

DOCUMENTO DE TRABAJO N° 392

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MARKET AND EXCHANGE RATE RETURNS:
A BAYESIAN APPROXIMATION**

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Diciembre, 2014

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<http://files.pucp.edu.pe/departamento/economia/DDD392.pdf>

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Stochastic Volatility in Peruvian Stock Market and Exchange Rate Returns:
a Bayesian Approximation
Lima, Departamento de Economía, 2014
(Documento de Trabajo 392)

PALABRAS CLAVE: Modelo de Volatilidad Estocástica, Estimación
Bayesiana, Gibbs Sampler, Mixture Sampler, Integration Sampler,
Mercado Bursátil, Mercado Cambiario, Modelos GARCH, Perú.

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Hecho el Depósito Legal en la Biblioteca Nacional del Perú N° 2015-02436.
ISSN 2079-8466 (Impresa)
ISSN 2079-8474 (En línea)

Impreso en Kolores Industria Gráfica E.I.R.L.
Jr. La Chasca 119, Int. 264, Lima 36, Perú.
Tiraje: 100 ejemplares

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Abstract

This study is one of the first to utilize the SV model to model Peruvian financial series, as well as estimating and comparing with GARCH models with normal and t-student errors. The analysis in this study corresponds to Peru's stock market and exchange rate returns. The importance of this methodology is that the adjustment of the data is better than the GARCH models using the assumptions of normality in both models. In the case of the SV model, three Bayesian algorithms have been employed where we evaluate their respective inefficiencies in the estimation of the model's parameters being the most efficient the Integration sampler. The estimated parameters in the SV model under the various algorithms are consistent, as they display little inefficiency. The Figures of the correlations of the iterations suggest that there are no problems at the time of Markov chaining in all estimations. We find that the volatilities in exchange rate and stock market volatilities follow similar patterns over time. That is, when economic turbulence caused by the economic circumstances occurs, for example, the Asian crisis and the recent crisis in the United States, considerable volatility was generated in both markets.

JEL Classification: C22.

Keywords: Stochastic Volatility Model, Bayesian Estimation, Gibbs Sampler, Mixture Sampler, Integration, Stock Market, Forex Market, GARCH Models, Peru.

Resumen

Este estudio es uno de los primeros en utilizar el modelo SV para modelar series financieras Peruanas, así como estimar y comparar con los modelos GARCH con errores normales y t-student. El análisis en este estudio corresponde a los mercados bursátiles y cambiarias de Perú. La importancia de esta metodología es que el ajuste de los datos es mejor que los modelos GARCH utilizando los supuestos de normalidad en ambos modelos. En el caso del modelo SV, se han empleado tres algoritmos Bayesianos donde evaluamos sus respectivas ineficiencias en la estimación de los parámetros del modelo siendo el algoritmo más eficiente y utilizado el *Integration Sampler*. Los parámetros estimados en el modelo SV muestran que los diversos algoritmos son consistentes, ya que muestran poco ineficiencia. Las cifras de las correlaciones de las iteraciones sugieren que no hay problemas en el momento del encadenamiento de las Cadenas de Markov en todas las estimaciones. Encontramos que las volatilidades en el mercado cambiario y la volatilidad del mercado bursátil siguen patrones similares en el tiempo. Es decir, cuando una turbulencia económica causada por las circunstancias económicas se produce, por ejemplo, la crisis asiática y la reciente crisis en los Estados Unidos, entonces la volatilidad se percibe en ambos mercados.

Clasificación JEL: C22.

Palabras Claves: Modelo de Volatilidad Estocástica, Estimación Bayesiana, *Gibbs Sampler*, *Mixture Sampler*, *Integration Sampler*, Mercado Bursátil, Mercado Cambiario, Modelos GARCH, Perú.

Stochastic Volatility in Peruvian Stock Market and Exchange Rate Returns: a Bayesian Approximation¹

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1 Introduction

Many financial series, especially stock market and exchange rate market returns, are characterized by their volatile behavior, where periods of major stability and times of high uncertainty can be appreciated -a phenomenon known in the literature as clustering. These clusters of volatility can be caused by economic or political factors that affect the perception of investors about the stock market and the agents of the economy with regard to the exchange rate.

Because it is worth modeling the variance by establishing its dynamic over time, two branches have emerged in the literature to model it: generalized autoregressive conditional heteroscedasticity (GARCH) models and stochastic volatility (SV) models. In the GARCH models it is assumed that the variance follows a single process between the return and the volatility (a single shock or error term component); in return, the SV models admit greater flexibility, given that their variance has a process that is independent from that of return.

An initial model for conditional variance was developed by Engle (1982), called Autoregressive Conditional Heteroscedasticity (ARCH), which was applied to inflation in the United Kingdom showing high persistence of the variance. Bollerslev (1986) presents the GARCH model whose conditional variance groups together the extensive lags of Engle's ARCH model (1982), developing an autoregressive moving averages (ARMA) structure for the variance. Along these lines, the model proposed by Nelson (1991), known as exponential GARCH (EGARCH), allows the leverage effect to be studied; that is, the asymmetrical relationship between returns and variance, which occurs, for example, when there is bad news in the stock market, thus generating volatility that is more than proportional to the shock that originally occurred. Another model along the same lines is that put forward by Glosten et al. (1993)-GJR (1993), which analyzes the leverage effect. When the variance is included in the equation for the mean, the model is denoted by GARCH-M.

Further contributions to the literature include Baillie et al. (1996) consisting of GARCH models with fractional integration, which allow the long-term dependencies of the conditional variances to be modeled. The authors apply these models to the US stock market returns and present results that are highly significant for the integration parameter by rejecting the extreme cases of GARCH and IGARCH³.

¹This paper is drawn from the Thesis of Willy Alanya (2014) at the Department of Economics, Pontificia Universidad Católica del Perú. We thank useful comments of Paul Castillo (Central Bank of Peru), Rodolfo Cermeño and Luis García (PUCP), Pierre Perron and Z. Qu (Boston University).

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³For a more complete review of the list of models in the GARCH family, see Andersen and Bollerslev (1998), Bollerslev (2008), Bollerslev, Chou and Kroner (1992), Engle and Bollerslev (1986), Bollerslev, Engle and Nelson (1994), De Arce (2004), Degiannakis and Xekalaki (2004), Engle (2001), Laurent et al. (2010), Talor (1994, 2005).

Another strand developed in the literature is provided by the SV models that also establish the mean equation, a stochastic process inherent to the volatility or variance that determines the values realized from the variance conditioned to the data. It is an unobservable process, and one that changes over time⁴. These models arise in the modeling of share prices. In a continuous version, Hull and White (1987), and Wiggins (1987) model option-pricing where variance follows a stochastic process. Hull and White (1987) find that the Black-Scholes model overestimates the price of an option in relation to the stochastic volatility model, and that this problem worsens if the option's time to maturity is greater⁵. In Wiggins (1987), this relationship is analyzed for different market options with similar results of those of Hull and White (1987), as only for options with an average duration of more than six months does the volatility model proposed offer further advantages for price valuation.

The SV model does not have analytical representation for the likelihood function. Therefore, a number of parameter estimation methods have been proposed. A first approach is the estimation through the method of moments analyzed by Wiggins (1987), and whose estimation method was given further prominence in Melino and Turnbull (1990). They consider the selection of moments in accordance with familiarity, identification, and efficiency, and apply their methodology to the exchange rate between the Canadian and US dollars. Harvey et al. (1994) employ the quasi-maximum likelihood model for the estimation of likelihood function, which is based on procedures filtered using the Kalman filter.

Moreover, Jaquier et al. (1994) develop a discrete version of the SV model and compare the estimation methodologies proposed with a new Bayesian approach. To do so, the authors simulate series and assess the efficiencies of these methods. Efficiency implies a smaller correlation in the iterations performed and a rapid convergence to the true values of the parameters, in pursuit of the model's objective distributions. The authors conclude that the Bayesian approximation is the best in terms of efficiency and generates better predictions due to the filtering procedure for estimating volatilities.

However, Kim et al. (1998) show the poor performance of the estimation in small samples, caused by poor approximation of the error term to a Normal distribution, and that the parameters are bounded to a predetermined range of values. For this same model, in Kim et al. (1998) new Bayesian algorithms are established that help to improve the quality of the estimators, starting off with the classic Gibbs sampler method, to later develop new algorithms such as the Mixture sampler and the Integration sampler. These algorithms serve to improve the quality of the estimations and their efficiency.

