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Stochastic Expected Utility Theory

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Abstract:

A new decision theory is proposed to explain the violations of expected utility theory through the role of random errors. The main premise of the new theory is that individuals make random errors when they compute the expected utility of a risky lottery. When being distorted by error, the expected utility of a lottery should neither exceed the utility of the highest possible outcome nor fall below the utility of the lowest possible outcome. This crucial assumption implies that the expected utility of a lottery is likely to be overvalued (undervalued) by random errors, when it is close to the utility of the lowest (highest) possible outcome. The new theory explains many stylized empirical facts such as the fourfold pattern of risk attitudes, the common consequence effect (Allais paradox), the common ratio effect and violations of betweenness. The model fits the data from ten well-known experimental studies at least as well as cumulative prospect theory.

Keywords: decision theory, stochastic utility, expected utility theory, cumulative prospect theory

JEL Classification codes: C91, D81

Stochastic Expected Utility Theory

> "Perhaps we should now spend some time on thinking about the noise, rather than about even more alternatives to EU?"
> Hey and Orme (1994), *Econometrica* **62**, p.1322

This paper proposes a new decision theory to describe individual decision making under risk, as defined by Knight (1921). A normative theory of choice under risk is expected utility theory, or EUT. However, persistent violations of EUT, such as the Allais paradox (Allais, 1953), make EUT a descriptively inadequate theory (Camerer, 1995). Many theories have been proposed to improve the descriptive fit of EUT (see Starmer (2000) for a recent review). EUT and nearly all non-expected utility theories are deterministic theories i.e. they predict that an individual always makes the same decision in identical choice situations (unless he or she is exactly indifferent between lotteries). In contrast, this paper proposes a stochastic decision theory to explain the violations of EUT through the role of random errors. The new model is motivated both by a recent revival of interest among economic theorists in stochastic decision theories (Loomes et al. 2002) and by the compelling empirical evidence of random variation in individuals' decisions (Ballinger and Wilcox, 1997).

For example, Camerer (1989, p.81) reports that 31.6% of the subjects reverse their preference, when presented with the same choice decision for the second time. Starmer and Sugden (1989) find that the observed preferences are reversed in 26.5% of cases. Wu (1994, p.50) reports that the reveled preferences change in 5%-45% of cases when the same binary choice problem is repeated. Hey and Orme (1994) find that around 25% of choice decisions are inconsistent, when an individual faces the same choice problem twice and he or she can declare indifference. Moreover, Hey (2001) provides experimental evidence that the variability of the subjects' responses is generally higher than the difference in the predictive error of various deterministic decision theories. Thus, a model predicting stochastic choice patterns can be a promising alternative to the deterministic non-expected utility theories.

Existing non-expected utility theories typically do not consider stochastic choice patterns (see, however, Machina, 1985, and Hey and Carbone, 1995). Only when the theoretical model is estimated are assumptions about error specification introduced. Effectively, the stochastic component plays only a secondary role being regarded as unimportant on the theoretical level (Hey, 2005). Camerer and Ho (1994) use a stochastic choice model in which the probability of choosing one lottery over another is simply a logit function of the difference in their utilities according to the deterministic underlying theory. Harless and Camerer (1994) assume that there is a constant probability with which an individual reverses his or her deterministic choice pattern. This probability is the same in all choice problems and it reflects the possibility of errors such as pure trembles. Hey and Orme (1994) obtain a stochastic choice pattern by means of a white noise (normally distributed zero-mean error term) additive on the utility scale. Such an error term reflects the average of various genuine errors that might obscure a deterministic choice pattern. Hey (1995) and Buschena and Zilberman (2000) go one step further and assume that this error term is heteroskedastic. The standard deviation of errors is higher in certain decision problems *e.g.* when the lotteries have many outcomes or when the subjects take more time to make a decision.

This paper proposes new and more elaborate structure of an error term. The stochastic component is introduced as a part of the decision theory, which makes explicit prediction in form of a stochastic choice pattern. Thus, econometric estimation of the proposed theory on the empirical data does not require any additional assumptions about error specification. Moreover, new theory assumes that individuals have a preference relation on the set of risky lotteries, which admits expected utility representation. Thus, the proposed theory is essentially a stochastic extension of neoclassical expected utility theory, so that its estimation is relatively simple compared to non-expected utility models.

Individuals are assumed to maximize their expected utility when choosing between risky lotteries. However, individuals make random errors when computing the expected utility of a lottery. The errors are additive on the utility scale, similarly as in Hey and Orme (1994). The distribution of random errors is essentially symmetric around zero with a restriction that the stochastic utility of a lottery cannot be lower (higher) than the utility of the lowest (highest) possible outcome for certain. This assumption reflects a rather obvious fact that there is a limit to a measurement error that an individual can commit. In particular, violations of obvious dominance, when a risky lottery is chosen over its highest possible outcome for sure, or when it is not chosen over its lowest possible outcome for sure, appear to be implausible. Hence, computational errors are naturally truncated by the highest and the lowest outcomes in the gamble.

This restriction implies that lotteries whose expected utility is close to the utility of the lowest possible outcome (*e.g.* unlikely gains or probable losses) are more likely to be overvalued rather than undervalued by random errors. Similarly, lotteries whose expected utility is close to the utility of the highest possible outcome (*e.g.* probable gains or unlikely losses) are likely to be undervalued by random errors. This offers an immediate explanation for the fourfold pattern of risk attitudes—a risk seeking behavior in face of unlikely gains or probable losses and a risk averse behavior in face of probable gains or unlikely losses (Tversky and Kahneman, 1992). Stochastic version of expected utility theory can also explain other empirical anomalies such as the common consequence effect and the Allais paradox (Allais, 1953), the common ratio effect and violations of betweenness (Camerer and Ho, 1994).

Apart from demonstration that many empirical paradoxes can be attributed to a simple stochastic version of expected utility theory, this paper also reexamines the data from ten well-known experimental studies. The proposed theory accommodates the experimental data with a remarkable success. Its goodness of fit is generally at least as good as the fit of such prominent non-expected utility models as cumulative prospect theory or rank-dependent expected utility theory. This suggests that a careful specification of the stochastic structure of the errors that subjects make in the experiments is a promising avenue for constructing a descriptive decision theory. Systematic errors that subjects commit when evaluating the expected utility of risky lotteries can account for many of the well-known empirical anomalies, which have been traditionally attributed to non-linear probability weighting, regret or disappointment aversion etc.

The remainder of this paper is organized as follows. Stochastic expected utility theory or StEUT is described in section 1. Section 2 demonstrates how StEUT explains many stylized empirical facts such as the fourfold pattern of risk attitudes and the Allais paradox. Section 3 tests the explanatory power of StEUT on the data from ten well-known experimental studies. Section 4 concludes.

1. Theory

Notation $L(x_1, p_1; \ldots x_n, p_n)$ denotes lottery *L* delivering a monetary outcome x_i with probability p_i, $i \in \{1,\ldots,n\}$. Let x_1 be the lowest possible outcome and let x_n be the highest possible outcome. The expected utility of lottery *L* according to deterministic preferences of an individual is $\mu_L = \sum_{i=1}^{n} p_i u(x_i)$. A subjective non-decreasing utility function $u: \mathbf{R} \rightarrow \mathbf{R}$ is defined over changes in wealth rather than absolute wealth levels, as proposed by Markowitz (1952) and later advocated by Kahneman and Tversky (1979). An individual makes random errors when calculating the expected utility μ_L of a risky lottery.[1]

Random errors are assumed to be additive on the utility scale, similarly as in Hey and Orme (1994, p.1301) and Gonzalez and Wu (1999). Thus, instead of maximizing deterministic expected utility μ_L, an individual behaves as if he or she maximizes *stochastic expected utility*

(1) $$U(L) = \mu_L + \xi_L .$$

For simplicity it is assumed that an error term ξ_L is independently distributed across lotteries. In other words, the error which occurs when an individual calculates the expected utility of one lottery is not correlated with an error when calculating the expected utility of another lottery.

The stochastic expected utility (1) of a lottery is assumed to be bounded from below and above. It cannot be less than the utility of the lowest possible outcome for certain (see, however, Gneezy et al., 2006). Similarly, it cannot exceed the utility of the highest possible outcome for certain. Formally, the internality axiom holds i.e. $u(x_1) \leq \mu_L + \xi_L \leq u(x_n)$, which imposes the following restriction on the cumulative distribution function $\Psi_L(v) = prob(\xi_L \leq v)$ of a random error ξ_L:

[1] Computational errors occur for a variety of reasons (Hey and Orme, 1994). An individual may be not sufficiently motivated to make a balanced decision. A subject can get tired during long experiment and lose attention (especially if lotteries do not involve losses). A subject can simply press a wrong key by accident or inertia. Wu (1994, p.50) suggests that subjects can suffer from fatigue and hurry up with their responses at the end of the experiment.

(2) $$\Psi_L(v)=0, \quad \forall v < u(x_1)-\mu_L \quad \text{and} \quad \Psi_L(v)=1, \quad \forall v \geq u(x_n)-\mu_L .$$

Assumption (2) implies that there is no error in choice between "sure things". A degenerate lottery delivers one outcome for certain, which is simultaneously its lowest possible and its highest possible outcome ($x_1 = x_n$). In this case, equation (2) immediately implies that $prob(\xi_L = 0)=1$ i.e. the utility of a degenerate lottery is not affected by random errors.

For non-degenerate lotteries, the random errors are assumed to be symmetrically distributed around zero as long as restriction (2) is not violated i.e. $prob(0 \leq \xi_L \leq v) = prob(-v \leq \xi_L \leq 0)$ for every $v \in [0, \min\{\mu_L - u(x_1); u(x_n) - \mu_L\}]$. Formally, this corresponds to the restriction

(3) $$\Psi_L(0)+\Gamma_L(-v)=\Gamma_L(0)+\Psi_L(v), \quad \forall v \in [0, \min\{\mu_L - u(x_1); u(x_n)-\mu_L\}],$$

where $\Gamma_L(v) = prob(\xi_L \geq v)$. Intuitively, random errors are non-systematic if they are within a reasonable range so that a lottery is not valued less than its worst possible outcome or more than its best possible outcome. In general, the cumulative distribution function of random errors for risky lotteries is unknown and it is likely to be lottery-specific (Hey, 1995).

Equations (1)-(3) complete the description of StEUT. Obviously, when $prob(\xi_L = 0)=1$ for every lottery *L*, StEUT coincides with the deterministic EUT. StEUT resembles the Fechner model of stochastic choice e.g. Becker et al. (1963). Both models introduce an error term, which is additive on the utility scale. However, they differ in two important aspects.

First, the error term in the Fechner model is a continuous random variable that is symmetrically distributed around zero and unbounded. In practical applications, it is typically assumed to be normally distributed (Hey and Orme, 1994; Loomes at al., 2002). In contrast, the error term in StEUT is bounded from below and above by a basic rationality requirement of the internality axiom. For practical estimations, such an error term can be drawn from a truncated normal distribution (see section 3).

