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# Stimulated scattering of electromagnetic waves carrying orbital angular momentum in quantum plasmas

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We investigate stimulated scattering instabilities of coherent circularly polarized electromagnetic (CPEM) waves carrying orbital angular momentum (OAM) in dense quantum plasmas with degenerate electrons and nondegenerate ions. For this purpose, we employ the coupled equations for the CPEM wave vector potential and the driven (by the ponderomotive force of the CPEM waves) equations for the electron and ion plasma oscillations. The electrons are significantly affected by the quantum forces (viz., the quantum statistical pressure, the quantum Bohm potential, as well as the electron exchange and electron correlations due to electron spin), which are included in the framework of the quantum hydrodynamical description of the electrons. Furthermore, our investigation of the stimulated Brillouin instability of coherent CPEM waves uses the generalized ion momentum equation that includes strong ion coupling effects. The nonlinear equations for the coupled CPEM and quantum plasma waves are then analyzed to obtain nonlinear dispersion relations which exhibit stimulated Raman, stimulated Brillouin, and modulational instabilities of CPEM waves carrying OAM. The present results are useful for understanding the origin of scattered light off low-frequency density fluctuations in high-energy density plasmas where quantum effects are eminent.

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## I. INTRODUCTION

Recently, there has been growing interest in investigating collective nonlinear processes [1–4] in dense quantum plasmas, which are ubiquitous in a variety of physical environments (e.g., the cores of Jupiter and white dwarf stars [5–9], neutron and quark stars [7], warm dense matter [10]), in compressed plasmas produced by intense laser beams [11], in pulsed thermonuclear fusion devices, as well as in processing devices for modern high-technology (e.g., semiconductors [12], in thin films and nanometallic structures [13], etc.). In fact, due to superdensity plasmas in the crust of dense neutron stars and in the cores of white dwarf stars, one [14] encounters the formation of ionic crystals (fully ionized carbon, oxygen, iron, etc.) embedded into a sea of degenerate Fermi gas of electrons (here the electron Fermi temperature exceeds the plasma electron and ion temperatures). There have also been suggestions [15,16] that strongly coupled quantum plasmas can be produced in laboratory devices by using laser cooling methods, so that the plasma electron temperature could be comparable with the electron Fermi temperature so that the quantum effects become significant at high plasma number densities. In quantum plasmas, degenerate electrons obey the Fermi-Dirac statistics.

Theoretical investigations [17–21] of nonlinear phenomena associated with both electrostatic and electromagnetic waves in quantum plasma fluids have been carried out previously. These nonlinear studies were based on the generalized

quantum hydrodynamical (GQHD) equations [22–25] for nonrelativistic degenerate electron fluids supplemented by Poisson's and Maxwell's equations. The generalized electron momentum equation in the GQHD model includes the quantum statistical pressure [26,27] and quantum forces due to electron tunneling through the quantum Bohm potential [22,24,25,28,29], spin magnetization of Bohr electrons [30], as well as the electron exchange and correlation effects [31,32] due to the electron spin [33]. Quantum mechanical effects are relevant for solid density plasmas, where the interelectron distance is of the order of the atomic dimensions. Here overlapping of electron wave functions occurs due to the Heisenberg uncertainty and Pauli's exclusion principles [27]. Accordingly, one encounters novel nonlinear high-frequency (hf) dispersive wave phenomena [2] at nanoscales. Furthermore, in quantum plasmas, the electron and ion coupling parameters are  $\Gamma_e = e^2/a_e k_B T_F$  and  $\Gamma_i = Z_i^2 e^2/a_i k_B T_i$ , respectively, where  $e$  is the magnitude of the electron charge,  $a_e \sim a_i = (3/4\pi n_0)^{1/3}$  the Wigner-Seitz radius,  $n_0$  the unperturbed electron number density,  $k_B$  the Boltzmann constant,  $Z_i$  the ion charge state,  $T_i$  the ion temperature,  $T_F = (\hbar^2/2m_0 k_B)(3\pi^2 n_0)^{2/3}$  the electron Fermi temperature in the nonrelativistic and zero-temperature limits, and  $m_0$  the electron rest mass. It turns out that  $\Gamma_i/\Gamma_e = Z_i^2 T_F/T_i \gg 1$ , since in quantum plasmas we usually have  $T_F > T_i$ . Recently, Glenzer *et al.* [34] have reported observations of enhanced electron plasma waves (plasmons) in solid density plasmas. Their measurements involved collective x-ray scattering techniques that are capable of measuring the high-frequency plasma wave spectra, revealing a signature of quantum effects associated with the quantum statistical electron pressure and the quantum recoil of electrons, at electron number densities  $\sim (1.5\text{--}4.5) \times 10^{23} \text{ cm}^{-3}$  and at

