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Stochastic analysis of double-skin façades subjected to imprecise seismic excitation

Federica Genovese[a], Alba Sofi [b*]

Department of Architecture and Territory, University "Mediterranea" of Reggio Calabria, Via dell'Università 25, 89124 Reggio Calabria, Italy

[a]federica.genovese@unirc.it, [b]alba.sofi@unirc.it

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Abstract. Double-skin façades (DSFs), widely used in buildings to provide specific thermal efficiency, acoustic isolation, and weather resistance properties, have been recently used as passive control systems. The present study focuses on the stochastic analysis of a shear-type frame equipped with a DSF subjected to ground motion acceleration modelled as a zero-mean stationary Gaussian random process fully characterized by an imprecise power spectral density function i.e., with interval parameters. The influence of imprecision of the seismic excitation on the performance of the DSF is investigated by evaluating the bounds of the interval reliability function for a selected displacement process of the frame structure, in the framework of the classical first-passage problem.

Introduction

A double-skin façade (DSF) is a multi-layered structure composed of a second "skin" placed in front of a regular building façade. DSFs are becoming increasingly popular for their capability of improving the energy performance and the aesthetics of buildings. Recently, the application of DSFs as vibration absorbers has been investigated assuming either deterministic [1] or stochastic excitation [2]. Previous studies mainly focused on the optimal design of the DSF by varying the layout of panels and design parameters such as the stiffness of links to the building.

The present study addresses the stochastic analysis of a shear-type frame equipped with a DSF subjected to ground motion acceleration taking into account epistemic uncertainties affecting the excitation. Following Pipitone et al. [2], the DSF is modelled as a set of independent panels, each one described as a mass lumped system connected to the main structure by elastic springs at the floor level. Within the *strong motion* phase, seismic excitation is modelled as a zero-mean stationary Gaussian process, fully characterized by an imprecise power spectral density (IPSD) function, recently proposed by Muscolino et al. [3]. The three parameters characterizing the assumed spectral model are described as interval variables by applying the so-called *Improved Interval Analysis* [4]. The bounds of the spectral parameters are estimated through the analysis of a set of accelerograms recorded on rigid soil deposits. Since the IPSD function has an interval nature, response statistics of the seismically excited system are described by intervals. To assess structural safety, the IPSD function of ground motion acceleration is incorporated into the formulation of the classical first-passage problem and the bounds of the interval reliability function for a selected response process are estimated.

Problem formulation

Let us consider a combined system (see Fig. 1) consisting of a $s-$storey shear-type frame (primary system) equipped with a DSF (secondary system) subjected to seismic excitation modelled as a zero-mean stationary Gaussian random process fully characterized by an IPSD i.e., with interval parameters. The primary system is characterized by storeys having equal mass m, lateral stiffness k and inter-storey height h. A constant viscous damping ratio ζ_0 is assumed for all modes of vibration. Following Pipitone et al. [2], the DSF is modelled as a set of N independent panels connected to the main structure by elastic springs at the floor level. The generic panel is modelled as a system composed by j lumped masses equally spaced by $h/2$. Each lumped mass has two degrees-of-freedom (DOFs), namely the horizontal displacement $x_j(t)$ and the rotation $\theta_j(t)$ (see Fig. 1). The flexural stiffness of the panels k_P and the stiffness of the springs are assumed proportional to the lateral stiffness *k* of the frame through the dimensionless coefficients υ and α, respectively. The same viscous damping ratio ζ_P is assumed for all the N panels.

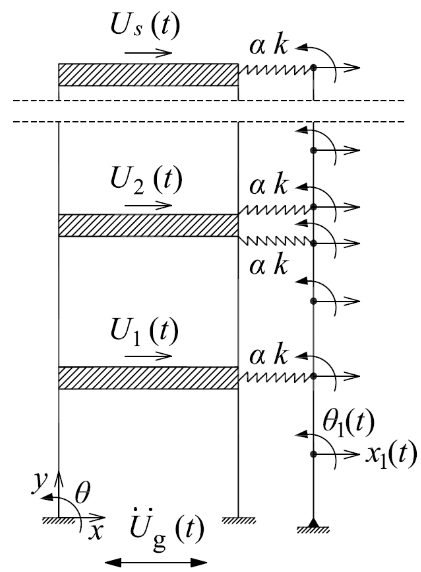

Figure 1: Shear-type primary structure with double-skin façade.

