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# STOCHASTIC BEHAVIOUR OF SUGAR PLANT WITH 2-OUT-OF-3:F BOILERS AND PRE-EMPITIVE RESUME REPAIR

**By**

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**ABSTRACT**

In this paper, the authors have studied stochastic behaviour of an industrial problem related to sugar plant. A sugar plant is complex system with various subsystems; viz., feeding system, cane cutters, crushing system, boilers and the mills. The system under consideration is Non-Markovian, supplementary variables have been used to convert this in Markovian. Mathematical model of considered system has been solved be using Laplace transform. Transition-state diagram has been shown in fig-1(b). Reliability, M.T.T.F and availability of the system have been computed. Ergodic behaviour and a particular case for the considered system have been obtained to improve practical utility of the model. A numerical illustration and its graphical representation have also been appended in the end to highlight important results of the study.

**KEY WORDS:** *Markovian System, Standby Redundancy, Supplementary Variables, Reliability, M.T.T.F., Availability, Ergodic Behaviour etc.*

## 1. INTRODUCTION

In the process of manufacturing of sugar, initially the sugar canes go to cutters (arranged in series). These cutters then start cutting of cane in fine small pieces and the pieces are crushed by chain conveyor and juice is obtained. This juice contains 78-86% reducing sugar, 0.3 – 0.77% ash etc. So, the refining of the juice is necessary and is an important procedure. After filtration, this juice goes to the boilers. As this cane juice contains many soluble and insoluble impurities, the juice is heated up to $68^0 C$ with the help of boilers. When these impurities are removed, purification is completed by sulphonation or carbonation process. The so obtained juice is again heated up to $102^0 C$ to get its concentrated from. By crystallizing the concentrated juice, a colored crystalline sugar is obtained and is called a raw sugar. Bleaching the concentrated juice further helps the production of white sugar. White sugar is directly packed.

In this model, the authors have considered the process up to boilers. The whole system has divided in two subsystems namely, A and B, connected in series. The subsystem A has three units in series and these units are cutting unit, crushing unit and refining unit. On failure of any one unit of subsystem A, the whole system goes to failed state. The subsystem B consists of three boilers connected in parallel redundancy. The subsystem B is of 2-out-of-3:F and 1-out-of-3:D nature. The whole system can also fail due to environmental reasons. System configuration has been shown in fig-1(a). All failures follow exponential time distribution whereas all repairs follow general time distribution. Pre-emptive resume policy has been adopted for repair purpose. The subsystem A has given priority in repairs over the subsystem B.

## 2. ASSUMPTIONS

The following assumptions have been taken care in this study:

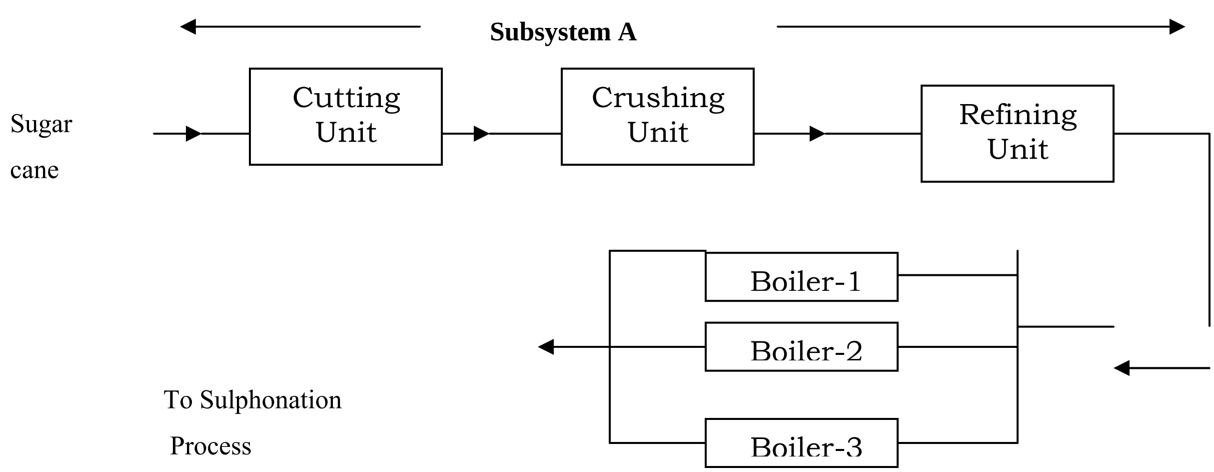
(1) Initially all the units are in operable condition of full efficiency.
(2) Failures are S-independent and nothing can fail from a failed state.
(3) All failures follow exponential time distribution.
(4) Repairs follow general time distribution. Repair facilities are always available and are perfect.
(5) Pre-emptive resume policy has been adopted for repair purpose. The subsystem A has given priority in repairs over the system B.
(6) The subsystem B is of 1-out-of-3:D and 2-out-of-3:F.
(7) The whole system can also fail due to environmental reasons.

