Stochastic Analysis of Multiconductor Cables and Interconnects

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Igor S. Stievano, *Senior Member, IEEE* Paolo Manfredi, *Student Member, IEEE,*
Flavio G. Canavero, *Fellow, IEEE,*

***Abstract*—This paper provides an effective solution for the simulation of cables and interconnects with the inclusion of the effects of parameter uncertainties. The problem formulation is based on the telegraphers equations with stochastic coefficients, whose solution requires an expansion of the unknown parameters in terms of orthogonal polynomials of random variables. The proposed method offers accuracy and improved efficiency in computing the parameter variability effects on system responses with respect to the conventional Monte Carlo approach. The approach is validated against results available in the literature, and applied to the stochastic analysis of a commercial multiconductor flat cable.**

***Index Terms*—EMC, Transmission-lines, Stochastic analysis, Tolerance analysis, Uncertainty, Circuit modeling, Circuit Simulation.**

I. Introduction

Nowadays, great attention is attributed to the availability of simulation techniques allowing for the analysis of cables or interconnects and including the effects of the variability of geometrical or electrical parameters of the structures. During the early design phase, the stochastic analysis of a circuit is a powerful tool that is extremely useful for the assessment of system performance and for setting realistic design margins.

Uncertainties like the position of wires within a cable bundle would require a large set of simulations with different samples of the random parameters to collect quantitative information on the statistical behavior of the structure. This solution, that is mainly based on possible enhancements of the well-known Monte Carlo (MC) method (e.g., see [1]), turns out to be extremely inefficient, thus leading to large simulation times and preventing us from its application to the analysis of complex realistic structures.

Recently, an effective solution that overcomes the previous limitations has been proposed. It is based on the so-called Polynomial Chaos (PC) technique, that assumes a series of orthogonal polynomials of random variables for the description of the solution of a stochastic problem [2], [3], [4][1]. This technique has been successfully applied to several problems in different domains, including the extension of the classical circuit analysis tools, like the modified nodal analysis (MNA), to the prediction of the stochastic behavior of circuits [6], [7], [8]. However, so far, the application has been mainly focused on the gaussian variability of model parameters and limited to dynamical circuits consisting only of lumped elements.

In this paper, the original contribution is twofold: (i) the Polynomial Chaos theory is extended to handle long and distributed interconnects described by multiconductor transmission-line equations [9] and (ii) the unknown parameters are described by uniform random variables, that seem to be the most suitable distributions for the description of the geometrical uncertainties of cables. For the specific application at hand, uncertainties like the position of wires in a bundle are known in terms of bounds available from the official documentation and datasheets.

The proposed approach has been validated against the results published in [10], that provides a probabilistic model of the crosstalk between two wires. It is worth noticing that the results of [10] are limited to the specific case of two lossless circular conductors over a ground plane and can be hardly extended to more complex structures. Also, the development in [10] is not fully analytical and its validity is limited to electrically short lines, as pointed out by [11] that provides an extension valid for the mean value and standard deviation of crosstalk.

The approach of this paper is more general than previous work and is well suited to account for multiconductor structures with arbitrary geometries. The feasibility and strength of the advocated approach are verified by applying it to the stochastic analysis of a commercial multiconductor flex-cable used for the communication between PCB cards.

II. Polynomial chaos overview

This Section provides a quick overview of the PC method, that has been established as a reference tool for the solution of stochastic equations. The idea underlying this technique is the spectral expansion of a stochastic process in terms of a truncated series of orthogonal polynomials. For a comprehensive and formal discussion of PC theory, the reader is referred to [2], [3], [12] and references therein.

In order to illustrate the main steps and characteristics of the proposed method, we refer to a simple yet representative example, consisting in a known nonlinear function $y = \ln(1 + \xi/4)$ of a random variable ξ with predefined (e.g., Gaussian) distribution. The functional dependence of this example is inspired from the class of relationships that can be found in the analytical computation of the per-unit-length parameters of canonical transmission-line structures (cfr. Eq. (14)). The mechanics of the method developed in

Igor Stievano, Paolo Manfredi and Flavio Canavero are with Dipartimento di Elettronica, Politecnico di Torino, 10129 Torino, Italy (e-mail: {igor.stievano,paolo.manfredi,flavio.canavero}@polito.it).

[1]In this context, the word Chaos is used in the sense originally defined by Wiener [5] as an approximation of a Gaussian random process by means of Hermite polynomials.

this Section will be generalized in Sect. III to the treatment of a differential equation with several stochastic parameters.