After a static condensation of the rotational DOFs of the panels, the equations of motion of the coupled system subjected to imprecise seismic excitation can be written as:

$$\mathbf{M}\ddot{\mathbf{U}}^I(t)+\mathbf{C}\dot{\mathbf{U}}^I(t)+\mathbf{K}\mathbf{U}^I(t)=-\mathbf{M}\boldsymbol{\tau}\ddot{U}_{\mathrm{g}}^I(t) \tag{1}$$

where a dot over a variable denotes differentiation with respect to time t; $\mathbf{U}^I(t)$ is the interval vector random process that collects the n dynamically significant nodal displacements, with the apex *I* denoting interval quantities; $\ddot{U}_{\mathrm{g}}^I(t)$ is the ground motion acceleration characterized by an IPSD function; $\boldsymbol{\tau}$ is the n-vector listing the influence coefficients; $\mathbf{M}$ and $\mathbf{K}$ are the $n\times n$ mass and stiffness block matrices of the coupled structure:

$$\mathbf{M}=\begin{bmatrix}\mathbf{M}_0 & \mathbf{O} & \cdots & \mathbf{O}\\ \mathbf{O} & \mathbf{M}_1 & \cdots & \mathbf{O}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{M}_N\end{bmatrix};\qquad \mathbf{K}=\begin{bmatrix}\mathbf{K}_0+\mathbf{K}_{S0} & \mathbf{K}_{01} & \cdots & \mathbf{K}_{0N}\\ \mathbf{K}_{01}^{\mathrm{T}} & \mathbf{K}_1+\mathbf{K}_{S1} & \cdots & \mathbf{O}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{K}_{0N}^{\mathrm{T}} & \mathbf{O} & \cdots & \mathbf{K}_N+\mathbf{K}_{SN}\end{bmatrix} \tag{2}$$

where $\mathbf{M}_0$ and $\mathbf{K}_0$ are the mass and stiffness matrices of the primary structure; $\mathbf{M}_i$ and $\mathbf{K}_i$ are the mass and stiffness matrices (after static condensation) of the i-th panel; $\mathbf{K}_{S0}$ represents the increment of the stiffness matrix of the primary structure due to the secondary system; $\mathbf{K}_{Si}$ and $\mathbf{K}_{0i}$ contain the stiffness of the elastic springs at each storey level; $\mathbf{O}$ is a zero matrix of appropriate dimensions. The primary building and the panels are individually considered as classically damped subsystems. In Eq. (1), the damping matrix $\mathbf{C}$ of the coupled system is defined as:

$$\mathbf{C} = \mathbf{\Gamma}^{-1}\,\mathbf{\Xi}\,\mathbf{\Gamma}^{-\mathrm{T}} \tag{3}$$

where $\mathbf{\Gamma}$ is a convenient transformation matrix while $\mathbf{\Xi}$ is the modal damping matrix.

Imprecise power spectral density function
The model of the IPSD function of ground motion acceleration recently proposed in [3] is assumed:

$$G^I_{\ddot{U}_g}(\omega) = \left(\sigma^2_{\ddot{U}_g}\right)^I \beta^I_0 \left(\frac{\omega^2}{\omega^2 + \left(\omega^I_H\right)^2}\right)\left(\frac{\left(\omega^I_L\right)^4}{\omega^4 + \left(\omega^I_L\right)^4}\right)\frac{\rho_0}{\pi}\left[\frac{1}{\left(\rho^I_0\right)^2 + (\omega + \Omega^I_0)^2} + \frac{1}{\left(\rho^I_0\right)^2 + (\omega - \Omega^I_0)^2}\right] \tag{4}$$

where β^I_0 is defined so as to ensure that the interval stochastic process $\ddot{U}^I_g(t)$ possesses variance $\left(\sigma^2_{\ddot{U}_g}\right)^I$, while $\omega^I_L = \Omega^I_0 + 0.8\,\rho^I_0$ and $\omega^I_H = 0.1\,\Omega^I_0$ are, respectively, the interval frequency control of the second order low-pass and first order high-pass butterworth filters, that in turn depend on the interval predominant circular frequency Ω^I_0 and frequency bandwidth ρ^I_0 of the filtered stationary process [5]. Since the one-sided PSD function in Eq. (4) depends linearly on $\left(\sigma^2_{\ddot{U}_g}\right)^I$, this variable could be set a posteriori.