## 3. LIST OF NOTATIONS

The list of notations is as follows:

| | | |
|---|---|---|
| $\lambda_i (i = 1,2,3)$ | : | Failure rate of $i^{th}$ unit of subsystem A. |
| $\lambda$ | : | $\sum_{i=1}^{3} \lambda_i$ |
| $\mu_1 / \mu_2$ | : | Failure rate of first/second unit of subsystem B. |
| $e_1, e_2$ | : | Failure rates due to environmental reasons. |
| $\alpha_1(x)\Delta / \alpha_2(m)\Delta$ | : | First order probability that subsystem A/environmental failure will be repaired in the time interval $(x, x+\Delta)/(m, m+\Delta)$, conditioned that it was not repaired up to the time x/m. |
| $\beta_1(y)\Delta / \beta_2(z)\Delta$ | : | The first order probability that one/two units of subsystem B will be repaired in the time interval $(y, y+\Delta)/(z, z+\Delta)$, conditioned that it was not repaired up to the time y/z. |
| $P_0(t)$ | : | Pr {at time t, system is all operable}. |
| $P_i(j,t)\Delta$ | : | Pr {at time t, system suffers with $i^{th}$ failure}. Elapsed repair time lies in the |

| | | |
|---|---|---|
| | | interval $(j, j+\Delta)$. |
| $P_{B_1}(y,t)\Delta$ | : | Pr {at time t, system works with reduced efficiency due to failure of any one unit of subsystem B}. Elapsed repair time lies in the interval $(y, y+\Delta)$. |
| $\overline{P}(s)$ | : | Laplace transform (L.T) of function P(t). |
| $S_{\alpha_i}(j)$ | : | $\alpha_i(j)\exp\cdot\{-\int\alpha_i(j)dj\}$ |
| $D_{\alpha_i}(j)$ | : | $1-\overline{S}_{\alpha_i}(j)/(j)$ |
| $M.T.T.F.$ | : | Mean time to failure. |
| $P_{B_1A}(x,y,t)\Delta$ | : | Pr {at time t, system is failed due to failure of subsystem A while one unit of subsystem B has already failed}. Elapsed repair time for A lies in the interval $(x, x+\Delta)$ and for B it lies within $(y, y+\Delta)$. |