Second, the error term in the Fechner model affects the *difference* in the expected utilities of two lotteries that are compared. We can think of it as a compound error equal to the difference between two computational errors that occur separately when an individual evaluates the expected utility of lotteries. Moreover, if computational errors are normally distributed, their difference is also normally distributed. In contrast, the error term in StEUT is a genuine computational error that affects the expected utility of a lottery. When two lotteries are compared, two corresponding computational errors are taken into account.

2. Stylized facts

2.1. The fourfold pattern of risk attitudes

The fourfold pattern of risk attitudes is an empirical observation that individuals exhibit risk aversion when dealing with probable gains or improbable losses, and risk seeking—when dealing with improbable gains or probable losses (Tversky and Kahneman, 1992). One illustration of the fourfold pattern of risk attitudes is a simultaneous purchase of insurance and public lottery tickets. Historically, it was the first descriptive challenge for the deterministic EUT (Friedman and Savage, 1948).

A conventional indication of risk averse (seeking) behavior is when the certainty equivalent of a lottery is smaller (greater) than the expected value of the lottery. In the context of deterministic decision theories, the certainty equivalent of a lottery is defined as a monetary outcome which is perceived exactly as good as the lottery itself. For stochastic decision theories, there is no established definition of a certainty equivalent in the literature. One can think of at least two intuitive definitions. First, the certainty equivalent of a lottery can be defined as a monetary outcome which is perceived exactly as good as the average stochastic utility of the lottery. Second, it can be defined as a monetary outcome which is equally likely to be chosen or to be rejected, when it is offered as an alternative to a lottery. StEUT is consistent with the fourfold pattern of risk attitudes when either of these two definitions is used (as shown below).

Definition 1 The certainty equivalent of lottery *L* is an outcome CE_L that is implicitly defined by equation

(4) $$u(CE_L) = \mu_L + E[\xi_L],$$

where the expected error $E[\xi_L]$ can be spelled out as $E[\xi_L] = \int_{u(x_1)-\mu_L}^{u(x_n)-\mu_L} v \, \mathrm{d}\Psi_L(v)$ due to assumption (2).

Assumption (3) implies that $E[\xi_L] = \underbrace{\int_{u(x_1)-\mu_L}^{\mu_L-u(x_1)} v \, \mathrm{d}\Psi_L(v)}_{=0} + \underbrace{\int_{\mu_L-u(x_1)}^{u(x_n)-\mu_L} v \, \mathrm{d}\Psi_L(v)}_{\geq 0}$ if $u(x_n) - \mu_L \geq \mu_L - u(x_1)$

and $E[\xi_L] = \underbrace{\int_{u(x_1)-\mu_L}^{\mu_L-u(x_n)} v \, \mathrm{d}\Psi_L(v)}_{\leq 0} + \underbrace{\int_{\mu_L-u(x_n)}^{u(x_n)-\mu_L} v \, \mathrm{d}\Psi_L(v)}_{=0}$ if $\mu_L - u(x_1) \geq u(x_n) - \mu_L$. Thus, the expected error is positive or zero, i.e. $u(CE_L) \geq \mu_L$, when the expected utility of a lottery is close to the utility of the lowest possible outcome, i.e. $\mu_L \leq (u(x_1) + u(x_n))/2$. These are improbable gains or probable losses in the terminology of Tversky and Kahneman (1992). The expected error is negative or zero for lotteries whose expected utility is close to the utility of the highest possible outcome, i.e. $\mu_L \geq (u(x_1) + u(x_n))/2$. These are probable gains or improbable losses in the terminology of Tversky and Kahneman (1992).

Let $EV_L = \sum_{i=1}^{n} p_i x_i$ denote the expected value of lottery *L*. Jensen's inequality $u(EV_L) \geq \mu_L$ holds if and only if an individual has a concave utility function. Thus, according to StEUT, the individual with a concave utility function exhibits risk averse behavior only when the expected utility of a lottery is close to the utility of the highest possible outcome. In this case, $u(CE_L) \leq \mu_L \leq u(EV_L)$ which is equivalent to $CE_L \leq EV_L$ because utility function $u(.)$ is non-decreasing. When the expected utility of a lottery is close to the lowest possible outcome, the individual with a concave utility function is not necessarily risk averse because it is possible that $u(CE_L) \geq u(EV_L) \geq \mu_L$ i.e. $CE_L \geq EV_L$.

Now consider an individual with a convex utility function $u(.)$, which implies that $u(EV_L) \leq \mu_L$. He or she exhibits risk seeking behavior, i.e. $CE_L \geq EV_L$, only when the expected utility of a lottery is close to the utility of the lowest possible outcome, i.e. when $u(CE_L) \geq \mu_L \geq u(EV_L)$. He or she may be risk averse when the expected utility of a lottery is close to the highest possible outcome, in which case it is possible that $u(CE_L) \leq u(EV_L) \leq \mu_L$ i.e. $CE_L \leq EV_L$. Thus, StEUT is consistent with the fourfold pattern of risk attitudes when the certainty equivalent is defined by equation (4).

Definition 2 The certainty equivalent of lottery *L* is an outcome CE_L^* that is implicitly defined by equation

(5) $$prob\left(u\left(CE_L^*\right) \geq \mu_L + \xi_L\right) = prob\left(\mu_L + \xi_L \geq u\left(CE_L^*\right)\right),$$

or, equivalently, by equation

(6) $$\Psi_L\left(u\left(CE_L^*\right) - \mu_L\right) = \Gamma_L\left(u\left(CE_L^*\right) - \mu_L\right).$$

Notice that $\Psi_L\left(u\left(CE_L^*\right) - \mu_L\right) \geq \Psi_L(0)$ and $\Gamma_L\left(u\left(CE_L^*\right) - \mu_L\right) \leq \Gamma_L(0)$ if and only if $u\left(CE_L^*\right) \geq \mu_L$. Thus, equation (6) implies that $\Psi_L(0) \leq \Gamma_L(0)$ if and only if $u\left(CE_L^*\right) \geq \mu_L$. At the same time, we can show that $\Psi_L(0) = \Psi_L(0) + \Gamma_L\left(u(x_1) - \mu_L\right) - 1 = \Gamma_L(0) + \Psi_L\left(\mu_L - u(x_1)\right) - 1 \leq \Gamma_L(0)$, with the first equality due to assumption (2), and the second equality due to assumption (3), if $\mu_L \leq \left(u(x_1) + u(x_n)\right)/2$. Thus, if the expected utility of L is close to the utility of its lowest possible outcome, it follows that $\Psi_L(0) \leq \Gamma_L(0)$ and $u\left(CE_L^*\right) \geq \mu_L$. A similar argument implies that $u\left(CE_L^*\right) \leq \mu_L$ if the expected utility of lottery L is close to the utility of the highest possible outcome. We already established that these two conclusions are consistent with the fourfold pattern of risk attitudes both for concave and convex utility functions.

Intuitively, the underlying assumptions about the distribution of random errors imply that errors are more likely to overvalue than undervalue the expected utility of lotteries, when the latter is close to the utility of the lowest possible outcome (e.g. improbable gains or probable losses). The stochastic utility of a lottery cannot be lower than the utility of its lowest possible outcome. Due to this constraint, it is relatively difficult to undervalue the expected utility of a lottery by mistake, when it is already close to the utility of the lowest possible outcome. At the same time, it is relatively easy to overvalue the expected utility of such lottery. Thus, in this case, random errors reinforce a risk seeking behavior.

Similarly, when the expected utility of a lottery is close to the utility of the highest possible outcome (e.g. probable gains or improbable losses), it is more likely to be undervalued by random errors. The stochastic utility of a lottery cannot be higher than the utility of its highest possible outcome. Thus, the overvaluation of the true expected utility by mistake is constrained when the latter is already close to the utility of the highest possible outcome. At the same time, there is plenty of room for random errors to undervalue the expected utility of a lottery. In this case, random errors reinforce a risk averse behavior.

2.2. Common consequence effect (Allais paradox)

There exist outcomes $x_1 < x_2 < x_3$ and probabilities $p > q > 0$ such that lottery $S_1(x_2, 1)$ is preferred to lottery $R_1(x_1, p-q; x_2, 1-p; x_3, q)$ and at the same time lottery $R_2(x_1, 1-q; x_3, q)$ is preferred to lottery $S_2(x_1, 1-p; x_2, p)$ (Slovic and Tversky, 1974; MacCrimmon and Larsson, 1979). This choice pattern is frequently found in the experimental data and it is known as the common consequence effect. The most famous example of the common consequence effect is the Allais paradox (Allais, 1953), which is a special case when $x_1 = 0, x_2 = 10^6, x_3 = 5 \cdot 10^6, p = 0.11$ and $q = 0.1$ (Starmer, 2000). Intuitively, when the probability mass is shifted from the medium outcome to the lowest possible outcome, the choice of a riskier lottery *R* becomes more probable.

Four lotteries in the common consequence effect are constructed so that $\mu_{R_1} - \mu_{S_1} = \mu_{R_2} - \mu_{S_2}$ and let us denote this difference by δ. Since the expected utilities of a riskier and a safer lottery always differ by the same amount δ, EUT cannot explain why the choice of the riskier lottery becomes more likely. In contrast, StEUT is compatible with the common consequence effect.

Lottery S_1 is a degenerate lottery and random errors do not affect its utility $\mu_{S_1} = u(x_2)$. In a binary choice, $prob(S_1 \succeq R_1) = prob(\mu_{S_1} \geq \mu_{R_1} + \xi_{R_1}) = \Psi_{R_1}(-\delta)$. Similarly, R_1 is (weakly) preferred to S_1 with probability $prob(R_1 \succeq S_1) = \Gamma_{R_1}(-\delta)$. Choice probabilities $prob(S_1 \succeq R_1)$ and $prob(R_1 \succeq S_1)$ depend only on the properties of the cumulative distribution function of a random error ξ_{R_1} that distorts the expected utility of R_1. In the previous subsection we established that $\Psi_{R_1}(0) \geq \Gamma_{R_1}(0)$, whenever the expected utility of R_1 is close to the utility of the highest possible outcome.[2] In addition, if the cumulative

[2] For example, in the Allais paradox, this condition is satisfied when the gain of one million starting from zero wealth position brings a higher increase in utility than the gain of additional four million.

distribution function of ξ_{R_1} is continuous, it is always possible to find small $\delta \geq 0$ such that $\Psi_{R_1}(-\delta) \geq \Gamma_{R_1}(-\delta)$, i.e. $prob(S_1 \succeq R_1) \geq prob(R_1 \succeq S_1)$.

The probability that lottery S_2 is (weakly) preferred to lottery R_2 is given by $prob(S_2 \succeq R_2) =$ $= prob(\mu_{S_2} + \xi_{S_2} \geq \mu_{R_2} + \xi_{R_2}) = \int_{u(x_1)-\mu_{S_2}}^{u(x_3)-\mu_{S_2}} \Psi_{R_2}(v-\delta)\,\mathrm{d}\Psi_{S_2}(v)$ and it depends on the properties of the cumulative distribution functions of random errors ξ_{R_2} and ξ_{S_2}. In general, these two errors can be drawn from different distributions. In the simplest possible case when ξ_{R_2} and ξ_{S_2} are drawn from the same distribution, $prob(S_2 \succeq R_2) = \int_{u(x_1)-\mu_{S_2}}^{u(x_3)-\mu_{S_2}} \Psi_{R_2}(v-\delta)\,\mathrm{d}\Psi_{S_2}(v) \leq \int_{u(x_1)-\mu_{S_2}}^{u(x_3)-\mu_{S_2}} \Psi_{R_2}(v)\,\mathrm{d}\Psi_{S_2}(v) = 0.5$ where the inequality holds if and only if $\delta \geq 0$. By analogy, we can also show that $prob(R_2 \succeq S_2) \geq 0.5$.