On the other hand, to estimate the logarithm of likelihood that will allow us to ascertain the adjustment of the model to the data and estimate the filtered volatilities, it is necessary to use the so-called particle filter. This sequential Monte Carlo algorithm generates approximate samples on the distribution of latent variables at each point of time, using a similar methodology to the Kalman filter, where a state-space structure is used for the model. Gordon et al. (1993) propose a bootstrap filter in the state-space framework by utilizing approximations or samples on the state vector. Moreover, Pitt and Shephard (1999) employ discrete auxiliary variables whose function consists of having a better sample on the densities of the stochastic volatilities. Meanwhile, Kim et al. (1998) establish a particular algorithm of the Pitt and Shephard's auxiliary filter of (1999),

⁴For more details in the definition, see Taylor (1994).

⁵Wiggins (1987), on the other hand, finds that the estimators for the SV model parameters do not differ from the Black-Scholes model.

where they perform approximations on the objective distributions by way of Taylor expansions.

Kim et al. (1998) show the best performance of the SV model in relation to the conditional heteroscedasticity models under three statistical tests. The first test is the likelihood ratio statistic that measures the model's degree of adjustment to the data. A second test is based on the Atkinson criterion (1986) that consists of simulations of series with SV and GARCH structures using the estimated parameters, to then contrast with the estimation performed on the models studied. The last criterion is that of Chib (1995), who includes both the posterior and prior estimations in the likelihood ratio statistic.

In Peru, there are no works that aim to model the volatility of different financial series. Humala and Rodríguez (2013) present and analyze the stylized facts of Peru's stock market and exchange rate returns and volatilities. On the basis of that paper, different lines of research are proposed, to which this paper seeks to contribute. We apply the SV model as per the discrete version employed by Kim et al. (1998), which consists of a first-order autoregressive SV model. For the estimation of parameters, the prior distributions of Kim et al. (1998) and Jaquier et al. (1994) are assumed. The samples for exchange rate returns covers the months of January 1994 to December 2010, and the stock market returns from January 1992 to December 2010. The frequency is daily for both time series.

Given the paucity of studies dedicated to Peru's exchange rate and stock market returns, it is important to point out that the estimation model and method proposed in this paper constitutes one of the first studies of this type for the Peruvian case⁶. Moreover, one of the academic impacts that can be taken into consideration is the determination of periods of volatility in the Peruvian economy. The estimated volatilities are important in estimating risk aversion for listed companies, as well as the possible effects on the process of economic growth, arbitrage, exchange rate, and decision-making of economic agents (e.g. the export sector).

The structure followed by this paper is as follows: Section 2 presents the structure of the SV models. Section 3 shows and analyzes the empirical results for the series studied, and the performance of the SV model is compared with the traditional N-GARCH and t-GARCH models. The conclusions are set out in the final section.

2 Methodology

The SV models assume that financial series are generated under a stochastic process, both for the mean equation and the variance. Moreover, at each point in time this process determines the volatilities realized, which follows a latent process; that is, they are not observable. For a y_t financial series corrected by the mean of each of the observations $\{t = 1, \dots, T\}$, the representation

⁶To our knowledge, there is only another ongoing study conducted by Lengua, Bayes and Rodríguez (2014), who apply an SV model with leverage and heavy-tailed errors to Latin-American Stock Returns using a GH Skew Student's t-Distribution. Though the algorithm applied is also Bayesian, the proposal is different as it modifies the structure of the distribution of the errors in the model.

of a general canonical SV model has the following structure:

$$\begin{aligned}
y_t &= \exp(h_t/2)\epsilon_t, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_n\eta_t, \\
\beta &= \exp(\mu/2), \\
h_0 &\sim N\left(\mu, \frac{\sigma^2}{1 - \phi^2}\right), \\
\epsilon_t &\sim N(0, 1), \\
\eta_t &\sim N(0, 1),
\end{aligned} \tag{1}$$

where h_t is the volatility of the return at moment t . It is assumed that h_t follows an AR(1) stationary process and σ_n represents the volatility of the process h_t , and the parameter β represents a factor of scale for the equation of volatility. Moreover, for the equation of volatility, we assume that the initial volatility h_0 follows a Normal distribution with the above-mentioned characteristics. Finally, the shocks ϵ_t and η_t are *i.i.d.*, thus the $cov(\epsilon_t, \eta_t) = 0$.⁷

One of the advantages of the SV model is that it allows a linear representation, and thus the use of estimation methods is feasible. The linearization is for the first equation in the system (1)⁸, which corresponds the mean-corrected return:

$$\begin{aligned}
y_t^2 &= [\exp(\mu/2 + h_t/2)]^2 (\epsilon_t)^2, \\
\log(y_t^2) &= \mu + h_t + \log(\epsilon_t^2), \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_n\eta_t.
\end{aligned} \tag{2}$$

Nonetheless, under this linearized scheme, another difficulty related to the approximation of the term $\log(\epsilon_t^2)$ is presented, as it is now a variable that is distributed as an χ^2 . A first proposal is Harvey et al. (1994), using the Kalman filter under the estimation scheme by quasi-maximum likelihood. Nonetheless, Kim *et al.* (1998) conclude that this approximation performs very badly in small samples. Kim et al. (1998) establish an approximation to this new error term by way of a combination of Normal distributions. Firstly, they perform a normalization technique that allows a linearization that is appropriate to the error term: $\epsilon_t^* \sim \sum_{i=1}^k q_i N(m_i, \sigma_i^2)$, where each distribution of the new error term ϵ_t^* has a probability of a mean m_i and a variance σ_i^2 . The values that are determined in Kim et al. (1998) and which approximate the distribution χ^2 as much as possible is when $k = 7$. In this way, a new return is considered, from which the expected value of the logarithm of χ^2 is subtracted to maintain equality in the mean equation in the system (2):

$$\begin{aligned}
x_t &= \log(y_t^2 + c) - E[\log(\epsilon_t^2)], \\
x_t &= \mu + h_t + \epsilon_t^*, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \sigma_n\eta_t.
\end{aligned} \tag{3}$$

where c is a constant (offset= 0.001) used to avoid the logarithm values close to zero. The model (3) is used for the estimations. The estimation of the SV models entails the estimation of two

⁷Other assumptions in the terms of error can also be assumed; see Kim et al. (1998). On the other hand, h_t can take more complex structures as a general ARMA process

⁸Also known as a transition equation, if we consider the structure of a state-space model.

groups of variables. First, the parameters of the model $\theta = \{\mu, \phi, \sigma_n\}$ are estimated. Second, based on these parameters, the filtered volatilities are obtained. In an SV model, the estimation of the θ parameters requires the construction of the likelihood function, to then be maximized. The likelihood function is: $f(y|\theta) = \int f(y|h, \theta)f(h|\theta)dh$. Under the SV model, the set of returns is conditioned to the vector of unobservable volatilities. This expression has to be integrated into each point of time in the sample $t = 1, 2, \dots, T$.

In Jaquier et al. (1994) it is argued that the likelihood function does not have an analytical representation. Nonetheless, in the literature, three approximations have been developed with the aim of overcoming this difficulty: approximation through the Kalman filter using techniques of quasi-maximum likelihood, method of moments, and the Bayesian approach. The first approach does not adequately estimate the non-linear approximation of the error, $\log(\epsilon_t^2)$, as well as a marked bias in finite samples. The second approach has the limitation of determining the number of moments, which if badly specified leads to a significant loss of information derived from the data; see Jaquier et al. (1994). The approach adopted in this study is the Bayesian, whose advantage lies in its efficiency in the estimation of the parameters, and the filtered procedure for the estimation of the latent processes. Jaquier et al. (1994) conclude that the Bayesian method offers an optimum solution in identifying the unobservable process for the variance in the context of the model (3).

In Bayesian econometrics, the aim is to find the posterior distributions. If we have the set of parameters, θ , the data y , then through the Bayes Theorem: $\pi(\theta|y) \propto \pi(y|\theta)\pi(\theta)$, where $\pi(\theta|y)$ is the posterior distribution conditioned by the data whose mean will be the Bayesian estimation of the parameters. Meanwhile, $\pi(y|\theta)$ is the likelihood function, and $\pi(\theta)$ are the prior distributions, which constitute the beliefs with regard to the parameter distributions. To calculate the likelihood function, the Markov chain Monte Carlo (MCMC) is employed, which allows a direct estimation with multiple simulations of this posterior distribution. Following on from Kim et al. (1998), we use three methods for estimating the model's parameters, sampling the significant improvements in the factor of inefficiency, which determines improvements in terms of the convergence of estimated values.