The PC expansion that *approximates* y in a mathematical sense writes

$$y = \ln(1+\xi/4) \approx \sum_{k=0}^{P} \alpha_k \cdot \phi_k(\xi) \tag{1}$$

where ϕ_k are suitable orthogonal polynomials expressed in terms of the random variable ξ. The above expression is defined by the class of the orthogonal basis, by the number of terms P and by the expansion coefficients α_k.

As an example, the first three orthogonal functions of the expansion based on Gaussian variables are the Hermite polynomials $\phi_0 = 1$, $\phi_1 = \xi$ and $\phi_2 = (\xi^2 - 1)$. The orthogonality property of Hermite polynomials is expressed by

$$< \phi_k, \phi_j > = < \phi_k, \phi_k > \delta_{kj} \tag{2}$$

where δ_{kj} is the Kronecker delta and $< \cdot, \cdot >$ denotes the inner product in the Hilbert space of the variable ξ with Gaussian weighting function, i.e.,

$$\begin{cases} < \phi_k, \phi_j > = \int_{-\infty}^{+\infty} \phi_k(\xi)\phi_j(\xi) W(\xi) d\xi \\ W(\xi) = \exp(-\xi^2/2)/(\sqrt{2\pi}). \end{cases} \tag{3}$$

With the above definitions, the expansion coefficients are computed via the projection of y onto the orthogonal components ϕ_k as follows:

$$\alpha_k = < y(\xi), \phi_k(\xi) > / < \phi_k(\xi), \phi_k(\xi) > . \tag{4}$$

It is worth noticing that relation (1), that turns out to be a known nonlinear function of the random vector ξ, can be used to predict the probability density function (PDF) of y via numerical simulation or analytical formulas [13].

In order to highlight the strengths of the proposed method, Fig. 1 collects the results of the approximation of the random variable y by means of Equation (1) with an increased number of terms. This Figure shows the reference and the predicted deterministic functions defining y and their corresponding probability distributions (top panels) along with the Gaussian distribution of the variable ξ (bottom panel). The accuracy of the PC expansion with a small number of terms can be clearly appreciated in the Figure. Also, this comparison confirms the flexibility of the proposed technique in reproducing the behavior of random variables with arbitrary distribution.

It is relevant to remark that the Hermite-based PC expansion uniformly converges for any arbitrary random process with finite second-order moments. However, the convergence rate is optimal for Gaussian processes [14]. This can be simply understood by observing that the weighting function $W(\xi)$ of (3) is assumed to coincide with the probability density function of the Gaussian random variables $f_\xi(\xi) = \exp(-\xi^2/2)/(\sqrt{2\pi})$. For different statistics, the convergence rate may be substantially slower and alternative orthogonal polynomials have been proven to provide better results. In particular, when the system

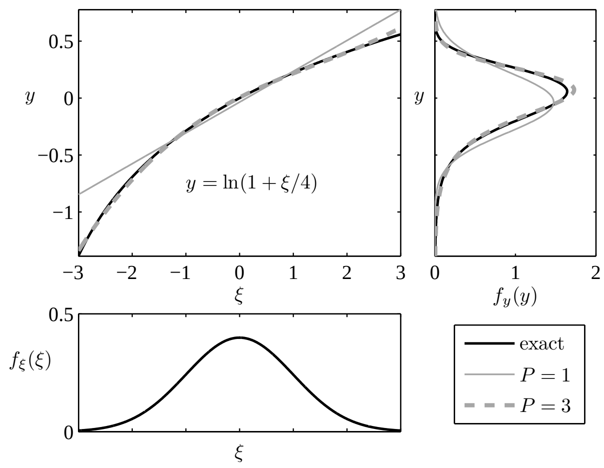

Fig. 1. Nonlinear function $y = \ln(1+\xi/4)$ of a random variable of known probability distribution (top left panel) and its corresponding probability density function $f_y(y)$ (right panel). The probability function $f_\xi(\xi)$ of the Gaussian random variable ξ is reported in the bottom panel. The curves marked "exact" are the reference curves; The other curves refer to the prediction obtained with a first and a third order polynomial chaos expansion.

parameters are known in terms of bounded uncertainties with uniform distribution, the most appropriate representation is the one based on Legendre orthogonal polynomials [3]. In this case, the random variable ξ in (1) corresponds to the normalized uniform random variable with support $[-1, 1]$ and probability density function

$$f_\xi(\xi) = \begin{cases} 1/2, & |\xi| \le 1 \\ 0, & |\xi| > 1. \end{cases} \tag{5}$$

According to what is done for the Gaussian variable, the weighting function in (3) is substituted by the above uniform distribution.

