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Stochastic acceleration by intense laser fields

Tatsufumi Nakamura[a)]
Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan

Susumu Kato and Mitsumori Tamimoto
Institute of Advanced Industrial Science and Technology, Umezono 1-1-1, Tukuba 305-8568, Japan

Tomokazu Kato
Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan

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A stochastic acceleration mechanism which is a direct acceleration mechanism effective for the intense laser case is studied by the Fokker–Planck approach. The Fokker–Planck equation of the electron distribution function is derived from the equation of motion of electrons which interact with filamented laser fields. The Fokker–Planck equation contains nonlinear coefficients and gives an anisotropic distribution in momentum space. The strong directionality of the acceleration is explained. The accelerated electrons tend to be collimated towards the direction of the wave vector. The effective temperature scales as $T \propto t^{\beta}$ with $\beta \simeq 1$.
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I. INTRODUCTION

Laser–plasma interaction in the relativistic regime is crucial for electron acceleration,[1–4] the fast ignitor scheme for inertial confinement fusion,[5] and x-ray lasers. For electron acceleration schemes, there exist different proposals depending on how the plasma wave is driven, such as, laser wake field acceleration,[6] self-modulated laser wake field acceleration,[7] and beat-wave acceleration,[8] etc. In those schemes longitudinal plasma waves are excited in order to accelerate electrons, since transverse and uniform electromagnetic fields can not directly accelerate electrons. In the case of sufficiently high laser intensity and plasma density, however, the wake field acceleration mechanism becomes less effective due to the wave breaking.[9] As a direct acceleration mechanism by the transverse laser fields, research has been done on the ponderomotive acceleration, or schemes using modified electromagnetic fields such as a solitary laser pulse,[10] rectified laser pulse,[11] etc.

In recent experiments, high energy electrons are observed without the existence of a clear electrostatic wave, where the laser pulse is split into filaments and the plasma is strongly disturbed,[12] and an direct laser acceleration mechanism, which is different from the previously mentioned ones, is confirmed to be effective for intense laser pulses.[13] An acceleration mechanism under electromagnetic fields with a random force was investigated by Meyer-ter-Vehn and Shen.[14] It is shown that the electrons dephased by a random force, which is transverse to accelerating direction, are directly accelerated by the laser field. Their theory was based on the Langevin approach, i.e., the equation of motion for numbers of electrons are numerically integrated by adding a random force.

In the present paper, we investigate the stochastic acceleration mechanism by the Fokker–Planck approach. A Fokker–Planck equation is derived for the system where the laser field has a multifilament structure. The laser field is assumed to be composed of filaments of uniform electromagnetic plane waves whose phases are randomly different from each other. The derived Fokker–Planck equation has nonlinear coefficients, which explains the directionality of electron acceleration towards the wave vector which is observed in experiments. Also, the time evolution of the effective temperature of electrons and the ejected angle of high energy electrons are obtained from the Fokker–Planck equation.

In Sec. II, an electron motion in uniform electromagnetic fields is reviewed. The solution is expressed with an initial condition to see the possibility of a direct acceleration by the electromagnetic fields. In Sec. III, the Fokker–Planck equation is derived to show how the electron distribution function becomes anisotropic in momentum space. The directionality of acceleration is explained from its nonlinear coefficients. The energy spectrum, electron temperature and ejected angle are obtained. The conclusion and discussion are given in Sec. IV.

II. ELECTRON MOTION IN A LINEARLY POLARIZED PLANE WAVE

We start from a review of the motion of an electron in a linearly polarized plane wave with relativistic intensity. The solution is given in a well-known book.[15] Here we use a slightly generalized form which is written with an initial condition,[16] since the electrons with initially relativistic electrons play an important role. The electromagnetic fields are assumed to be polarized in the x-direction and propagating in the z-direction, i.e., $\mathbf{A} = A \sin(\omega t - kz)\mathbf{e}_x$, where ω and k are the laser frequency and wave number in vacuum. When the radiative reaction is neglected, the equation of motion of an electron in the field is written as follows:

[a)]Electronic mail: tnaka@mn.waseda.ac.jp

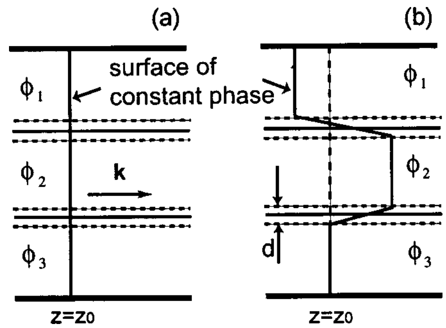

FIG. 1. Schematic figure of the phase for (a) uniform laser field, (b) filamented laser field. In uniform laser field the position of the same phase of the laser field is in a plane which is perpendicular to the wave vector, i.e., $\phi_1(z_0)=\phi_2(z_0)=\phi_3(z_0)$. In filamented laser field the phases of the laser field are different at the same z coordinate, i.e., $\phi_1(z_0)\neq\phi_2(z_0)\neq\phi_3(z_0)$.

$$mc\frac{du_x}{d\tau}=-e(\gamma-u_z)E, \tag{1}$$

$$mc\frac{du_z}{d\tau}=-eu_xE, \tag{2}$$

$$mc\frac{d\gamma}{d\tau}=-eu_xE, \tag{3}$$

where $\mathbf{E}=-(\partial\mathbf{A}/\partial t)$, m is the electron rest mass, c is the speed of light, $\mathbf{u}=\mathbf{p}/mc$, τ is proper time, i.e., $d\tau=dt/\gamma$, and γ is the Lorentz factor. There are two constants of motion in this system. The first one is $\gamma^2-u_x^2-u_z^2=1$, which is satisfied, since $u_iu^i=1$. The second constant is written as

$$\gamma-u_z=\frac{d\chi}{d\eta}\equiv\alpha, \tag{4}$$

where $\chi=\omega t-kz$ and $\eta=\omega\tau$. The constant α is obtained from the fact that the field considered here is a plane wave. The analytic expressions of the electron momentum with initial values are written as follows:

$$u_x=u_{x0}+a(\sin(\chi)-\sin(\chi_0)), \tag{5}$$

$$u_z=u_{z0}+u_{x0}\frac{a(\sin(\chi)-\sin(\chi_0))}{\alpha}+\frac{a^2(\sin(\chi)-\sin(\chi_0))^2}{2\alpha}, \tag{6}$$

where u_{x0}, u_{z0}, and χ_0 are the initial values of u_x, u_z, and χ, respectively, and $a=eA/mc$ is the normalized vector potential.

For electrons which are initially at rest (i.e., $u_{x0}=u_{z0}=0$), $\alpha=1$. This means that the maximum u_z is $a^2/2$ and the average momentum is $(a/2)^2$. As far as considering the electrons which are initially at rest, the electron energy is just the order of the ponderomotive energy. For electrons whose initial values are $u_{x0}=0$ and $u_{z0}\gg 1$, on the other hand, $1/(2\alpha)\simeq u_{z0}$. Then the time average of u_z becomes $\simeq u_{z0}a^2/2$ and the electrons have substantial energy, which is much larger than the ponderomotive energy.

When considering the interaction of an electron with a laser pulse whose length is much longer than the wavelength, the electron motion is still periodic. After it has been passed by the laser pulse its momentum returns to u_{z0}. Thus no energy transfer occurs. However, if the laser field suddenly disappears or the phase of the laser field discontinuously changes, the electron can have higher energy than the initial one due to the truncation of the interaction. This is more effective for initially relativistic electrons than initially at rest electrons, since they have substantial energy at the maxima of the periodic motion.

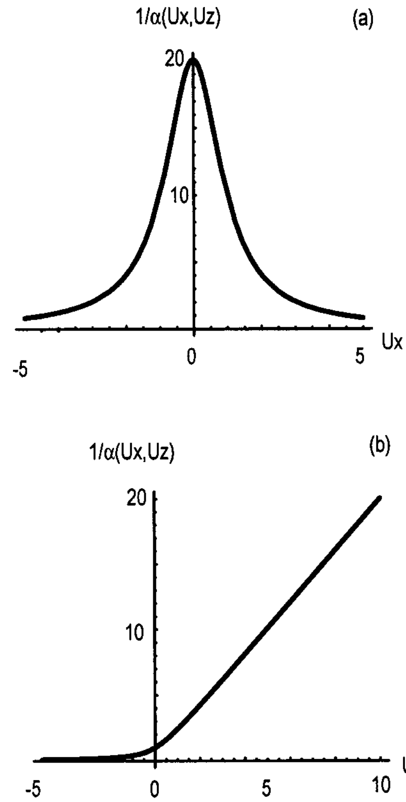

FIG. 2. Profile of $\alpha(u_x,u_z)^{-1}$ for (a) $u_z=10$ and (b) $u_x=0$. When $u_x\simeq 0$ and $u_z\gg 1$, $1/\alpha\gg 1$.