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electron temperatures below 25 eV (typically the electron temperature  $T_e$  was 12 eV in the experiments). The plasmon spectrum provides a sensitive measure of the electron number densities.

Large amplitude high-frequency electromagnetic (EM) waves are used for heating inertially confined fusion plasmas [35], as well as for diagnostic purposes [34] in solid density plasmas that are created by intense laser and charged particle beams. The hf EM pulses also appear as localized bursts of x rays and  $\gamma$  rays from compact astrophysical objects. Furthermore, the generation of coherent hf EM waves is of great importance in the context of free-electron lasers involving EM wigglers [36,37]. Therefore, studies of nonlinear phenomena (e.g., parametric instabilities [17] and hf EM wave localizations [19]) associated with large amplitude hf EM waves in dense quantum plasmas are of practical interest.

In this paper, we present an investigation of stimulated scattering instabilities of coherent circularly polarized electromagnetic (CPEM) waves carrying orbital angular momentum (OAM) in an unmagnetized dense quantum plasma composed of nonrelativistic degenerate electron fluids and mildly coupled nondegenerate ion fluids. It should be noted that a recent work [38] has proposed a scheme to generate intense coherent light that carries OAM at the fundamental wavelength of an x-ray free-electron laser (FEL). Our results reveal that quantum forces acting on degenerate electrons in quantum plasmas greatly modify the frequency spectra of both the electron and ion plasma oscillations that are driven at nonthermal levels by the CPEM wave pressure. Accordingly, the growth rates of stimulated Raman, stimulated Brillouin, and modulational instabilities are significantly affected when a large amplitude CPEM pump wave carrying OAM is present in an unmagnetized quantum plasma.

## II. FORMULATION

The nonlinear interactions between the CPEM wave and the background dense quantum plasma are governed by the EM wave equation [17,39]

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_p^2\right) \mathbf{A} + \omega_p^2 N \mathbf{A} = 0, \quad (1)$$

which is derived from the Maxwell equation and the electron equation of motion with the electromagnetic fields  $\mathbf{E} = -c^{-1} \partial \mathbf{A} / \partial t$  and  $\mathbf{B} = \nabla \times \mathbf{A}$ , with the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ . Here  $\mathbf{A}$  is the vector potential,  $c$  the speed of light in vacuum,  $\omega_p = (4\pi n_0 e^2 / m_0)^{1/2}$  the electron plasma frequency,  $N = n_{e1} / n_0 \ll 1$ , and  $n_{e1}$  the electron number density perturbation associated with low-frequency electrostatic plasma oscillations (EPOs) that are reinforced by the ponderomotive force of the CPEM waves. In the absence of nonlinear couplings between the latter and the EPOs, the paraxial EM wave solution  $\mathbf{A}(r, z) \exp(-i\omega t + ikz)$ , where  $\omega = (k^2 c^2 + \omega_p^2)^{1/2}$  and  $k$  are the frequency and the wave

number, respectively, of Eq. (1) is [40,41]

$$\mathbf{A}(r, z) = \mathbf{A} F_{p,l}(r, z) \exp(il\varphi). \quad (2)$$

Here we have denoted

$$F_{p,l}(r, z) = \frac{1}{2\sqrt{\pi}} \left[ \frac{(l+p)!}{p!} \right]^{1/2} X^{|l|} L_p^{|l|}(X) \exp(-X/2), \quad (3)$$