In Eq. (4), the generic interval variable z^I_i is expressed by using the *Improved Interval Analysis* [4] as follows:

$$z^I_i = \left[\underline{z}_i, \overline{z}_i\right] \equiv z_{\mathrm{mid},i}(1 + \delta^I_i) = z_{\mathrm{mid},i}(1 + \Delta\delta_i \hat{e}^I_i) \tag{5}$$

where the symbols $\underline{z}_i$ and $\overline{z}_i$ denote the lower bound (*LB*) and the upper bound (*UB*) of the interval, respectively; $\hat{e}^I_i = [-1,1]$ is the so-called *extra unitary interval* (*EUI*) associated with the *i*-th interval variable. In Eq. (5), $z_{\mathrm{mid},i}$ and $\Delta\delta_i$ are the *midpoint value* (*mid*) and the normalized *deviation amplitude* of z^I_i, given, respectively, by:

$$z_{\mathrm{mid},i} = \frac{\underline{z}_i + \overline{z}_i}{2}; \quad \Delta\delta_i = \frac{\Delta z_i}{z_{\mathrm{mid},i}} = \frac{\overline{\delta}_i - \underline{\delta}_i}{2} > 0 \tag{6}$$

where $\Delta z_i = (\overline{z}_i - \underline{z}_i)/2$ is the *deviation amplitude* (*dev*) of z^I_i. In Eq. (5), $\delta^I_i = \Delta\delta_i \hat{e}^I_i$ denotes the dimensionless interval fluctuation around $z_{\mathrm{mid},i}$ such that $\Delta\delta_i < 1$.

Interval reliability analysis
The interval displacement vector ruled by Eq. (1) is described by an interval zero-mean stationary Gaussian random vector process, completely characterized in the frequency domain by the knowledge of the interval one-sided PSD function matrix, given by:

$$\mathbf{G}^I_{\mathbf{U}}(\omega) = \mathbf{H}^*(\omega)\mathbf{p}\mathbf{p}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\omega)G^I_{\ddot{U}_g}(\omega); \;\; \mathbf{H}(\omega) = \left[\mathbf{K} - \omega^2\mathbf{M} + \mathrm{j}\,\omega\mathbf{C}\right]^{-1}; \;\; \mathbf{p} = -\mathbf{M}\boldsymbol{\tau} \tag{7}$$

where $\mathbf{H}(\omega)$ is transfer matrix; $\mathrm{j} = \sqrt{-1}$; the asterisk means complex conjugate.

To perform structural safety assessment under seismic excitation, the IPSD function is herein incorporated into the formulation of the classical first-passage problem. Let U_h^I be the interval displacement process of interest and $U_{\max,h}^I(T) = \max_{0 \le t \le T} \left| U_h^I(t) \right|$ the associated *extreme value* process. Adopting the Vanmarcke's failure criterion [6], the *interval cumulative distribution function* (ICDF) or *interval reliability function* $L_{U_{\max,h}}^I(b,T)$ can be expressed as [3]:

$$L_{U_{\max,h}}^I(b,T) \equiv L_{U_{\max,h}}\left(b,T;\Omega_0^I,\rho_0^I,(\sigma_{\ddot{U}_g}^2)^I\right) = \Pr\left[U_{\max,h}^I(T) \le b\right]$$
$$\cong \exp\left[-T\frac{1}{\pi}\sqrt{\frac{\tilde{\lambda}_{2,U_h}^I}{\tilde{\lambda}_{0,U_h}^I}}\left[\frac{1-\exp\left(-b\left(\tilde{\delta}_{U_h}^I\right)^{1.2}\sqrt{\dfrac{\pi}{2(\sigma_{\ddot{U}_g}^2)^I\,\tilde{\lambda}_{0,U_h}^I}}\right)}{\exp\left(\dfrac{b^2}{2(\sigma_{\ddot{U}_g}^2)^I\,\tilde{\lambda}_{0,U_h}^I}\right)-1}\right]\right] \quad (8)$$

where $\tilde{\lambda}_{i,U_h}^I$ $(i = 0,1,2)$ are the interval spectral moments of the response process $\tilde{U}_h^I(t) = U_h^I(t)/\sigma_{\ddot{U}_g}^I$ under unit variance seismic acceleration; $\tilde{\delta}_{U_h}^I$ is the *interval bandwidth parameter*; b represents the barrier level; T is the observation time. The *LB* and *UB* of the ICDF can be evaluated by performing global optimization for each value of the barrier level b under the constraint that the uncertain spectral parameters range within the pertinent intervals. Alternatively, accurate estimates of the bounds of the ICDF can be efficiently obtained as [3]:

$$\underline{L}_{U_{\max,h}}(b,T) = L_{U_{\max,h}}\left(b,T;\overline{\sigma}_{\ddot{U}_g}^2,\overline{\tilde{\lambda}}_{0,U_h},\overline{\tilde{\lambda}}_{1,U_h},\overline{\tilde{\lambda}}_{2,U_h}\right);$$
$$\overline{L}_{U_{\max,h}}(b,T) = L_{U_{\max,h}}\left(b,T;\underline{\sigma}_{\ddot{U}_g}^2,\underline{\tilde{\lambda}}_{0,U_h},\underline{\tilde{\lambda}}_{1,U_h},\underline{\tilde{\lambda}}_{2,U_h}\right) \quad (9)$$