Efficiency in the simulation is determined by the relationship between the standard square error of the parameter and the variance in the iterations performed in the process. Thus, this factor directly depends on the standard errors of the MCMC, whose mathematical definition is

as follows: $R_{BM} = 1 + \frac{2BM}{BM-1} \sum_{i=1}^{BM} K(\frac{i}{BM})\rho(i)$, where R_{BM} represents the standard MCMC error associated with the estimation of the parameter; BM represents the bandwidth, which is the range of frequencies for performing the Parzen kernel which is denoted by $K(\cdot)$, and the term $\rho(i)$ is the autocorrelation in the delay i .

The first method is the MCMC Gibbs sampler algorithm, which assumes that the error behaves under a standard Normal distribution, with consequences for the estimation of the parameters. The Gibbs sampler method seeks to find the posterior distribution set of parameters based on the conditional posterior distributions, taking the following steps: (i) the initial values to volatility h_t and the parameters β or μ , σ_η^2 , ϕ are obtained; (ii) sample values for h_t are obtained from the conditional posterior distribution $\pi(h_t|h_{(-t)}, \beta, \mu, \sigma_\eta^2, \phi, y)$ for each point of the process; (iii) likewise, for the parameter σ_η^2 based on $\pi(\sigma_\eta^2|h_t, \beta, \mu, \phi, y)$; (iv) moreover, for ϕ with the posterior conditional $\pi(\phi|h_t, \beta, \mu, \sigma_\eta^2)$; (v) finally, μ under $\pi(\mu|h_t, \sigma_\eta^2, \phi)$. In this way, the steps (ii)-(v) are repeated under multiple simulations until the estimation of the joint posterior distribution, and

thus the marginal distributions, are reached.

In the second method, called Mixture sampler, the term of error of the linearized model is approximated through this combination of Normal distributions, and the realization of this new error term is called ω_t . In turn, x_t is determined by the series transformed in the equation (3). The process of convergence to the joint posterior distribution is performed under the following steps: (i) the initial values are given for ω_t , ϕ , σ_η^2 , μ , x_t ; (ii) the distribution is obtained for h_t based on $\pi(h_t|\omega_t, \beta, \mu, \sigma_\eta^2, \phi, x_t)$; (iii) the distributions for the mixture ω_t are obtained based on $\pi(\omega_t|x_t, h_t)$; (iv) the conditional posterior distributions are taken for steps (iii)-(v) of the Gibbs sampler method. The iteration and update process occurs between steps (ii)-(iv). Step (ii) differs from the Gibbs sampler ratio in this case, due to the approximation of the error in linear terms.

The third method is the Integration Sampler method: an extension of the Mixture sampler algorithm that consists of the integration or separation of the volatilities in the sampling process with the aim of improving randomness, and less correlation between the parameters and the volatilities. The process contains the following steps: (i) the initial values are given for ω_t , ϕ , σ_η^2 , μ_t , x ; (ii) the distributions are obtained for ϕ , σ_η^2 based on $\pi(\phi, \sigma_\eta^2|\omega_t, x_t)$; (iii) the distributions of h_t and μ are obtained from $\pi(h_t, \mu|\omega_t, \sigma_\eta^2, \phi, x_t)$; (iv) finally, the distributions of ω_t are obtained on the basis of $\pi(\omega_t|x_t, h_t)$. The iteration and update follow the steps sequentially (ii)-(iv). What is new about this procedure is the obtention of posterior distributions at the margin of the volatilities in the step (ii). This is possible as a consequence of the Metropolis-Hastings algorithm, which consists of whether or not to accept the values for the distribution, based on the probability of rejection $g(\phi, \sigma_\eta^2)$. Step (iii) is similar to the Mixture sampler method.

The volatilities are estimated after the SV model parameters. Given that volatility is a latent variable, its estimation can be achieved through methods known as particle filters. These methods allow the subsequent density of the volatilities at each point of time to be estimated. In this study, the filters of Kim et. al. (1998), and Pitt and Shephard (1999) are employed.

3 Empirical Results

In this section we present the results of the SV model estimations: the parameters and the volatilities. Moreover, we describe the efficiency gain in the estimation of parameters in utilizing the three algorithms presented in the previous section, as well as the filtered and smoothed estimations of the stochastic volatility. Finally, we compare the adjustment to the SV model data, with which the N-GARCH and t-GARCH models are obtained.

3.1 The Data

The data used for stock market and exchange rate returns are end-of-day and comprise the period of analysis from January 1992 to December 2010 in the first case, while for the exchange rate data from January 1994 to December 2010 is utilized⁹. The returns are calculated as $r_t = [\log(P_t) - \log(P_{t-1})] \times 100$, where P_t represents the closure price that takes the variable in its original form in the period t . Following the literature in general, we use the squared returns logarithm of as a proxy of volatility. For practical effects, $x_t = \log(r_t^2 + 0.001)$ is used with the aim of correcting the returns close to zero.

⁹The data on the Lima Stock Market is provided by Bloomberg, while that for the exchange rate comes from the Central Reserve Bank of Peru (BCRP).

The main descriptive statistics for both financial series is presented in Table 1. The mean for stock market returns is 0.001; for exchange rate returns it is practically a value of zero. This implies that there are clusters of data around zero. On the other hand, the standard deviation for exchange rate returns is 0.015, greater than the deviation of 0.002 for exchange rate returns¹⁰, which implies that the stock market returns display more volatile behavior than the exchange rate returns; see Figure 1.

Moreover, the asymmetry of stock market and exchange rate returns are 0.012 and 0.243 respectively, from which it is inferred that the observations of the exchange rate returns are biased to one side of density. The fourth moment, or kurtosis, provides evidence of expected results for the financial series, as the observations group together and extend at the tails of the densities. For stock market returns, it is 10.179 and for exchange rate returns it is 15.820.

The initial values and the prior distributions are based on Kim et al. (1998). Thus, we have $\sigma_\eta^2 \sim IG(\sigma_r/2, S_\sigma/2)$, where IG denotes the Inverse-Gamma distribution. It is assumed that $\sigma_r = 5$ and $S_\sigma = 0.01 \times \sigma_r$. For the case of the parameter of persistence ϕ it is specified that $\phi = 2\phi^* - 1$, where ϕ^* is distributed in accordance with a Beta distribution with parameters $(\phi^{(1)}, \phi^{(2)})$. In this way, the prior for ϕ is $\pi(\phi) \propto \left\{\frac{(1+\phi)}{2}\right\}^{\phi^{(1)}-1} \left\{\frac{(1-\phi)}{2}\right\}^{\phi^{(2)}-1}$ where $\phi^{(1)}, \phi^{(2)} > 1/2$ and has support on the interval $(-1, 1)$ with a prior mean of $\{2\phi^{(1)}/(\phi^{(1)} + \phi^{(2)}) - 1\}$. This study utilizes $\phi^{(1)} = 20$ and $\phi^{(2)} = 1.5$.

3.2 Gibbs Sampler Estimation

Table 2 shows the estimation of the parameters ϕ, σ_η, β for the series employed. The initial values and the prior distributions are the same as in Kim et al. (1998); nonetheless, the results presented are robust to the different initial iteration values¹¹. We have employed 1 000 000 iterations and discard the first 50 000 iterations. The associated number for determining the inefficiency statistic is determined with the values established in Kim et al. (1998); that is, a bandwidth of 2000 for the parameters ϕ and β , and a value of 4000 for σ_η , determining the standard errors on the estimation of these parameters.

The results of the estimations for stock market returns are as follows. The mean of ϕ is 0.957. Moreover, the estimation of the parameter β , which represents the scale factor, shows a mean of 1.082. Finally, the parameter σ_η has a posterior mean distribution of 0.322. The estimation of the parameters by way of the Gibbs sampler is quite inefficient, in the sense that convergence is very slow and thus there are probabilities of a biased estimation. In this case, the estimation of the parameter ϕ has an inefficiency of 97.655, the estimate of σ_η has 183.86 and β has 3.921. Another statistic that is used to determine whether the estimation converges adequately is the randomness of the iterations. In this case, Figure 2 (upper panel) presents the autocorrelation functions of the parameters. It is appreciated that the autocorrelations decay more slowly in the parameters ϕ and σ_η , which at the same time shows the highest levels of inefficiency.

