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Stock index realized volatility forecasting

Dimitrios P. Louzis[a,b], Spyros Xanthopoulos-Sisinis[a,]* and Apostolos P. Refenes[a]

In this article, we account for the presence of heterogeneous leverage effects and the persistence in the volatility of stock index realized volatility. The Heterogeneous Autoregressive (HAR) realized volatility model is extended in order to account for asymmetric responses to negative and positive shocks occurring at distinct frequencies, as well as, for the long range dependence in the heteroscedastic variance of the residuals. Compared with established HAR and Autoregressive Fractionally Integrated Moving Average (ARFIMA) realized volatility models, the proposed model exhibits superior in-sample fitting, as well as, out-of-sample volatility forecasting performance. The latter is further improved when the realized power variation is used as a regressor, while we show that our analysis is also robust against microstructure noise.

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I. Introduction

"A volatility model must be able to forecast volatility; this is the central requirement in almost all financial applications.", Engle and Patton (2001). Indeed, everyday core business functions such as Basel II capital adequacy calculations, risk management, capital allocation, derivatives pricing and hedging, rely on accurate volatility estimation and forecasting. A plethora of volatility implementations have been proposed in the extant literature, e.g. see Poon and Granger (2003) for a good review. In Andersen and Bollerslev (1998), the authors showed that the daily unobserved volatility could be adequately approximated by the sum of squared intraday returns, the so-called, realized volatility[1]. As evidence appeared that realized volatility possessed long memory, a number of researchers employed the Autoregressive Fractionally Integrated Moving Average (ARFIMA) specification for its modelling (e.g. see Andersen *et al*., 2003; Giot and Laurent, 2004; Koopman *et al*., 2005; Noh and Kim, 2006; Degiannakis, 2008; Martens *et al*., 2009).

An alternative implementation, based on the Heterogeneous Market Hypothesis and Muller's *et al*. (1997) HARCH model, the Heterogeneous Autoregressive Realized Volatility model (HAR-RV, referred to as HAR henceforth) was also proposed by Corsi (2009). The HAR model utilized volatility components of different time resolutions in order to capture the long memory property of realized volatility in a more straightforward manner. Its tractable estimation and good volatility forecasting performance encouraged its use in several econometric studies e.g., see Andersen *et al*. (2007) on stock, exchange rate and bond price volatility forecasting, Forsberg and

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[1] The sum of squared intraday returns is actually the realized variance. Realized volatility is defined as the square root of realized variance. However, many authors use the term realized volatility interchangeably with the term realized variance.

Ghysels (2007) and Martens *et al*. (2009) on volatility forecasting and Clements *et al*.

(2008) on Value at Risk applications.

We contribute to this growing literature by introducing a logarithmic HAR model

with asymmetries, or leverage effects[2], modelled here as lagged standardized returns

and absolute standardized returns (analogous to an EGARCH-type structure[3]), occurring

at distinct time horizons: daily, weekly and monthly. Moreover, in order to capture any

remaining long range dependence in the volatility of realized volatility, we propose a

Fractionally Integrated GARCH (FIGARCH) implementation for the conditional

heteroscedasticity of the residuals. We also apply the Realized Power Variation (RPV),

proposed by Barndorff-Nielsen and Shephard (2004) as a regressor, which has been

shown to be a robust to jumps, more persistent and accurate predictor of future volatility

than realized volatility. As far as we are aware, this is the first time a HAR model with

RPV regressors is combined with heterogeneous asymmetric effects and a FIGARCH

specification for the residuals. Finally, the robustness of our findings to microstructure

noise is assessed using the Two-Times Scale (TTS) volatility estimator of Zhang *et al*.

(2005) which consistently estimates the integrated variance in the presence of

microstructure noise.

The proposed model is initially estimated using two ten year data sets from the S&P

500 and DJIA stock indices. We find that against eight alternative HAR and ARFIMA

models, the proposed model produces superior in-sample fitting revealing that not only

past negative daily, but also weekly and monthly negative shocks yield a greater impact

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[2] "Bad news" in a stock market (i.e. negative returns) tend to increase future volatility more than "good news" (i.e. positive returns). This asymmetry between negative and positive returns is referred to as asymmetric or leverage effect. In theory, the leverage of the company increases as its stock price goes down, i.e. the company uses more debt than owned capital to finance its business activities. This increases the risk of investing in this stock which in turn increases its volatility.
[3] The asymmetric behavior between returns and volatility is well documented in the GARCH literature. For the use of asymmetric GARCH models in forecasting volatility see Kisinbay (2008).

on current volatility than positive ones, suggesting a heterogeneous component structure in asymmetric effects. Moreover, an interesting contribution of past monthly positive shocks is also identified. Although the inclusion of leverage effects in the HAR regression reduces both the skewness and the heteroskedasticity of the error term, it does not eliminate the ARCH effects. Through Exact Local Whittle (ELW) and Maximum Likelihood Estimation (MLE) integration order estimations, the suspected long range dependence in the volatility of realized volatility is also verified.

The out-of-sample one day ahead, five and twenty-two days ahead forecasting performance is then evaluated for seven established loss functions, as well as with Hansen's (2005) Superior Predictive Ability (SPA) test. The proposed specification minimizes the majority of the loss functions, for both indices and for all the forecasting horizons. Its volatility forecasting performance is further improved when the RPV is included as a regressor, while its superiority is also confirmed by the SPA test *p*-values. Finally, the TTS estimated realized volatility forecasting results underline its robustness against the microstructure noise in the returns process.

