from the load-deformation diagram of the actual device (Figure 8) when the diameter of the material and the length of the workpiece are known and measured by using the extensometer.

2.4. Experimental Result. From Figure 8, Young's modulus $E = 200$ GPa of steel 45 can be calculated and, then, it is substituted into formula (10). Then, the measured data are fitted with Matlab so that the error between the curve and the experimental data is as small as possible. As shown in Figure 9, the characteristic relationship between the deformation and pressure of the fitted joint surface is as follows:

$$\lambda_{\text{joint}} = 6.8 \times 10^{9} \cdot p_{\text{normal}}^{0.499}. \quad (11)$$

The relationship between joint stiffness and pressure is as follows:

$$k_{\text{joint}} = 2.947071 \times 10^{8} \cdot p_{\text{normal}}^{0.501}. \quad (12)$$

As shown in Figure 9, the experimental value is compared with the theoretical value. From Figure 9, it can be seen that the experimental curve is in good agreement with the fitting theoretical curve and the error is a little large within 500 N. However, after that, the two are very close and the fitting is very successful as a whole.

3. Analysis of Stiffness Effect of Joint Surface

3.1. Cylinders and Its Joint Surface. The material of the cylinder is 45 steel and its elastic modulus is 200,000 MPa. As shown in Figure 10, the height of cylinder1 is $H_1 = 30$ mm and that of cylinder2 is $H_1 = 23$ mm. The normal (axial) compressive stiffness of the two structures is studied to study the effect of the characteristics of the joint surface on the overall axial stiffness.

As can be seen from Figure 11, the results of the traditional method without considering the effect of surface roughness are larger than those of the proposed method.

With the increase of the load on the joint surface, the calculation results of the method presented in this paper gradually increase, and gradually approach the calculation results of the traditional theory.

In order to further study the influence of the stiffness of the surface roughness on the overall stiffness, the thickness

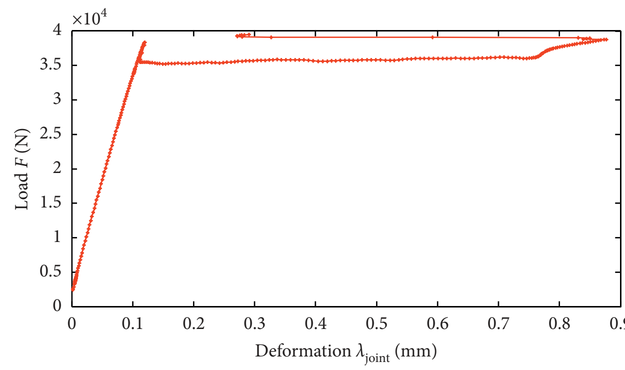

Figure 8: Load-deformation curve of 45 steel.

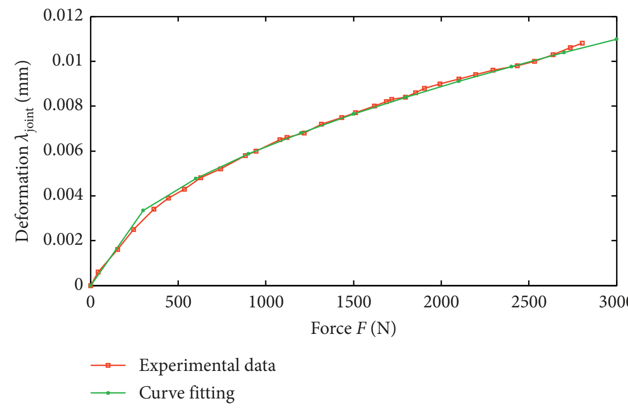

Figure 9: Fitting of test results.