where  $X = r^2/w^2(z)$ ,  $w(z)$  is the beam waist, and the associated Laguerre polynomials  $L_p^{|l|}(X)$  are defined by the Rodriguez formula  $L_p^{|l|}(X) = (X^l p!)^{-1} \exp(X) d^p [X^{l+p} \exp(-X)] / dX^p$ ,  $p$  and  $l$  are the radial and angular mode numbers of the photon orbital angular momentum states, respectively,  $\varphi$  is the azimuthal angle, and  $r = (x^2 + y^2)^{1/2}$  is the radial of the cylindrical coordinates  $(r, \varphi, z)$ , so that  $\nabla^2 = \nabla_{\perp}^2 + \partial_z^2$ , where  $\nabla_{\perp}^2 = (1/r)(\partial/\partial r)(r\partial/\partial r) + (1/r^2)\partial^2/\partial\varphi^2$ . The Laguerre-Gauss (LG) solutions (2) describe CPEM waves with a finite OAM.

The dynamics of low-frequency (in comparison with the CPEM wave frequency  $\omega$ ) plasma oscillations involving degenerate electron and nondegenerate ion fluids is governed by a set of equations composed of the electron continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0, \quad (4)$$

the electron momentum equation [13,32]

$$m_0 n_e \frac{d\mathbf{u}_e}{dt} - en_e \nabla(\phi - \phi_p) + \nabla P_e - n_e \nabla V_{xc} - n_e \nabla V_B = 0, \quad (5)$$

and Poisson's equation

$$\nabla^2 \phi = 4\pi\rho, \quad (6)$$

together with the ion continuity equation and the generalized viscoelastic ion momentum equation [2]. Here  $n_e$  ( $n_i$ ) is the electron number density,  $\mathbf{u}_e$  the electron fluid velocity,  $d/dt = \partial/\partial t + \mathbf{u}_e \cdot \nabla$ ,  $\rho = e(n_e - n_i)$ ,  $\phi$  the electric potential, and  $\phi_p = e|\mathbf{A}|^2/m_0 c^2$  the ponderomotive potential of the CPEM waves. The light ponderomotive force [35,39]  $-e\nabla\phi_p$  comes from the averaging (over the light wave period) of the advection and nonlinear Lorentz force involving the electron quiver velocity and the laser wave magnetic field. We have denoted the quantum statistical electron pressure [27]  $P_e = (n_0 m_0 V_*^2 / 5)(n_e / n_0)^{5/3}$ , where  $V_* = \hbar(3\pi^2)^{1/3} / m_0 r_0$  is the electron Fermi speed and  $r_0 = n_0^{-1/3}$  represents the Wigner-Seitz radius, and the sum of the electron exchange and electron correlation potential is [31,32]  $V_{xc} = -0.985e^2 n_e^{1/3} [1 + (0.034/a_B n_e^{1/3}) \ln(1 + 18.37 a_B n_e^{1/3})]$ , where  $a_B = \hbar^2 / m_0 e^2$  represents the Bohr radius. The quantum Bohm potential is [2,22]  $V_B = (\hbar^2 / 2m_0)(1/\sqrt{n_e}) \nabla^2 \sqrt{n_e}$ . We have thus retained the desired quantum forces that act on degenerate electrons in a nonrelativistic quantum plasma. Equation (5) is valid [2–4,23] if the plasmonic energy density  $\hbar\omega_{pe}$  is smaller than or comparable to the electron Fermi energy  $k_B T_F$ , and the electron-ion collision relaxation time is larger than the electron plasma period.