To summarize, it is possible to find a small $\delta \geq 0$ such that $prob(S_1 \succeq R_1)$ is higher or equal to $prob(R_1 \succeq S_1)$ (if the expected utility of R_1 is close to the utility of the highest possible outcome) and at the same time $prob(R_2 \succeq S_2)$ is higher or equal to $prob(S_2 \succeq R_2)$ (if random errors that distort the expected utilities of S_2 and R_2 are drawn from the same or similar distributions). Thus, under fairly plausible assumptions, StEUT is consistent with the common consequence effect.

Intuitively, when the probability mass is allocated to the medium outcome, which is close to the highest possible outcome in terms of utility, an individual prefers a degenerate lottery S_1 to risky lottery R_1 even when the expected utility of R_1 is (slightly) higher. Utility of S_1 is not affected by random errors but random errors are likely to undervalue the expected utility of R_1 because it is close to the utility of the highest possible outcome. When probability mass is shifted to the lowest possible outcome, random errors distort the expected utility of both S_2 and R_2. If the distorting effect of random errors is similar for both lotteries, an individual opts for the lottery with higher expected utility i.e. lottery R_2.

StEUT predicts that the common consequence effect can disappear if lottery S_1 is not degenerate. Conlisk (1989) and Camerer (1992) find experimental evidence confirming this prediction. StEUT is also compatible with the so called generalized common consequence effect (Wu and Gonzalez, 1996) but the theoretical analysis is rather cumbersome and hence it is omitted (see working paper Blavatskyy, 2005).

2.3. Common ratio effect

The common ratio effect is the following empirical finding. There exist outcomes $x_1 < x_2 < x_3$ and probability $\theta \in (0,1)$ such that $S_3(x_2,1)$ is preferred to $R_3(x_1,1-\theta;x_3,\theta)$ and at the same time $R_4(x_1,1-\theta r;x_3,\theta r)$ is preferred to $S_4(x_1,1-r;x_2,r)$ when probability *r* is close to zero (Starmer, 2000). Intuitively, when the probabilities of medium and highest possible outcome are scaled down in the same proportion (hence the name of the effect), the choice of a riskier lottery *R* becomes more probable. Notice that $\mu_{R_4} - \mu_{S_4} = r(\mu_{R_3} - \mu_{S_3})$ and EUT cannot explain the common ratio effect. StEUT explains the common ratio effect by analogy to its explanation of the common consequence effect.

On the one hand, $prob(S_3 \succeq R_3) = prob(\mu_{S_3} \geq \mu_{R_3} + \xi_{R_3}) = \Psi_{R_3}(-\Delta)$, where $\Delta = \mu_{R_3} - \mu_{S_3}$. When $\theta \geq 0.5$ the expected utility of lottery R_3 is close to the utility of the highest possible outcome, i.e. $\mu_{R_3} \geq 0.5u(x_1) + 0.5u(x_3)$, and $\Psi_{R_3}(0) \geq \Gamma_{R_3}(0)$. If the cumulative distribution function of a random error ξ_{R_3} is continuous, it is possible to find small $\Delta \geq 0$ such that $\Psi_{R_3}(-\Delta) \geq \Gamma_{R_3}(-\Delta)$, i.e. $prob(S_3 \succeq R_3) \geq prob(R_3 \succeq S_3)$. On the other hand, $prob(S_4 \succeq R_4) = prob(\mu_{S_4} + \xi_{S_4} \geq \mu_{R_4} + \xi_{R_4}) = $ $= \int_{u(x_1)-\mu_{S_4}}^{u(x_3)-\mu_{S_4}} \Psi_{R_4}(v - r\Delta)\, \mathrm{d}\Psi_{S_4}(v)$. If random errors ξ_{R_4} and ξ_{S_4} are drawn from the same distribution and Δ is non-negative, we can conclude that $prob(S_4 \succeq R_4) \leq 0.5 \leq prob(R_4 \succeq S_4)$.

In summary, an individual chooses S_3 more often than R_3 even though R_3 has a (slightly) higher expected utility because random errors are more likely to undervalue than overvalue the expected utility of R_3, when $\theta \geq 0.5$. In contrast, utility of S_3 is not affected by random errors. In binary choice between

R_4 and S_4, the expected utility of both lotteries is affected by random errors. If random errors ξ_{R_4} and ξ_{S_4} are drawn from the same or similar distribution, an individual chooses the lottery with a higher expected utility (R_4) more often. Thus, the common ratio effect is observed. Notice that StEUT cannot explain the common ratio effect if $\theta < 0.5$, which is consistent with the experimental evidence.[3]

2.4. Violation of betweenness

According to the betweenness axiom, if an individual is indifferent between two lotteries then any probability mixture of these lotteries is equally good e.g. Dekel (1986). Systematic violations of the betweenness have been reported in Coombs and Huang (1976), Chew and Waller (1986), Battalio et al. (1990), Prelec (1990) and Gigliotti and Sopher (1993). There exist lotteries *S*, *R* and a probability mixture $M = \theta \cdot S + (1-\theta) \cdot R$, $\theta \in (0,1)$, such that significantly more individuals exhibit a quasi-concave preference $M \succeq S \succeq R$ than a quasi-convex preference $R \succeq S \succeq M$, or vise versa. Preferences are elicited from a binary choice between *S* and *R* and a binary choice between *S* and *M*. Asymmetric split between quasi-concave and quasi-convex preferences is taken as evidence of a violation of the betweenness.

In the context of stochastic choice, an individual reveals the quasi-concave preference $M \succeq S \succeq R$ with probability $prob(M \succeq S) \cdot prob(S \succeq R) = prob(S \succeq R) \cdot (1 - prob(S \succeq M))$. Similarly, the same individual reveals the quasi-convex preference $R \succeq S \succeq M$ with probability $prob(R \succeq S) \cdot prob(S \succeq M) =$ $= prob(S \succeq M) \cdot (1 - prob(S \succeq R))$. Thus, a quasi-concave preference is observed more (less) often than a quasi-convex preference if and only if $prob(S \succeq R)$ is greater (smaller) than $prob(S \succeq M)$. According to StEUT, $prob(S \succeq R) = prob(\mu_S + \xi_S \geq \mu_R + \xi_R)$ and $prob(S \succeq M) = prob(\mu_S + \xi_S \geq \mu_M + \xi_M)$.

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[3] Bernasconi (1994) finds the common ratio effect when $\theta = 0.8$ and $\theta = 0.75$. Loomes and Sugden (1998) find evidence of the common ratio effect when $\theta \in \{0.6, 2/3, 0.8\}$, and no such evidence when $\theta = 0.4$ and $\theta = 0.5$.

Notice that $\mu_S - \mu_M = (1-\theta)\cdot(\mu_S - \mu_R)$ because lottery *M* is a probability mixture of *S* and *R*. In the simplest possible case when random errors ξ_R and ξ_M are drawn from the same distribution, we can write $prob(S \succeq R) = \int_{u(x_1)-\mu_S}^{u(x_n)-\mu_S} \Psi_R(v + \mu_S - \mu_R)\,\mathrm{d}\Psi_S(v) = \int_{u(x_1)-\mu_S}^{u(x_n)-\mu_S} \Psi_M(v + \mu_S - \mu_R)\,\mathrm{d}\Psi_S(v) \geq$

$\geq \int_{u(x_1)-\mu_S}^{u(x_n)-\mu_S} \Psi_M(v + (1-\theta)\cdot(\mu_S - \mu_R))\,\mathrm{d}\Psi_S(v) = prob(S \succeq M)$ when $\mu_S > \mu_R$. Similarly, $prob(S \succeq R) \leq prob(S \succeq M)$ when $\mu_S < \mu_R$. Thus, an individual is more (less) likely to reveal quasi-concave preferences when the expected utility of *S* is higher (lower) than the expected utility of *R*.

The intuition behind the asymmetric split between quasi-concave and quasi-convex preferences is very straightforward. By construction, mixture *M* is located between lotteries *S* and *R* in terms of expected utility. Two cases are possible. If the expected utility of *S* is higher than the expected utility of *R*, random errors are more likely to reverse preference $S \succeq M$ than $S \succeq R$. To reverse preference $S \succeq M$, random errors only need to overcome the difference between the expected utility of *S* and the expected utility of *M*. This difference is smaller than the difference between the expected utility of *S* and the expected utility of *R*. Hence, an individual is more likely to exhibit preference $S \succeq R$ than $S \succeq M$, which implies a higher likelihood of the quasi-concave preference $M \succeq S \succeq R$. Similarly, if the expected utility of *R* is higher than the expected utility of *S*, random errors are more likely to reverse preference $M \succeq S$ than preference $R \succeq S$. In this case, an individual is more likely to exhibit preference $S \succeq M$ than $S \succeq R$, which implies a higher likelihood of the quasi-convex preference $R \succeq S \succeq M$.

StEUT can also explain the violation of the betweenness documented in Camerer and Ho (1994) and Bernasconi (1994) who elicited preferences from three binary choices: between *S* and *R*, between *S* and *M* and between *M* and *R*. In fact, Blavatskyy (2006a) shows that the violations of the betweenness are compatible with any Fechner-type model of stochastic choice with error term additive on the utility scale.

3. Fit to experimental data

This section presents a parametric estimation of StEUT using the data from ten well-known experimental studies. Experimental datasets do not allow for non-parametric estimation of StEUT. StEUT admits the possibility that the distribution or random errors is lottery-specific. Thus, many observations involving the same lotteries are required to estimate the cumulative distribution function of random errors for every lottery. Parametric estimation allows reducing the number of estimated parameters.

3.1. Parametric form of StEUT

A natural assumption for an economist to make is that an error ξ_L in equation (1) is drawn from the normal distribution with zero. To satisfy assumption (2), normal distribution of ξ_L must be truncated so that $u(x_1) \leq \mu_L + \xi_L \leq u(x_n)$. Specifically, the cumulative distribution function of ξ_L is given by

(7) $$\Psi_L(v) = \frac{\Phi_L(v) - \Phi_L(u(x_1) - \mu_L)}{\Phi_L(u(x_n) - \mu_L) - \Phi_L(u(x_1) - \mu_L)}, \quad u(x_1) - \mu_L \leq v \leq u(x_n) - \mu_L$$

where $\Phi_L(.)$ is the cumulative distribution function of the normal distribution with zero mean and standard deviation σ_L. Obviously, the cumulative distribution function (7) satisfies equation (3).