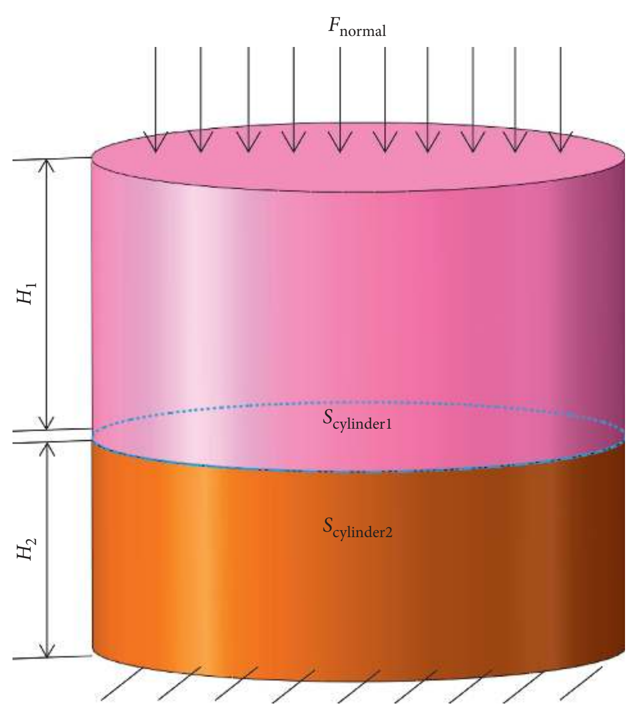

Figure 10: Rough-surface cylindrical joint surface.

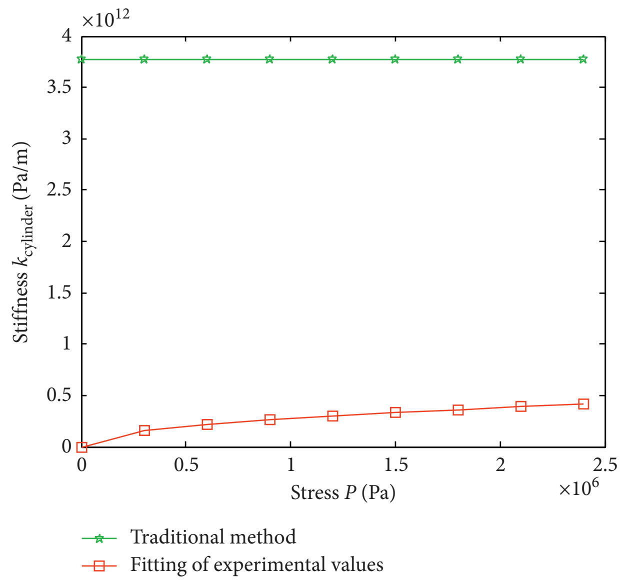

FIGURE 11: Comparison between fitting values of experimental values and traditional theoretical values.