III. FOKKER–PLANCK EQUATION

In this paper we consider the situation where the laser field is divided into some regions in a transverse plane, such as a multifilament structure. The structure is observed in the experiments of electron acceleration by intense strong laser fields, such that the laser field becomes nonuniform, e.g., the laser pulse is split into filaments due to the interaction between the laser field and the plasma. The illustration of the phase of the laser field is drawn in Fig. 1. In the multifilament structure the surface of a constant phase is modified as Fig. 1(b). For simplicity we assume $d=0$, then the phase sharply changes at the filament edge. The phase of the laser field in each region is different, while it is uniform inside one region, i.e., the laser field consists of sectionally divided plane waves. An electron inside an each region executes a motion expressed with Eqs. (5) and (6). It leaves the region at $t=t_1$ with $u_x=u_{x1}$ and $u_z=u_{z1}$ and enters an adjacent region. Then the electron again executes the periodic motion with u_{x1} and u_{z1} as new initial values until it moves to a

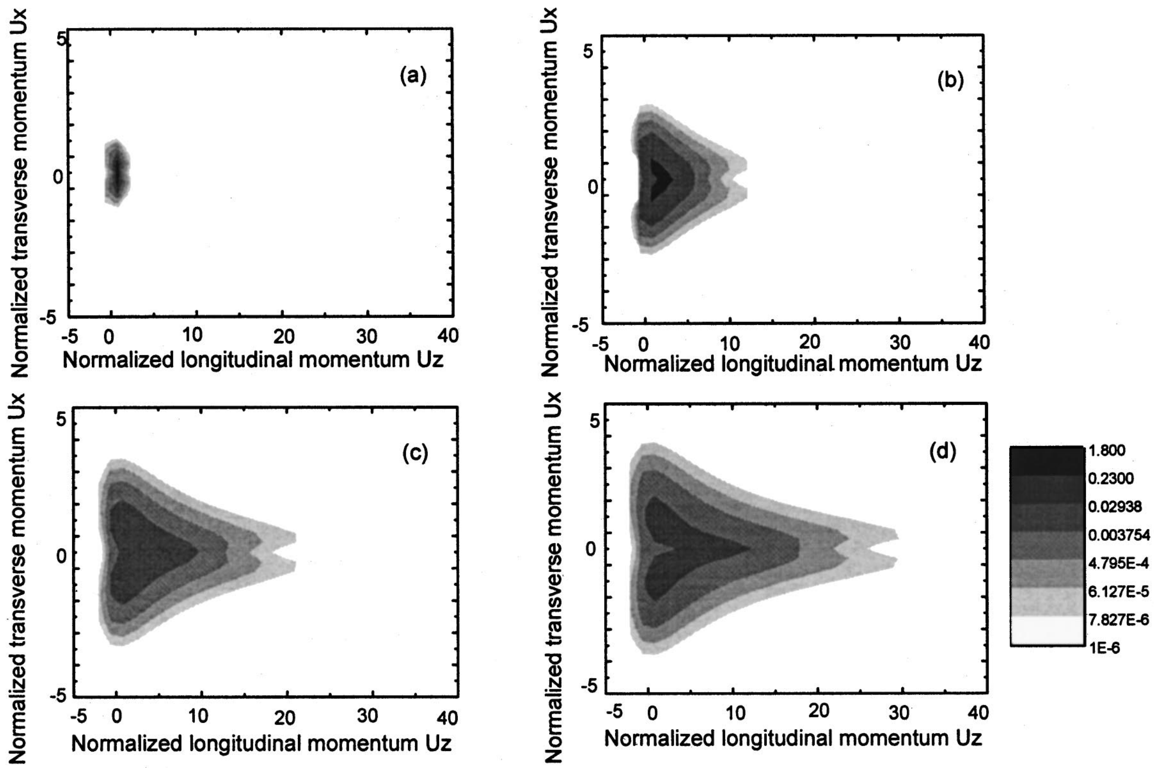

FIG. 3. Profile of electron distribution in momentum space at (a) $\omega t = 100$, (b) $\omega t = 1000$, (c) $\omega t = 2000$, and (d) $\omega t = 3000$.