The standard deviation σ_L is lottery-specific (Hey, 1995). It captures the fact that for some lotteries the error of miscalculating the expected utility is more volatile than for the other lotteries. First of all, it is plausible to assume that σ_L is higher for lotteries with a wider range of possible outcomes. In other words, when possible outcomes of a lottery are widely dispersed, there is more room for error. Second, since there is no error in choice between "sure things", it is natural to assume that σ_L converges to zero for lotteries converging to a degenerate lottery, i.e. $\lim_{p_i \to 1} \sigma_L = 0, \forall i \in \{1,...,n\}$. A simple function that captures these two effects (and fits very well the empirical data) is

(8) $$\sigma_L = \sigma \cdot (u(x_n) - u(x_1)) \sqrt{\prod_{i=1}^{n} (1 - p_i)} .$$

where σ is constant across all lotteries. Coefficient σ captures the standard deviation of random errors that is not lottery-specific. For example, in the experiments with hypothetical incentives, σ is expected to be higher than in the experiments with real incentives because real incentives tend to reduce the number of errors (Smith and Walker, 1993; Harless and Camerer, 1994). In the limiting case when coefficient $\sigma \to 0$ we obtain a special case of the expected utility theory: $prob(|\xi_L| > \varepsilon) \to 0$, for any $\varepsilon > 0$.

Finally, a subjective utility function is defined over changes in wealth by

(9) $$u(x) = \begin{cases} (x+1)^{\alpha} - 1, & x \geq 0 \\ 1 - (1-x)^{\beta}, & x \leq 0 \end{cases}$$

where $\alpha > 0$ and $\beta > 0$ are constant. Coefficients α and β capture the curvature of utility function correspondingly for positive and negative outcomes. Utility function (9) resembles the value function of prospect theory proposed by Kahneman and Tversky (1979). However, unlike the value function, utility function (9) is constructed so that the marginal utility of a gain (loss) of one penny does not become infinitely high (low), which appears as a counterintuitive property for a Bernoulli utility function. Since none of ten experimental datasets reexamined below includes mixed lotteries involving both positive and negative outcomes, we abstract from the possibility of loss aversion (Kahneman and Tversky, 1979).

Equations (7)-(9) complete the description of the parametric form of StEUT. This parametric form is estimated below on the data from ten well-known experimental studies. For every dataset, the fit of StEUT is also compared with the fit of cumulative prospect theory or CPT (Tversky and Kahneman, 1992), which coincides with the rank-dependent expected utility theory (Quiggin, 1981) when lotteries involve only positive outcomes. A detailed discussion why rank-dependent expected utility theory is a good representative non-expected utility theory is offered in Loomes et al. (2002).

3.2. Experiments with certainty equivalents

This section presents the reexamination of experimental data from Tversky and Kahneman (1992) and Gonzalez and Wu (1999). Both studies elicited the certainty equivalents of two-outcome lotteries to measure individual risk attitudes. Tversky and Kahneman (1992) recruited 25 subjects to elicit their certainty equivalents of 28 lotteries with positive outcomes and 28 lotteries with negative outcomes[4]. The obtained empirical data provides strong support for the fourfold pattern of risk attitudes.

Definition (4) is used to calculate the certainty equivalent of every lottery. Specifically, for cumulative distribution function (7), the certainty equivalent of lottery L is implicitly defined by

$$u(CE_L) = \mu_L + \frac{\sigma_L}{\sqrt{2\pi}} \frac{e^{-\frac{(u(x_1)-\mu_L)^2}{2\sigma_L^2}} - e^{-\frac{(u(x_n)-\mu_L)^2}{2\sigma_L^2}}}{\Phi_L(u(x_n)-\mu_L) - \Phi_L(u(x_1)-\mu_L)} \qquad (10)$$

where σ_L has functional form (8) and utility function $u(.)$ is given by equation (9). Thus, the predicted certainty equivalent CE_L is in fact a function of two parameters: coefficient α (or β) of the power utility function and the standard deviation of random errors σ. For every subject, these two parameters are estimated to minimize the weighted sum of squared errors $WSSE = \sum_L \left(CE_L / \overline{CE}_L - 1\right)^2$, where $\overline{CE}_L$ is the certainty equivalent of lottery L that was actually elicited in the experiment.[5]

For comparison, the prediction of a parametric form of CPT proposed by Tversky and Kahneman (1992) is also calculated.[6] For every subject, two parameters of CPT (power coefficient α (or β) of the value function and coefficient γ (or δ) of the probability weighting function) are estimated to minimize the weighted sum of squared errors $WSSE = \sum_L \left(CE_L^{CPT} / \overline{CE}_L - 1\right)^2$, where CE_L^{CPT} is CPT's prediction.

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[4] Tversky and Kahneman (1992) also used 8 decision problems involving mixed lotteries with positive and negative outcomes. Unfortunately, Richard Gonzalez, who conducted the experiment for Tversky and Kahneman (1992) could not find the raw data on these mixed lotteries and no reexamination was possible.

[5] Non-linear unconstrained optimization was implemented in the *Matlab* 6.5 package (based on the Nelder-Mead simplex algorithm).

[6] Specifically, the utility of lottery $L(x_1, p_1; \ldots x_n, p_n)$ with outcomes $x_1 < \ldots < x_m < 0 \le x_{m+1} < \ldots < x_n$ is $\tilde{u}(L) = \sum_{i=1}^{m} u^-(x_i)\left(w^-\left(\sum_{j=1}^{i} p_j\right) - w^-\left(\sum_{j=1}^{i-1} p_j\right)\right) + \sum_{i=m+1}^{n} u^+(x_i)\left(w^+\left(\sum_{j=i}^{n} p_j\right) - w^+\left(\sum_{j=i+1}^{n} p_j\right)\right)$, where $u^-(x) = -\lambda(-x)^\beta$, $u^+(x) = x^\alpha$, $w^+(p) = p^\gamma / \left(p^\gamma + (1-p)^\gamma\right)^{1/\gamma}$ and $w^-(p) = p^\delta / \left(p^\delta + (1-p)^\delta\right)^{1/\delta}$.

[Insert Table 1 and Table 2 here]

Table 1 and Table 2 present the best fitting parameters of StEUT and CPT for all 25 subjects, as well as the achieved minimum weighted sum of squared errors. Table 1 presents the results for lotteries with positive outcomes and Table 2 — for lotteries with negative outcomes. For 19 out of 25 subjects, the utility function of StEUT has the same shape as the value function of prospect theory: concave for positive outcomes i.e. $\alpha \in (0,1)$ and convex for negative outcomes i.e. $\beta \in (0,1)$. Standard deviation of random errors σ varies from 0.0125, which indicates that an individual behaves according to EUT, to 3.4419, which indicates that an individual assigns certainty equivalents essentially at random. For 16 out of 25 subjects, standard deviation of random errors σ is lower when lotteries have negative outcomes than when lotteries have positive outcomes. One interpretation of this finding might be that the subjects are more diligent (less vulnerable to error) when making decisions involving losses.

The prediction of StEUT and CPT are very similar (correlation coefficient is 0.95 for lotteries with positive outcomes and 0.93 for lotteries with negative outcomes). Nevertheless, Table 1 shows that StEUT fits better than CPT for 15 out of 25 subjects, in the dataset where lotteries involve gains. Similarly, Table 2 shows that StEUT achieves a lower weighted sum of squared errors for 14 out of 25 subjects, in the dataset where lotteries involve losses.

[Insert Table 3 here]

Gonzalez and Wu (1999) conducted a similar experiment to Tversky and Kahneman (1992). They recruited 10 subjects to elicit their certainty equivalents of 165 lotteries with positive outcomes. For this dataset, the prediction of StEUT is estimated along the procedure already outlined above. Gonzalez and Wu (1999) estimated CPT with a probability weighting function $w^+(p) = \delta \cdot p^\gamma / \left(\delta \cdot p^\gamma + (1-p)^\gamma\right)$ and I use this functional form as well to estimate the prediction of CPT. For every subject, three coefficients of CPT (power coefficient α of the value function, curvature coefficient γ and elevation coefficient δ of the probability weighting function) are estimated to minimize the corresponding weighted sum of squared errors. The results of parametric fitting for StEUT and CPT are presented in Table 3. StEUT fits better than CPT for all 10 subjects in the sample. A possible explanation for such superior explanatory power of StEUT is that the dataset is quite noisy. Gonzalez and Wu (1999) report themselves that weak monotonicity is violated in 21% of the pairwise comparisons of the elicited certainty equivalents. Therefore, it is not really surprising that the model with an explicit noise structure fits the data very well.

3.3. Experiments with repeated choice

This section reexamines the experimental data from Hey and Orme (1994) and Loomes and Sugden (1998). In both studies the subjects faced a binary choice under risk and every decision problem was repeated again after a short period of time. Hey and Orme (1994) recruited 80 subjects to make 2x100 choice decisions between two lotteries with a possibility of declaring indifference. Hey and Orme (1994) constructed the lotteries using only four outcomes: £0, £10, £20 and £30. This convenient feature of the dataset allows us to estimate the utility function of StEUT without committing to a specific functional form (9). Since von Neumann-Morgenstern utility function can be arbitrary normalized for two outcomes, we can fix $u(£0)=0$ and $u(£10)=1$. The remaining parameters $u_1 = u(£20)$ and $u_2 = u(£30)$ capture the curvature of utility function and they are estimated from the observed choices.

The probability that lottery *S* with the lowest outcome x_1^S and the highest outcome x_n^S is preferred to lottery *R* with the lowest outcome $x_1^R \leq x_1^S$ and the highest outcome $x_n^R \geq x_n^S$ is equal to

$$(11) \qquad prob(S \succ R) = \frac{\dfrac{\int_{u(x_1^S)-\mu_S}^{u(x_n^S)-\mu_S} \Phi_R(v+\mu_S-\mu_R)\, d\Phi_S(v)}{\Phi_S\left(u(x_n^S)-\mu_S\right)-\Phi_S\left(u(x_1^S)-\mu_S\right)} - \Phi_R\left(u(x_1^R)-\mu_R\right)}{\Phi_R\left(u(x_n^R)-\mu_R\right)-\Phi_R\left(u(x_1^R)-\mu_R\right)}.$$

Explicit derivation of equation (11) can be found in the working paper Blavatskyy (2005). For every subject, three parameters of StEUT (σ, u_1 and u_2) are estimated to maximize log-likelihood

$$(12) \qquad \sum_S \sum_R \left(a \cdot \log prob(S \succ R) + b \cdot \log(1-prob(S \succ R)) + c \cdot \frac{\log prob(S \succ R) + \log(1-prob(S \succ R))}{2} \right),$$

where a is the number of times the subject has chosen lottery S over lottery R, b is the number of times the subject preferred R to S and c is the number of times the subject declared that he or she does not care which lottery to choose.

An individual, who expresses indifference, is assumed to be equally likely to choose either lottery S or lottery R (i.e. each lottery is chosen with probability one-half). This interpretation of indifference is motivated by popular experimental procedures. For subjects, who reveal indifference, a choice decision is typically delegated to an arbitrary third party (e.g. a coin toss or a random number generator). Thus, if individuals reveal no preference for either lottery S or lottery R, they typically end up facing a 50%-50% chance of playing either lottery S or lottery R, which is equivalent to the situation when they deliberately choose each lottery with probability one-half. Alternatively, indifference in revealed choice can be treated as an event when the difference in stochastic utilities of two lotteries does not exceed the threshold of a just perceivable difference as modeled in Hey and Orme (1994).