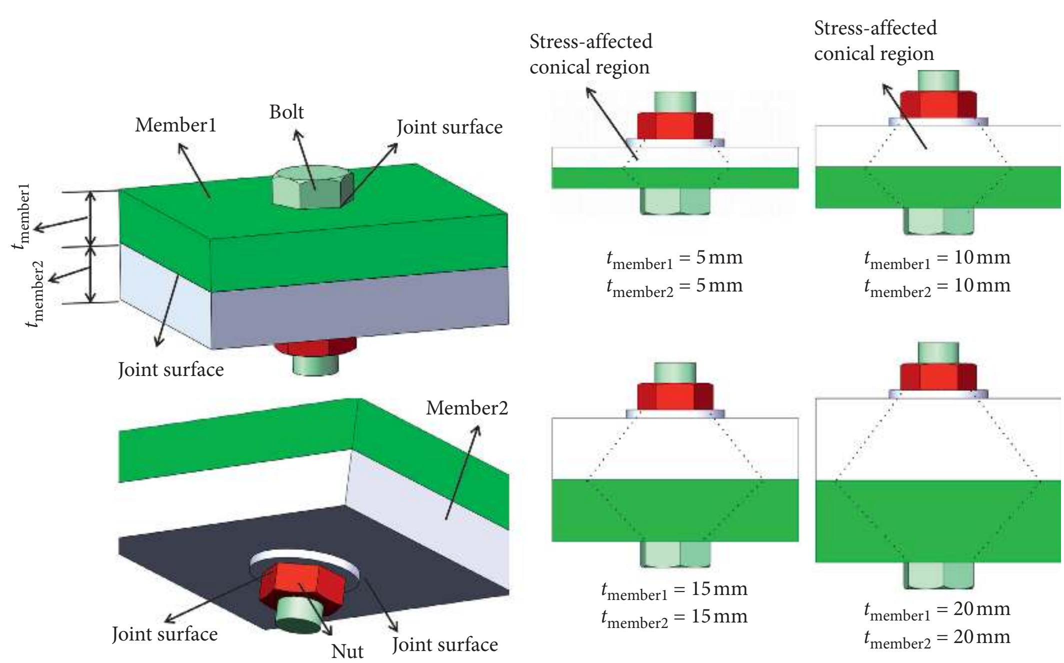

FIGURE 12: Bolted connectors with different thicknesses on rough surfaces.

of the upper member $H_{\text{cylinder1}} = 23$ mm, $H_{\text{cylinder2}} = 18$ mm, $H_{\text{cylinder3}} = 13$ mm, $H_{\text{cylinder4}} = 8$ mm are taken, respectively. The values of other parameters are the same as those before. The elastic modulus of material $E_{\text{cylinder}} = 200{,}000$ MPa, the diameter of cylinder $R_{\text{cylinder}} = 20$ mm, and the normal characteristic parameters of the surface roughness joint surface are obtained from experiments.

The influence of rough joint surface characteristics on the stiffness of bolted joints is studied. The thickness of the members is the same, and they are all 60 mm, the diameter of bolts is 10 mm, and the diameter of members is 11 mm. The traditional calculation method of the stiffness of the members is compared with the current calculation method. The material of the members is 45 steel, and the width of the member is enough to bolt to form the influence cone.

3.2. Bolted Members and Their Joint Surface. In order to further study the influence of stiffness characteristics of rough joints on bolted joints, the following research schemes are designed: as shown in Figure 12, M10 bolts are used with a bolt hole diameter of 11 mm and the thickness of bolted joints is $t_{\text{member1}} = t_{\text{member2}} = 5$ mm, $t_{\text{member1}} = t_{\text{member2}} = 10$ mm, $t_{\text{member1}} = t_{\text{member2}} = 15$ mm, and $t_{\text{member1}} = t_{\text{member2}} = 20$ mm,

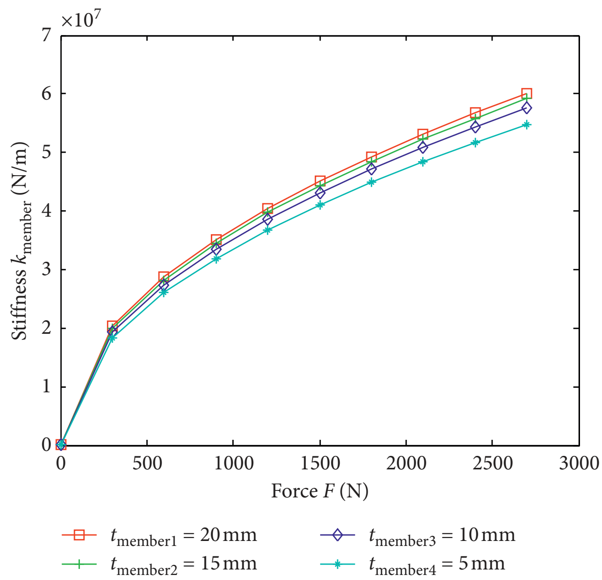

FIGURE 13: Stiffness of bolted joints with different thicknesses on rough surfaces.