Table I summarizes the basic definitions and properties of Legendre polynomials and extends the basic results of the PC theory for one random variable to the case of an arbitrary number of random variables. As an example, Table II collects the first ten terms of the Legendre-based expansion for two random variables and a third order expansion. Briefly speaking, the orthogonality relations allow the construction of the higher dimension polynomials as the product combination of the polynomials in one variable. This holds true both for Hermite- and Legendre-based expansions [15].

III. Stochastic transmission-line model

This section discusses the modification to the transmission-line equations, allowing to include the effects of the statistical variation of the per-unit-length (p.u.l.) parameters via the PC theory. For the sake of simplicity, the discussion is based on the multiconductor transmission-line structure shown in Fig. 2 that represents the typical problem of two wires whose heights above ground and separation are not known exactly, thus leading to a probabilistic definition of crosstalk between the wires.

In the structure of Fig. 2, the height h and the separation d are assumed to be defined by

TABLE I
LEGENDRE POLYNOMIAL CHAOS DEFINITIONS AND PROPERTIES.

Object	e.g., parameter or variable y that depends on $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_n]^T$
Expansion	$y(\boldsymbol{\xi}) = \sum_{k=0}^{P} \alpha_k \cdot \phi_k(\boldsymbol{\xi})$
Orthogonal basis	Legendre polynomials $\{\phi_k(\boldsymbol{\xi})\}$ (e.g., see Tab. II for the case $n = 2$)
Inner product	$< \phi_k, \phi_j >= \int_{\Re^n} \phi_k(\boldsymbol{\xi})\phi_j(\boldsymbol{\xi})W(\boldsymbol{\xi})d\boldsymbol{\xi}$
Weighting function	$W(\boldsymbol{\xi}) = \begin{cases} \frac{1}{2^n}, & \lvert\xi_j\rvert \le 1 \\ 0, & \lvert\xi_j\rvert > 1 \end{cases}, j = 1, \ldots, n$
Orthogonality	$< \phi_k, \phi_j >=< \phi_k, \phi_k > \delta_{kj}$
Expansion Coefficients	$\alpha_k =< y, \phi_k > / < \phi_k, \phi_k >$
Mean	α_0

TABLE II
LEGENDRE POLYNOMIALS FOR THE CASE OF TWO INDEPENDENT RANDOM VARIABLES ($n = 2$, $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$) AND A THIRD ORDER EXPANSION ($p = 3$).

index k	order p	k-th basis ϕ_k	$< \phi_k, \phi_k >$
0	0	1	1
1	1	ξ_1	$\frac{1}{3}$
2	1	ξ_2	$\frac{1}{3}$
3	2	$\frac{3}{2}\xi_1^2 - \frac{1}{2}$	$\frac{1}{5}$
4	2	$\xi_1\xi_2$	$\frac{1}{9}$
5	2	$\frac{3}{2}\xi_2^2 - \frac{1}{2}$	$\frac{1}{5}$
6	3	$\frac{5}{2}\xi_1^3 - \frac{3}{2}\xi_1$	$\frac{1}{7}$
7	3	$\frac{3}{2}\xi_1^2\xi_2 - \frac{1}{2}\xi_2$	$\frac{1}{15}$
8	3	$\frac{3}{2}\xi_1\xi_2^2 - \frac{1}{2}\xi_1$	$\frac{1}{15}$
9	3	$\frac{5}{2}\xi_2^3 - \frac{3}{2}\xi_2$	$\frac{1}{7}$

$$\begin{cases} h &= \bar{h} + (\Delta_h/2)\xi_1 \\ d &= \bar{d} + (\Delta_d/2)\xi_2 \end{cases} \tag{6}$$

where ξ_1 and ξ_2 are independent normalized uniform random variables, with $\bar{h}$ and $\bar{d}$ mean values and Δ_h and Δ_d supports.

A. Transmission line model

The electrical behavior in frequency-domain of the line of Fig. 2 is described by the well-known telegraph equations,

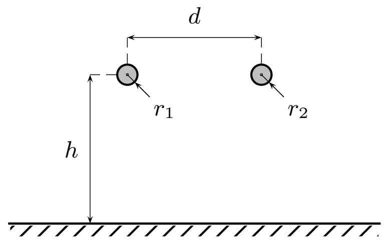

Fig. 2. Cross-section of two coupled lines, whose height above ground and wire separation are uncertain parameters.

$$\frac{d}{dz}\begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} = -s \begin{bmatrix} 0 & \mathbf{L} \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} \tag{7}$$

where s is the Laplace variable, $\mathbf{V} = [V_1(z,s), V_2(z,s)]^T$ and $\mathbf{I} = [I_1(z,s), I_2(z,s)]^T$ are vectors collecting the voltage and current variables along the multiconductor line (z coordinate) and $\mathbf{C}$ and $\mathbf{L}$ are the p.u.l. capacitance and inductance matrices depending on the geometrical and material properties of the structure [9]. It is important to remark that, for notational convenience, a lossless assumption is done, leading to Equation (7) that includes the inductive and capacitive terms only. However, the proposed method is general and holds for the lossy case as well. Equation (7) and the forthcoming equations can be suitably modified by including the conductance and resistance matrices.