different region. Therefore, the electron gained $\delta u_x = u_{x1} - u_{x0}$ and $\delta u_z = u_{z1} - u_{z0}$ in moving through this region. The details of this gain are discussed in the next paragraph. The electron experiences different phases as it enters the adjacent region with time step τ_c. The phase of the laser field in the different regions is assumed to be randomly different. This allows us to treat the electron motion as a random process, where $\phi = \sin\chi - \sin(\chi_0)$ is a random variable with $-2 \leqslant \phi \leqslant 2$. The equation of motion of an electron for n-th step is written in a same manner such as,

$$u_{x,n+1} = u_{x,n} + a\phi, \tag{7}$$

$$u_{z,n+1} = u_{z,n} + \frac{u_{x,n} a\phi}{\alpha_n} + \frac{a^2\phi^2}{2\alpha_n}, \tag{8}$$

where $\alpha_n = \gamma(u_{x,n}, u_{z,n}) - u_{z,n}$. In the time scale of $\eta \gg \tau_c$, Eqs. (7) and (8) are written in differential forms as follows:

$$\frac{\partial u_x}{\partial \eta} = \frac{a\phi}{\tau_c}, \tag{9}$$

$$\frac{\partial u_z}{\partial \eta} = \frac{u_x a\phi}{\tau_c \alpha} + \frac{a^2\phi^2}{2\tau_c \alpha}. \tag{10}$$

The Fokker–Planck equation for the processes is obtained as[17]

$$\frac{\partial f_e}{\partial \eta} = \frac{a^2 D}{2\tau_c^2}\left(\frac{\partial^2 f_e}{\partial u_x^2} - 3\frac{\partial}{\partial u_z}\frac{f_e}{\alpha} + u_x^2 \frac{\partial^2}{\partial u_z^2}\frac{f_e}{\alpha^2}\right), \tag{11}$$

where f_e denotes the electron distribution function. Here the stochastic variable ϕ is assumed to be the Gaussian with $\langle\phi\rangle = 0$ and $\langle\phi(t)\phi(t')\rangle = D\delta(t-t')$, where $D \ll 1$. From the first term in Eq. (11) it is seen that the time evolution of f_e in u_x is a normal diffusion process and is estimated as $\langle u_x\rangle \simeq a/\tau_c\sqrt{Dt}$. The second and third terms correspond to the drift and diffusion processes of f_e in u_z, which include $\alpha(u_x, u_z)$. In both terms $\alpha(u_x, u_z)$ plays an important role, since $\langle u_z\rangle$ is roughly estimated as $\langle u_z\rangle \simeq (a/\tau_c\sqrt{Dt})/\alpha = \langle u_x\rangle/\alpha$. The profile of α^{-1} is plotted in Fig. 2. From Fig. 2(a) it is seen that $1/\alpha$ is small unless $u_x \simeq 0$. In Fig. 2(b) $\alpha(u_x = 0, u_z)$ is plotted. It is clear that $1/\alpha$ is proportional to u_z for $u_z \gg 1$ and $1/\alpha \simeq 0$ for $u_z \leqslant 0$. Therefore, the electron distribution drifts and diffuses in the positive u_z direction and not in negative direction. The diffusion effect in u_z is much larger than that in u_x. These facts explain the strong directionality of the acceleration towards u_z.

Here we discuss the relation between the momentum gain and the stochastic variable ϕ. Since we introduced the stochastic variable ϕ, which expresses the random momentum gain δu_x in one step, it is obvious that the electron distribution diffuses in u_x and also diffuses in u_z due to the Lorentz force. But it is noticed that electrons are accelerated only in positive u_z not in negative u_z. This is a significant feature of electron acceleration by electromagnetic fields with relativistic intensity.

The time evolution of f_e is obtained by numerically solving the Fokker–Planck equation, which is plotted in Fig. 3. The initial conditions are the Maxwell distribution with $k_B T = 5$ keV, $a = 3$, $D = 0.05$, and $\tau_c = 10$. In the u_z direction, the electrons with large u_z are strongly accelerated only in the positive direction. The distribution slowly diffuses in u_x. In Figs. 3(b), 3(c), and 3(d) there are two peaks in the high

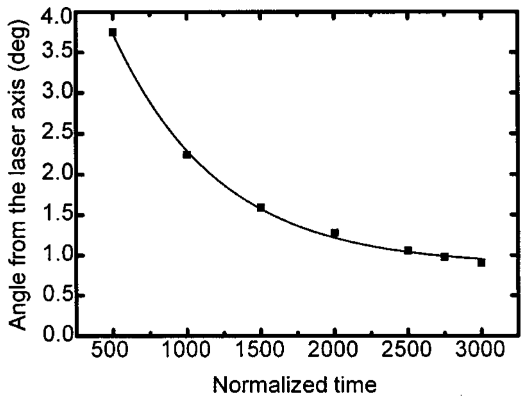

FIG. 4. Angular distribution of high energy electrons. The solid square is numerical result and the solid line indicates the exponential decay.