← **Subsystem B** →

**Fig-1(a) (System Configuration)**

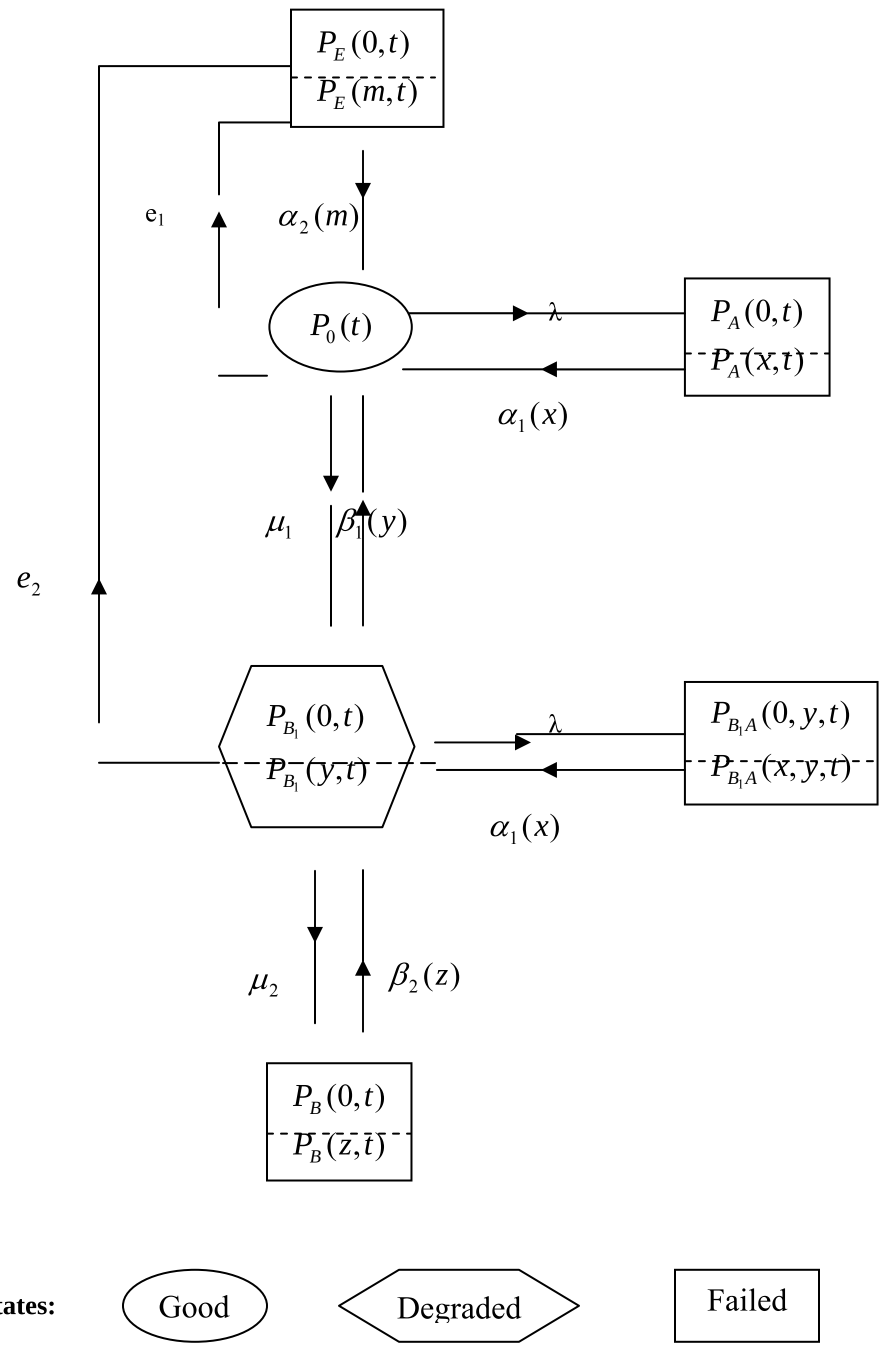



**Fig-1(b): Transition-state diagram**

## 4. FORMULATION OF MATHEMATICAL MODEL

By using probability consideration and limiting procedure, we obtain the following set of difference-differential equations, which is continuous in time, discrete in space and governing the behaviour of considered system:

$$\left[\frac{d}{dt}+\lambda+\mu_1+e_1\right]P_0(t)=\int_0^\infty P_A(x,t)\alpha_1(x)dx+\int_0^\infty P_E(m,t)\alpha_2(m)dm+\int_0^\infty P_{B_1}(y,t)\beta_1(y)dy \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\alpha_1(x)\right]P_A(x,t)=0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda+\mu_2+e_2+\beta_1(y)\right]P_{B_1}(y,t)=\int_0^\infty P_{B_1A}(x,y,t)\alpha_1(x)dx \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\alpha_1(x)\right]P_{B_1A}(x,y,t)=0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\beta_2(z)\right]P_B(z,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\alpha_2(m)\right]P_E(m,t)=0 \quad \ldots(6)$$

Boundary conditions are:

$$P_A(0,t)=\lambda P_0(t) \quad \ldots(7)$$

$$P_{B_1}(0,t)=\mu_1P_0(t)+\int_0^\infty P_B(z,t)\beta_2(z)dz \quad \ldots(8)$$

$$P_{B_1A}(0,y,t)=\lambda P_{\mathrm{B}_1}(y,t) \quad \ldots(9)$$

$$P_B(0,t)=\mu_2P_{B_1}(t) \quad \ldots(10)$$

$$P_E(0,t)=e_1P_0(t)+e_2P_{B_1}(t) \quad \ldots(11)$$

Initial conditions are:

$$P_0(0)=1,\ \text{otherwise zero} \quad \ldots(12)$$

## 5. SOLUTION OF THE MODEL

In order to solve above mathematical model, we have to compute transition-state probabilities of fig -1(b). We shall use Laplace transform to solve above mathematical model. Taking Laplace transforms of equations (1) through (11) subjected to initial conditions (12) and then on solving them one by one, we obtain:

$$\overline{P}_0(s)=\frac{1}{C(s)} \quad \ldots(13)$$

$$\overline{P}_A(s)=\frac{\lambda D_{\alpha_1}(s)}{C(s)} \quad \ldots(14)$$

$$\overline{P}_{B_1}(s)=\frac{B(s)}{C(s)} \quad \ldots(15)$$

$$\overline{P}_{B_1A}(s)=\frac{\lambda D_{\alpha_1}(s)}{C(s)}\left[\mu_1+\mu_2 B(s)\overline{S}_{\beta_2}(s)\right]D_{\beta_1}(A) \quad \ldots(16)$$

$$\overline{P}_B(s)=\frac{\mu_2 B(s)}{C(s)}D_{\beta_2}(s) \quad \ldots(17)$$

$$\overline{P}_E(s)=\frac{D_{\alpha_2}(s)}{C(s)}\left[e_1+e_2 B(s)\right] \quad \ldots(18)$$

where, $A=s+\lambda+\mu_2+e_2-\lambda\overline{S}_{\alpha_1}(s)$ …(19)

$$B(s)=\frac{\mu_1 D_{\beta_1}(A)}{1-\mu_2\overline{S}_{\beta_2}(s)D_{\beta_1}(A)} \quad \ldots(20)$$

and, $$C(s)=s+\lambda+\mu_1+e_1-\lambda\overline{S}_{\alpha_1}(s)-\left[e_1+e_2B(s)\right]\overline{S}_{\alpha_2}(s) -\left[\mu_1+\mu_2B(s)\overline{S}_{\beta_2}(s)\right]\overline{S}_{\beta_1}(A) \quad \ldots(21)$$

**VERIFICATION**

It is worth nothing that

Sum of equations (13) through (18) $=\dfrac{1}{s}$ …(22)