In order to account for the uncertainties affecting the guiding structure, we must consider the p.u.l. matrices $\mathbf{C}$ and $\mathbf{L}$ as random quantities, with entries depending on the random vector $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$. In turn, (7) becomes a stochastic differential equation, leading to randomly-varying voltages and currents along the line.

B. PC expansion of the p.u.l. parameters

For the current application, the random p.u.l. matrices in (7) are represented through the Legendre expansion as follows,

$$\mathbf{C} = \sum_{k=0}^{P} \mathbf{C}_k \cdot \phi_k(\boldsymbol{\xi}), \quad \mathbf{L} = \sum_{k=0}^{P} \mathbf{L}_k \cdot \phi_k(\boldsymbol{\xi}) \tag{8}$$

where $\{\mathbf{C}_k\}$ and $\{\mathbf{L}_k\}$ are expansion coefficients matrices with respect to the orthogonal components $\{\phi_k\}$ defined in Tab. II. For a given number of random variables n and order p of the expansion (that corresponds to the highest order of the polynomials in (8) and generally lies within the range $2 \div 5$ for practical applications), the total number of terms is

$$(P+1) = \frac{(n+p)!}{n!p!} \tag{9}$$

that turns out to be equal to ten for the case $n = 2$ and $p = 3$.

C. Stochastic model

The randomness of the p.u.l parameters reflects into stochastic values of the voltage and current unknowns and makes us decide to use expansions similar to (8) for the electrical variables. This yields a modified version of (7), whose second row is provided below in extended form for $P = 2$, as an exemplification

$$\frac{d}{dz}(\mathbf{I}_0(z,s)\phi_0(\boldsymbol{\xi}) + \mathbf{I}_1(z,s)\phi_1(\boldsymbol{\xi}) + \mathbf{I}_2(z,s)\phi_2(\boldsymbol{\xi})) = \\ -s(\mathbf{C}_0\phi_0(\boldsymbol{\xi}) + \mathbf{C}_1\phi_1(\boldsymbol{\xi}) + \mathbf{C}_2\phi_2(\boldsymbol{\xi}))(\mathbf{V}_0(z,s)\phi_0(\boldsymbol{\xi}) + \\ +\mathbf{V}_1(z,s)\phi_1(\boldsymbol{\xi}) + \mathbf{V}_2(z,s)\phi_2(\boldsymbol{\xi})) \tag{10}$$

where the interpretation of the new variables is straightforward.

Projection of (10) on the first three Legendre polynomials leads to the following set of equations, where the explicit dependence on variables is dropped for notational convenience,

$$\frac{d}{dz}(\mathbf{I}_0{<}\phi_0,\phi_j{>} + \mathbf{I}_1{<}\phi_1,\phi_j{>} + \mathbf{I}_2{<}\phi_2,\phi_j{>}) \\ = -s(\mathbf{C}_0{<}\phi_0\phi_0,\phi_j{>}\mathbf{V}_0 + \mathbf{C}_0{<}\phi_0\phi_1,\phi_j{>}\mathbf{V}_1 + \\ + \ldots + \mathbf{C}_2{<}\phi_2\phi_2,\phi_j{>}\mathbf{V}_2), \quad j = 0,1,2 \tag{11}$$

The above equation, along with the companion relation arising from the first row of (7), can be further simplified by using the orthogonality relations of Tab. I and Tab. II for the computation of the inner products $<\phi_k,\phi_j>$ and $<\phi_k\phi_l,\phi_j>$, leading to the following augmented system, where the random variables collected in vector $\boldsymbol{\xi}$ do not appear, due to the integration process:

$$\frac{d}{dz}\begin{bmatrix} \tilde{\mathbf{V}}(z,s) \\ \tilde{\mathbf{I}}(z,s) \end{bmatrix} = -s\begin{bmatrix} 0 & \tilde{\mathbf{L}} \\ \tilde{\mathbf{C}} & 0 \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{V}}(z,s) \\ \tilde{\mathbf{I}}(z,s) \end{bmatrix} \tag{12}$$

In the above equation, the new vectors $\tilde{\mathbf{V}} = [\mathbf{V}_0, \mathbf{V}_1, \mathbf{V}_2]^T$ and $\tilde{\mathbf{I}} = [\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2]^T$ collect the coefficients of the PC expansion of the unknown variables.