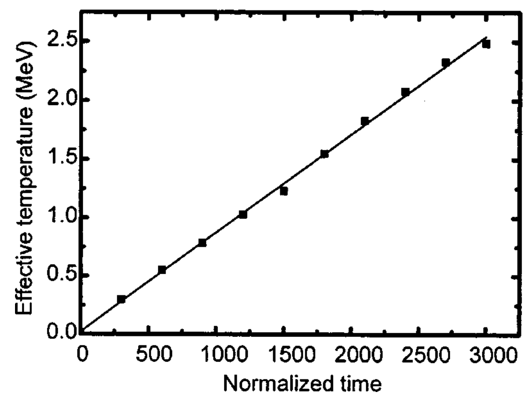

FIG. 6. Time dependence of effective energy. The solid square is numerical result and the solid line indicates $T \simeq t^{0.998}$.

energy tail, which means that the high energy electrons will be observed in a direction which is slightly deviated from the laser propagation axis. This is explained from the third term in Eq. (11) such that the electrons with small u_x are slightly more accelerated than electrons with $u_x = 0$. The time evolution of the ejecting angle of the fast electrons, which is defined as $u_x/u_z|_{z=z_p}$, where z_p is the u_z coordinate of the position of peaks, is plotted in Fig. 4. The high energy electron beam tends to be collimated towards the direction of the wave vector. The angle exponentially decreases in time. The energy distributions at $\omega_0 t = 600$, 1800, and 3000 are plotted in Fig. 5. It is seen that the distribution is quasithermal distribution and its effective energy increases up to 2.8 MeV at $\omega_0 t = 3000$, while the ponderomotive energy is 1 MeV. The time dependence of the effective energy is plotted in Fig. 6. It roughly scales as $T \propto t^{\beta}$ with $\beta \simeq 1$, whereas $0.5 \leqslant \beta \leqslant 1$ for the simulation result.[14] The energy spectrum of electrons accelerated in the direction of 0° and 30° to the laser opagation axis are shown in Fig. 7. The maximum energy is about 10 keV at 0° and 2 keV at 30°, respectively. A similar energy spectrum having a hump in a high energy region is observed in experiments of the solid-laser interaction using the Pettawatt laser by Cowan *et al.* at Lawrence Livermore National Laboratory.[18]

IV. DISCUSSION AND CONCLUSION

The stochastic acceleration mechanism is analyzed by the Fokker–Planck approach. The derived Fokker–Planck equation contains the nonlinear coefficient with the factor $\alpha(u_x, u_z)$. It is clear from the equation that the electrons with $u_x \cong 0$ and $u_z \geqslant 0$ are accelerated towards the positive u_z direction. The time evolution of the electron distribution function shows the directionality of the acceleration, which is the important feature of electron acceleration by the intense laser field. The accelerated electrons tends to be collimated towards the wave vector as the interaction takes place. The effective temperature roughly scales as $T \propto t^{\beta}$ where $\beta \simeq 1$.

The stochastic approach has been used by many authors to explain electron acceleration by electromagnetic fields. The stochastic acceleration by electrostatic fields is suggested as a possible mechanism of the generation of the extremely high energy cosmic ray emanating from a gamma ray burst.[19] In the research of charged particle acceleration in the universe, the stochastic acceleration has been studied, as well as the Fermi acceleration.[20,21] The Fokker–Planck equation is used and the diffusion coefficients are determined phenomenologically from the measurement. Since their diffusion coefficients are constant, electrons are accelerated both in the positive and negative direction, which is not true in the case of acceleration by the laser field. In the research

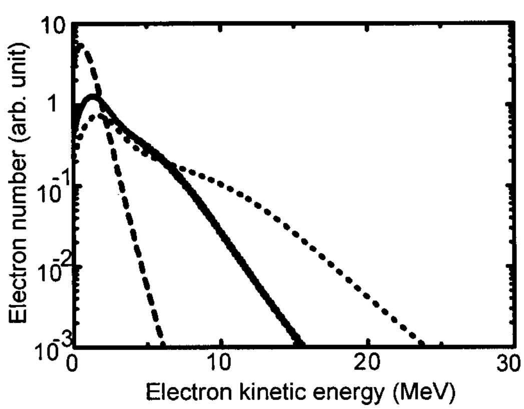

FIG. 5. Energy distribution of accelerated electrons at $\omega t = 600$ (dashed line), 1800 (solid line), and 3000 (dotted line).