**6. ERGODIC BEHAVIOUR**

Using final value theorem of Laplace transform, viz., $\underset{t\to\infty}{Lim}\, P(t)=\underset{s\to 0}{Lim}\, s\ \overline{P}(s)=P$ (say), provided the limit on L.H.S exists , we obtain the following ergodic behaviour of considered system from equations (13) through (18):

$$P_0=\frac{1}{C'(0)} \quad \ldots(23)$$

$$P_A=\frac{\lambda M_{\alpha_1}}{C'(0)} \quad \ldots(24)$$

$$P_{B_1} = \frac{B(0)}{C'(0)} \qquad \ldots(25)$$

$$P_{B_1A} = \frac{\lambda M_{\alpha_1}}{C'(0)}\left(\mu_1 + \mu_2 B(0)\right) D_{\beta_1}\left(\mu_2 + e_2\right) \qquad \ldots(26)$$

$$P_B = \frac{\mu_2 B(0)}{C'(0)} M_{\beta_2} \qquad \ldots(27)$$

and $P_E = \dfrac{M_{\alpha_2}}{C'(0)}\left[e_1 + e_2 B(0)\right]$ …(28)

where, $C'(0) = \left[\dfrac{d}{ds}C(s)\right]_{s=0}$

$M_i = -\overline{S}'_i(0) =$ Mean time to repair i$^{th}$ failure.

and $\mathrm{B}(0) = \dfrac{\mu_1 D_{\beta_1}\left(\mu_2 + e_2\right)}{1 - \mu_2 D_{\beta_1}\left(\mu_2 + e_2\right)}$

## 7. PARTICULAR CASE

### When repairs follow exponential time distribution

In this case, setting $\overline{S}_{\alpha_i}(j) = \dfrac{\alpha_i}{(j + \alpha_i)}$ $and\, \overline{S}_{\beta_i}(j) = \dfrac{\beta_i}{(j + \beta_i)}$ etc. in equations (13) through (18) we obtain the following Laplace transforms of transition- state probabilities of fig-1(b):

$$\overline{P}_0(s) = \frac{1}{E(s)} \qquad \ldots(29)$$

$$\overline{P}_A(s) = \frac{\lambda}{E(S)(s + \alpha_1)} \qquad \ldots(30)$$

$$\overline{P}_{B_1}(s) = \frac{B_1(s)}{E(s)} \qquad \ldots(31)$$

$$\overline{P}_{B_1A}(s) = \frac{\lambda}{E(s)(s + \alpha_1)}\left[\mu_1 + \mu_2 B_1(s)\frac{\beta_2}{s + \beta_2}\right]\frac{1}{A + \beta_1} \qquad \ldots(32)$$

$$\overline{P}_B(s) = \frac{\mu_2 B_1(s)}{E(s)(s + \beta_2)} \qquad \ldots(33)$$

and $\overline{P}_E(s) = \dfrac{1}{E(s)(s + \alpha_2)}\left[e_1 + e_2 B_1(s)\right]$ …(34)

where, $$\mathrm{B}_1(s) = \frac{\mu_1}{A + \beta_1 - \dfrac{\mu_2 \beta_2}{s + \beta_2}} \qquad \ldots(35)$$

and $$E(s) = s + \lambda + \mu_1 + e_1 - \frac{\lambda \alpha_1}{s + \alpha_1} - [e_1 + e_2 B_1(s)] \frac{\alpha_2}{s + \alpha_2} - \left[\mu_1 + \mu_2 B_1(s) \frac{\beta_2}{s + \beta_2}\right] \frac{\beta_1}{A + \beta_1} \qquad \ldots(36)$$

## 8. RELIABILITY AND M.T.T.F. OF THE SYSTEM

From equation (13), we have

$$\overline{R}(s) = \frac{1}{s + \lambda + \mu_1 + e_1}$$

Taking inverse L.T., we get

$$R(t) = \exp.\{-(\lambda + \mu_1 + e_1)\ t\} \qquad \ldots(37)$$

Also, $$M.T.T.F. = \lim_{s \to 0} \overline{R}(s)$$

$$= \frac{1}{\lambda + \mu_1 + e_1} \qquad \ldots(38)$$

## 9. AVAILABILITY OF CONSIDERED SYSTEM

We have from equations (13) and (15)