The ion number density perturbation  $n_{i1}$  is obtained from the ion continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0, \quad (7)$$

where the ion fluid velocity  $\mathbf{u}_i$  is determined from the generalized viscoelastic ion momentum equation [2]

$$\left(1 + \tau_m \frac{D}{Dt}\right) \left[ m_i n_i \frac{D\mathbf{u}_i}{Dt} + \nabla P_i + Z_i e n_i \nabla \phi \right] - \eta \nabla \cdot \nabla \mathbf{u}_i - \left( \xi + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{u}_i) = 0, \quad (8)$$

where  $D/Dt = \partial/\partial t + \mathbf{u}_i \cdot \nabla$ ,  $\tau_m$  is the viscoelastic relaxation time for ion correlations,  $P_i = \mu_i k_B T_i n_i$  the ion thermal pressure involving strong ion coupling effects,  $m_i$  the ion mass,  $\mu_i = (1/k_B T_i)(\partial P_i/\partial n_i)_{k_B T_i} \equiv 1 + U(\Gamma_i)/3 + (\Gamma_i/9)\partial U(\Gamma_i)/\partial \Gamma_i$  the isothermal compressibility factor for nondegenerate ion fluids [42], and the function  $U(\Gamma_i)$  a measure of the excess internal energy of the system, which is related to the correlation energy  $E_c$  by  $U(\Gamma_i) = E_c/n_i k_B T_i$  ( $\equiv \Gamma_i(0.9 + 1.5r_i^2/a_i^2)$ , where  $r_i$  is the ion core radius which depends on the degree of ion stripping [43]). For a one-component plasma model, one usually adopts [44,45]  $U(\gamma_i) \approx -0.9\Gamma_i$  for  $\Gamma_i \gg 1$ . Furthermore, the coefficients of the shear and bulk ion fluid viscosities are denoted by  $\eta$  and  $\xi$ , respectively. Thus, Eq. (8) is similar to that used by Frenkel [46] and Ichimaru and co-workers [44,45] in the context of ordinary fluids and one-component strongly coupled plasmas, respectively. Kaw and Sen [47] adopted a generalized viscoelastic dust momentum equation for studying the properties of dust acoustic waves [48] in multicomponent dusty plasmas with highly correlated coupled charged dust grains. Furthermore, we note that the low-frequency ponderomotive force (i.e., the gradient of the ion ponderomotive potential) of the high-frequency CPEM waves acting on the ion fluid is smaller by a factor  $m_0/m_i$  as compared to  $-\nabla \phi_p$ , and therefore it has been neglected in Eq. (8).

Let us now derive the governing equations for the electron and ion plasma oscillations in the presence of the ponderomotive force of the CPEM wave in a quantum plasma. First, we consider the driven electron plasma oscillations on the time scale of the electron plasma period, so that the ions do not have time to respond and the ion density perturbation is zero. Letting  $n_e = n_0 + n_{e1}$ , where  $n_{e1} \ll n_0$ , we linearize Eqs. (4)–(6) and combine the resultant equations to obtain the electron plasma wave equation

$$\left( \frac{\partial^2}{\partial t^2} + \omega_p^2 - U_*^2 \nabla^2 + \frac{\hbar^2}{4m_0^2} \nabla^4 \right) N = \frac{e^2}{2m_0^2 c^2} \nabla^2 |\mathbf{A}|^2, \quad (9)$$

where we have denoted  $U_* = (V_*^2/3 + C_{xc}^2)^{1/2}$ , with  $C_{xc} = (0.328e^2/m_0 r_0)^{1/2} [1 + 0.62/(1 + 18.36a_B n_0^{1/3})]^{1/2}$ .

Second, we consider driven ion oscillations by supposing that  $|\partial^2 N/\partial t^2| \ll |U_*^2 \nabla^2 N - (\hbar^2/4m_0)\nabla^4 N|$ . Here, one can neglect the right-hand side in Eq. (5), and use the resultant equation to eliminate the electric field  $-\nabla \phi$  from Eq. (8) to obtain, after linearization of the resultant equation under the quasineutral approximation  $n_{e1} = n_{i1}$ , the driven ion