The remaining of this article is organized as follows: In Section II we introduce the realized volatility measures and the mathematical notations and definitions used throughout this article. In Section III we present the HAR and ARFIMA based models, while the data set, descriptive statistics and the in-sample maximum likelihood models estimation are shown in Section IV. In Section V, we present the out-of sample realized volatility forecasting evaluation methodology and results. Section VI summarizes and concludes this article.

II. Realized volatility measures

In Andersen and Bollerslev (1998), the authors defined realized variance as the sum of squared intraday returns and proved it is an unbiased and less noisy estimator for the daily unobserved volatility, than the squared daily returns proxy (for a good review on realized volatility see McAller and Medeiros, 2008). Let us define the mth intraday return for day t as $r_{m,t} = 100 \times \left(\log\left(P_{m,t}\right) - \log\left(P_{m-1,t}\right) \right)$ with $m = 1,...,M$, where M is the total number of intraday returns. The first and last price levels observed at day t are denoted as $P_{0,t}$ and $P_{M,t}$ respectively, while the overnight return, or "sleepage", is defined as $r_{0,t} = 100 \times \left(\log\left(P_{0,t}\right) - \log\left(P_{M,t-1}\right) \right)$. Since these close-to-open price levels are often in practice quite different, the overnight returns could bias the realized variance estimation and hence the following scaling is applied:

$$\sigma^2_{RV,t} = \left[\left(\sigma^2_{oc} + \sigma^2_{co} \right) \Big/ \sigma^2_{oc} \right] \sum_{m=1}^{M} r^2_{m,t} \tag{1}$$

where σ^2_{oc} and σ^2_{co} are the "open-to-close" and "close-to-open" sample variances respectively, which in turn are computed from $\sigma^2_{oc} = (1/T) \sum_{t=1}^{T} \left[\log\left(P_{M,t}\right) - \log\left(P_{0,t}\right) \right]^2$ and $\sigma^2_{co} = (1/T) \sum_{t=1}^{T} \left[\log\left(P_{0,t}\right) - \log\left(P_{M,t-1}\right) \right]^2$ (see Martens, 2002; Koopman *et al.*, 2005 and Degiannakis, 2008). The realized volatility is simply the square root of the realized variance, i.e. $\sigma_{RV,t} = \sqrt{\sigma^2_{RV,t}}$. Since realized volatility is an observable variable, standard time series techniques can be used for forecasting purposes.

The intraday sampling frequency used in this article is five minutes, which for liquid assets like the S&P 500 and the DJIA stock indices it has been found to be the highest sampling frequency with acceptable market microstructure bias (see Andersen *et al.*, 2001a; Koopman *et al.*, 2005; Corsi *et al.*, 2008). Moreover, in order to verify the robustness of our findings against microstructure noise, we repeat our analysis

calculating the realized variance with the Zhang *et al.* (2005) Two-Times Scale (TTS)

volatility estimator, which has been shown to produce consistent realized volatility

estimates in the presence of microstructure noise in the returns process. Here, the TTS

volatility estimates are computed using a fifteen minute sampling interval as in Martens

et al. (2009).

III. The realized volatility models

Heterogeneous Autoregressive (HAR) models for realized volatility

Based on the Heterogeneous Market Hypothesis[4] and the HARCH model[5], Corsi

(2009) proposed an approximate long memory model for realized volatility, the

Heterogeneous Autoregressive (HAR) Realized Volatility model. The author suggested

that a significant contributor to the market's heterogeneity was the presence of three

types of market agents with different time investment horizons: short (daily), medium

(weekly) and long term (monthly) investment horizons. Short-term traders (such as

hedge funds, FX and statistical arbitrage traders) typically adjust their market positions

intradaily, swiftly reacting to any relevant new information. Medium and long-term

investors (such as commercial banks and pension funds) have longer holding periods

and restructure their trading portfolios according to lower frequency information flow.

This asymmetry leads to a hierarchic structure of volatility components, where low (e.g.

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[4] The Heterogeneous Market Hypothesis (Muller *et al.*, 1993) states that market agents differ with respect to their investment horizon, risk aversion, degree of available information, institutional constraints, transaction costs, etc. This diversity is identified as the root cause of asset volatility, as market agents aim to settle at different asset valuations, according to their individual market view, preferences and expectations.

[5] For practical applications of the HARCH model see McMillan and Speight (2006a) and McMillan and Speight (2006b).

monthly) frequency volatility components should yield a greater impact on the overall

volatility than high (e.g. daily) frequency volatility components. The economic rationale

is that short-term investors interpret the level of long-term volatilities as predictions of

future volatility and adjust their trading strategies accordingly, while short-term

volatility is irrelevant to investors with longer holding periods. Corsi showed that by

aggregating daily, weekly and monthly volatility components in an autoregressive

structure, one could capture the heterogeneity of realized volatility, whilst

approximating its long range dependence properties.