The utility of a lottery according to CPT is calculated using the probability weighing function $w^+(p) = p^\gamma / \left(p^\gamma + (1-p)^\gamma\right)^{1/\gamma}$ and the value function $u^+(£0) = 0$, $u^+(£10) = 1$, $u_1 = u^+(£20)$ and $u_2 = u^+(£30)$. Since CPT is a deterministic theory, it has to be embedded into a stochastic choice model to yield a probabilistic prediction. Similar as Hey and Orme (1994), I estimate CPT embedded in the Fechner model.[7] Specifically, the probability that lottery S is preferred to lottery R according to CPT is

(13) $$prob(S \succ R) = 1 - \Phi_{0,\rho}\left(\tilde{u}(R) - \tilde{u}(S)\right),$$

where $\Phi_{0,\rho}(.)$ is the cumulative distribution function of the normal distribution with zero mean and standard deviation ρ, and $\tilde{u}(.)$ is the utility of a lottery according to CPT. For every subject, four parameters of CPT (u_1, u_2, γ and ρ) are estimated to maximize the corresponding log-likelihood (12).

[7] I also estimated CPT with a stochastic choice model $prob(S \succ R) = 1/\left(1 + \exp\{\tau \cdot (\tilde{u}(R) - \tilde{u}(S))\}\right)$, $\tau = const$, proposed by Luce and Suppes (1965, p.335) and used by Camerer and Ho (1994) and Wu and Gonzalez (1996). The result of this estimation was nearly identical to the estimation of CPT with the Fechner model.

For all 80 subjects, the estimated best fitting parameters of StEUT satisfy weak monotonicity, i.e. $u_2 \geq u_1 \geq 1$. However, for 14 subjects the estimated parameters are $u_2 = u_1 = 1$, which suggest that these subjects simply maximize the probability of "winning at least something". For 19 subjects the estimated parameter γ of a probability weighting function of CPT is greater than one, which contradicts to the psychological foundations of CPT (Tversky and Kahneman, 1992). Additionally, for one subject the estimated value function of CPT violates weak monotonicity. For these 20 subjects, whose unconstrained best fitting parameters of CPT are inconsistent with the theory, the parameters of CPT are estimated subject to the constraints $\gamma \leq 1$ and $u_2 \geq u_1 \geq 1$.

StEUT and CPT are non-nested models that can be compared through Vuong's adjusted likelihood ratio test (Vuong, 1989). Loomes et al. (2002, p.128) describe the application of Vuong's likelihood ratio test to the selection between stochastic decision theories. Vuong's statistic z has a limiting standard normal distribution if StEUT and CPT make equally good predictions. A significant positive value of z indicates that StEUT fits the data better and a significant negative value—that CPT makes more accurate prediction. Figure 1 demonstrates that for the majority of subjects the predictions of StEUT and CPT (embedded into the Fechner model) are equally good. The number of subjects for whom the prediction of CPT is significantly better (worse) than the prediction of StEUT appears to be higher if we use Akaike (Schwarz) information criterion to adjust for the lower number of parameters in StEUT.

[Insert Figure 1 here]

Loomes and Sugden (1998) recruited 92 subjects and asked them to make 2x45 binary choice decisions designed to test the common consequence effect, the common ratio effect and the dominance relation. The subjects faced a choice between lotteries with only three possible outcomes. For 46 subjects these outcomes were £0, £10 and £20, and for the other 46 subjects—£0, £10, and £30. Therefore, the utility function of StEUT is normalized so that $u(£0) = 0$, $u(£10) = 1$ and the remaining utility $u_1 = u(£20)$ or $u_1 = u(£30)$ (as appropriate) is estimated from the observed choice decisions. The same normalization is used for the value function of CPT. For every subject, two parameters of StEUT (σ and

u_1) and three parameters of CPT embedded in the Fechner model (u_1, γ and ρ) are estimated to maximize the log-likelihood (12) as already described above.

Estimated best fitting parameter u_1 of StEUT satisfies strong monotonicity, i.e. $u_1 > 1$, for all 92 subjects. However, 38 subjects have S-shaped probability weighting function of CPT, i.e. the estimated best fitting parameter γ is greater than one, which is at odds with the psychological foundations of the prospect theory. Among these 38 subjects, 4 individuals also have a non-monotone value function, i.e. $u_1 < 1$. For these 38 subjects, the best fitting parameters of CPT are estimated subject to the constraints $\gamma \leq 1$ and $u_1 \geq 1$. The predictive power of StEUT and CPT (embedded in the Fechner model) is compared based on Vuong's adjusted likelihood ratio test. Figure 2 demonstrates that for the majority of subjects the predictions of StEUT and CPT are not significant different from each other. Thus, StEUT fits the experimental data in Loomes and Sugden (1998) and Hey and Orme (1994) at least as good as CPT.

[Insert Figure 2 here]

3.4. Other experiments

This section reexamines the experimental results reported in Conlisk (1989), Kagel et al. (1990), Camerer (1989, 1992), Camerer and Ho (1994) and Wu and Gonzalez (1996). In these experimental studies subjects were asked to make a non-repeated choice between two lotteries without the possibility to declare indifference[8]. For every binary choice problem, the prediction of StEUT is calculated through equation (11) using functional forms (8)-(9) and the prediction of CPT—through equation (13) using functional form proposed by Tversky and Kahneman (1992) (see footnote 6). For every experimental dataset, two parameters of StEUT (either α or β, and σ) and three parameters of CPT embedded into the Fechner model (either α, γ and ρ or β, δ and ρ) are estimated to maximize the corresponding

[8] Kagel et al. (1990) allowed the subjects to express indifference but do not report how many subjects actually used this possibility. Camerer (1989) allowed indifference in one experimental session. Camerer (1989) reports that three subjects revealed indifference in almost every decision problem, and the rest never expressed indifference.

log-likelihood (12), where *a* now denotes the number of individuals who have chosen lottery *S* over *R* and *b* denotes the number of individuals who preferred *R* to *S*. Since there is no possibility of declaring indifference, *c* is set to zero for every dataset. Of course, individuals do not share identical preferences. However, a single-agent stochastic model is a simple method for integrating data from many studies, where individual estimates have low power, e.g. when one subject makes only a few decisions (Camerer and Ho, 1994, p.186). Such approach is also relevant in an economic sense because it describes the behavior of a "representative agent" (Wu and Gonzalez, 1996).[9]

[Insert Table 4 here]

Table 4 presents 5 binary choice problems from Conlisk (1989). Conlisk (1989) replicates the Allais paradox in problems #1 and #2. Problems #3 and #4 constitute a common consequence problem without a degenerate lottery that delivers one million for certain. Table 4 shows that the incidence of the Allais paradox completely disappears in problems #3 and #4. Finally, problems #1 and #5 constitute a variant of the Allais paradox, when a probability mass is shifted from the medium to the highest (not lowest) outcome. Table 4 shows that the switch in preferences between lotteries *S* and *R* across problems #1 and #5 is comparable to that in problems #1 and #2 (the original Allais paradox).

Maximum likelihood estimates of the parameters of StEUT are $\alpha = 0.6711$ and $\sigma = 0.8764$. The best fitting parameters of CPT are $\alpha = 0.4882$, $\gamma = 0.4713$ and $\rho = 208.0832$. CPT predicts very well the original Allais paradox; however, it also predicts the common consequence effect for problems #3 and #4, which is not found in the data. StEUT makes a less accurate prediction for the original Allais paradox but it predicts no common consequence effect for problems #3 and #4. Vuong's likelihood ratio statistic adjusted through Schwarz criterion is $\mathrm{z} = -1.0997$, which suggests that the predictions of CPT and StEUT are not significantly different from each other according to conventional criteria.

[9] There is also a practical constraint why the reexamination of individual choice patterns is not feasible. Many of the experimental studies reexamined in this section were conducted over a decade ago and several authors, whom I contacted, could not find raw experimental data.

[Insert Table 5 here]

Table 5 presents experimental results for human subjects from Kagel et al. (1990). The upper number in every cell shows the number of subjects who revealed each of four choice patters that are theoretically possible in the experiment. Kagel et al. (1990) found frequent violations of EUT that are consistent with both fanning-out (higher risk aversion for stochastically dominating lotteries) and fanning-in (higher risk seeking for stochastically dominating lotteries) of indifference curves.

The second number in the second row of every cell shows the prediction of StEUT. Maximum likelihood estimates of StEUT parameters are $\alpha = 0.7112$, and $\sigma = 0.2549$. StEUT predicts fanning-out in the first set of lotteries, fanning-in—in the second set of lotteries and non-systematic violations of EUT—in the third set of lotteries. In contrast, CPT explains these choice patterns only when its probability weighting function has an atypical S-shaped form (estimated parameter $\gamma > 1$). The first number in the second row of every cell in Table 5 shows the prediction of unrestricted CPT. When the parameters of CPT are restricted, i.e. $\gamma \leq 1$, its fit (log likelihood -125.839) is worse than the fit of StEUT (log likelihood -125.095) even though CPT embedded in the Fechner model has more parameters.

[Insert Table 6 here]

Table 6 presents the results of estimation of CPT and StEUT on the experimental data reported in Camerer (1989, 1992). In both studies, binary choice problems are constructed to test the betweenness axiom, the common consequence effect and the fourfold pattern of risk attitudes. The important feature of the experimental design in Camerer (1992) is that all lotteries have the same range of possible outcomes (lotteries are located inside the probability triangle e.g. Machina, 1982). Camerer (1992) finds no significant evidence of the common consequence effect. This result is apparent in Table 6. For Camerer (1992) dataset, the best fitting parameter σ of StEUT is close to zero, which is a special case when StEUT coincides with EUT. When lotteries involve small outcomes, the parameter of probability weighting function of CPT is close to one, which is a special case when CPT coincides with EUT.

We compare the fit of CPT and StEUT, as before, using Vuong's adjusted likelihood ratio statistic z (significant positive values indicate that StEUT explains better the observed choice patterns). Table 6 shows that CPT explains significantly better than StEUT the choices over lotteries with large positive outcomes from Camerer (1989). StEUT explains significantly better than CPT the choices over lotteries with small positive and negative outcomes from Camerer (1992). For the remaining experimental data, the predictions of CPT and StEUT are not significantly different. Interestingly, for experimental data from Camerer (1989), parameter σ of StEUT is lower when real rather than hypothetical incentives are used suggesting that monetary incentives reduce random variation in the experiments (Hertwig and Ortmann, 2001). It is also lower when lotteries involve negative outcomes suggesting that subjects are more diligent when faced with the possibility of losses. These observations support the interpretation of parameter σ as the standard deviation of random errors, which are specific to the experimental treatment.

[Insert Table 7 here]

Camerer and Ho (1994) designed an experiment to test for the violations of the betweenness axiom. Table 7 presents the frequency with which all theoretically possible choice patterns are actually observed in their experiment, as well as the predicted frequencies according to CPT (embedded into the Fechner model) and StEUT. The predictions of CPT and StEUT are correspondingly the first and the second number in the second line of every cell. Estimated CPT parameters are $\alpha = 0.5555$, $\gamma = 0.9324$ and $\rho = 1.0689$, and estimated StEUT parameters are $\alpha = 0.4812$ and $\sigma = 0.1178$.