On the other hand, the result for the exchange rate returns are that the mean of ϕ is 0.969; the scale factor, β , obtains a value of 0.140 as a mean of marginal posterior distribution. Moreover, the mean of the parameter σ_η is 0.372. The inefficiency is 53.503, 133.14 and 2.021 for ϕ, σ_η and β , respectively. Just like the exchange rate returns, the inefficiency statistics associated with the estimation of these parameters is large on average and for the majority of the parameters. In

¹⁰This may be due to the frequent interventions in the exchange rate market by the BCRP.

¹¹In the results reported, we have used the bias in the generation of random numbers by default in Oxmetrics 6.0.

addition, it is evidenced in Figure 2 (lower panel) that the autocorrelations of the parameters with greater inefficiency show problems of correlation in a greater lag horizon.

The estimations of the parameter of persistence ϕ show that the mean life of the shocks is 15.77 and 22.01 days in the stock market and exchange rate markets, respectively.

3.3 Mixture Sampler Estimation

This method approximates the errors of the system variance equation (3) to the correct distribution corresponding to an χ^2 , and the approximation occurs under a combination of Normal densities. The results (Tabla 2) show that this approximation to the model proposed provides better levels of inefficiency overall. In this case, we use 750 000 iterations and discard the first 10000. The bandwidth associated with the standard parameter estimates is 2000 for the parameters ϕ and σ_η ; and 100 for the parameter β .

In the case of stock market returns, the parameter ϕ has a mean of its marginal posterior distribution of 0.957; moreover, σ_η has a mean of 0.316 and finally, the scale factor β with a mean of 1.086. The inefficiency of the parameters are 54.303, 97.452 and 1.475 for the parameters ϕ , σ_η , and β , respectively. The improvement in the efficiency of all parameters, in relation to the levels presented by the Gibbs sampler estimation, is important. Moreover, Figure 3 (upper panel) shows that the correlation is significantly smaller in all cases; in other words, the autocorrelation decays more rapidly than in the results obtained by the Gibbs sampler. This algorithm is more useful, as it provides better convergence in the pursuit of marginal subsequent distributions.

Meanwhile, the results for exchange rate returns have the same implications. The persistence, ϕ , has a mean of 0.976. The scale factor for the variance β whose mean is 0.152 and its volatility σ_η with 0.293. Unlike the stock market returns, the estimates are slightly different in relation to the Gibbs sampler. Nonetheless, the efficiency gains are similar, in that the inefficiencies are 30.542, 76.073 and 1.183 for the parameters ϕ , σ_η , and β , respectively. Moreover, a significant improvement can be noted in the autocorrelations of the iterations performed in Figure 3 (lower panel).

Although the mean life of stock market shocks is similar to that found in the Gibbs Sampler method, in the case of the volatility of exchange rate returns, this duration increases to 28.53 days.

3.4 Integration Sampler Estimation

The Integration sampler process entails integrating or separating the convergence process of the volatility from that of the parameters. In this way, an improvement in the quality and speed of the estimation and convergence of the estimated value is expected. In this case, 250 000 iterations were used, with the first 250 discarded. Equivalently, the quantity BM is used in Kim *et. al.* (1998), who determine a value of 1000 for all parameters.

The estimations of the parameters for the stock market returns by Integration sampler are very similar to those obtained by the Mixture Sampler. The estimates imply a mean of 0.957 for ϕ ; moreover, the scale factor β has a mean of 1.086. Finally, the mean for the volatility of the equation for the variance is 0.315. The efficiency continues to improve, and this time posts 11.398, 17.351 and 5.885 for ϕ , σ_η , and β , respectively. Nonetheless, a degree of efficiency is lost for the parameter β with relation to the results by way of the Gibbs sampler and Mixture sampler; but the level of inefficiency for this and the other parameters is low. Moreover, in Figure 4 (upper panel) the improvement in the autocorrelation of the iterations is evident; thus, for the persistence ϕ and the scale factor β , the correlation decays rapidly in delay 50 in both cases, and in delay 25,

approximately for σ_η , unlike the previous autocorrelations where these decay with a delay of 200 to 250 for β and σ_η , respectively.

For the case of exchange rate returns, the mean of persistence ϕ is 0.976, for the factor β of 0.152; finally, σ_η whose mean is 0.293. Efficiency also improves significantly, and are 8.613, 14.964 and 5.236 for the parameters ϕ , β , and σ_η , respectively. Equivalently, in the case of the stock market, there is also an improvement for the efficiency of the two first estimates with relation to the Gibbs and Mixture sampler algorithms. In Figure 4 (lower panel), the autocorrelations of the parameters improve and decay rapidly in lag 40 for the parameters ϕ and σ_η , which previously, for the Mixture Sampler, were prolonged up to 100, approximately.

Based on the estimates of the ϕ parameter, we can say that the mean life of a shock (or the persistence of the volatility) in the stock market has a duration of 16 days. In the case of the exchange rate market, the duration of these shocks rises to 28 days.

3.5 Model Volatility Estimations

The unobservable stochastic volatilities of the model are estimated through sequential Monte Carlo simulations or particles filters. Because of the greater efficiency presented, only the Integration sampler parameters are considered¹². Two filtration methods are employed: the algorithm proposed by Kim et al. (1998) and that of Pitt and Shephard (1999).

Figure 5 shows the estimated volatilities of both returns for the two algorithms by utilizing 12500 iterations¹³. Using the filter of Kim et al. (1998), in both cases the presence of atypical observations can be appreciated, and this is because this approximation is quite sensitive to observations very close to zero; in particular, this problem is heightened for exchange rate returns. In consequence, the logarithm of likelihood as a result of this filter is not well estimated. In return, the estimations due to the Pitt and Shephard (1999) filter are robust to the data of returns, reflecting the patterns of volatility of our data and generating an unbiased estimate of the SV model's logarithm of likelihood; see Figure 5.

There is an alternative for viewing the h_t volatilities of the model, which is known as smoothed volatility. The smoothed estimation is a simple average $\sum_{t=1}^H h_t/H$, where H is the quantity of iterations. This process determines the value of the volatility in time t using the information from the entire sample and the estimated parameters in the H iterations. The smooth estimation of the volatility is presented in Figure 6 (upper panel) for the Gibbs, Mixture, and Integration sampler methods for the series of stock market returns. Analogously, in Figure 6 (lower panel) the smooth estimations of the volatility for the exchange rate returns are presented. The results presented are realized with 12 500 iterations for all cases. For stock market returns, the estimations for the volatility using these methods are practically the same, which signifies that there is a strong convergence towards the real values of the parameters. In the case of the exchange rate returns, it is appreciated that the Gibbs sampler method provides greater values for volatility, especially in the peaks of high uncertainty in relation to the Mixture and Integration sampler, which almost do not differ in the estimation of the volatility across the sample.

Likewise, Figure 7 shows the filtered and smoothed estimations of the volatilities (upper and lower panels). The filtered estimations tend to reflect greater volatility because the filtered estima-

¹²Though the results are similar when the parameters estimated by Gibbs y Mixture sampler.

¹³For the auxiliary filter, 62 500 auxiliary components were employed.

tion uses information up to the period t , while the smoothed estimation utilizes the information from the entire period (T). For purposes of the following analysis, we only consider the estimated volatilities by using the Integration sampler process in both series. Figure 8 establishes a visual comparison of the evolutions of volatility and the absolute value of the returns. In both cases, the series shows similar patterns at times of low, medium and high uncertainty, reflected in greater values for the estimated stochastic volatility.

Based on these estimations, the Integration sampler method is used to perform a brief analysis of the main economic and financial facts that had repercussions on the volatility of both series. The variances of stock market and exchange rate returns are clearly affected by the international crises between 1997 and 1999. Both volatilities reflect high volatility due to the economic problems that occurred in the countries of Asia and Russia. Equally, the recent US financial crisis has had severe repercussions in both financial markets, while in 2010 there were no major scares in the markets; indeed, a period of stability was established. On the other hand, internal circumstances have also been a source on uncertainty, as in the political instability of the 1990s. In particular, the stock market of that decade shows permanently erratic behavior, initially because of a market reaction to the structural reforms, and later due to somewhat complex electoral processes. Between both variables there is a coefficient of correlation of 0.4, which confirms the similar relative dynamic of financial markets in the face of systematic shocks in the economy.

3.6 Comparing SV and GARCH Models

Because the SV and GARCH models do not possess the same structure, it is necessary to resort to so-called non-nested tests. In this paper, we resort to three tests: the likelihood ratio test, the second through simulations following Atkinson (1986); and the third, which is the marginal likelihood criteria proposed by Chib (1995). The likelihood ratio, which compares both models, is determined by $LR = 2 \{ \log f^1(y|M_1, \theta_1^*) - \log f^0(y|M_0, \theta_0^*) \}$, where $\log f(y|M, \theta^*)$ is the logarithm of likelihood of the M model conditioned to the estimated parameters θ^* . In this case, if the LR statistic is greater than zero, it is an indication in favor of the SV model denoted by M_1 ; moreover, if it is negative it will favor the GARCH model denoted by M_0 .