oscillation equation

$$\begin{aligned} & \left(1 + \tau_m \frac{\partial}{\partial t}\right) \left( \frac{\partial^2}{\partial t^2} - C_s^2 \nabla^2 + \frac{\hbar^2}{4m_0 m_i} \nabla^4 \right) N \\ & - \frac{\eta}{m_i n_0} \nabla \cdot \nabla \frac{\partial N}{\partial t} - \frac{(\xi + \frac{\eta}{3})}{m_i n_0} \nabla^2 \frac{\partial N}{\partial t} \\ & = \left(1 + \tau_m \frac{\partial}{\partial t}\right) \frac{Z_i^2 e^2}{2m_0 m_i c^2} \nabla^2 |\mathbf{A}|^2, \end{aligned} \quad (10)$$

where we have used Eq. (7) to eliminate  $\nabla \cdot \mathbf{u}_i$  and introduced  $C_s^2 = (V_{Ti}^2 + U_*^2)^{1/2}$ , with  $V_{Ti} = (\mu_i k_B T_i / m_i)^{1/2}$ .

Equations (1), (9), and (10) are the desired equations for studying the generation of wakefields [49] and nonlinear effects (viz., parametric instabilities [35] and localization of light pulses [39]) associated with LG CPEM beams in quantum plasmas at nanoscales.

### III. NONLINEAR DISPERSION RELATIONS AND THEIR ANALYSES

In the following, we present an investigation of stimulated Raman, stimulated Brillouin, and modulational instabilities [50] of LG CPEM waves. Accordingly, we decompose the vector potential as

$$\begin{aligned} \mathbf{A} = & \mathbf{A}_{0+} \exp(-i\omega_0 t + i\mathbf{k}_0 \cdot \mathbf{r}) + \mathbf{A}_{0-} \exp(i\omega_0 t - i\mathbf{k}_0 \cdot \mathbf{r}) \\ & + \sum_{+,-} \mathbf{A}_{\pm} \exp(-i\omega_{\pm} t + i\mathbf{k}_{\pm} \cdot \mathbf{r}), \end{aligned} \quad (11)$$

where the subscripts 0 and  $\pm$  denote the CPEM pump and CPEM sidebands, respectively, and  $\omega_{\pm} = \Omega \pm \omega_0$  and  $\mathbf{k}_{\pm} = \mathbf{K} \pm \mathbf{k}_0$  are the frequency and wave vectors of the CPEM sidebands that are created by the beating of the pump  $(\omega_0, \mathbf{k}_0)$  and electrostatic oscillations  $(\Omega, \mathbf{K})$ .

Inserting (11) into (1), (9), and (10), and supposing that  $N$  is proportional to  $\exp(-i\Omega t + i\mathbf{K} \cdot \mathbf{r})$ , we Fourier decompose the resultant equations to obtain the nonlinear dispersion relations

$$S_R = \frac{\omega_p^2 e^2 K^2}{2m_0^2 c^2} \sum_{+,-} \frac{|\mathbf{A}_0 F_{p,l}|^2}{D_{\pm}}, \quad (12)$$

and

$$S_B = \frac{\omega_p^2 Z_i^2 e^2 K^2}{2m_0 m_i c^2} \sum_{+,-} \frac{|\mathbf{A}_0 F_{p,l}|^2}{D_{\pm}}. \quad (13)$$

Here we have denoted  $S_R = \Omega^2 - \omega_p^2 - K^2 U_*^2 - \hbar^2 K^4 / 4m_0^2$  and  $S_B = \Omega^2 - K^2 C_s^2 - \hbar^2 K^4 / 4m_0 m_i + i\Omega(\xi + 4\eta/3)K^2 / m_i n_0 (1 - i\Omega\tau_m)$ , and  $D_{\pm} = \omega_{\pm}^2 - k_{\pm}^2 c^2 - \omega_p^2 \approx \pm 2\omega_0(\Omega - \mathbf{K} \cdot \mathbf{V}_g \mp \delta)$ , where  $\mathbf{V}_g = c^2 \mathbf{k}_0 / \omega_0$  is the group velocity of the CPEM pump,  $\omega_0 = (\omega_p^2 + k_0^2 c^2)^{1/2}$  the pump frequency, and  $\delta = K^2 c^2 / 2\omega_0$  the small frequency shift arising from the nonlinear interaction between the CPEM pump and the electrostatic plasma oscillations in a quantum plasma. In the absence of the pump, we have from (12) and (13)  $S_R = 0$  and  $S_B = 0$ , which give the frequencies of the electron and ion plasma oscillations in quantum plasmas. We have