Here, in order to mitigate any positivity restrictions on the model's parameters and

error term (e.g. see Andersen *et al.*, 2003), we will use the logarithm of the realized

volatility in the HAR implementation:

$$lrv_t^{(d)} \;=\; a_0 + a_{(d)} lrv_{t-1}^{(d)} + a_{(w)} lrv_{t-1}^{(w)} + a_{(m)} lrv_{t-1}^{(m)} + u_t \qquad (2)$$

where $lrv_t^{(d)} = \log\left(\sigma_{RV,t}^2\right)$ is the daily logarithmic realized variance, u_t are the errors

distributed as $N\left(0, \sigma_u^2\right)$ since $lrv_t^{(d)}$ is approximate normal (e.g. see Andersen *et al.*,

2003), $lrv_{t-1}^{(w)} = \left(\frac{1}{5}\right)\sum_{i=1}^{5} lrv_{t,t-i}^{(d)}$ and $lrv_{t-1}^{(m)} = \left(\frac{1}{22}\right)\sum_{i=1}^{22} lrv_{t,t-i}^{(d)}$ are the lagged weekly and

monthly volatility components respectively, computed for each day *t* as a moving

average of the previous five and twenty-two days daily logarithmic realized volatilities

respectively. We anticipate that some informational content in the individual lags of the

logarithm of realized volatility could as a result of the averaging be lost, however

empirical evidence has shown that the embedded long lag structure of the HAR model,

equivalent to a restricted AR(22)[6], is capable of reproducing the long memory behavior

of realized volatility (Corsi, 2009; Martens *et al.*, 2009). Moreover, the simple

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[6] For the use of AR models in realized volatility forecasting see Hooper *et al.* (2009)

autoregressive functional form of the HAR model requires no more than OLS for the estimation of its parameters.

In Corsi *et al.* (2008), the authors proposed a GARCH(p,q) error process in order to account for the time varying conditional heteroscedasticity of the normally distributed HAR errors, i.e. the so called "volatility of realized volatility":

$$lrv_t^{(d)} = a_0 + a_{(d)}lrv_{t-1}^{(d)} + a_{(w)}lrv_{t-1}^{(w)} + a_{(m)}lrv_{t-1}^{(m)} + u_t \qquad (3)$$

$$u_t = \sigma_{u,t}\varepsilon_t \text{ and } \sigma_{u,t}^2 = \omega + \alpha(L)u_t^2 + \beta(L)\sigma_{u,t}^2 \qquad (4)$$

where $\varepsilon_t \mid I_{t-1} \sim N(0,1)$, I_{t-1} is the information available until $t-1$, L is the lag operator $(Ly_t = y_{t-1})$, $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_p L^p$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q$ are the lag polynomials of orders p and q respectively.

Corsi and Reno (2009) also included past negative daily, weekly and monthly returns as regressors in the HAR model, aiming to capture the leverage effects in the volatility process, plus a jump component. Finally, the authors in Andersen *et al.* (2007) proposed a HAR model with a jump component and found that the latter had restricted persistence compared with the continuous part of the quadratic variation, i.e. its contribution to forecasting volatility was limited.

The Asymmetric HAR-(FI)GARCH models

In this article, we propose extending the HAR specification towards three directions. Firstly, we adopt a more flexible EGARCH-type structure for implementing the asymmetries in the volatility process. We expand the HAR model of Equation (2) in order to include standardized and absolute standardized returns aggregated over different time resolutions. Here, we consider the complete returns dataset in the

analysis, thus allowing for asymmetric responses to both negative and positive shocks. Secondly, through a FIGARCH specification, we account for the long memory of the residual's variance in Equation (4). Finally, we use the Realized Power Variation (RPV) as a regressor, which has been shown to be robust to jumps and a more persistent and accurate predictor of future volatility than realized volatility, see Ghysels *et al.* (2006) and Forsberg and Ghysels (2007).

Initially, the asymmetric dynamics of past daily positive and negative returns are introduced. The Asymmetric (daily) HAR (hereafter A*d*HAR) model with daily leverage effects is defined as follows:

$$lrv_t^{(d)} = a_0 + a_{(d)}lrv_{t-1}^{(d)} + a_{(w)}lrv_{t-1}^{(w)} + a_{(m)}lrv_{t-1}^{(m)} + \vartheta_{(d)}z_{t-1}^{(d)} + \gamma_{(d)}\left|z_{t-1}^{(d)}\right| + u_t \quad (5)$$

where $z_t^{(d)} = r_t/\sigma_{RV,t}$ are the daily standardized returns. Equation (5) can be extended in order to account for the heterogeneity in asymmetric effects, i.e. asymmetric volatility reactions not only to past daily but also to weekly and monthly standardized returns. The Asymmetric HAR (AHAR) is given by:

$$\begin{aligned}
lrv_t^{(d)} &= a_0 + a_{(d)}lrv_{t-1}^{(d)} + a_{(w)}lrv_{t-1}^{(w)} + a_{(m)}lrv_{t-1}^{(m)} + \\
&\quad + \vartheta_{(d)}z_{t-1}^{(d)} + \vartheta_{(w)}z_{t-1}^{(w)} + \vartheta_{(m)}z_{t-1}^{(m)} + \\
&\quad + \gamma_{(d)}\left|z_{t-1}^{(d)}\right| + \gamma_{(w)}\left|z_{t-1}^{(w)}\right| + \gamma_{(m)}\left|z_{t-1}^{(m)}\right| + u_t
\end{aligned} \quad (6)$$

where $z_t^{(h)} = \sum_{i=1}^{h}r_{t-i+1}/\sqrt{\sum_{i=1}^{h}\sigma_{RV,t-i+1}^2}$ are the daily ($h = d = 1$), weekly ($h = w = 5$) and monthly ($h = m = 22$) standardized returns. The response of the logarithmic realized variance to past positive and negative standardized returns is given by:

$$\frac{\partial lrv_t^{(d)}}{\partial z_{t-1}^{(\cdot)}} = \begin{cases} \vartheta_{(\cdot)} + \gamma_{(\cdot)}, & \text{if } z_{t-1}^{(\cdot)} > 0 \\ \vartheta_{(\cdot)} - \gamma_{(\cdot)}, & \text{if } z_{t-1}^{(\cdot)} < 0 \end{cases} \quad (7)$$

The leverage effects are captured by the coefficient $\vartheta_{(\cdot)}$ which is expected to be negative and statistically different from zero, should past negative shocks yield a greater impact on future volatility.