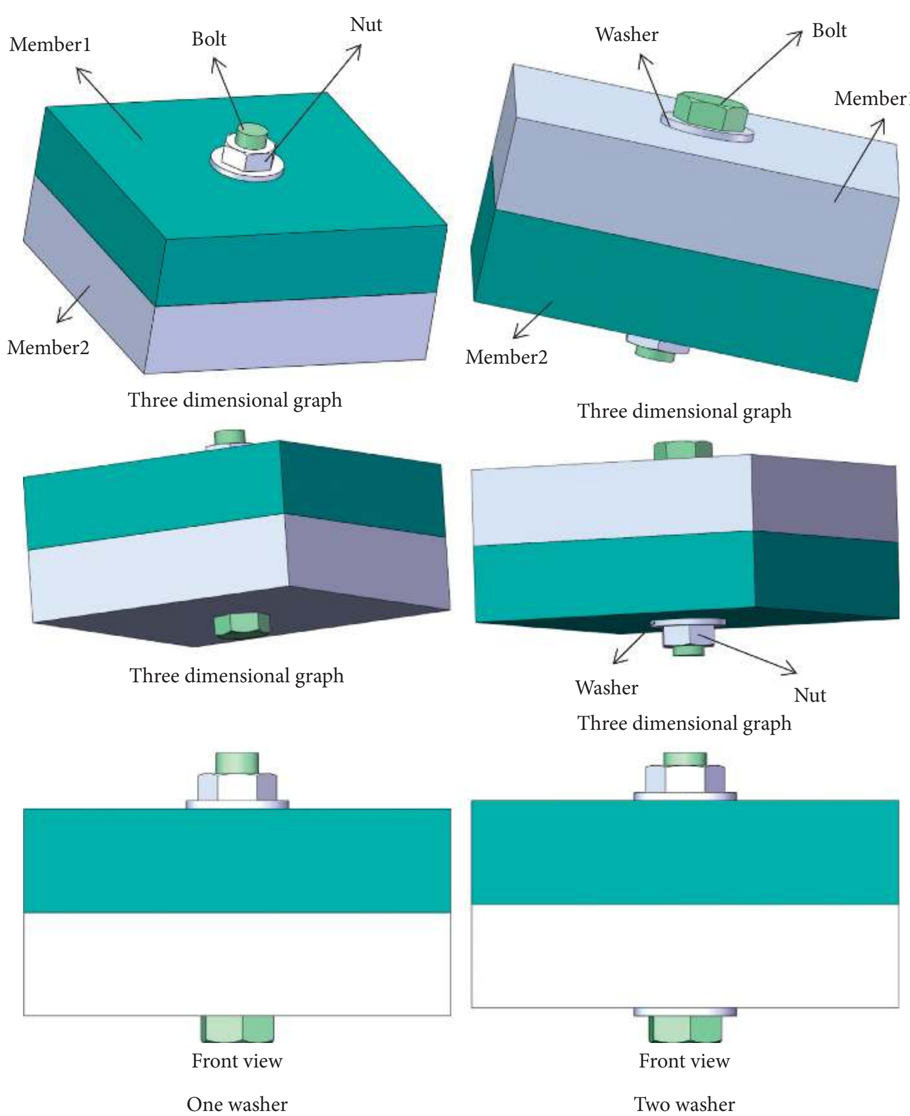

FIGURE 14: Bolted connections with members.

respectively. According to the theory of influence cone, the main load-bearing areas of the joints can be calculated and the elastic modulus of the materials of the members is 200,000 MPa. The analysis results are shown in Figure 13.

3.3. Influence of Washer and Joint Surface. In order to study the influence of the member on the overall stiffness, the research model shown in Figure 14 is designed. The thickness of the two members is 25 mm. The bolts of M10 are used to connect the members. The elastic modulus of the member material is $E = 200{,}000$ MPa. The length and width of the members are 100 mm. The diameter of the bolt holes of the members is 11 mm. The bolts are of standard specifications.

There are two ways of connection: one is to install a member at the nut end and the other is to install a member at the bolt head and the nut end, respectively.

