STOCHASTIC FAULT ANALYSIS OF BALANCED SYSTEMS

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ABSTRACT
A sequence coordinates approach for fault calculations is extended to take into account the uncertainty of the network input data. The probability of a fault current on a bus exceeding its short circuit current is determined. These results would be of importance in determining the protective philosophy of any network. The simple 6-bus Saskatchewan Power Corporation System is used to demonstrate the features of this new development.

NOTATION

AEP	American Electric Power Corporation
Diag	diagonal of a matrix
erf	error function
I_{fi}	short-circuit fault current of bus i
$I_{fi}^{s/c}$	short-circuit current capacity of bus i
J	Jacobian of the load flow equations
P_G	real power generation
P_L, Q_L	real and reactive power
V	expected variance of the input quantities
v_{fi}	Prefault voltage at bus i
V_G	generator voltage
X	Variable
$\bar{X}$	expected value of X
Z_{ii}	Driving point impedance of the faulted bus i
δ	Voltage angle
σ	Standard diviation
σx	standard deviations of the input quantities

INTRODUCTION

The analysis of power systems under fault conditions plays an important role in the design and operation of protection schemes. Methods using the sequence components for the three phase and single phase-to-ground faults have been developed and used successfully for many years [1,2]. These faults are commonly considered to be the severest and most frequent respectively. For unbalanced faults another treatment based on phase coordinates has been proposed by Laughton [3-6].

Generally the methods which use symmetrical components are characterised by the ability to obtain savings in computer time and storage. The phase coordinate approach uses extra computer time and storage demands due to the 3 by 3 matrix representations of the network elements. Moreover, most of the fault data in many utilities are still in terms of the sequence impedances. Reitan [7,8) has extended the capabilities of the symmetric components approach by proposing method for inclusion of coupled line impedances into the bus impedance matrix and the solution of simultaneous faults Podmore [9] developed a technique which combines the advantages of the symmetrical components and phase coordinates techniques. The network is partitioned into a balanced sub-network and a number of unbalanced subsystem. The Solution of the balanced subnetwork is initially obtained in terms of the sequence components and transformed to phase coordinates. The phase coordinates solution of all the subsystems are then combined to form the solution of the original unpartioned network.

The above deterministic methods assume a precise input data as a result of measurement error, forecast inaccuracy or demand variation. The fault evaluation is, therefore, stochastic with the bus fault currents having associated probabilities of occurrence. In this paper, the problem is solved by obtaining the initial system conditions before the fault by stochastic loadflow analysis [10]. The bus admittance matrix is formed and inverted with the system loads and their variation due to network un- certainty taken into consideration. Assuming zero impedance faults, the fault currents are determined on the buses and the probability that the short circuit current capacity is not exceeded is evaluated. El-Kadi [11] showed the use of Monte-Carlo Simulation for probabilistic short-circuit analysis but the method described here will be a faster approach.

2. SOLUTION PROCEDURE

The following solution steps have been employed for balanced three phase fault analysis using the symmetrical coordinate technique. The simple 6-bus Saskatchewan Power Corporation network [12] whose data is given in tables 3 to 4 in Appendix A is analysed. Input variances have also been assumed and are given in table 5 in Appendix A.

Step 1

With the input variances given in table 5, the AEP Stochastic load-flow analysis is solved to obtain the pre-fault bus voltages with .The network uncertainty considered. (The AEP stochastic loadflow problem and its solution are given in Appendix B). The AEP stochastic loadflow model assumes that the load and generation forecasts are normally distributed

Step 2

The bus admittance matrix is formed and inverted with the loads (and their variation) represented as constant admittances. The usual simplifying assumptions for fault analysis some of which are:

(i) flat voltage profile prior to fault and

(ii) effects of loads neglected

are no longer tenable. The pre fault bus voltages are obtained in Step 1 whereas the loads (and their variation) are considered in this step.

It should be noted that the network state is not fixed but varies statistically with time and due to uncertainty.

Step 3

From the diagonal elements of the bus impedance matrix, the fault currents on the buses are computed as

$$I_{fi} = V_{fi}/Z_{ii} \qquad (1)$$

for a three-phase fault on bus i.

Step 4

Since the estimation of the output variables (say voltages on load buses)

have been obtained in Step 1 and the expected values are assumed to be normally distributed, the fault currents obtained from equation (1) above can be assumed to be normally distributed

The density function of a normal distribution f(x) is given by [13]:

$$f(\times) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(\times - \hat{\times})}{2\sigma^2}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

the distribution function F(X) is

$$F(\times) = \int_{-\theta}^{\times} f(\xi)\, d\xi = 0.5 + \int_{0}^{\times} f(\xi) d\xi \ldots\ldots\ldots\ldots.(3)$$

$$\text{defining erf}\left\{\frac{\times - \hat{\times}}{\sigma}\right\}$$

$$\equiv \int_{0}^{\times} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(\xi - \times)^2}{2^2}\right\} d\xi \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$$

Then equation (3) becomes

$$F(\times) = 0.5 - \text{erf}\left\{\frac{\times - \hat{\times}}{\sigma}\right\} \ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

The probability of finding X greater than its maximum limit X^{max} is given by PR where

$$PR = 0.5 - \text{erf}\left\{\frac{\times^{max} - \times}{\sigma}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Hence the probility of the fault current I_{fi} exceeding its rate capacity of a bus $i, I_{fi}\ s/c\ is$

$$p_i = 0.5 - \text{erf}\left\{\frac{I_{fi}\ s/c\ - I_{fi}}{\sqrt{i}}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Considerable savings in computing time were effected by making use of the tabulated values of erf(X) from X = 0 to X = 3 in Papoulis [14].