Although accounting for leverage effects in Equation (6), may lead to some
reduction in the skewness of the errors, the heteroscedasticity in the residuals is
expected to remain due to the variance of the realized volatility estimator (Corsi *et al.*,
2008). A straightforward approach, is to implement a GARCH(*p,q*) error process to
account for the conditional heteroscedasticity of the HAR residuals, in an AHAR-
GARCH model.

We suspect however that the residuals could still retain the long memory property of
realized volatility. Motivated by the findings of Beltratti and Morana (2005) (see
ARFIMA models below), we propose to model the residuals with a
FIGARCH(m,d_u,q) specification (see Baillie *et al.*, 1996), implemented as:

$$\sigma_{u,t}^2 = \omega + \beta(L)\sigma_{u,t}^2 + \left\{1 - \beta(L) - \varphi(L)(1-L)^{d_u}\right\}u_t^2 \qquad (8)$$

The FIGARCH model captures the long memory behavior of the variance process
through the long memory, or fractional differencing parameter, d_u, and is essentially an
ARFIMA implementation of the squared residuals in Equation (6). For values of the
differencing parameter d_u between 0 and 1, the autocorrelation of the volatility process
exhibits a slow hyperbolic rate of decay. As the term $(1-L)^{d_u}$ in Equation (8) is an
infinite summation, the FIGARCH obtains an infinite order specification, which in
practice is truncated at 1000 lags, as suggested in Baillie *et al.* (1996).

The Realized Power Variation

Recently, the Realized Power Variation (RPV) proposed by Barndorff-Nielsen and
Shephard (2004), has been found to produce superior realized volatility forecasts when
implemented as a reggresor in a HAR model. The RPV of order p, is defined as:

$$RPV_t(p) = \mu_z^{-1} M^{-(1-p/2)} \sum_{m=1}^{M} \left| r_{m,t} \right|^p \qquad (9)$$

where $0 < p < 2$, $\mu_z = E|z|^p = 2^{p/2} \Gamma\left(\frac{1}{2}(p+1)\right) / \Gamma\left(\frac{1}{2}\right)$ with $z \sim N(0,1)$ [7]. Forsberg

and Ghysels (2007), Ghysels *et al.* (2006) and Ghysels and Sinko (2006) demonstrated

the ability of realized absolute variation, i.e. RPV(1), to produce superior volatility

forecasts compared to the squared return volatility measures. They argued that the RPV

is a better predictor of realized volatility because of its robustness to jumps, its smaller

sampling error and its improved predictability. Here, following Liu and Maheu (2009)

and Fuertes *et al.* (2009), who showed that an RPV of order other than one can

significantly improve the accuracy of volatility forecasts, we use a RPV of order 1.5 as

a regressor in the HAR models presented above. Hence, the simple HAR-RPV model is

defined as:

$$lrv_t^{(d)} = a_0 + a_{(d)} lrpv_{t-1}^{(d)} + a_{(w)} lrpv_{t-1}^{(w)} + a_{(m)} lrpv_{t-1}^{(m)} + u_t \qquad (10)$$

where $lrpv_t^{(d)} = \log\left(RPV_t\right)$ is the RPV daily logarithm and $lrpv_{t-1}^{(w)}$ and $lrpv_{t-1}^{(m)}$ are

the respective five and twenty days logarithm RPV moving average components for

each day *t*. The other HAR models are analogously defined.

ARFIMA models for realized volatility

The application of ARFIMA models for realized volatility modelling and

forecasting purposes was first proposed by Andersen *et al.* (2001a) and Andersen *et al.*

(2003), based on the analysis of Granger (1980) and Granger and Joyeux (1980). Since

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[7] Note that when $p = 2$, the RPV is by definition equal to the realized volatility (i.e. $RPV_t(2) \equiv \sigma_{RV,t}^2$). In this case, the RPV is not robust to jumps and converges to the integrated volatility plus the jump component.

then, a number of ARFIMA based models have been proposed for volatility forecasting applications, most with good results (e.g. see Andersen *et al*., 2003; Pong *et al*., 2004 and Koopman *et al*., 2005). We will thus include in this study some of the more relevant ARFIMA models, in order to provide a straightforward comparison to the HAR based model proposed above.

ARFIMA models are genuine long memory time series models which describe the stochastic behavior of a fractionally integrated variable using autoregressive and moving average components. The basic $\text{ARFIMA}(r, d_{RV}, l)$ model for the logarithm of the realized volatility, in terms of deviations from the mean, μ, is defined as:

$$(1 - \psi(L))(1 - L)^{d_{RV}} \left(lrv_t^{(d)} - \mu \right) = (1 + \delta(L)) u_t \qquad (11)$$

where d_{RV} is the fractionally differencing parameter of the logarithmic realized variance and $u_t \sim N(0, \sigma_u)$ as in (2). Beltratti and Morana (2005) first considered the conditional heteroscedasticity in the ARFIMA errors by proposing an $\text{ARFIMA}(r, d_{RV}, l) - \text{FIGARCH}(m, d_u, q)$ model:

$$(1 - \psi(L))(1 - L)^{d_{RV}} \left(lrv_t^{(d)} - \mu \right) = (1 + \delta(L)) u_t \qquad (12)$$