$$\Omega(K) = \left( \omega_p^2 + K^2 U_*^2 + \frac{\hbar^2 K^4}{4m_0^2} \right)^{1/2} \equiv \Omega_L \quad (14)$$

for the electron plasma oscillations, and

$$\Omega^2 + i\Omega \frac{\Omega_v}{(1 - i\Omega\tau_m)} - K^2 C_s^2 - \frac{\hbar^2 K^4}{4m_0 m_i} = 0 \quad (15)$$

for the ion plasma oscillations. Here we have denoted  $\Omega_v = (\xi + 4\eta/3)K^2/m_i n_0$ . In the hydrodynamic limit, viz.,  $\Omega\tau_m \ll 1$ , we have viscous damping of the quantum ion mode. The real and imaginary parts of the frequencies ( $\Omega = \Omega_r + i\Omega_i$ ) are, respectively,

$$\Omega_r(K) = \left[ K^2 C_s^2 + \frac{\hbar^2 K^4}{4m_0 m_i} - \Omega_i^2 \right]^{1/2} \quad (16)$$

and

$$\Omega_i = -\frac{\Omega_v}{2}. \quad (17)$$

Furthermore, in the kinetic regime characterized by  $\omega\tau_m \gg 1$ , we have from (15)

$$\Omega(K) = \left( \frac{\Omega_v}{\tau_m} + K^2 C_s^2 + \frac{\hbar^2 K^4}{4m_0 m_i} \right)^{1/2} \equiv \Omega_I(K). \quad (18)$$

Generally,  $\tau_m = \tau_0 Y_G(K)$ , where  $\tau_0 = 1/\Omega_v [1 - \mu_i + 4U(\Gamma_i)/15]$  and  $Y_G(K) = \exp(-K/K_G)$  for a Gaussian distribution, and  $Y_G(k) = (1 + K^2/K_L^2)^{-1}$  for a Lorentzian distribution. Here  $K_G$  and  $K_L$  are the scale factors [45].

We now present a summary of formulas for the growth rates of stimulated Raman (SR) and stimulated Brillouin (SB) scattering instabilities, as well as of modulational instabilities of a constant amplitude pump that is scattered off a quantum electron plasma wave, a quantum ion mode, and a spectrum of nonresonant electron and ion density perturbations. For three-wave decay interactions, one assumes that  $D_- = 0$  and  $D_+ \neq 0$ . Thus, one ignores  $D_+$  from Eqs. (12) and (13). Letting  $\Omega = \mathbf{K} \cdot \mathbf{V}_g - \delta + i\gamma_{R,B}$ , and  $\Omega = \Omega_L + i\gamma_R$  and  $\Omega = \Omega_r(\Omega_I) + i\gamma_B$  in the resonant equations, we obtain the growth rates for Raman and Brillouin backscattering ( $|\mathbf{K}| = 2k_0$ ) instabilities, respectively,

$$\gamma_R = \frac{\omega_p k_0 e |\mathbf{A}_0 F_{p,l}|}{\sqrt{2\omega_0 \Omega_R m_0 c}}, \quad (19)$$

and

$$\gamma_B = \frac{\omega_p k_0 Z_i e |\mathbf{A}_0 F_{p,l}|}{\sqrt{2\omega_0 \Omega_B m_0 m_i c}}, \quad (20)$$

where  $\Omega_R = \Omega_L(K = 2k_0)$ ,  $\Omega_B = \Omega_{r,I}(K = 2k_0)$ , and  $|\mathbf{2k}_0 \cdot \mathbf{V}_g - \delta| \sim \Omega_R, \Omega_B$ . Since the growth rates of SR and SB scattering instabilities, given by Eqs. (19) and (20), respectively, are proportional to  $\Omega_R$  and  $\Omega_B$ , one notices that quantum and strong ion correlation effects significantly affect the  $e$ -folding time of the instabilities. Furthermore, the growth rates, which are proportional to  $F_{p,l}$ , are minimum at the center of the vortex pump wave with OAM.