Table 7 shows that the predictions of CPT and StEUT are remarkably similar. Vuong's adjusted likelihood ratio statistic is $z = -0.4521$ based on Akaike Information Criterion and $z = +0.636$ based on Schwarz Criterion. Although both theories fit the experimental data in Camerer and Ho (1994) quite well, they fail to explain a modal quasi-concave preference in the last lottery triple, which is a replication of a hypothetical choice problem originally reported in Prelec (1990). Apparently, the parameterizations of StEUT (and CPT) compatible with an asymmetric split between quasi-concave and quasi-convex preferences, when a modal choice pattern is consistent with the betwenness axiom, cannot explain such asymmetric split when a modal choice pattern violates betwenness.

Wu and Gonzalez (1996) study the common consequence effect using 40 binary choice problems grouped into 5 blocks ("ladders"). Eight problems grouped within one block can be derived from each other by shifting the same probability mass from the lowest to the medium outcome. Wu and Gonzalez (1996) find that the fraction of subjects choosing a more risky lottery *R* first increases and then decreases when the probability mass is shifted from the lowest to the medium outcome (Figure 3 and Figure 4).

[Insert Figure 3 and Figure 4 here]

Figure 3 and Figure 4 demonstrate the predictions of CPT (embedded in the Fechner model) and StEUT about the fraction of subjects who choose a more risky lottery *R*. The predictions of CPT and StEUT replicate the generalized common consequence effect, though the predicted effect appears to be not as strong as in the actual experimental data. According to Vuong's likelihood ratio test adjusted though Akaike Information Criterion, the predictions of CPT and StEUT are not significantly different from each other. Vuong's likelihood ratio test adjusted though Schwarz Criterion shows that the prediction of StEUT is closer to actual choice data than the prediction of CPT in ladders 2 and 5.

5. Conclusion

New decision theory—stochastic expected utility theory (StEUT)—is proposed to describe individual decision making under risk. Existing experimental evidence demonstrates that individuals often make inconsistent decisions when they face the same binary choice problem several times. This empirical evidence can be interpreted that individual preferences over lotteries are stochastic and represented by a random utility model e.g. Loomes and Sugden (1995). Alternatively, an observed randomness in revealed choice under risk can be due to errors that occur when individuals execute their deterministic preferences. This paper follows the latter approach. Individual preferences are fully captured by a non-decreasing Bernoulli utility function defined over changes in wealth rather than absolute wealth levels. However, individuals make random errors when calculating the expected utility of a risky lottery.

Simple models of random errors have already been proposed in the literature when the probability of an error (Harless and Camerer, 1994) or the distribution or errors (Hey and Orme, 1994) was assumed to be constant for every choice problem. Such assumptions are clearly too simplistic because individuals obviously make no errors when choosing between "sure things" (degenerate lotteries) and very few errors—when one of the lotteries (transparently) first-order stochastically dominates the other lottery (Loomes and Sugden, 1998). On the other hand, when individuals choose between more complicated lotteries they switch their revealed preferences in nearly one third of all cases (Camerer, 1989).

StEUT assumes that although individuals make random errors when calculating the expected utility of a lottery, they do not make transparent errors and always evaluate the lottery as at least as good as its lowest possible outcome and at most as good as its highest possible outcome. In other words, the internality axiom is imposed on the stochastic expected utility of a lottery, which is defined as expected utility of the lottery plus an error additive on the utility scale. Apart from this restriction, the distribution of random errors is assumed to be symmetric around zero.

These intuitive assumptions about the distribution of random errors immediately imply that the lotteries whose expected utility is close to the utility of its lowest (highest) possible outcome are likely to be overvalued (undervalued) by random errors. Therefore, on the one hand, random errors reinforce risk-seeking behavior when the utility of a lottery is close to the utility of its lowest outcomes (e.g. unlikely gains or probable losses). On the other hand, random errors reinforce risk averse behavior when the utility of a lottery is close to the utility of its highest outcomes (e.g. probable gains or unlikely losses). Thus, StEUT can explain the fourfold pattern of risk attitudes. The paper also shows that StEUT is consistent with the common consequence effect, the common ratio effect, and the violations of the betweenness.

To assess the descriptive merits of StEUT, the experimental data from ten well-known empirical studies are reexamined. Ten selected studies are Conlisk (1989), Kagel et al. (1990), Camerer (1989, 1992), Tversky and Kahneman (1992), Camerer and Ho (1994), Hey and Orme (1994), Wu and Gonzalez (1996), Loomes and Sugden (1998) and Gonzalez and Wu (1999). Within-subject analysis shows that for the majority of individual choice patterns there is no significant difference between the predictions of

StEUT and CPT. Between-subject analysis shows that StEUT explains the aggregate choice patterns at least as well as does CPT (except for the experiment with large hypothetical gains reported in Camerer, 1989). Thus, a descriptive decision theory can be constructed by modeling the structure of an error term rather than by developing deterministic non-expected utility theories. For the brevity of exposition, StEUT is contested only against CPT (or rank-dependent expected utility theory), similar as in Loomes et al. (2002). A natural extension of this work is to evaluate the goodness of fit of several decision theories as it is done, for example, in Carbone and Hey (2000) and to compare their performance with the fit of StEUT.

StEUT does not explain satisfactorily all available experimental evidence such as the violation of betweenness when a modal choice pattern is inconsistent with the betweenness axiom (see the last column of Table 7). Interestingly, CPT does not explain this phenomenon either, though it is able to predict such violations theoretically (Camerer and Ho, 1994). StEUT and CPT embedded into the Fechner model also predict too many violations of transparent stochastic dominance than are actually observed in the experiment. Loomes and Sugden (1998) argue that any stochastic utility model with an error term additive on the utility scale predicts, in general, too many violations of dominance. Thus, a natural extension of the present model is to incorporate a mechanism that reduces error in case of a transparent first-order stochastic dominance. Blavatskyy (2006b) develops such model by reducing the standard deviation of random errors in decision problems where one choice option transparently dominates the other alternative.

To summarize, there is a potential for constructing even a better descriptive model than StEUT (and CPT) that explains the above mentioned choice patterns. The contribution of this paper is to demonstrate that this hunt for a descriptive decision theory can be successful with modeling the effect of random errors. The latter approach makes clear prediction about the consistency rates (test-retest reliability) when an individual faces the same decision problem on two different occasions. This is a promising avenue for future research, which received little attention so far (see, however, Hey, 2001).

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Subject	CPT			StEUT		
	Value function parameter (α)	Probability weighting function parameter (γ)	Weighted sum of squared errors	Utility function parameter (α)	Standard deviation of random errors (σ)	Weighted sum of squared errors
1	1.0512	0.9710	1.2543	1.0971	0.0125	1.2376
2	0.9627	0.7428	0.7066	0.9572	0.4039	0.4868
3	0.9393	0.6804	1.1799	0.8863	0.5443	1.1507
4	0.7633	0.4858	2.1941	0.4722	1.5406	2.9207
5	0.7204	0.6943	1.0540	0.6248	0.5401	0.8587
6	0.9673	0.6630	1.2134	0.7776	0.8996	1.1326
7	0.7566	0.5566	0.7596	0.5539	0.9143	1.4563
8	0.7291	0.5759	1.3861	0.5821	0.7226	1.6746
9	0.6791	0.7646	0.9218	0.6386	0.3194	0.6807
10	0.4994	0.3079	11.789	-0.0040	3.2733	8.3627
11	1.2238	0.6344	0.7594	1.0124	0.9579	0.7094
12	0.9941	0.6921	0.7624	0.8420	0.7252	0.7563
13	0.6588	0.4210	4.2171	0.2738	1.8278	3.7672
14	0.8643	0.5843	1.9677	0.6772	0.9226	1.9173
15	0.4802	0.4000	6.7237	0.0387	1.4860	6.8369
16	0.6632	0.7258	1.2451	0.5406	0.4920	1.1556
17	0.7527	0.6830	3.2389	0.5497	0.8728	3.0933
18	1.0497	0.6088	1.0080	0.8656	0.9472	1.0523
19	0.6222	0.6908	3.1512	0.4823	0.5230	3.1211
20	0.7973	0.5264	1.3734	0.5413	1.2739	1.5855
21	1.0185	0.4987	1.2101	0.7265	1.2014	1.9130
22	0.8550	0.6057	1.0114	0.6337	1.1372	0.7605
23	1.1555	0.7893	2.3968	1.4594	0.0127	2.5917
24	0.5399	0.5205	3.7401	0.2231	1.0190	4.7727
25	0.7559	0.4530	1.3065	0.3818	1.3125	2.6873

Table 1 Tversky and Kahneman (1992) dataset (lotteries with positive outcomes)

Subject	CPT			StEUT		
	Value function parameter (β)	Probability weighting function parameter (δ)	Weighted sum of squared errors	Utility function parameter (β)	Standard deviation of random errors (σ)	Weighted sum of squared errors
1	0.7629	0.7027	1.1354	0.7568	0.4104	0.7768
2	0.7797	0.6996	1.2220	0.6040	0.7227	1.2889
3	0.8269	0.7415	1.0416	0.6150	0.7351	1.1325
4	0.9189	0.9458	2.4068	0.9259	0.1821	2.1180
5	0.7982	0.7517	0.6756	0.7262	0.4417	0.6646
6	0.8449	0.6817	1.9138	0.5873	1.0136	1.7034
7	0.7053	0.6314	1.0289	0.5187	0.8566	0.8882
8	0.8753	0.7742	1.3920	0.7922	0.4488	1.3531
9	0.7893	0.8257	0.6382	0.8132	0.2324	0.2464
10	0.5341	0.3220	11.191	0.1034	3.4419	7.1512
11	0.8241	0.4502	1.4381	0.4549	1.2060	2.6437
12	0.8769	0.6459	0.7200	0.8937	0.4759	0.6199
13	0.7339	0.6012	1.8306	0.5225	0.9917	1.5751
14	0.5424	0.7152	4.0507	0.4466	0.3131	4.6246
15	0.5127	0.4544	4.7260	0.0834	1.2485	5.1786
16	0.5113	0.3275	11.438	0.0858	3.1535	7.7726
17	0.7617	0.6792	1.0847	0.5815	0.7726	1.1324
18	0.8759	0.7498	1.7615	0.6140	0.8818	1.7514
19	0.7251	0.7260	2.6886	0.6438	0.4799	2.5950
20	0.9872	0.5670	1.1429	0.8858	0.7376	1.6388
21	0.9205	0.8139	1.7946	0.6580	0.7205	1.9273
22	1.4422	0.5445	0.5073	1.2752	0.8179	0.7485
23	0.9146	0.4978	1.1196	0.6455	0.9473	1.8291
24	0.5043	0.3730	3.6980	-0.0680	1.4884	5.6860
25	0.6932	0.5648	4.5404	0.4262	1.2306	3.8524

Table 2 Tversky and Kahneman (1992) dataset (lotteries with negative outcomes)