where $\underline{\tilde{\lambda}}_{i,U_h}$ and $\overline{\tilde{\lambda}}_{i,U_h}$ are the bounds of the interval spectral moments $\tilde{\lambda}_{i,U_h}^I$ $(i = 0,1,2)$. Since the monotonic behaviour is not guaranteed, such bounds might correspond to intermediate values of the interval spectral parameters Ω_0^I and ρ_0^I, and can be evaluated by optimization.

Numerical application and discussion

A six-storey shear-type frame equipped with a DSF consisting of two independent panels is considered [2]. The primary building, having a fundamental period of vibration $T_1 = 0.582\,\text{s}$, is characterized by the following parameters: floor mass $m = 20000\,\text{kg}$, lateral stiffness $k = 4\times10^7$ N/m; inter-storey height $h = 3.32$ m; constant viscous damping ratio $\zeta_0 = 0.02$ for all modes of vibration. The total mass of the two panels is assumed as 10% of the mass of the primary building [2]. For the two panels, $\upsilon = 0.392$ and $\zeta_P = 0.13$ are considered. Different values of the stiffness of the links to the frame αk are considered such that $\alpha_{\min} \le \alpha \le \alpha_{\max}$, with $\alpha_{\min} = 1.72\times10^{-3}$ and $\alpha_{\max} = 5.16\times10^{-3}$. The value $\alpha_0 = 3.44\times10^{-3}$ is assumed as the nominal one. The displacement of the first floor of the primary structure, $U_1^I(t)$, is selected as response quantity of interest. The bounds of the interval parameters Ω_0^I, ρ_0^I, $\left(\sigma_{\ddot{U}_g}^2\right)^I$ entering the IPSD function in Eq. (4) are estimated by analysing 10 site-compatible accelerograms, recorded on rigid soil deposits, downloaded from PEER [8] and Engineering Strong Motion [9] database. All the selected accelerograms are scaled to the target peak ground acceleration value $a_g = 3.33\,\mathrm{m/s^2}$. Table 1 lists the *LB*, *UB*, *mid* and percentage ratio (*dev/mid*)% of the spectral parameters.

Table 1: Main characteristics of the interval spectral parameters.

Parameter	*LB*	*UB*	*mid*	*(dev/mid) %*
$\sigma^2_{\ddot{U}_g}\,[\mathrm{m^2/s^4}]$	0.64	1.47	1.06	39.09
Ω_0 [rad/s]	23.91	45.22	34.57	30.82
ρ_0[rad/s]	12.10	22.56	17.33	30.17

The realizations of the IPSD function $\tilde{G}^I_{\ddot{U}_g}(\omega) = G^I_{\ddot{U}_g}(\omega)/\left(\sigma^2_{\ddot{U}_g}\right)^I$ pertaining to the extreme values of the interval parameters Ω^I_0 and ρ^I_0 along with the nominal spectrum are reported in Fig. 2a. The ratio $\tilde{J}_{U_1} = \tilde{\sigma}^2_{U_1,\mathrm{c}} / \tilde{\sigma}^2_{U_1,\mathrm{u}}$ between the variance of the displacement $\tilde{U}^I_1(t) = U^I_1(t)/\sigma^I_{\ddot{U}_g}$ of the primary structure with and without the DSF [2] is herein taken as representative of structural performance under seismic excitation. The subscripts "c" and "u" stand for controlled and uncontrolled. Fig. 2b displays the ratio $\tilde{J}_{U_1}$ versus the coefficient α for all possible combinations of the endpoints of the interval spectral parameters Ω^I_0 and ρ^I_0 as well as for the nominal values. The value of the dimensionless coefficient α which minimizes $\tilde{J}_{U_1}$ is nearly $\alpha_M = 0.00392$ in all the considered cases. Furthermore, over the whole range of α, the smallest value of the ratio $\tilde{J}_{U_1}$ pertains to $\overline{\Omega}_0$ and $\underline{\rho}_0$ which indeed provide the *LB*, $\underline{\tilde{\lambda}}_{0,U_1}$, of $\tilde{\lambda}^I_{0,U_1}$.