It is worth noticing that Equation (12) belongs to the same class of (7) and plays the role of the set of equations of an extended multiconductor transmission line, whose number of conductors is $(P+1)$ times larger than in the original line. Despite the increased system size, for limited values of P (as typically occurs in practice), the additional overhead in handling the augmented equations is much less than the time required to run a large number of MC simulations. Additional results are collected in Table III of Sec. V, summarizing the quantitative information on the speed-up introduced by the proposed approach for the stochastic simulation of a commercial cable.

The extension of the proposed technique to different multiconductor structures that possibly include losses and to a larger number of random variables is straightforward.

D. Boundary conditions and simulation

For the deterministic case, the simulation of an interconnect like the one of Fig. 3 amounts to combining the port electrical relations of the two terminal elements defining the source and the load with the transmission line equation, and solving the system. This is a standard procedure as illustrated for example in [9]. The port equations of the terminations of Fig. 3 in the Laplace domain become

$$\begin{cases} \mathbf{V}_a(s) &= \mathbf{E}(s) - \mathbf{Z}_S(s)\mathbf{I}_a(s) \\ \mathbf{V}_b(s) &= \mathbf{Z}_L(s)\mathbf{I}_b(s) \end{cases} \tag{13}$$

where $\mathbf{Z}_S = \mathrm{diag}([Z_{S1}, Z_{S2}])$, $\mathbf{Z}_L = \mathrm{diag}[Z_{L1}, Z_{L2}]$ and $\mathbf{E} = [E_1, 0]^T$. Also, in the above equation, the port voltages and currents need to match the solutions of the differential Equation (7) at line ends (e.g., $\mathbf{V}_a(s) = \mathbf{V}(z{=}0, s)$, $\mathbf{V}_b(s) = \mathbf{V}(z{=}\mathcal{L}, s)$).

Similarly, when the problem becomes stochastic, the augmented transmission-line Equation (12) is used in place of (7) together with the projection of the characteristics of the source and the load elements (13) on the first P Legendre polynomials. It is worth noticing that in this specific example, no variability is included in the terminations and thus the augmented characteristics of the source and load turn out to have a diagonal structure.

Once the unknown voltages and currents are computed, the quantitative information on the spreading of circuit responses can be readily obtained from the analytical expression of the unknowns. As an example, the frequency-domain solution of the magnitude of voltage V_{a1} with $P = 2$, leads to $|V_{a1}(j\omega)| = |V_{a10}(j\omega)\phi_0(\boldsymbol{\xi}) + V_{a11}(j\omega)\phi_1(\boldsymbol{\xi}) + V_{a12}(j\omega)\phi_2(\boldsymbol{\xi})|$. As already outlined in the introduction, the above relation turns out to be a known nonlinear function of the random vector $\boldsymbol{\xi}$ that can be used to compute the PDF of $|V_{a1}(j\omega)|$ via standard techniques as numerical simulation or analytical formulae [13].

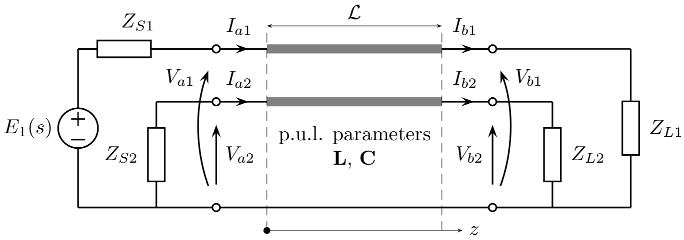

Fig. 3. Test setup considered to demonstrate the proposed approach.

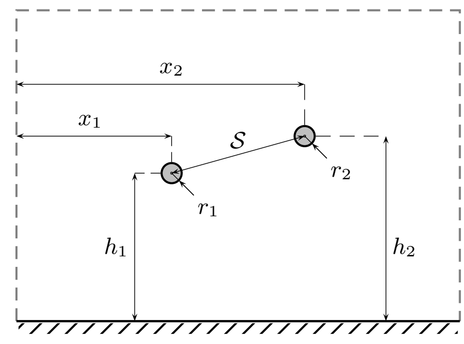

Fig. 4. Cross-section of the coupled lines setup used for validation. The heights above ground and separation of wires are selected as uncertain parameters.

IV. VALIDATION

This Section summarizes the validation of the proposed technique by reproducing the results published in [10], where the frequency-domain analysis of the test setup of Fig. 3 with the cross-section geometry of Fig. 4 is considered.