Subject	CPT				StEUT		
	Value function parameter (α)	Curvature of probability weighting function (γ)	Elevation of probability weighting function (δ)	Weighted sum of squared errors	Utility function parameter (α)	Standard deviation of random errors (σ)	Weighted sum of squared errors
1	0.5426	0.2253	0.3799	44.774	0.0955	2.1386	37.392
2	0.4148	0.3314	1.0153	27.241	0.3305	1.5108	19.162
3	0.5575	0.2665	1.4461	10.145	0.7155	1.7907	10.126
4	0.6321	0.2058	0.1523	40.382	-0.056	3.3712	26.255
5	0.3853	0.2351	0.915	17.368	0.2052	2.0611	13.435
6	1.3335	1.1966	0.4634	14.621	0.7546	0.3539	12.229
7	0.5306	0.2349	0.4106	25.176	0.1123	3.382	17.076
8	0.5184	0.4773	0.1263	61.97	-0.171	1.4185	37.992
9	1.1011	0.9363	0.2209	15.747	0.3776	0.6134	10.165
10	0.5991	0.5634	0.4315	36.291	0.2197	0.8115	28.311

Table 3 Gonzalez and Wu (1999) dataset

#	Lottery *S*	Lottery R	Choice of *S*	$prob(S \succ R)$ predicted by	
				CPT	StEUT
1	$(10^6,1)$	$(0,0.01;10^6,0.89;5*10^6,0.1)$	0.5127	0.5012	0.4225
2	$(0,0.89;10^6,0.11)$	$(0,0.9; 5*10^6,0.1)$	0.1441	0.1714	0.2403
3	$(0,0.01;10^6,0.89;5*10^6,0.1)$	$(0,0.02;10^6,0.78;5*10^6,0.2)$	0.4651	0.5334	0.4904
4	$(0,0.71;10^6,0.19;5*10^6,0.1)$	$(0,0.72;10^6,0.08;5*10^6,0.2)$	0.4651	0.4269	0.4947
5	$(0,0.01;10^6,0.11;5*10^6,0.88)$	$(0,0.02; 5*10^6,0.98)$	0.2500	0.2275	0.2805
Log-likelihood			**0**	**-689.7011**	**-697.7902**

Table 4 Conlisk (1989) dataset: the fraction of subjects choosing *S* over *R* in the experiment and the prediction of CPT ($\alpha = 0.4628, \gamma = 0.4553, \rho = 133.381$) and StEUT ($\alpha = 0.5314, \sigma = 1.8367$).

Choice pattern	Pattern consistent with	Pairs of lotteries		
		S_1(-\$14,1) R_1(-\$20,0.7;\$0,0.3) S_2(-\$14,0.2;\$0,0.8) R_2(-\$20,0.14;\$0,0.86)	S_1(-\$20,0.74;-\$14,0.2;\$0,0.06) R_1(-\$20,0.88;\$0,0.12) S_2(-\$14,0.9;\$0,0.1) R_2(-\$20,0.63;\$0,0.37)	S_1(-\$20,0.6;-\$14,0.4) R_1(-\$20,0.88;\$0,0.12) S_2(-\$14,0.9;\$0,0.1) R_2(-\$20,0.63;\$0,0.37)
$S_1 \succ R_1$, $S_2 \succ R_2$	EUT	10 4 4	7 5 7	3 4 4
$R_1 \succ S_1$, $R_2 \succ S_2$	EUT	5 10 11	11 12 10	17 12 13
$R_1 \succ S_1$, $S_2 \succ R_2$	Fanning Out	10 10 9	1 6 6	5 6 8
$S_1 \succ R_1$, $R_2 \succ S_2$	Fanning In	4 5 5	15 11 11	7 10 7
		N=29	N=34	N=32

Table 5 Kagel et al. (1990) dataset: the upper number in every cell is the number of subjects who revealed a corresponding choice pattern in the experiment; the lower numbers in every cell are the predicted numbers of subjects according to CPT (first number) with best fitting parameters $\alpha = 0.4$, $\gamma = 2.0127, \rho = 1.6165$ and StEUT (second number) with parameters $\alpha = 0.7112$, $\sigma = 0.2549$.

Experiment	Incentives	Cumulative Prospect Theory (embedded into Fechner model)				Stochastic Expected Utility Theory			Vuong's adjusted likelihood ratio	
		Value function parameter (α or β)	Probability weighting function parameter (γ or δ)	Standard deviation of random errors (ρ)	Log likelihood	Utility function parameter (α or β)	Standard deviation of random errors (σ)	Log likelihood	Akaike Infor-mation Criterion	Schwarz Criterion
Camerer (1989), large positive outcomes	Hypothetical	0.4316	0.7101	13.7896	-883.842	0.2949	0.5065	-895.551	-2.629 **	-1.986 *
Camerer (1989), small positive outcomes	Random lottery incentive scheme	0.9881	0.9975	0.0516	-945.091	0.5190	0.3383	-947.523	-0.4985	+0.4155
Camerer (1989), small negative outcomes	Random lottery incentive scheme	0.0000	0.8285	0.4141	-908.124	0.8772	0.0433	-911.541	-1.1949	+0.1041
Camerer (1992), large positive outcomes	Hypothetical	0.0141	0.6177	0.3508	-502.552	0.585	0.0868	-505.623	-0.7238	+0.2034
Camerer (1992), small positive outcomes	Hypothetical	0.9847	0.9981	0.1063	-490.652	0.8729	0.0914	-490.618	+3.248 ***	+10.47 ***
Camerer (1992), small negative outcomes	Hypothetical	0.9520	0.9912	0.1236	-521.269	0.6951	0.0917	-522.543	+4.407 ***	+6.544 ***

* Significant at 5% (one-sided test)
** Significant at 1% (one-sided test)
*** Significant at 0.1% (one-sided test)

Table 6 Camerer (1989, 1992) dataset

Choice pattern	Revealed preference	Lottery triples			
		S($0,0.3;$80,0.4;$200,0.3) M($0,0.4;$80,0.2;$200,0.4) R($0,0.5;$200,0.5)	S($0,0.4;$80,0.6) M($0,0.5;$80,0.4;$200,0.1) R($0,0.6;$80,0.2;$200,0.2)	S($0,0.5;$80,0.4;$200,0.1) M($0,0.6;$80,0.2;$200,0.2) R($0,0.7;$200,0.3)	S($0,0.66;$120,0.34) M($0,0.67;$120,0.32;$200,0.01) R($0,0.83;$200,0.17)
$S \succ R$, $S \succ M$, $M \succ R$	Betweenness	37 26 22	29 20 24	33 23 21	17 46 46
$S \succ R$, $S \succ M$, $R \succ M$	Quasi-convex	9 15 14	6 13 15	10 13 14	0 5 6
$R \succ S$, $M \succ S$, $R \succ M$	Betweenness	14 3 4	10 4 3	8 3 3	3 0 0
$R \succ S$, $M \succ S$, $M \succ R$	Quasi-concave	1 5 6	7 7 5	1 5 6	4 4 3
$S \succ R$, $M \succ S$, $M \succ R$	Quasi-concave	6 15 15	21 15 14	6 14 14	76 42 41
$S \succ R$, $M \succ S$, $R \succ M$	Intransitive	9 9 10	8 9 9	13 9 9	4 4 5
$R \succ S$, $S \succ M$, $R \succ M$	Quasi-convex	6 5 6	2 6 5	9 5 5	1 1 1
$R \succ S$, $S \succ M$, $M \succ R$	Intransitive	4 8 9	0 9 8	1 9 9	1 4 4
		N = 86	N = 83	N = 81	N = 106

Table 7 Camerer and Ho (1994) dataset: the upper number in every cell is the number of subjects who revealed a corresponding choice pattern in the experiment; the lower numbers in every cell are the predicted numbers of subjects according to CPT (first number) with best fitting parameters $\alpha = 0.5555, \gamma = 0.9324, \rho = 1.0689$ and StEUT (second number) with parameters $\alpha = 0.4812, \sigma = 0.1178$.

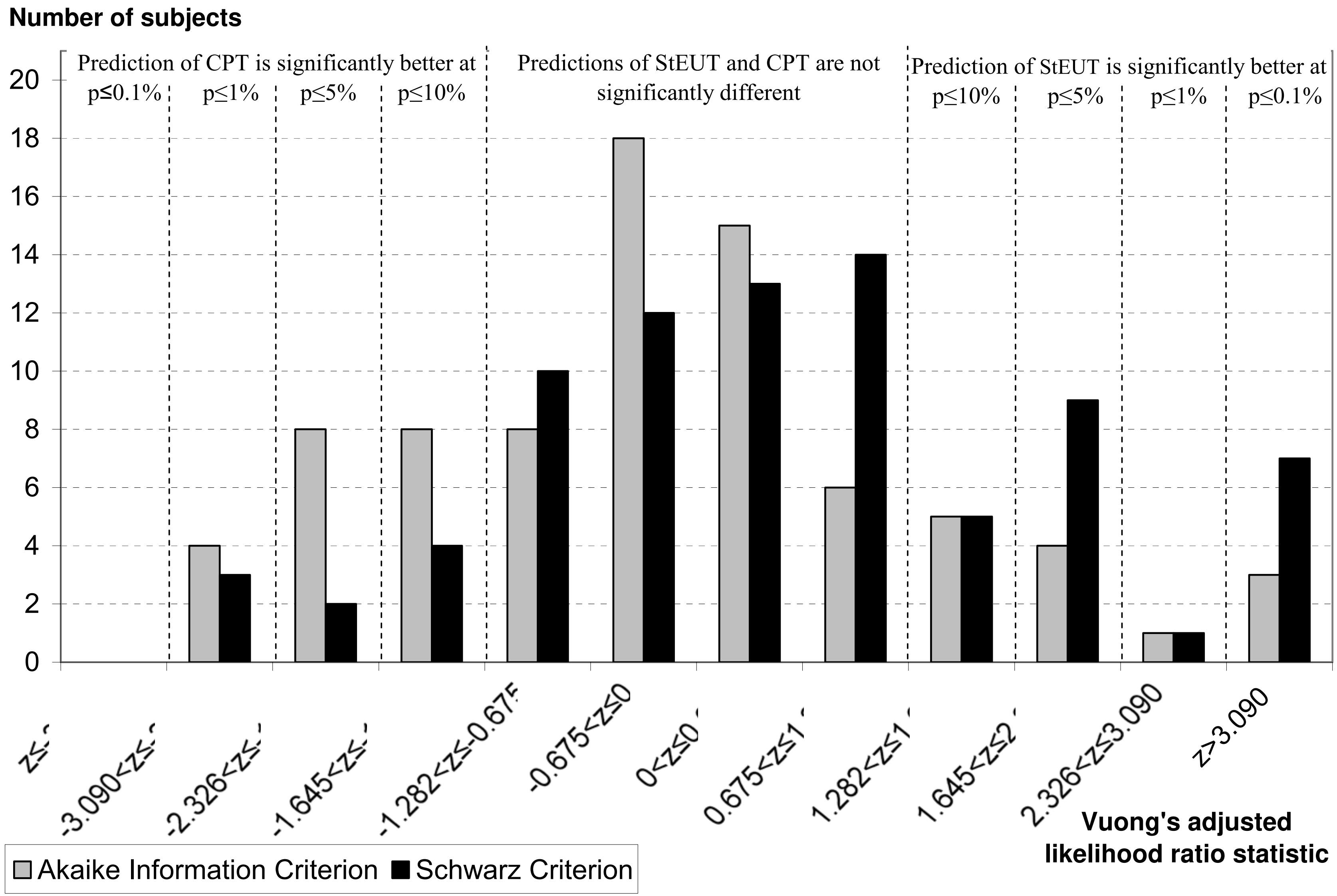

Figure 1 Hey and Orme (2004) dataset (N=80)