$$\overline{P}_{up}(s) = \frac{1}{s + \lambda + \mu_1 + e_1} \left[1 + \frac{\mu_1}{s + \lambda + \mu_2 + e_2}\right]$$

taking inverse L.T., we have

$$P_{up}(t) = \left[1 + \frac{\mu_1}{\mu_2 + e_2 - \mu_1 - e_1}\right] \exp\{-(\lambda + \mu_1 + e_1)\ t\} - \frac{\mu_1}{\mu_2 + e_2 - \mu_1 - e_1} \exp\{-(\lambda + \mu_2 + e_2)\ t\} \qquad \ldots(39)$$

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## 10. NUMERICAL ILLUSTRATION

For numerical illustration, let us consider the values $\mu_1 = 0.004$, $\mu_2 = 0.007$, $e_1 = 0.002$, $e_2 = 0.004$, $\lambda = 0.05$ *and* $t = 0,1,2............$ Using these values in the equations (37), (38) and (39), we compute the tables (1), (2) and (3) respectively. Corresponding graphs have been shown in fig-2, 3 and 4, respectively.

## 11. RESULTS AND DISCUSSION

Fig-2 shows the graph of "Reliability Vs. Time" and its values have given in table-1. Analysis of fig-2 yields that reliability of system decreases approximately in a constant manner.

Fig-3 is the graph "M.T.T.F. Vs. $\lambda$" and its values have given in table-2. Examination of fig-3 reveals that M.T.T.F. of considered system decreases as we make increases in the value of failure rate $\lambda$ of subsystem A.

Fig-4 is the graph "Availability Vs. Time" and its values have given in table-3. Critical examination of fig-4 concludes that availability decreases catastrophically in the beginning but thereafter it decreases approximately in constant manner. It should be noted that there are no sudden jumps in the values of R(t), M.T.T.F. and $P_{up}(t)$.

| **t** | **R(t)** |
|---|---|
| 0 | 1 |
| 1 | 0.945539 |
| 2 | 0.894044 |
| 3 | 0.845354 |
| 4 | 0.799315 |
| 5 | 0.755784 |
| 6 | 0.714623 |
| 7 | 0.675704 |
| 8 | 0.638905 |
| 9 | 0.604109 |
| 10 | 0.571209 |

**Table-1**

| $\lambda$ | **M.T.T.F.** |
|---|---|
| 0 | 166.6667 |
| 0.01 | 62.5 |
| 0.02 | 38.46154 |
| 0.03 | 27.77778 |
| 0.04 | 21.73913 |
| 0.05 | 17.85714 |
| 0.06 | 15.15152 |
| 0.07 | 13.15789 |
| 0.08 | 11.62791 |
| 0.09 | 10.41667 |
| 0.10 | 9.433962 |