4. Results and Discussion

Figure 11 shows that under the condition of uniform normal load, the results calculated by the traditional method are larger than those obtained by experiment. Therefore, it shows that the stiffness of the rough surface formed by the machined surface has a certain influence on the overall stiffness of equipment. It is unreasonable that the stiffness calculated by the traditional method does not consider the effect of surface roughness on the stiffness. The results show that the stiffness of the joint reduces the overall stiffness of the mechanism.

As can be seen from Figure 11, the results of the traditional method without considering the effect of surface roughness are larger than those of the proposed method. With the increase of the load on the joint surface, the calculation results of the method presented in this paper gradually increase and gradually approach the calculation results of the traditional theory.

Figure 15 shows that when the cylinder is subjected to uniform load, the height of the cylinder has an effect on the overall stiffness. The effect is that the overall stiffness increases with the decrease of thickness, but the increase is relatively small.

Figure 16 shows that for bolted connections with rough surfaces, the calculated values of traditional theory are larger than those of the present theory. The stiffness characteristics of rough joints have an effect on the overall stiffness of the structure. With the increase of normal load, the overall stiffness gradually increases and gradually approaches the traditional theoretical calculation value. The larger the normal load, the closer the calculation value of this method is to the traditional calculation value.

Figure 13 shows that the thicker the bolted connection is, the greater the overall stiffness is. This is contrary to the conclusion of the uniformly loaded cylindrical members (Figure 15). From Figure 13, it can be seen that the overall stiffness of uniformly loaded cylindrical members decreases with the increase of the thickness of the members. The two conclusions are contrary because the effective area of the

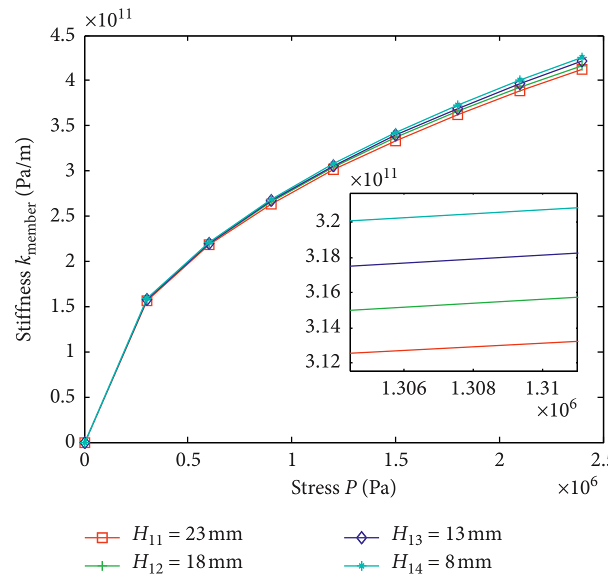

Figure 15: Effect of cylinder thickness on global Stiffness.

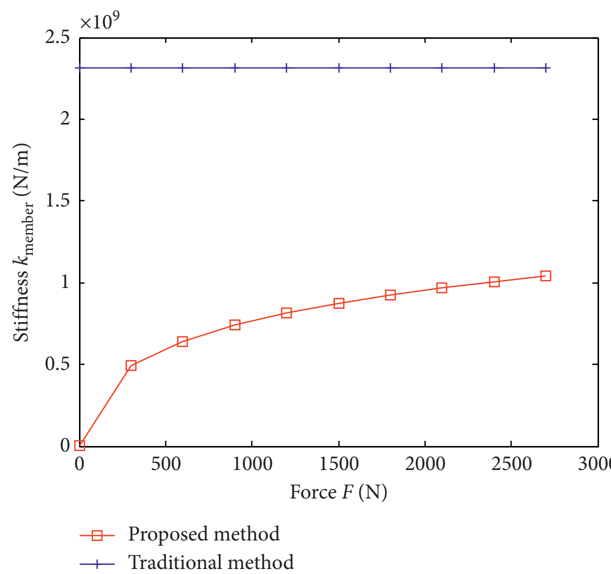

Figure 16: Stiffness comparison of bolted joint surface members with the rough surface.

interface between the bolted members and the members decreases with the thickness of the members, so the stiffness of the interface between the members and the members decreases. Although the thickness of the members decreases, the stiffness of the members increases, but the increase is slow, that is to say, the stiffness of the joints without members decreases rapidly, so the thicker the structure is, the greater the overall stiffness is.