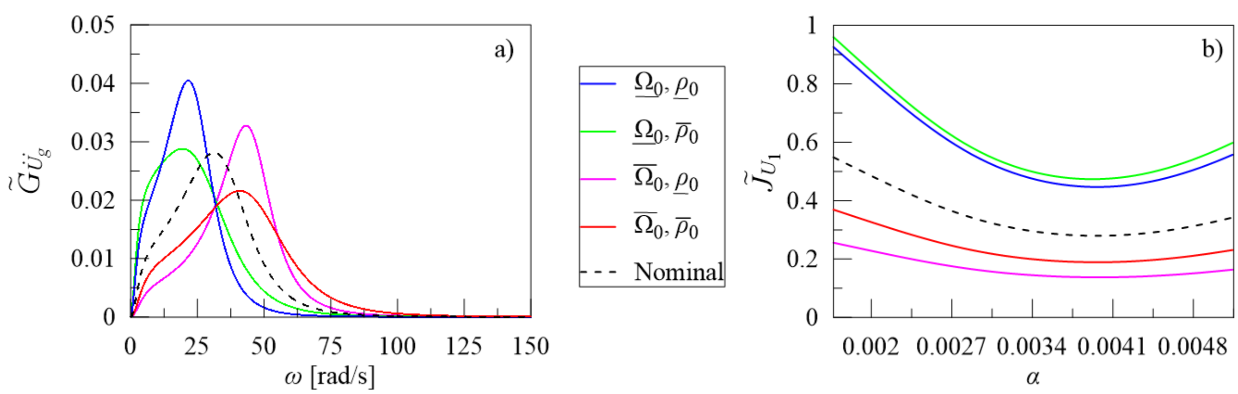

Figure 2: a) some realizations of the IPSD function of ground motion acceleration; b) associated ratios between the variance of $\tilde{U}^I_1(t) = U^I_1(t)/\sigma^I_{\ddot{U}_g}$ with and without DSF versus α.

Fig. 3a shows the *LB* and *UB* of the ICDF, $L^I_{U_{\max,1}}(b,T)$, of the *extreme value* process $U^I_{\max,1}(T)$, along with the nominal solution for $T = 30\,\mathrm{s}$ and $\alpha = \alpha_0$. Notice that the proposed bounds (Eq. (9)) are in excellent agreement with the "Exact" ones obtained by applying the scanning method. As expected, neglecting the uncertainties affecting the main parameters of the IPSD function of ground motion acceleration may lead to serious overestimation of the safety level. To ensure a conservative design, the worst-case scenario, corresponding to the *LB* of the ICDF, needs to be considered [3]. To assess the influence of the DSF on structural reliability, in Fig. 3b the bounds of the ICDF, $L^{I}_{U_{\max,1}}(b,T)$, provided by Eq. (9) for $\alpha=\alpha_{\min},\alpha_M,\alpha_{\max}$ are contrasted with the ones pertaining to the frame without DSF. It can be noticed that the DSF significantly improves the seismic performance of the primary structure. In particular, the DSF with link stiffness $k\alpha_M$ ensures the highest safety level.

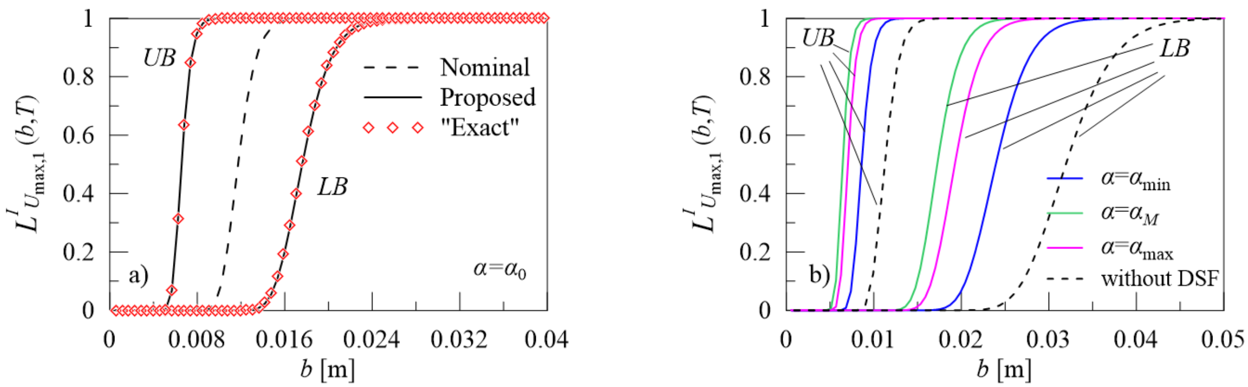

Figure 3: ICDF of $U^{I}_{\max,1}(T)$: a) comparison between the proposed and "Exact" bounds ($\alpha=\alpha_0$); b) proposed bounds for different configurations.

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