Given that the function of likelihood of the SV model requires simulations based on an approximated density of the likelihood logarithm, Pitt and Shephard's auxiliary particle filter (1999) is employed. Thus, Table 3 shows the estimated likelihoods of the SV and GARCH models for both series of returns¹⁴. According to this test in both series of returns, the SV model is superior to the N-GARCH model. In the case of the stock market, the LR statistic is 208.932, and for the case of the exchange rate, it is 354.824, which categorically favors the SV model. The results of this test for the case of the t-GARCH do not favor the SV model, with the LR statistics of -68.218 and -52.998 for stock market and exchange rate returns, respectively¹⁵.

¹⁴The estimated GARCH (1,1) model has the following structure:

$$\begin{aligned} r_t|_{t-1} &\sim N(0, \sigma_t^2), \\ \sigma_t^2 &= \alpha_0 + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2, \end{aligned} \tag{4}$$

with σ_t^2 the conditional variance, α_1 a parameter related to the past values of the returns and α_2 the persistence of variance. When a t-student specification $r_t|_{t-1}$ is considered, it is distributed with a $t(0, \sigma_t^2)$.

¹⁵Note that the mean life of the shocks to the variance of stock market returns is between 53 and 46 days, according to the N-GARCH and t-GARCH model, respectively. In the case of the volatility of exchange rate returns, the sum of the parameters is slightly greater than 1, giving evidence of an IGARCH process where the mean life of the shocks

The test of Atkinson (1986) consists of simulations of series based on the estimated parameters of the SV and GARCH models. Based on these series, the LR statistics are calculated exactly as is set out above. These statistics are a sample of the observed LR, which is calculated by employing the original series. Based on the value observed, its position or ranking inside the simulated LR is determined. The criteria, according to Atkinson (1986), is that if the ranking is close to the extreme values of the sample, it is a result that goes against the null hypothesis model. Meanwhile, if the ranking is relatively far from these values, the null hypothesis model is superior. In our case, we have performed 99 simulations for each series in the cases of SV and GARCH; that is, when the null hypothesis is the SV model, as well as when the GARCH is. For the series simulated under the SV model, this null hypothesis is favored, with the ranking located in positions 64 for stock market returns, and position 36 for exchange rate returns. Moreover, when the series is simulated using the GARCH model parameters, the rankings were 2 and 92 for stock market and exchange rate returns, respectively. Given that the ranking is close to the limit values, this hypothesis was rejected in favor of the SV model. Nonetheless, the case in which the null hypothesis is the SV model and the alternative is a t-GARCH model is also studied. The results for the stock market returns favor the SV model, but in the case of the exchange rate, it is not possible to determine which model is better, because the null hypotheses are rejected when the data is simulated under the two models. This is because the ranking for the exchange rate returns are far from the extreme values and occupy position 21 for the SV case, and position 82 in the t-GARCH case.

Finally, the marginal likelihood test of Chib (1995) consists of estimating the Bayes factor through marginal likelihood, as follows: $VM = \log f(y|M_1, \theta_1^*) + \log f(\theta_1^*) - \log f(\theta_1^*|M_1, y)$, where the first term of the equation is the logarithm of likelihood for the model M_1 , the term $\log f(\theta_1^*)$ is the logarithm of the prior distribution evaluated in the mean of the subsequent distribution θ_1^* and the final term is the logarithm of subsequent distributions evaluated at this point through a Gaussian kernel. The difference in the marginal likelihoods of the two models is the Bayes factor. The analysis of this factor follows the same criterion as the likelihood ratio test. The main input in this test is the logarithm of likelihood, which is estimated by the filtered procedure. Therefore, the GARCH models are estimated using Bayesian inference; more specifically, the algorithms of Gilks and Wild (1992) and the results of this estimation are presented in Table 4. The results are similar to those obtained by the estimation of maximum likelihood. The marginals are -7473.98 and -7312.51 for the stock market (N-GARCH and t-GARCH, respectively) and 1262.03 and 1471.37 for the exchange rate market¹⁶. The difference in these likelihoods allow the Bayes factor to be obtained. The entire procedure has been conducted under 12 500 iterations, using the Integration sampler algorithm. By calculating the Bayes factor, it can be concluded that the SV model is superior to the N-GARCH model. In effect, the Bayes factor is 157.34 and 196.10 for the stock market and exchange rate markets, respectively. On the other hand, this cannot be affirmed when the SV model is compared with the t-GARCH model. In effect, the Bayes factor is -4.164 and -13.23 for the stock and exchange rate markets, respectively. Note that the differences between the SV and a t-GARCH model are not high, which suggests that an SV model with t-Student innovations would overcome a t-GARCH model. This, nonetheless, is the topic of an ongoing investigation.

to volatility is infinite.

¹⁶Note, nonetheless, that unlike the difference in the estimation due to maximum likelihood (Table 3), now the mean life of shocks to the volatility of stock market returns is approximately 15 days. In the case of the volatility of exchange rate returns, the mean life is almost 231 days. This is evidence of greater persistence in the exchange rate market.

4 Conclusions

The SV model is an alternative to the conditional heteroscedasticity models for the estimation of the volatilities of financial series. This study is one of the first to utilize the SV model to model Peruvian financial series, as well as estimating and comparing with GARCH models with normal and t-student errors. The analysis in this study corresponds to Peru's stock market and exchange rate returns. The importance of this methodology is that the adjustment of the data is better than the GARCH models using the assumptions of normality in both models.

In the case of the SV model, three Bayesian algorithms have been employed where we evaluate their respective inefficiencies in the estimation of the model's parameters. The most efficient and used algorithm is the Integration sampler. With respect to the GARCH models, they are all estimated by Bayesian inference, with the aim of rendering them comparable with the SV model for the marginal likelihood test of Chib (1995). The main input for this test is the logarithm of likelihood, which is estimated using the auxiliary estimate of Pitt and Shephard (1999), which also determines the filtered likelihoods of our model.

The estimated parameters in the SV model under the various algorithms are consistent, as they display little inefficiency. The Figures of the correlations of the iterations suggest that there are no problems at the time of Markov chaining in all estimations. On making a simple correlation between the stochastic volatilities, this was found to be significant, though not highly so (0.40). We therefore find that the volatilities in exchange rate and stock market volatilities follow similar patterns over time. That is, when economic turbulence caused by the economic circumstances occurs, for example, the Asian crisis and the recent crisis in the United States, considerable volatility was generated in both markets.

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Table 1. Descriptive Statistics

Values	Stock		Forex	
	Returns	Volatility	Returns	Volatility
	(1992:01–2010:12)		(1994:01–2010:12)	
Mean	0.001	-4.890	0.000	-6.189
Median	0.006	-4.863	0.000	-6.287
Maximum	0.128	-2.046	0.022	-3.728
Minimum	-0.114	-6.907	-0.023	-6.907
Standard Deviation	0.015	0.911	0.002	0.554
Skewness	0.012	-0.015	0.243	0.926
Kurtosis	10.179	2.591	15.820	3.648
Jarque-Bera	9831.276	31.960	23205.82	543.796
Observations	4577		3384	