$$u_t = \sigma_{u,t} \varepsilon_t \qquad (13)$$

where $\sigma_{u,t}$ is modeled as in Equation (8). In order to capture the asymmetric effects, lagged returns and indicator functions have been utilized as explanatory variables of the mean (ARFIMAX models), e.g. see Andersen *et al*. (2001a), Thomakos and Wang (2003), Giot and Laurent (2004), Angelidis and Degiannakis (2008). Here, we implement lagged (absolute) daily standardized returns, along with a FIGARCH structure for the conditional heteroscedasticity of the errors, thus defining the $\text{ARFIMAX}(r, d_{RV}, l) - \text{FIGARCH}(m, d_u, q)$ as:

$$(1 - \psi(L))(1 - L)^{d_{RV}} \left(lrv_t^{(d)} - \mu - \vartheta_{(d)} z_{t-1}^{(d)} - \gamma_{(d)} \left| z_{t-1}^{(d)} \right| \right) = (1 + \delta(L)) u_t \quad (14)$$

where $z_t^{(d)} = r_t / \sigma_{RV,t}$ are the daily standardized returns and $\sigma_{u,t}$ is a FIGARCH

specification (see Equation (8)). The asymmetric effect is captured by $\vartheta_{(d)}$, which is

expected to be negative and statistically different from zero.

IV. The data set, descriptive statistics and model estimation

The data set was obtained from Tick Data and consists of five minutes previous tick

interpolated prices, $P_{m,t}$, for the S&P 500 and the DJIA cash indices over a ten year

period, from 1.1.1997 to 12.31.2006. After adjustments for holidays and half-holidays,

there were $T = 2,508$ trading days per index, with six and a half trading hours per day,

interpreted as $M = 78$ intraday returns. Each full data set, was divided into $T^* = 1,508$

in-sample observations, from 31.01.1997 to 30.12.2002 and $n = T - T^* = 1,000$ out-of-

sample observations.

Descriptive statistics and stylized facts

The descriptive statistics for the daily logarithmic returns[8], daily standardized

returns, realized variance and logarithmic realized variance for the two full data sets are

shown in Table 1. Both original return series have negative skewness and fat tails, a

departure from normality which can be attributed to mainly negative price shocks near

the end of 1997 and 1998, all through 2000 and towards the end of 2002. The skewness

and kurtosis of the standardized returns and of the logarithmic variance series as well as

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[8] The daily logarithmic returns are calculated as $r_t = 100 \times \left(\log\left(P_{M,t}\right) - \log\left(P_{M,t-1}\right) \right)$ where $P_{M,t}$, $\left(P_{M,t-1}\right)$ is the closing price of day t, (t-1).

the normality tests suggest that the respective distributions are approximately normal, a

conclusion similar as in Andersen *et al.* (2001a) and Andersen *et al.* (2001b).

Insert **Table 1**

The fractional integration order of the logarithmic realized variance series is calculated

using the Exact Local Whittle (ELW) estimator proposed by Shimotsu and Philips

(2005). The estimator relies on the frequency domain representation of the observed

series, as expressed by its Discrete Fourier Transform (DFT) and evaluated at m

Fourier frequencies from the spectrum's origin. A widely adopted functional form for m

is $m = T^{\mu}$ with $0 < \mu < 1$, while the empirical evidence suggests that values for μ in the

interval $[0.5, \ 0.6]$ limit the bias and variance of the integration order estimate. The

logarithmic realized variance ELW integration order estimates for both indices are

shown in Fig. 1 for $\mu \in [0.4, \ 0.7]$. The d_{RV} estimates vary between 0.55 and 0.6,

suggesting that the realized variance follows a covariance non-stationary fractionally

integrated process.

Insert **Fig. 1**

Finally, the descriptive statistics and stylized facts for the two indices for the TTS

estimated realized variance show no significant departure from the aforementioned

squared intraday returns observations and are available from the authors upon request.

Estimation of the HAR based models

Before proceeding with the parameters estimation[9], the optimum lag order for the

AHAR-FIGARCH model was first determined. The lag structure combinations which

minimized the AIC and SIC criteria were an AHAR-FIGARCH($1,d_u,1$) and an AHAR-

FIGARCH($0,d_u,0$) for the S&P 500 and DJIA indices respectively. The coefficient

estimates for all the HAR based models, as well as the respective in-sample diagnostics

are summarized in Tables 2 and 3 for the S&P 500 and DJIA indices respectively.

Insert **Table 2**

Insert **Table 3**

For the HAR, HAR-GARCH and AdHAR models, the coefficient of the lagged

weekly volatility component, $a_{(w)}$, bears the greatest impact on current volatility,

followed by the daily volatility component, while the monthly one influences the total

volatility significantly less. Hence, the day-ahead volatility appears to be the aggregate

effect of short and mostly medium term volatility components and much less of longer

term volatility factors. However, when the heterogeneity of the leverage effects is taken

into consideration in the AHAR based models (i.e. in the AHAR, AHAR-GARCH and

AHAR-FIGARCH models), the balance shifts drastically. Now, past longer horizon

volatility events (weekly and monthly volatility components) appear to primarily shape

the indices future volatility, a conclusion which is in agreement with the economic

rationale laid out in Section III. The discussion of the estimation results in terms of the

leverage effects and the presence of long memory in the residuals is next presented.

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[9] All estimates were deduced by numerical optimization of the log likelihood function (Maximum Likelihood Estimation, MLE) and they were conducted with the Ox Metrics G@RCH 4.2 package developed by Laurent and Peters (2002).