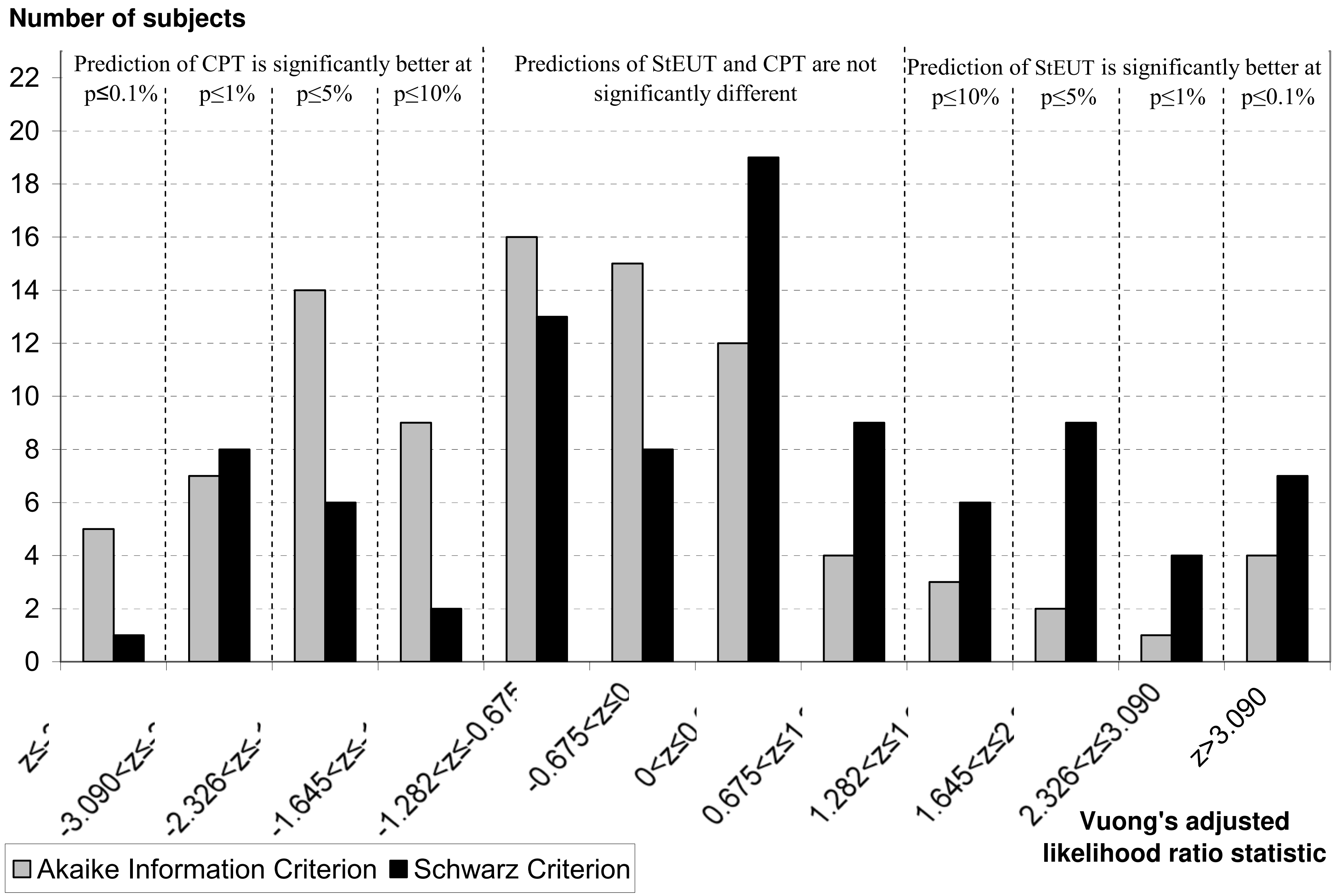

Figure 2 Loomes and Sugden (1998) dataset (N=92)

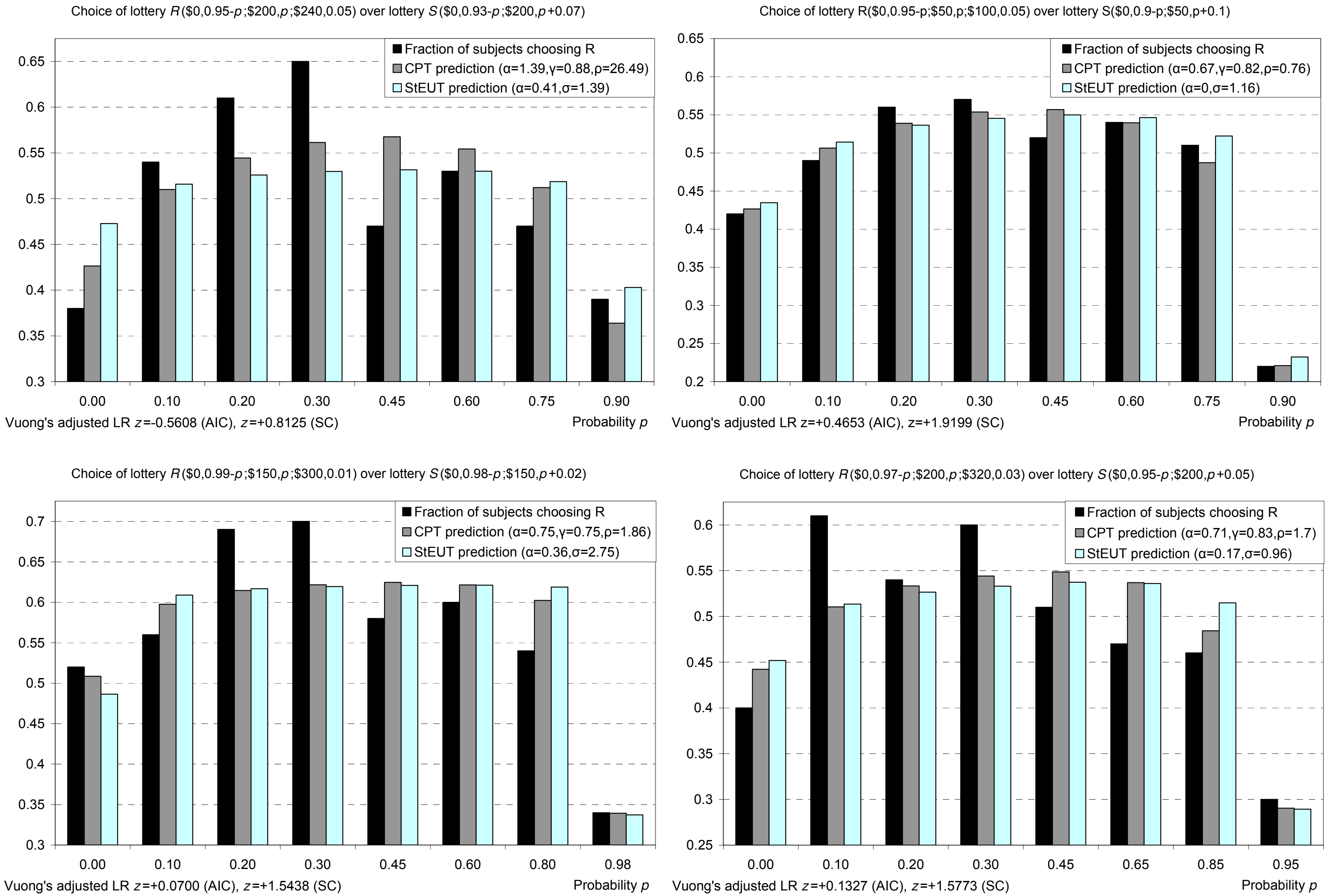

Figure 3 Wu and Gonzalez (1996) dataset (ladders 1-4)

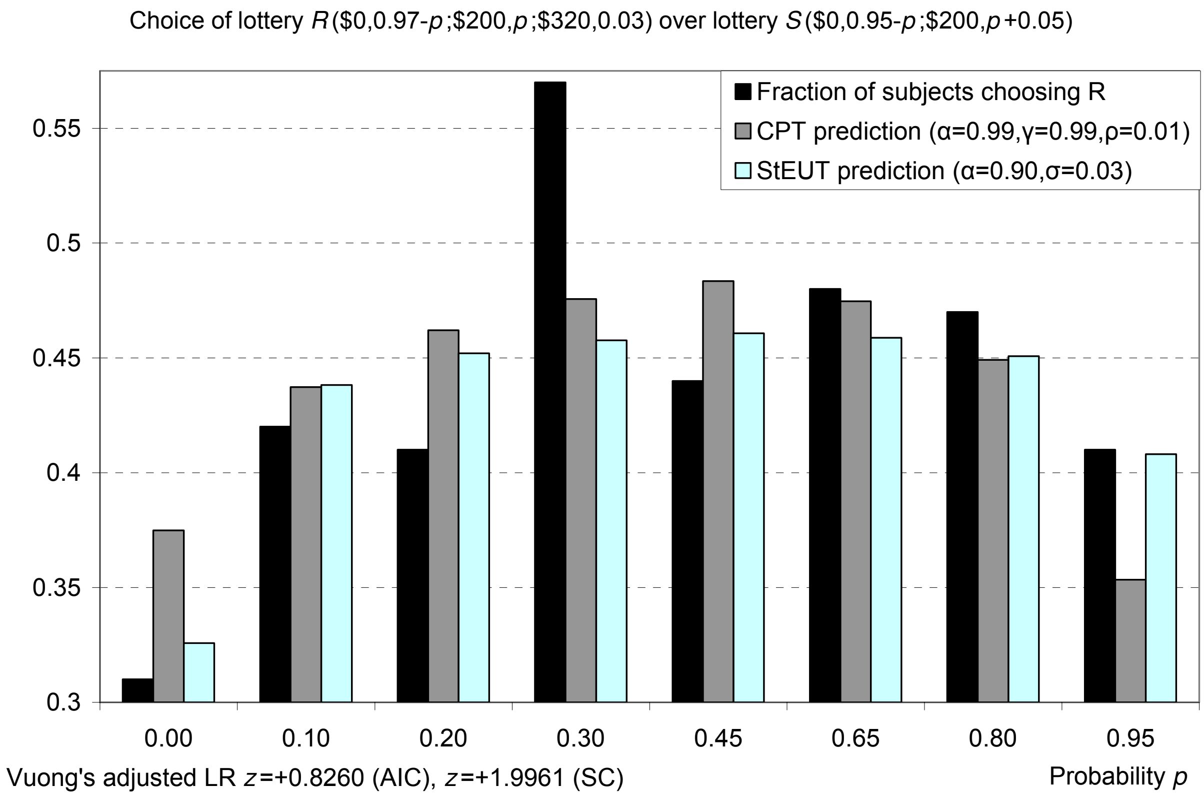

Figure 4 Wu and Gonzalez (1996) dataset (ladder 5)