This case is an extension of the illustrative example of Sect. III, where the location of each wire is defined by a pair of independent random variables (the wire heights $h_{1,2}$ above the ground plane and the horizontal distances $x_{1,2}$ from a conventional origin), thus leading to a problem that involves a total of four uniformly-distributed random variables. However, the expressions of the entries of the p.u.l. inductance matrix $\mathbf{L}$ ([9]):

$$\begin{cases} l_{ii} = \dfrac{\mu_0}{2\pi}\ln(2h_i/r_i) & i=1,2 \\ l_{12} = \dfrac{\mu_0}{4\pi}\ln(1+4\dfrac{h_1 h_2}{\mathcal{S}^2}) & (=l_{21}) \end{cases} \quad (14)$$

suggest that only three random variables are needed to compute the actual p.u.l. matrix, the third variable being the separation $\mathcal{S}$ between the wires, given by

$$\mathcal{S} = \sqrt{(h_1-h_2)^2 + (x_1-x_2)^2}. \quad (15)$$

The dependence of $\mathcal{S}$ from the other random variables and the additional constraint of a minimal separation between wires (2 mm) in order to avoid overlapping, imply that $\mathcal{S}$ is not uniformly distributed, but this fact does not jeopardize the validity of PC technique, as stated in Sect. II. The previous consideration applies also to the p.u.l. capacitance matrix, which is determined by $\mathbf{C} = \mu_0\epsilon_0\mathbf{L}^{-1}$, since no dielectric coating on the wires is assumed.

The computation of this validation case was performed for the following values: $\mathcal{L} = 10\,\text{m}$, $r_1 = r_2 = 0.4\,\text{mm}$, $Z_{S1} = 0\,\Omega$, $Z_{S2} = (1+j0)\,\Omega$, $Z_{L1,2} = (10+j0)\,\Omega$ and $f = 10\,\text{kHz}$. The wire heights and the horizontal distances were defined as uniform variables in the range $[0.1, 2]$ cm and $[0, 2]$ cm, respectively. The definitions and properties of Tab. I are directly applicable to this case, whereas the entire set of basis functions can be readily obtained by an extension of Tab. II to the case of three random variables.

Figure 5 shows a comparison between the probability density function of the crosstalk $|V_{a2}/E_1|$ reproduced from [10], and the results computed via the advocated PC method with a fifth order expansion; the results of the MC procedure with a large number of simulations ($40,000$ in this case) are also shown. The comparison of Fig. 5 validates the advocated PC method by demonstrating that the results agree with the general (although computationally expensive) MC approach and with analytical/numerical results published in [10] (although limited to the specific two-wire case with random cross-section). In addition, Fig. 5 confirms the potential of the proposed PC method to be efficiently used in practical applications dealing with parameters characterized by large variability with non-uniform statistical distributions.

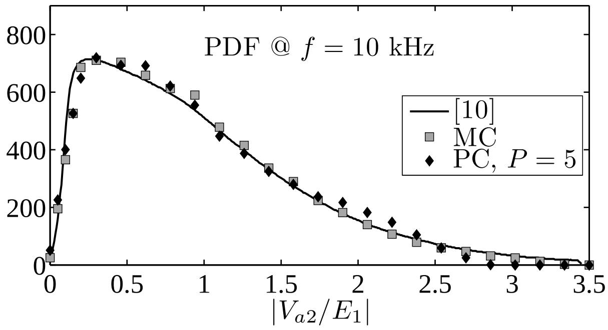

Fig. 5. Probability distribution of $|V_{a2}/E_1|$ for the test structure described in Fig.s 3 and 4 (see text for details).

V. Application

As a proof of the capabilities of the proposed technique, the analysis of the test structure depicted in Fig. 6 is presented. The structure represents a .050" High Flex Life Cable in a standard 9-wire configuration. Figure 6 collects both the key parameters defining the geometry of the wires as well as the information on the two-terminal circuit elements connected at the near-end of the cable. The cable length is 80 cm and the far-end terminations are defined by identical RC parallel elements ($R = 10\,\text{k}\Omega$, $C = 10\,\text{pF}$) connecting the wires #1,. . . ,#8 to the reference wire #0.

In this example, the goal is to estimate the response variability of the near-end crosstalk between two adjacent wires in a bundle of many wires. As highlighted in Fig. 6, line #4 is energized by the voltage source E_4 and the other lines are quiet and kept in the low state via the R_S resistors. The variability is introduced by the relative permittivity ε_r of the coating and by the separations d_{34} and d_{45} between the active and its immediately adjacent lines. These quantities are assumed to behave as independent uniform random variables as follows: $\varepsilon_r \in [2.8, 4]$ and $d_{34}, d_{45} \in [48, 52]$ mils. The adopted values of variability correspond to the tolerance limits available from the official datasheet of the cable. The other separations are considered to be equal to the nominal value of 50 mils, since their possible variations have negligible effects on the crosstalk. For this comparison a third order PC expansion of the p.u.l. parameters is computed via numerical integration based on the method described in [9].

