Stochastic Programming with Integer Recourse

Maarten H. van der Vlerk

To Wilma

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door

Maarten Hendrikus van der Vlerk

geboren op 28 oktober 1961
te Assen

Promotor: Prof. dr. W.K. Klein Haneveld
Referent: Dr. L. Stougie

Preface

This thesis reflects the main part of my work during the four years I worked at the Department of Econometrics of the University of Groningen. The National Operations Research Network in The Netherlands (LNMB) is gratefully acknowledged for financing this position.

Looking back, the recent past is maybe best characterized by the term 'global optimization'. First of all, it refers to the excellent work of my supervisors Wim Klein Haneveld and Leen Stougie, who skillfully removed constraints of many kinds that threatened to obstruct my work. I am grateful for the personal relationships that we developed.

Secondly, during this period I obtained the opportunity to make contact with the international research community working on stochastic programming and related fields. In particular, I mention the cooperation with Rüdiger Schultz, which I am pleased to know will continue for some time to come. I am especially indebted to the members of my committee: Professors Jan Karel Lenstra, Werner Römisch, Caspar Schweigman, and Roger Wets.

In the third place, in global optimization it is acceptable, if not necessary, to settle for sub-optimal outcomes of a search procedure that is often disrupted because of a time constraint. Therefore, the analogy makes it easier to accept the fact that the results reported in this thesis are sub-optimal and incomplete.

I thank my former colleagues for providing such a pleasant working environment. To a large extend this is due to Daan Brand, with whom I shared a working room. His friendly attitude and wide interest and knowledge made it a pleasure to work there. Jointly with Wietse Dol and Erik Frambach I initiated what is now called the 4TeX workbench. Due to their enormous efforts this computer program is nowadays widely used and appreciated. I do not know if the amount of time that I saved by using 4TeX to write this thesis compensates for the hours that I invested in developing it. Anyway, the joy of working together made it well-worth. Lies Huizenga and Evert Schoorl of the Graduate School / Research Institute Systems, Organisations and Management have been very helpful in arranging many practical things. The publication of this thesis is financed by SOM.

Finally, I apologize to my family and friends: often a seemingly harmless question about my work triggered an over-enthusiastic, lengthy, and mostly incomprehensible answer. By far the greatest burden has been on Wilma. To her, I dedicate this thesis.

Groningen, March 1995.

Contents

List of notations

Let $s \in \mathbb{R}$, $t \in \mathbb{R}^n$, ξ a random variable in $\mathbb{R}$, τ a random vector in $\mathbb{R}^n$, φ a real function, and $\alpha \in [0, 1)$.

$\mathbb{R}$	set of reals
$\mathbb{R}_+$	set of non-negative reals
$\mathbb{Z}$	set of integers
$\mathbb{Z}_+$	set of non-negative integers
$\mathbb{Z}_-$	set of non-positive integers ($\mathbb{Z}_+ \cap \mathbb{Z}_- = \{0\}$)
t_i	ith element of the vector $t = (t_1, \ldots, t_n)$
$t(i)$	the vector $(t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \in \mathbb{R}^{n-1}$
cx, $\langle c, x \rangle$	inner product of the vectors c and x
n_1	dimension of vector of first-stage decision variables
m_1	number of first-stage constraints
n_2	dimension of vector of second-stage decision variables
m_2	number of second-stage constraints
g	integer expected surplus function
g_α	α-approximation of g
$\bar{g}$	expected surplus function
h	integer expected shortage function
h_α	α-approximation of h
$\bar{h}$	expected shortage function
Q	integer expected value function (EVF)
Q_α	α-approximation of integer EVF
Q^{**}	convex hull of integer EVF
$\bar{Q}$	expected value function
$\tilde{Q}$	one-dimensional EVF

$\hat{Q}$	one-dimensional integer EVF
$\hat{Q}^c$	convex approximation of one-dimensional integer EVF
$\hat{Q}_\alpha$	α-approximation of one-dimensional integer EVF
$\hat{Q}^{**}$	convex hull of one-dimensional integer EVF
v	integer second-stage value function
$\bar{v}$	second-stage value function
$(s)^+$	$\max\{0, s\}$
$(s)^-$	$\max\{0, -s\}$
q^+, q^-	vectors of simple recourse cost coefficients ($+$ and $-$ are indices)
y^+, y^-	vectors of simple recourse variables ($+$ and $-$ are indices)
$\lceil s \rceil$	integer round up of s
$\lfloor s \rfloor$	integer round down of s
$\lceil s \rceil^+$	$\max\{0, \lceil s \rceil\}$
$\lfloor s \rfloor^-$	$\max\{0, -\lfloor s \rfloor\}$
$\lceil s \rceil_\alpha$	round up of s with respect to $\alpha + \mathbb{Z}$
$\lfloor s \rfloor_\alpha$	round down of s with respect to $\alpha + \mathbb{Z}$
F	cumulative distribution function (cdf), $F(s) = \Pr\{\xi \leq s\}$
$\hat{F}$	left continuous cdf, $\hat{F}(s) = \Pr\{\xi < s\}$
F_α	α-approximation of the cdf F
f	probability density function (pdf)
f_α	α-approximation of the pdf f
F_ξ, f_ξ	marginal cdf/pdf of ξ
$F_{\xi\mid\tau=t}$, $f_{\xi\mid\tau=t}$	conditional cdf/pdf of ξ given $\tau = t$
f_+	right continuous version of f
f_-	left continuous version of f
$\mathcal{E}(\lambda)$	exponential distribution with parameter λ
$\mathcal{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$\mathcal{P}(\theta)$	Poisson distribution with parameter θ
$\mathcal{U}(a, b)$	uniform distribution with support $[a, b]$
ξ^k	kth mass point of the discrete random variable ξ
ξ_α	α-approximation of ξ
E_ξ	expectation with respect to ξ
μ^+	expectation of $(\xi)^+$
μ^-	expectation of $(\xi)^-$
φ^*	conjugate function of φ
φ^{**}	biconjugate function of φ
φ'_+	right derivative of φ

φ'_-	left derivative of φ
$\partial\varphi$	subdifferential of φ
$\Delta^+\varphi$	total increase of φ
$\Delta^-\varphi$	total decrease of φ
$\lvert\Delta\rvert\varphi$	total variation of φ