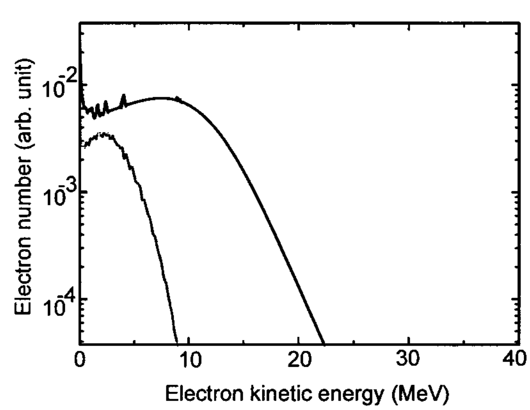

FIG. 7. Energy distribution of accelerated electrons emitted in 0° (solid line), 30° (dotted line).

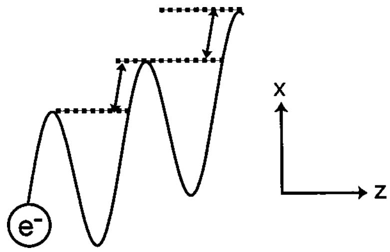

FIG. 8. Illustration of an electron trajectory in a uniform laser field. The phase of electrons which move into an adjacent region is limited in the narrow range.

of laser–plasma interaction the direct electron acceleration by the intense laser field was studied by the Langevin approach.[14] In their work equations of motion of numbers of electrons are numerically integrated by adding a random force to obtain an energy distribution. The time evolution of effective temperature is obtained as $T \simeq t^{\beta}$ with $0.5 \leqslant \beta \leqslant 1$.

In the experiment of Gahn *et al.* the laser pulse is 200 fs with 1.2 TW, correspondingly $a=1.4$, and the plasma density is $n=4\times10^{20}$ cm^{-3}. The measured effective plasma temperature is 2.2 MeV, where the observed channel length is about 400 μm. The same temperature is obtained at $\omega_0 t = 2850$ in our result as is shown in Fig. 6. Their result is recovered in our theory by adjusting the time scale, namely $a^2D/2\tau_c^2$ in Eq. (11). Since the radius of the filament is approximately c/ω_p, τ_c is estimated as $\tau_c \simeq c/(\omega_p v_x)\omega_0$ where $v_x \simeq c\sqrt{u_x^2}$ is the average velocity when an electron moves to an adjacent region. From Eq. (7) the average velocity is estimated as $a\sqrt{D}$. Therefore, τ_c is estimated as $\tau_c \simeq \omega_0/(\omega_p a\sqrt{D})$. The coefficient of the Fokker–Planck equation becomes $a^2D/2\tau_c^2 \simeq (a^2D\omega_p/\omega_0)^2/2$. By setting the coefficient $D=0.07$, the effective temperature at $\omega_0 t = 3100$, which corresponds to the channel length of 400 μm, is obtained as 2.2 MeV. In their result of angular distribution of electrons, there exists two peaks which are at $\theta=0°$ and $\theta \simeq 5°$. The result of the second peak might be explained by our theory. In our case, the peak at $\theta=0$ does not appear since the indirect acceleration mechanism such as the wakefield acceleration is not taken into account.

The coefficient D is chosen as $D \ll 1$ in our calculation and $D=0.07$ for Gahn's result. This seems to be too small considering it is defined as $-2 \leqslant \phi \leqslant 2$. This is understood by considering how an electron moves into an adjacent region. The phase of the electron which moves into adjacent region is limited, as is shown in Fig. 8. As the velocity is smallest at the turning point, most electrons moving into an adjacent region are close to the turning point. Therefore, the correlation of ϕ becomes quite small.

The model that the laser field is divided into regions with randomly different phases is suitable for the understanding of the direct laser acceleration experiment.[12,13] It is observed that the laser pulse propagating through underdense plasma splits into the filaments when its power well exceeds $P_c[\mathrm{PW}]=17(\omega_0^2/\omega_p^2)$, which is a critical power for self-focusing.[22] Also the plasma enters a turbulent state when the laser intensity is in the relativistic regime. In those cases, the electron experiences the laser field with different phases during the interaction, and the stochastic acceleration plays the important role.

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