The leverage effects

For both indices, accounting for daily leverage effects in the A*d*HAR model leads to a reduction in the residuals' skewness and a significant improvement in the goodness of fit indicators (i.e. A(S) Information Criteria). However, there is still evidence of ARCH effects in the residuals of both indices, as the ARCH-LM tests suggest. For the DJIA index, the inclusion of weekly and monthly standardized returns in the AHAR model reduces the skewness of the errors, while there is also evidence in favor of the rejection of the ARCH effects hypothesis. This is also reflected in the GARCH coefficients estimates of the AHAR-GARCH model for the DJIA index, where none of them are statistical significant at a 5% significance level.

The coefficients of the lagged daily, weekly and monthly standardized returns, $\vartheta_{(.)}$, in the AHAR based models are all statistically significant at a 1% significance level, confirming that future market volatility will react asymmetrically not only to yesterday's negative returns, but also to past weekly and monthly returns. Their negative weighting also suggests that past negative shocks induce more market volatility than past positive ones. This is clearly depicted in Fig. 2 below, where the impact of past daily, weekly and monthly shocks on future realized volatility is shown.

Insert **Fig. 2**

It is clear that past negative return events, irrespective of the time horizon, subscribe to future volatility variations more than positive ones. The volatility contribution hierarchy is analogous to that of the volatility components, with the weekly

standardized return being the prevailing contributor to the overall volatility, followed by

the monthly one. However, it is interesting to note that past positive monthly shocks

will also tend to increase volatility, while past positive daily and weekly shocks will

have a negative impact on volatility. As far as we are aware, this is a novel finding,

underlining the importance of including in the analysis the complete returns dataset and

not just past negative returns.

Long range dependence in the residuals

The presence of long memory in the residuals' variance is depicted in Fig. 3, where

the ELW fractional differencing parameter estimates, d_u, for the HAR model squared

residuals are shown for $\mu \in [0.4,\ 0.7]$.

Insert **Fig. 3**

For both indices, the ELW d_u estimates are statistically significant, thus confirming

that not only realized volatility, but also the "volatility of realized volatility" is

autocorrelated for longer time periods. The presence long memory process in the HAR

residuals confirms the suitability of the proposed FIGARCH implementation for their

modelling. The AHAR-FIGARCH residuals d_u MLE estimates shown in Tables 2 and

3 are very close to the respective HAR squared residuals d_u ELW estimates, as

expected.

Finally for both indices, there is no noteworthy difference between the TTS realized

volatility model coefficient estimates and the ones presented above, confirming the

robustness of our findings in the presence of microstructure noise (the estimation results

are available from the authors upon request).

Estimation of the ARFIMA based models

The estimation results for the ARFIMA based models are shown in Table 4. The lag

structures minimizing the A(S)IC information criteria for the ARFIMA models were

again determined through testing various lag combinations for $r = \{0,1,2\}$, $l = \{0,1,2\}$,

$m = \{0,1\}$ and $q = \{0,1\}$. For both indices, we adopted the following specifications for

the respective models: an $\text{ARFIMA}(1, d_{RV}, 0)$, an

$\text{ARFIMA}(1, d_{RV}, 0) - \text{FIGARCH}(0, d_u, 0)$ and an $\text{ARFIMAX}(0, d_{RV}, 1) -$

$\text{FIGARCH}(0, d_u, 0)$.

Insert **Table 4**

The MLE fractional differencing parameter, d_{RV}, estimates vary between 0.49 and

0.58 for both indices, confirming the respective ELW estimation results shown in Fig. 1.

Moreover, the long range dependence in the conditional volatility of the ARFIMA

residuals is also verified by the respective d_u MLE estimates, which for the ARFIMA-

FIGARCH and ARFIMAX-FIGARCH models range from 0.07 to 0.09 and are

statistically significant. Finally, the ARFIMAX-FIGARCH $\vartheta_{(d)}$ and $\gamma_{(d)}$ estimates

confirm the asymmetric impact of past negative shocks on future volatility, as $\vartheta_{(d)}$ is

negative and statistically significant.

Overall, amongst the nine HAR and ARFIMA realized volatility models estimated here, the proposed model exhibits for both indices the best overall goodness of fit, as measured by the AIC and SIC criteria. Moreover, for both the HAR and ARFIMA models, when the conditional heteroscedasticity of the residuals and more significantly so the leverage effects are accounted for, the in-sample fitting as determined by the AIC and SIC criteria is considerably improved. Finally, the excess kurtosis values suggest that a more fat tailed distribution than the normal, might have been more appropriate.

V. Realized volatility forecasting and evaluation

In order to evaluate the realized volatility forecasting performance, a rolling window of T^* observations was used to re-estimate the models and produce n out-of-sample day-ahead realized volatility forecasts calculated as:

$$E\left(\sigma_{RV,t}^{j} / I_{t-1}\right) \; = \; \hat{\sigma}_{RV,t/t-1}^{j} \; = \; \left(\exp\left(l\hat{r}v_{t/t-1}^{j} + 0.5\sigma_{u,t/t-1}^{2j}\right)\right)^{1/2} \quad (15)$$

where $l\hat{r}v_{t/t-1}^{j}$ is the day-ahead logarithmic realized variance forecast and $\sigma_{u,t/t-1}^{2j}$ is the model j residuals variance[10], while n=1,000, 996 and 979 observations for the day ahead, five days and twenty-two days ahead realized volatility forecasts respectively, spanning from the 31st December 2002 to the 29th December 2006. For the five and twenty two days ahead realized volatility forecasts, the corresponding realized volatilities were computed as the square root of the sum of daily realized variances over each forecasting period.