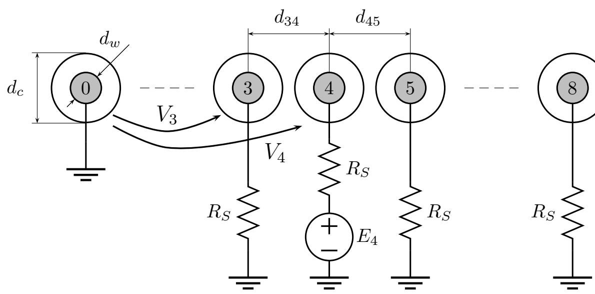

Fig. 6. Application test structure: 80 cm long commercial flex cable (.050" High Flex Life Cable, 28 AWG Standard, PVC, 9-wire configuration). $R_S = 50\,\Omega$, $d_w = 15$ mils, $d_c = 35$ mils. The nominal value of the distance between adjacent wires (e.g., d_{34} and d_{45}) is 50 mils.

Figure 7 shows a comparison of the Bode plot (magnitude) of the transfer function $H(j\omega) = V_3(j\omega)/E_4$ defining the near-end crosstalk computed via the advocated PC method and determined by means of the MC procedure. The solid black thin curves of Fig. 7 represent the $\pm 3\sigma$ interval of the transfer function, where σ indicates the standard deviation, determined from the results of the proposed technique. For comparison, the deterministic response with nominal values of all parameters is reported in Fig. 7 as a solid black thick line; also, a limited set of MC simulations (100, out of the 40,000 runs, in order not to clutter the figure) are plotted as gray lines. Clearly, the thin curves of Fig. 7 provide a qualitative information of the spread of responses due to parameters uncertainty. A better quantitative prediction can be appreciated in Fig. 8, comparing the PDF of $|H(j\omega)|$ computed for different frequencies with the distribution obtained via the analytical PC expansion. The frequencies selected for this comparison correspond to the dashed lines shown in Fig. 7. The good agreement between the actual and the predicted PDFs and, in particular, the accuracy in reproducing the tails and the large variability of non-uniform shapes of the reference distributions, confirm the potential of the proposed method. For this example, it is also clear that a PC expansion with $P = 3$ is already accurate enough to capture the dominant statistical information of the system response.

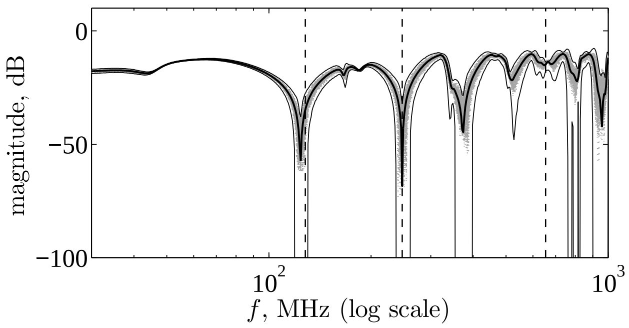

Fig. 7. Bode plots (magnitude) of the near-end crosstalk transfer function $H(j\omega)$ of the example test case (see text for details). Solid black thick line: deterministic response; solid black thin lines: 3σ tolerance interval of the third order polynomial chaos expansion; gray lines: a sample of responses obtained by means of the MC method (limited to 100 curves, for graph readability).

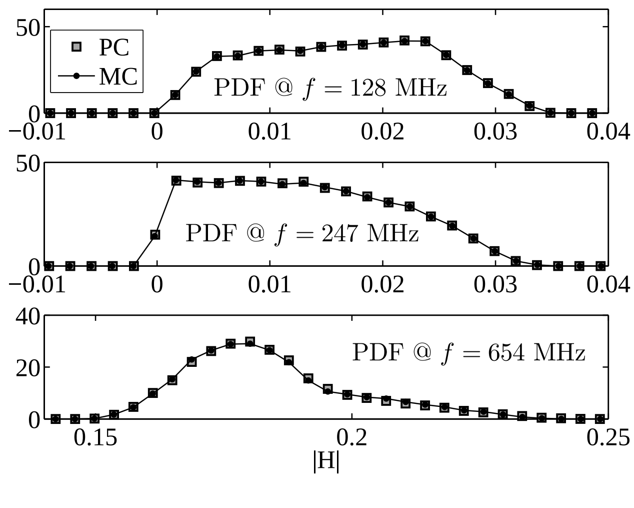

Fig. 8. Probability density function of $|H(j\omega)|$ for the example of this study, computed at different frequencies. Of the two distributions, the one marked MC refers to 40000 MC simulations, and the one marked PC refers to the response obtained via third order polynomial chaos expansion.

