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Stochastic Processes in Polymeric Fluids

Tools and Examples for Developing
Simulation Algorithms

With 34 Figures and 3 Tables



Springer

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