Stochastic Vortices in periodically reclassified populations

Gracinda Rita Diogo Guerreiro
Dpt. of Mathematics
UNL - FCT
grg@fct.unl.pt

João Tiago Praa Nunes Mexia
Dpt. of Mathematics
UNL - FCT
jtm@fct.unl.pt

April 2008

Abstract

In our model we consider an open population divided into sub-populations. These populations play a relevant part in many problems. For instance, we may consider the drivers which are clients of an insurance company. According to their records they are placed in one of the *Bonus-Malus* classes. A similar example is given by the clients of a bank each of which is placed in a *Credit-Rating* level. Another example can be given by the population within a Pension Plan. The beneficiaries of the fund can be grouped into subgroups, namely, "Active", "Retired", "Invalid" and even more.

It is easily seen that to manage these populations it is very important to have information about the relative sizes of the sub-populations.

With our model, we obtain limit results for these relative sizes assuming that:
a) the entries, reallocations and departures of the population elements occur at equally spaced times;
b) the probabilities of reallocation of the population elements are stable;
c) the entries are given by a Poisson process;
for populations with a finite number of sub-populations.

We consider the possibility that a new element can be initially placed into any of the sub-populations.

Our treatment will be based on finite, discrete parameter, homogeneous Markov chains. We consider the possibility of more than one transient class as well as more than one recurrent class.

The stability of relative sizes of sub-populations, despite entrances, departures and re-allocations, shows the existence of a structure. We call these structures stochastic vortices. Using the terminology of Prigogine there will clearly be dissipative structures.

An interesting problem occurs when, for the one step transition matrix of a recurrent class, we have more than one module 1 eigenvalues. Then, from the Frobenius theorem, it may be shown that there is a limit cycle for the transition probabilities between states in that class. Nevertheless, under general conditions the relative sizes of the corresponding sub-populations will be stable.

Keywords: Markov Chains, Stochastic Vortices

References

[1] Guerreiro,G.& Mexia,J., *An Alternative Approach to Bonus Malus*, Discussiones Mathematicae, Probability and Statistics, 24, pp. 197 - 213 , 2004

[2] Healy,M., *Matrices for Statistics*, Oxford Science Publications, 1986

[3] Parzen,E., *Stochastic Processes*, Holden Day, San Francisco, 1962

[4] Rolski, T.,Schmidli, H.,Schmidt,V. & Teugels, J., *Stochastic processes for insurance and finance*, John Wiley & Sons, 1999

[5] Schott,J.R., *Matrix Analysis for Statistics*, New York, John Wiley and Sons, Inc., 1997