As can be seen from Figure 17, the stiffness of both bolts increases with the increase of bolt preload and the change speed decreases with the increase of load.

From Figure 17, it can be seen that the stiffness of one member is greater than that of two members. Under the same preload, the stiffness of one member is much greater

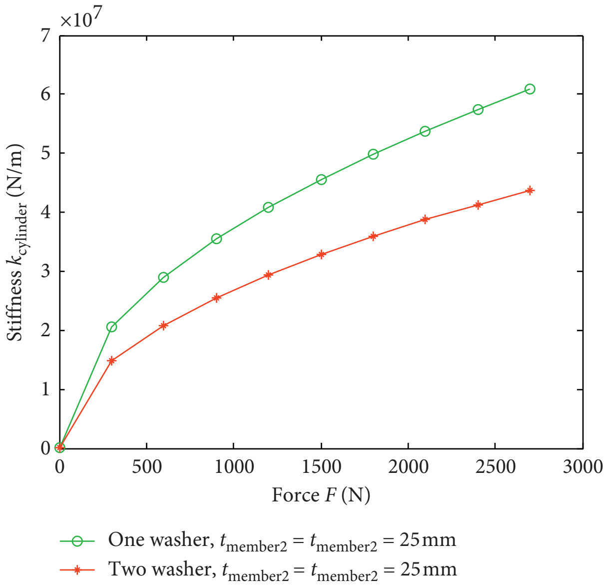

Figure 17: Bolted connection with member.

than that of two members, indicating that the number of members has a greater impact on the overall stiffness.

5. Conclusion

The calculation model proposed in this paper is more practical than in the previous method. The results show that the stiffness characteristics of machined surface roughness have a great influence on the overall stiffness. The more the number of surface joints, the smaller the overall stiffness of the structure and the easier the structure is to deform.

The study also shows that the stiffness of the joint and the whole structure increases with the increase of the pretightening force of the bolt and the greater the pretightening force, the greater the overall stiffness. The results also show that the gap between the joints reduces the stiffness of bolted connections. This paper theoretically verifies the conjecture that the larger the bolt pretension force, the greater the stiffness of the structure, the more the joint surface, and the smaller the stiffness of the mechanism.