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[10] The transformation in Equation (15) is derived from the realized variance lognormality assumption: A random variable y_t is lognormally distributed if $x_t = \log y_t$ is normally distributed. Then, the expectation of y_t is $E(y_t) = \exp(\mu + 0.5\sigma^2)$, with μ and σ^2 denoting the mean and the variance of y_t respectively, e.g. see Beltratti and Morana (2005) and Giot and Laurent (2004).

Realized volatility forecasting evaluation

In order to evaluate the model's out-of sample realized volatility forecasting performance over the three forecasting horizons, we used the seven loss functions shown in Table 5 below.

Insert **Table 5**

MSE and MAE stand for the Mean Square Error and Mean Absolute Error standard loss functions respectively. MAPE is the Mean Absolute Percentage Error and MLAE is the Mean Logarithm of Absolute Errors. The QLIKE criterion proposed by Bollerslev *et al.* (1994) is the loss implied by a Gaussian likelihood. The R2LOG loss function of Pagan and Schwert (1990) is equivalent to the MSE, but for the logarithm of realized volatility. This loss function applies a greater penalty when forecasting errors occur in low volatility periods, than when they occur in high volatility periods. Using the MSE (R2LOG) criteria is equivalent to testing whether the R-square of the Mincer-Zarnowitz regressions for the (logarithmic) realized volatility of the benchmark model, is always greater than that of its counterparts (Marcucci, 2005). Finally, the HMSE is the Heteroscedastic Mean Squared Error proposed by Bollerslev and Ghysels (1996).

The predictive ability of the realized volatility models was also assessed via Hansen's (2005) Superior Predictive Ability (SPA) test. The SPA test relies on a predetermined loss function in order to test whether the null hypothesis that the benchmark model is not outperformed by any of its competitors, is rejected or not. The forecasting performance of the benchmark model, model 0, with respect to model k is

deduced from: $f_{t,k} = L_{t,0} - L_{t,k}$, $k = 1...l$ and $t = 1...n$. Where $L_{t,j} \equiv L\left(\sigma_{RV,t}, \hat{\sigma}_{RV,t/t-1}^{j}\right)$

with $j = 0$, k is the predetermined forecast loss function of the benchmark model and

of model k respectively. Under the null hypothesis and assuming stationarity for $f_{t,k}$,

we expect that on average the forecasting loss function of the benchmark model will be

smaller, or at least equal to that of model k. Thus, the null hypothesis can be stated as:

$H_0 : \max_{k=1...l} \mu_k = E\left(f_{t,k}\right) \leq 0$ and can be tested through the following test statistic:

$$T_n^{SPA} = \max_{k=1...l} \frac{\sqrt{n}\bar{f}_k}{\sqrt{\text{var}\left(\sqrt{n}\bar{f}_k\right)}} \qquad (16)$$

where $\bar{f}_k = (1/n)\sum_{t=1}^{n} f_{t,k}$ and $\text{var}\left(\sqrt{n}\bar{f}_k\right)$ is the variance of $\sqrt{n}\bar{f}_k$. Both

$\text{var}\left(\sqrt{n}\bar{f}_k\right)$ and the test statistic p-values are consistently estimated via stationary

bootstrapping as in Politis and Romano (1994).

The SPA test analysis focuses only on the MSE and QLIKE loss functions as these

two measures have been shown to be robust against volatility proxy noise (see Patton,

2006). Since realized volatility is a proxy for the true unobservable volatility, the

aforementioned two loss functions yield consistent model rankings, without however

diminishing the informative power of the other loss functions in Table 5.

The models' loss function performance

In Tables 6 and 7, the Table 5 loss functions results, as well as each model's relative

performance rankings (in parenthesis) are shown for the S&P 500 and DJIA stock

indices respectively. Across all forecasting horizons, the proposed AHAR-RPV-

FIGARCH model nearly always ranks first amongst the alternative models, minimizing

the respective loss functions, with the exception of the DJIA index twenty-two days

ahead forecast where the AHAR-RPV-GARCH model ranks first. Note, that for the DJIA index where the persistence in the residuals' variance is smaller than in the S&P 500 index (e.g. see the Tables 2 and 3 and Fig. 3), the benefit of implementing an explicit long memory volatility specification for the residuals is moderated, especially for longer term forecasts. The AHAR-RPV-GARCH model according to most of the loss functions ranks second for the one and five days ahead forecasting horizons, followed by the AHAR-FIGARCH and the AHAR-GARCH models. For the S&P 500 index and for the twenty-two days ahead horizon, the AHAR-FIGARCH ranks second (first for the QLIKE and HMSE loss functions), followed by the AHAR-RPV-GARCH and AHAR-GARCH models.

As for the rest HAR and ARFIMA model variations, the HAR models with leverage effects (A*d*HAR, AHAR) typically outperform the more advanced ARFIMA models (ARFIMA-FIGARCH and ARFIMAX-FIGARCH). However, for the S&P500 index twenty-two days ahead forecasts, the aforementioned performance ranking is reversed. In turn, the AHAR model consistently outperforms the A*d*HAR one, underlining again the importance of considering the heterogeneity in the leverage effects.

It is nonetheless clear that even though the inclusion of daily, weekly and monthly (absolute) standardized returns in the HAR regression reduces the heteroscedasticity in the residuals (see the ARCH-LM tests in Tables 2 and 3), a GARCH, or significantly more so, a FIGARCH implementation for the residuals invariably enhances the forecasting performance. Finally, the AFRIMA model performs, par from a few exceptions, better than the HAR-GARCH model, while the basic HAR model is the overall worst performing specification.

