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Stochastic Differential Equations in Infinite Dimensional Spaces

(An extended version of the lectures delivered by Gopinath Kallianpur as part of 1993-Barrett Lectures at the University of Tennessee, Knoxville, TN)



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