Nomenclature

k_{member}: Overall stiffness of members (N/m)
$L_{member\text{-}total}$: Total thickness of members (m)
E_{member}: Young's modulus of members (Pa)
D_{hole}: Member hole diameter (m)
d_{washer}: Diameter of washer (m)
α: Influencing cone angle (radian)
$k_{member1}$: Stiffness of member1 (N/m)
$E_{member1}$: Young's modulus of member1 (Pa)
d_{bolt}: Bolt diameter (m)
$k_{member21}$: Stiffness of the first part of member2 (N/m)
$E_{member2}$: Young's modulus of member2 (Pa)
$t_{member2}$: Thickness of member2 (mm)
$t_{member1}$: Thickness of member1 (mm)
$k_{member22}$: Stiffness of the second part of member2 (N/m)
λ_{joint}: Deformation of joint surface (mm)
m_{normal}: Normal characteristic parameters of joint surface
c_{normal}: Normal characteristic parameters of joint surface
p_{normal}: Normal pressure of joint surface (Pa)
k_{joint}: Stiffness of joint surface (N/m)
$S_{cylinder}$: Area of cylindrical joint surface (m^2)
$H_{cylinder1}$: Height of cylinder1 (m)
$E_{cylinder1}$: Elastic modulus of cylinder1 (Pa)
$H_{cylinder2}$: Height of cylinder2 (m)
$E_{cylinder2}$: Elastic modulus of cylinder2 (Pa)
$E_{cylinder}$: Modulus of elasticity of cylinder (Pa)
$k_{cylinder}$: Stiffness of cylinder (N/m)
$s_{cylinder}$: Joint area (m^2).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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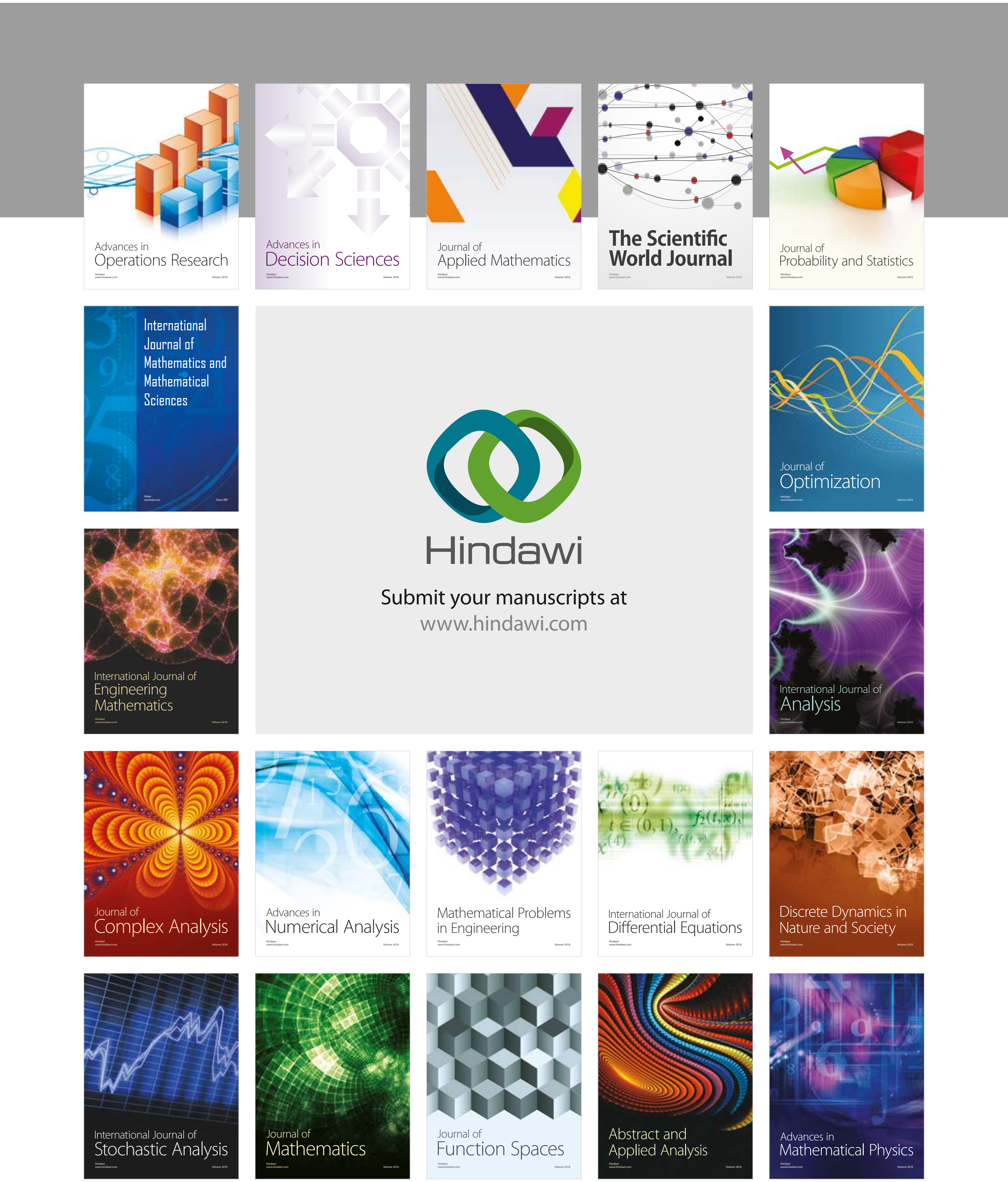
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