Table 2. Estimations

	Mean	MCMC Standard Error	Inefficiency	Var-Cov Matrix		
Gibbs Sampler						
Stock Returns						
ϕy	0.957	6.7652e-005	97.655	4.4523e-005	-0.00010263	1.8442e-005
$\sigma_\eta y$	0.322	0.00029937	183.86	-0.00010263	0.00046308	-8.6000e-005
βy	1.082	0.00012745	3.9206	1.8442e-005	-8.6000e-005	0.0039360
Forex Returns						
ϕy	0.969	4.2009e-005	53.503	3.1335e-005	-8.7707e-005	4.7550e-006
$\sigma_\eta y$	0.372	0.00029696	133.14	-8.7707e-005	0.00062922	-2.0438e-005
βy	0.140	2.2889e-005	2.0213	4.7550e-006	-2.0438e-005	0.00024624
Mixture Sampler						
Stock Returns						
ϕy	0.957	5.6888e-005	54.303	4.4101e-005	-0.00010267	1.6253e-005
$\sigma_\eta y$	0.316	0.00024641	97.452	-0.00010267	0.00046105	-7.9943e-005
βy	1.086	8.8757e-005	1.4751	1.6253e-005	-7.9943e-005	0.0039519
Forex Returns						
ϕy	0.976	3.1152e-005	30.542	2.3513e-005	-5.9172e-005	3.7994e-006
$\sigma_\eta y$	0.293	0.00020558	76.073	-5.9172e-005	0.00041110	-1.3642e-005
βy	0.152	2.1593e-005	1.1825	3.7994e-006	-1.3642e-005	0.00029179
Integration Sampler						
Stock Returns						
ϕy	0.957	4.4575e-005	11.398	4.3538e-005	-9.9104e-005	1.7526e-005
$\sigma_\eta y$	0.315	0.00017581	17.351	-9.9104e-005	0.00044488	-6.7189e-005
βy	1.086	0.00030554	5.8851	1.7526e-005	-6.7189e-005	0.0039619
Forex Returns						
ϕy	0.976	2.8332e-005	8.613	2.3276e-005	-5.7679e-005	1.6331e-006
$\sigma_\eta y$	0.293	0.00015557	14.964	-5.7679e-005	0.00040390	-1.0475e-005
βy	0.152	7.8126e-005	5.236	1.6331e-006	-1.0475e-005	0.00029114

Table 3. Maximum Likelihood Estimation of SV and GARCH Models

Models	α_0	$\alpha_1 + \alpha_2$	v	Log-Likelihood
Stock Returns				
SV			—	-7305.667
N-GARCH	0.077	0.987	—	-7410.133
t-GARCH	0.078	0.985	5.890	-7271.558
Forex Returns				
SV				1469.234
N-GARCH	0.0003	1.043	—	1291.822
t-GARCH	0.0003	1.053	4.523	1495.733

The GARCH models have been estimated using Eviews while the SV models used the Integration Sampler

Table 4. Bayesian Estimations of GARCH Models

Models	$\frac{\alpha_0}{1-\alpha_1-\alpha_2}$	$\alpha_1 + \alpha_2$	$\frac{\alpha_2}{\alpha_1+\alpha_2}$	v	<i>Log. Lik</i>
Stocks Returns					
N-GARCH	0.982	0.953	0.847	–	–7462.4
t-GARCH	0.975	0.953	0.847	7.335	–7299.8
Forex Returns					
N-GARCH	0.169	0.997	0.809	–	1274.1
t-GARCH	0.108	0.997	0.821	5.375	1484

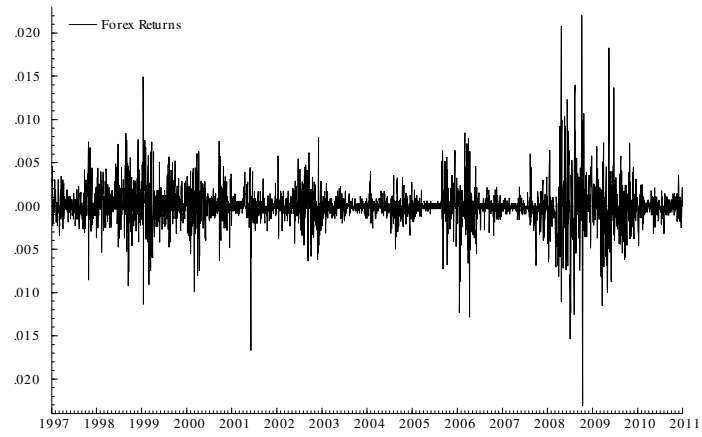
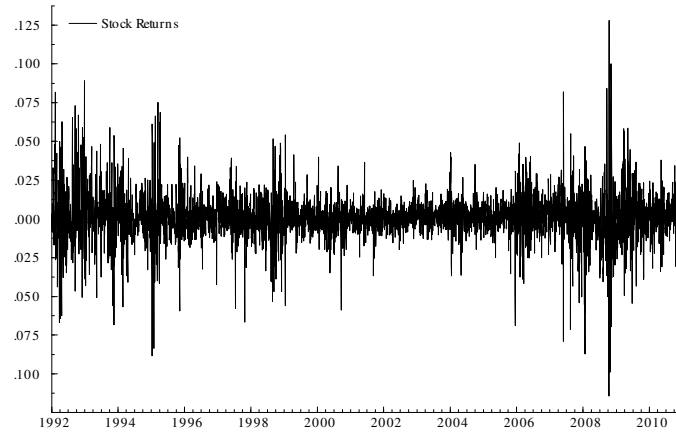


Figure 1. Stock and Forex Returns Series

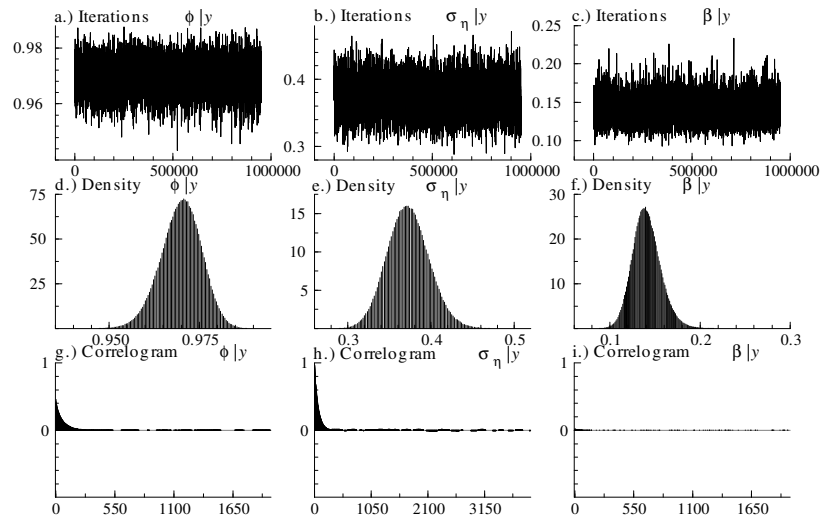
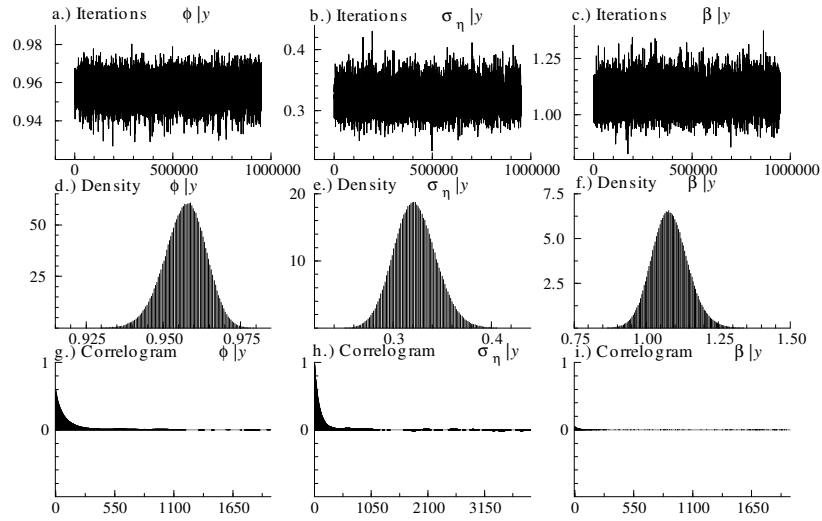


Figure 2. Estimation by Gibbs Sampler. Stock Market (Top Panel) and Forex Market (Bottom Panel). Inside of each Panel (from left to right): a) iterations for $\phi|y$, b) iterations for $\sigma_\eta|y$, c) iterations for $\beta|y$, d) density of $\phi|y$, e) density of $\sigma_\eta|y$, f) density of $\beta|y$, g) correlogram of $\phi|y$, h) correlogram of $\sigma_\eta|y$, i) correlogram of $\beta|y$.