Next, for the modulational instabilities, we have  $D_{\pm} \neq 0$  and  $S_{R,B} \neq 0$ . Here, we have to retain both upper and lower CPEM sidebands on an equal footing in (12) and (13), and

write them as

$$S_R[(\Omega - \mathbf{K} \cdot \mathbf{V}_g)^2 - \delta^2] = \frac{\delta \omega_p^2 e^2 K^2 |\mathbf{A}_0 F_p(p,l)|^2}{2\omega_0 m_0^2 c^2} \quad (21)$$

and

$$S_B[(\Omega - \mathbf{K} \cdot \mathbf{V}_g)^2 - \delta^2] = \frac{\delta \omega_p^2 Z_i^2 e^2 K^2 |\mathbf{A}_0 F_p(p,l)|^2}{2\omega_0 m_0 m_i c^2}. \quad (22)$$

Equations (21) and (22) can be analyzed numerically to obtain the growth rates of the modulational instabilities. However, some analytical results follow for  $\mathbf{K} \cdot \mathbf{V}_g = 0$ , in which case we have from (21) and (22), respectively,

$$\begin{aligned} \Omega^2 &= \frac{1}{2}(\Omega_L^2 + \delta^2) \\ &\pm \frac{1}{2} \left[ (\Omega_L^2 - \delta^2)^2 + \frac{2\delta \omega_p^2 e^2 K^2 |\mathbf{A}_0 F_p(p,l)|^2}{\omega_0 m_0^2 c^2} \right]^{1/2} \end{aligned} \quad (23)$$

and

$$\begin{aligned} \Omega^2 &= \frac{1}{2}(\Omega_{r,I}^2 + \delta^2) \\ &\pm \frac{1}{2} \left[ (\Omega_{r,I}^2 - \delta^2)^2 + \frac{2\delta \omega_p^2 Z_i^2 e^2 K^2 |\mathbf{A}_0 F_p(p,l)|^2}{\omega_0 m_0 m_i c^2} \right]^{1/2}, \end{aligned} \quad (24)$$

which exhibit oscillatory modulational instabilities.

#### IV. SUMMARY AND CONCLUSIONS

In summary, we have considered nonlinear interactions of large amplitude CPEM waves carrying OAM with electron and ion plasma modes in an unmagnetized quantum plasma, accounting for quantum forces that act on a degenerate electron fluid. On the time scale of the electron plasma period, the ions do not respond to electrostatic electron plasma waves that are driven by the CPEM wave pressure. It is shown that the electron plasma wave spectrum is significantly modified by the inclusion of the quantum statistical electron pressure, perturbations of quantum forces associated with the electron exchange and electron-correlation potentials due to spin effects, as well as that of the quantum Bohm potential through which electrons tunnel at nanoscales. Furthermore, inclusion of the dynamics of strongly correlated nondegenerate ions provides the possibility of low-frequency electrostatic oscillations that are supported by restoring forces coming from gradients of the quantum statistical electron pressure as well as electron-exchange and electron correlations potentials, and the quantum electron Bohm potential, while the ion mass provides the inertia to sustain the wave. The viscoelastic relaxation of ion correlations and ion fluid shear and bulk viscosities introduce the damping of low-frequency electrostatic ion oscillations. Both electron and ion plasma oscillations are excited by large amplitude CPEM waves due to stimulated Raman and Brillouin scattering instabilities. We also have the possibility of modulational instabilities of the CPEM waves via which nonresonant electron density perturbations are created. Hence, there are enhanced electrostatic fluctuations at nanoscales in dense quantum plasmas. In conclusion, we stress that the results of the present investigation are useful for

understanding the salient features of enhanced density fluctuations and inhomogeneities which are nonlinearly created by large amplitude CPEM waves with OAM in laser created solid density compressed plasmas and in compact astrophysical objects.

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