**Table-2**

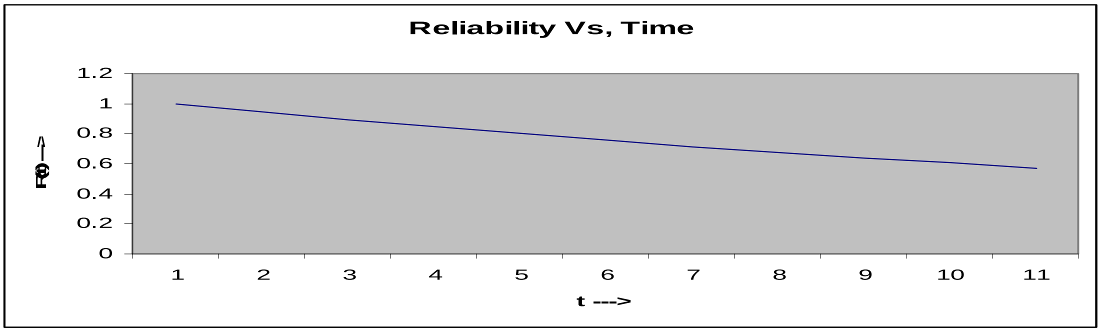


Fig-2

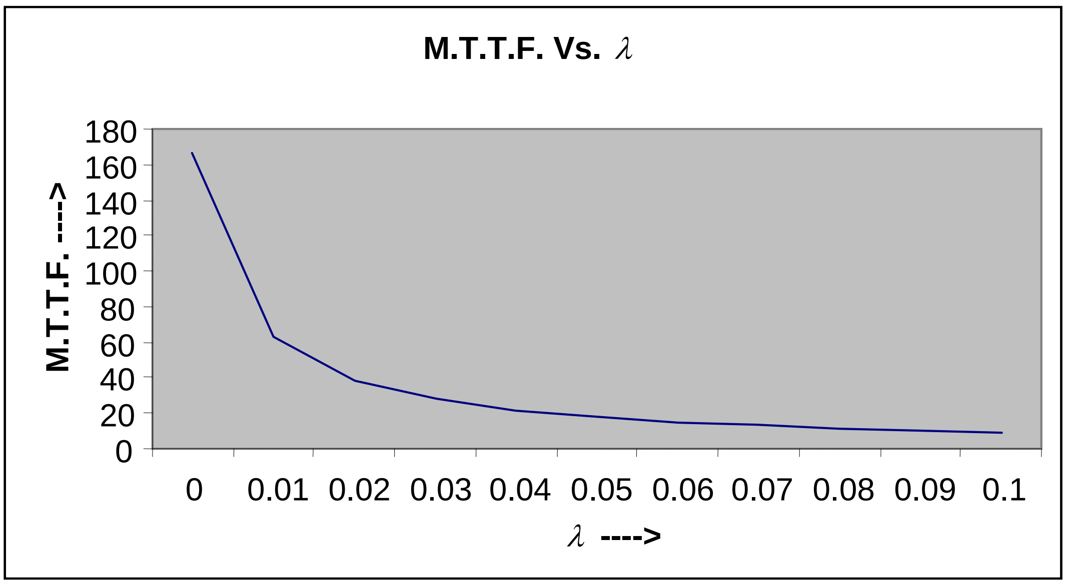


Fig-3

| t | $P_{up}(t)$ |
|---|---|
| 0 | 1 |
| 1 | 0.949312 |
| 2 | 0.901161 |
| 3 | 0.855422 |
| 4 | 0.811977 |
| 5 | 0.770712 |
| 6 | 0.731519 |
| 7 | 0.694297 |
| 8 | 0.658946 |
| 9 | 0.625375 |
| 10 | 0.593496 |

Table-3

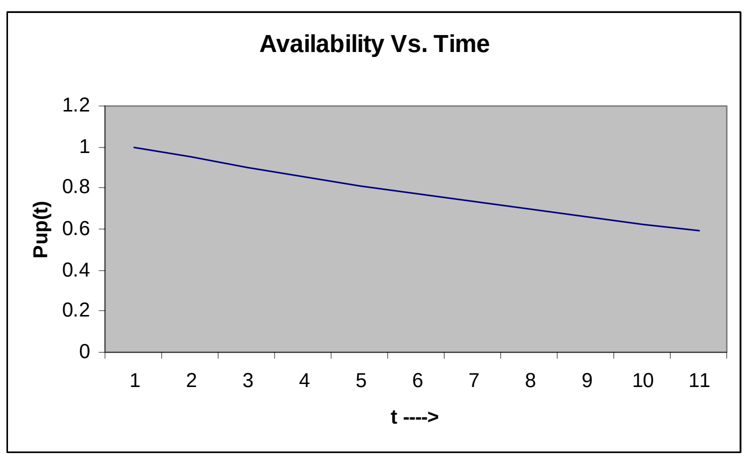


Fig-4

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