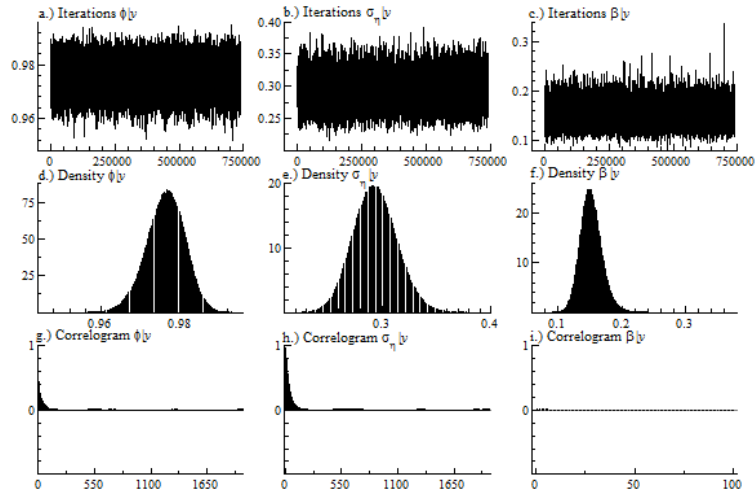
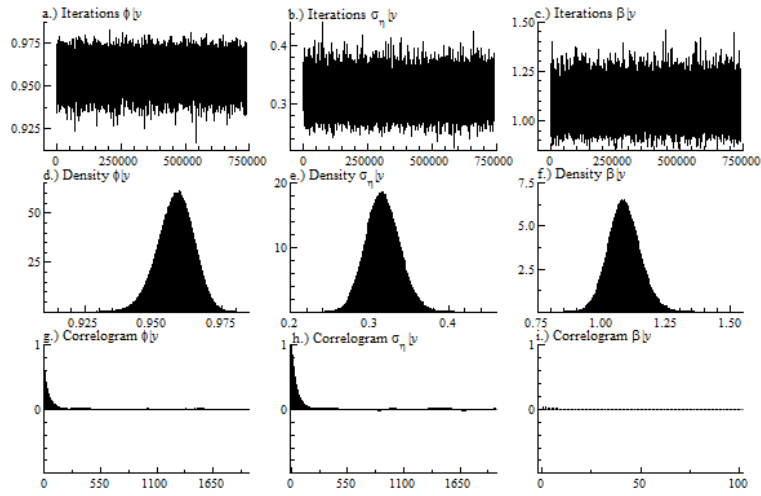


Figure 3. Estimates by Mixture Sampler. Stock Market (Top Panel) and Forex Market (Bottom Panel). Inside of each Panel (from left to right): a) iterations for $\phi|y$, b) iterations for $\sigma_\eta|y$, c) iterations for $\beta|y$, d) density of $\phi|y$, e) density of $\sigma_\eta|y$, f) density of $\beta|y$, g) correlogram of $\phi|y$, h) correlogram of $\sigma_\eta|y$, i) correlogram of $\beta|y$.

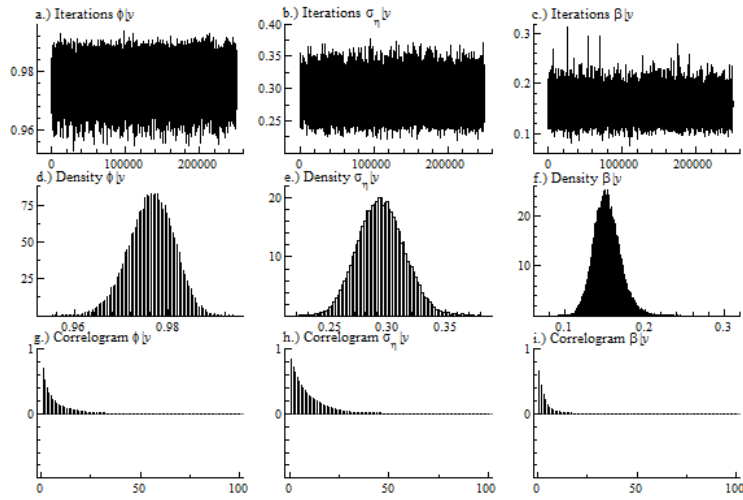
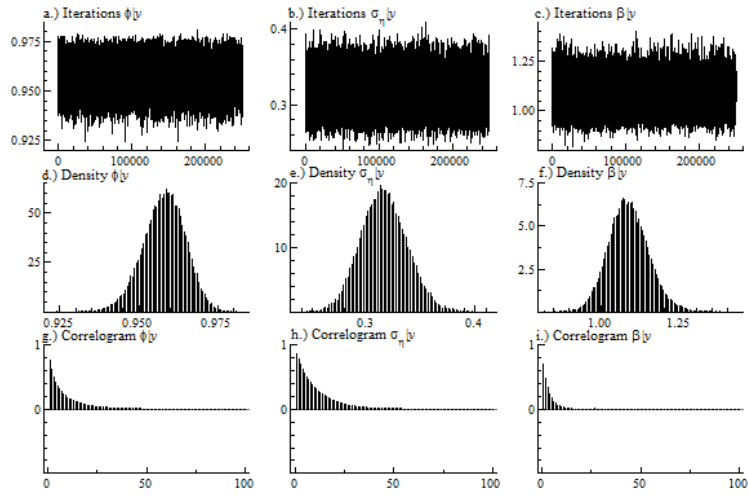


Figure 4. Estimates by Mixture Sampler. Stock Market (Top Panel) and Forex Market (Bottom Panel). Inside of each Panel (from left to right): a) iterations for $\phi|y$, b) iterations for $\sigma_\eta|y$, c) iterations for $\beta|y$, d) density of $\phi|y$, e) density of $\sigma_\eta|y$, f) density of $\beta|y$, g) correlogram of $\phi|y$, h) correlogram of $\sigma_\eta|y$, i) correlogram of $\beta|y$.

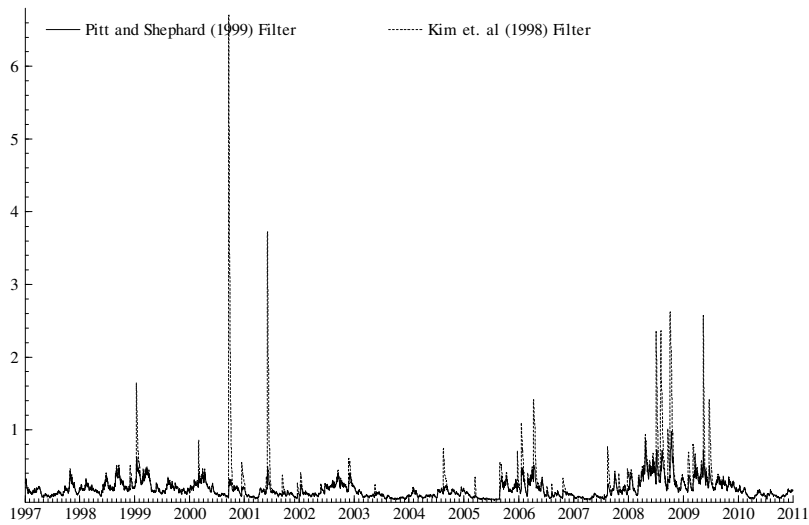
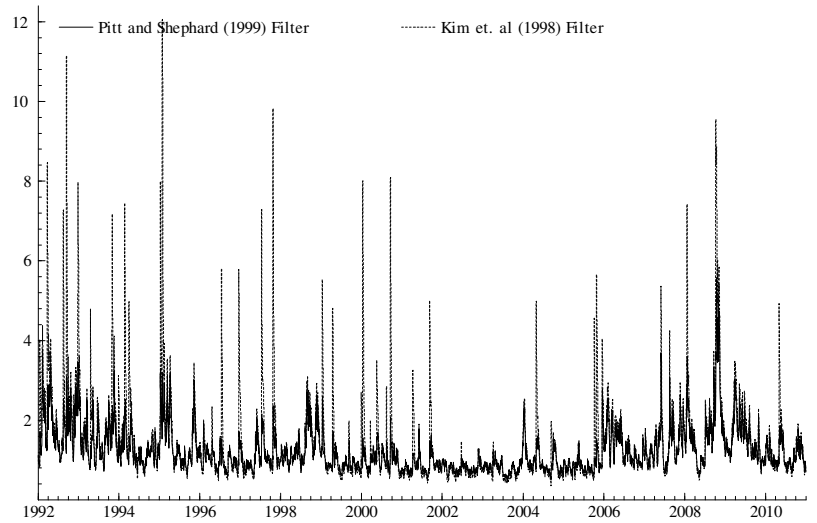


Figure 5. Estimates of Filtered Stochastic Volatility. Stock Market (Top Panel) and Forex Market (Bottom Panel)