From equation (7) all buses with p_i greater than 0.5 have exceeded their short-circuit current capacity and should be considered very seriously

3. RESULTS AND DISCUSSION

The probabilities of the short-circuit current capacities being exceeded on the buses are shown in Table 1; these are buses 1, 3 and 6 thus requiring adequate protective mechanism.

Table 1

Bus	Probability of exceeding the short-circuit current capacity
1	0.9792
2	0.0
3	1.0
4	0.2351
5	0.1946
6.	0.9836

As the paper stands, the balanced faults have been assumed to occur with certainty at those busbars. Practical data on the assumed quantities should be taken from operating records and analysed over a wide range of situations and for various types of faults. Hence the results in Table 1 do not represent the typical behaviour of the 'SPC network because of the assumptions made in
Tables 2 and 5.

4. CONCLUSION

A sequence components approach for fault calculations has been extended to take into accunt the un-certainty of the netweork input data as a result of measurement error, forecast inacurracy or demand variation. Only three phase balanced faults are considered but this technique can be extended to any type of fault. The probability associated with each faults type would be evaluated given adequate fault statistics of the network; this would be of much use in determining the appropriate protective mechanism required in a given utility. Much work needs to be done in this area.

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Appendix A

Network Data for the 6-bus Saskatchewan Power Corporation System [1.2]
Base MVA: 100
Table 2: Nodal Data in per unit

Bus No.	Load		Generation	Voltage	$\#_{x'_d}$	$\#_{I_f S/s}$
	P	Q				
1	0.0	0.0	1.5	1.05	0.35	0.92
2	0.5634	0.0390	0.0	1.00	—	1.52
3	0.9845	0.1025	0.0	1.00	–	1.59
4	2.4713	0.5249	3.8	1.05	0.21	0.72
5	0.9167	0.1500	0.5	1.05	0.18	1.87
6*	1.6998	0.0904	0.0	1.05	0.18	1.00

Slack bus *
assumed data

Table 3: Line Data in per unit

Line	R	X	B/2
1-2	0.0250	0.1682	0.1297
2-3	0.1494	0.3692	0.0206
2-6	0.0238	0.2108	0.1509
3-4	0.2130'	0.8957	0.1203
3-6	0.1191	0.2704	0.0164
4-5	0.1021	0.4980	0.2492
5-6	0.0328	0.1325	0.0163

Table 4: Voltage Correction Devices (Rectors) in per unit

Bus	Susceptance
2	0.30
4	0.30
6	0.95

Table 5: Input Variances (assumed data)

Load	PL	= 0.1*PL/3.0
Buses	QL	= 0.1*QL/3.0
Generator	Gi	= 0.05*P Gi/3.0
Buses	Vi	= 0.01*VGi /3.0

Appendix B

Review of the AEP Stochastic Loadflow Method

The AEP stochastic loadflow problem [10] can be stated as

On solving a deterministic loadflow problem

P
$=$ J
Q V/V ......(B1)

the variances of the input quantities are calculated as

$\times^2 = \text{diag}\,(j^t V^{-1} \text{J})^{-1}$ .... (B2)

Under the assumption of a normal distribution there is 99% probability of enclosing the true values within a range of ±3 times the standard deviations thus:

For the input quanties $X_t = x\ 3\ _x$ (B3)

It has been argued [15] that if the Jacobian J is decoupled into B' and B" matrices of the fast decoupled loadflow method [16], the resulting ranges are good estimates of the expected variations formulation of the stochastic loadflow is employed: two independent variance though the 99% confidence limits cannot be guaranteed. Because of the consequent reduction in storage requirements and computing time the decoupled equations result for the real power equations and the reactive and voltage subproblems as:

$$X_1^2 = \text{diag}\begin{pmatrix} 1 \\ \text{Bv} & -1_{\text{B}}\ 1 \\ 1 \end{pmatrix} - 1 \quad \textbf{(B4)}$$

$$\times_2\, 2 = diag\begin{pmatrix} 11 \\ B\,V & -1_B 11 \\ 2 \end{pmatrix} - 1$$

The B^1 and B^{11} matrices are real and symmetric, V_I and V_2 are the variance for the input quantities for the (PQ + PV) buses and the PQ buses respectively.