Finally, Tab. III collects the main figures related to the efficiency of the proposed PC method vs. conventional MC. Table data indicate that the PC computation of the curves of Fig. 7 is faster by almost two orders of magnitude with respect to MC computation. This holds even if for fairness we consider the computational overhead required by the solution of the augmented set of TL Equations (12). The above comparison confirms the strength of the proposed method, that allows to generate accurate predictions of the statistical behavior of a realistic interconnect with a great efficiency improvement.

TABLE III
CPU TIME REQUIRED BY THE SIMULATION OF THE SETUP OF FIG. 6 (FOR A SINGLE FREQUENCY SAMPLE) BY THE MC AND THE PROPOSED PC-BASED METHODS.

Method	Order p	Overhead	Simulation time
MC	-	0 sec	3 min 10 sec
PC	3	2 sec	1 sec

VI. CONCLUSIONS

This paper derives an enhanced multiconductor transmission-line equation allowing to include parameter uncertainties into interconnect structures. The proposed method, that enables to compute the quantitative information on the transmission-line response sensitivity to parameters variability, is based on the expansion of the voltage and current variables into a sum of a limited number of orthogonal polynomials. The advocated technique, while providing accurate results, turns out to be more efficient than conventional solutions like Monte Carlo, also in presence of several random variables. The method has been validated against reference results available in the literature and has been applied to the stochastic analysis of a commercial multiconductor flex cable with three uncertain parameters described by independent Uniform random variables.

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REFERENCES

[1] Q. Zhang, J. J. Liou, J. McMacken, J. Thomson, P. Layman, "Development of robust interconnect model based on design of experiments and multiobjective optimization," IEEE Transactions on Electron Devices, Vol. 48, No. 9, pp. 1885 – 1891, Sep. 2001.

[2] R. G. Ghanen, P. D. Spanos, "Stochastic Finite Elements. A Spectral Approach," Springer-Verlag, 1991 (Ch. 2).

[3] D. Xiu, G. E. Karniadakis, "The Wiener-Askey polynomial chaos for stochastic differential equations," SIAM, Journal of Sci. Computation, Vol. 24, No. 2, pp. 619–622, 2002.

[4] B. J. Debusschere et Al., "Numerical challenges in the use of polynomial chaos representations for stochastic processes," SIAM Journal on Scientific Computing, Vol. 26, No. 2, pp. 698–719, 2005.

[5] N. Wiener, "The homogeneous chaos", Amer. J. Math., Vol. 60, pp. 897–936, 1938.

[6] K. Strunz, Q. Su, "Stochastic formulation of SPICE-type electronic circuit simulation using polynomial chaos," ACM Transactions on Modeling and Computer Simulation, Vol. 18, No. 4, Sep. 2008.

[7] Y. Zou at Al., “Practical implementation of stochastic parametrized model order reduction via Hermite polynomial chaos,” Proc. of the *2007 Asia and South Pacific Design Automation Conference,* pp. 367–372, 2007.

[8] I. S. Stievano, F. G. Canavero, “Response Variability of High-Speed Interconnects via Hermite Polynomial Chaos,” Proc. of the *14th IEEE Workshop on Signal Propagation on Interconnects,* Hildesheim, Germany, pp. 3-6, May 09-12, 2010.

[9] C. R. Paul, “Analysis of Multiconductor Transmission Lines,” Wiley, 1994.

[10] S. Shiran, B. Reiser, H. Cory, “A Probabilistic Model for the Evaluation of Coupling Between Transmission Lines,” IEEE Transactions on Electromagnetic Compatibility, Vol. 35, No. 3, pp. 387–393, Aug. 1993.

[11] D. Bellan, S.A. Pignari, G. Spadacini, “Characterisation of crosstalk in terms of mean value and standard deviation,” IEE Proc.-Sci. Meas. Technol., Vol. 150, No. 6, pp. 289–295, Nov. 2003,

[12] W. Schoutens, “Stochastic Processes and Orthogonal Polynomials”, Springer, 2000.

[13] A. Papoulis, “Probability, Random Variables and Stochastic Processes,” 3^{rd} edition, McGraw-Hill, 1991.

[14] R. Cameron, W. Martin, “The orthogonal development of nonlinear functionals in series of Fourier-Hermite functionals”, Ann. of Math., No. 2, Vol. 48, pp. 385–392, 1947.

[15] R. Courant, D. Hilbert, Methods of Mathematical Physics. Volume 1. Wiley-Interscience: New York, 1989.