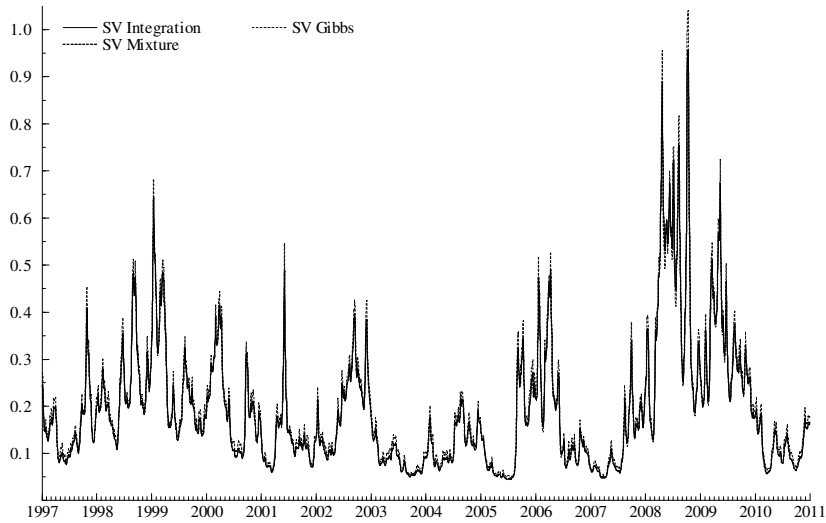
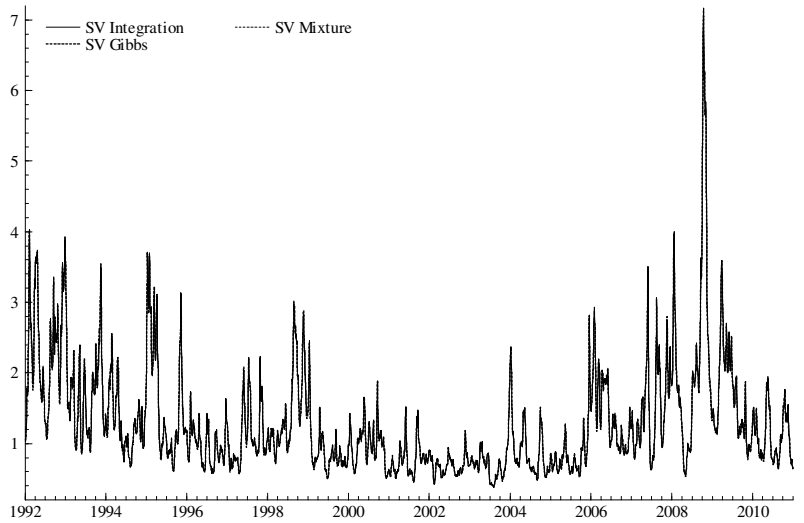


Figure 6. Different Estimates of Stochastic Volatility. Stock Market (Top Panel) and Forex Market (Bottom Panel)

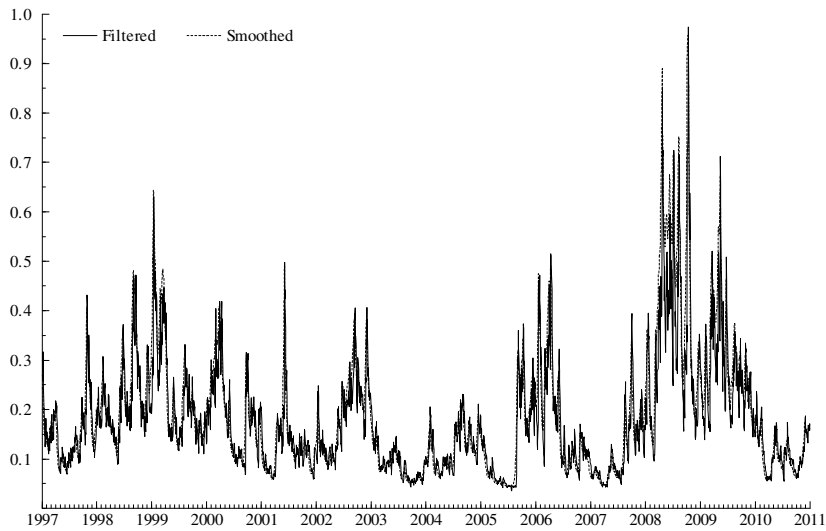
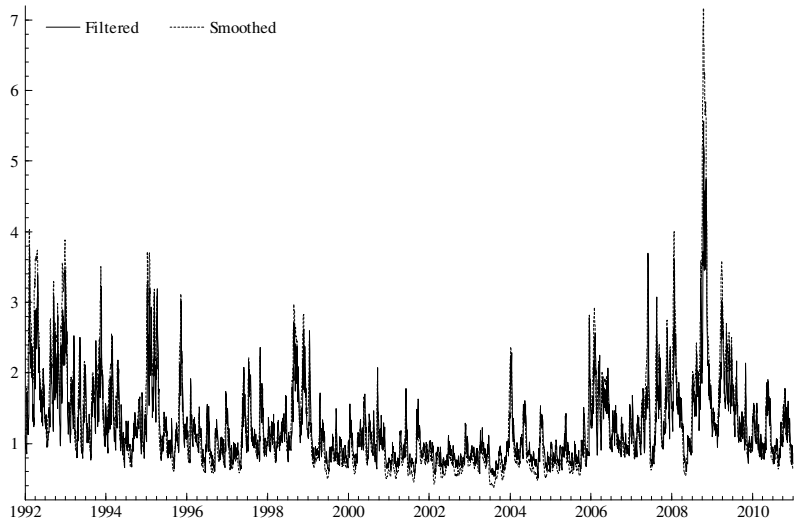


Figure 7. Filtered and Smoothed Stochastic Volatility. Stock Market (Top Panel) and Forex market (Bottom Panel)

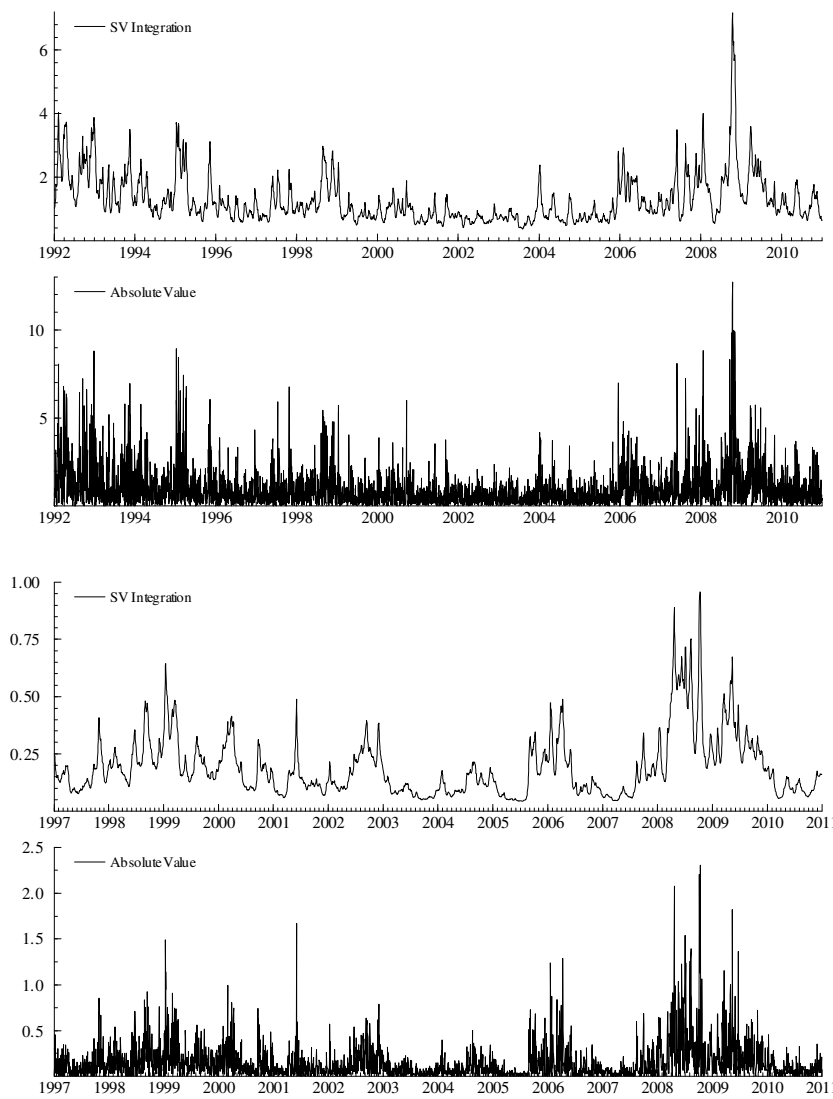


Figure 8. Estimates of Stochastic Volatility and Absolute Value of Returns. Stock Market (Top Panel) and Forex Market (Bottom Panel)

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