Insert **Table 6**

Insert **Table 7**

From the results synopsis presented above, the following conclusions can be drawn: Firstly, regardless if an RPV regressor is used or not, accounting for asymmetric effects is as important as accounting for their heterogeneity: the AHAR model always outperforms the A*d*HAR one and the latter consistently outperforms the HAR. Secondly, as expected, we confirm that considering the conditional heteroscedasticity of the realized volatility residuals is essential in volatility modelling and forecasting applications, even with the presence of asymmetric effects in the volatility equation. However, it is now evident that the heteroscedasticity is better accounted for with a FIGARCH implementation which captures the long memory of the variance residuals. The models with a FIGARCH specification for their residuals outperform the respective models with GARCH ones, as almost all the loss functions results suggest. Finally, the RPV is a better predictor of realized volatility than the squared returns measure, significantly improving the volatility forecasting performance when added as a regressor.

The TTS volatility forecast rankings

The out-of-sample volatility forecasting analysis for the HAR based models is once again evaluated using the TTS realized volatility estimates (see Table 8). For both indices and across all forecasting horizons, the AHAR-FIGARCH model outperforms all the other models as it minimizes almost all the loss functions. These results confirm that the aforementioned findings are robust to microstructure noise bias. However, in

tests run by the authors but not shown here, we noted that the inclusion of the RPV

regressor did not significantly improve the model's forecasting ability, when the

dependent variable is the TTS realized volatility.

Insert **Table 8**

The SPA test results

The SPA test *p*-values for the MSE and QLIKE loss functions are shown in Table 9

and align with the aforementioned findings. When the AHAR-RPV-FIGARCH or the

AHAR-RPV-GARCH model is the benchmark model, the null hypothesis of superior

performance is strongly accepted (at a 10% significance level), for both indices and both

loss functions, for the short and mid term forecasting horizons (one and five days

ahead). When an alternative realized volatility model is chosen as the benchmark model,

then the null hypothesis is rejected, implying that another model, or models, produces

statistically significant better forecasts.

For the twenty-two days ahead forecast horizon, the null hypothesis is also accepted

for the AFRIMA-FIGARCH and ARFIMAX-FIGARCH models, indicating that for

longer term forecasts the ARFIMA models are competitive to the HAR models,

potentially benefiting by their genuine long memory structure. Overall, the asymmetric

RPV models exhibit the best out-of-sample forecasting performance, yielding for both

functions the highest *p*-values for almost all the forecast horizons.

Insert **Table 9**

VI. Conclusions

Building on the HAR realized volatility model proposed by Corsi (2009), we
captured the leverage effects in the volatility process using (absolute) standardized
returns of daily, weekly and monthly frequencies. The HAR error's heteroscedasticity
and long memory was also accounted for with a FIGARCH implementation. Moreover,
we introduced the RPV as a regressor, which has been shown to be robust to jumps, has
a smaller sampling error and is more predictable. In order to examine the robustness of
our findings to microstructure noise, we also calculated the realized variance with a TTS
estimator and then re-evaluated our models.

The proposed AHAR-FIGARCH model produced the best in-sample fitting against
the alternative HAR and ARFIMA based realized volatility models. The estimation
results confirmed the appropriateness of our modelling as heterogeneity in the
asymmetric effects was established, along with a long range dependence in the
volatility's residuals. Rankings of each model's forecasting performance for seven
established loss functions were also produced. Overall, the proposed model with the
RPV as a regressor (i.e. the AHAR-RPV-FIGARCH), minimized the majority of the
forecast loss functions, across all forecasting horizons and indices. The SPA test *p*-
values also confirmed that the AHAR-RPV-GARCH model was not, for the most part,
outperformed by any other model. The TTS estimated realized volatility forecasting
results demonstrated that the proposed model specification is also robust against the
microstructure noise in the returns process.

The published evidence so far concurs that despite the predominant economic
significance of producing accurate volatility forecasts, there is no single "ideal"
volatility model for all markets and for all financial applications. Here, we showed that

the proposed HAR specification can significantly improve the stock index volatility forecasting performance. We have no reason to doubt that similar improvements can also be realizable for other liquid stock indices. However, further investigation is necessary into the performance of the proposed specification in key financial applications like risk management, but also for capital allocation, derivatives pricing and hedging and for other asset classes, such as bonds and currencies.

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Table 1: Descriptive statistics and stylized facts for the S&P 500 and DJIA indices

	S&P 500				DJIA			
	r_t	$r_t/\sigma_{RV,t}$	$\sigma_{RV,t}^2$	$\log(\sigma_{RV,t}^2)$	r_t	$r_t/\sigma_{RV,t}$	$\sigma_{RV,t}^2$	$\log(\sigma_{RV,t}^2)$
Mean	0.0261	0.0882	0.9718	-0.433	0.0263	0.0742	1.0537	-0.3416
Median	0.0492	0.0835	0.6224	-0.4741	0.0336	0.0458	0.6888	-0.3728
Maximum	5.308	3.5254	13.2346	2.5828	6.2002	3.0564	17.6652	2.8716
Minimum	-7.1127	-3.208	0.0543	-2.9133	-7.439	-3.0309	0.0555	-2.8915
Std. Dev.	1.1459	1.1499	1.1658	0.8648	1.1075	1.0608	1.2942	0.8443
Skewness	-0.0777	0.0977	4.4241	0.277	-0.1945	0.0789	4.6922	0.3079
Kurtosis	6.015	2.5655	32.235	3.0848	7.0752	2.608	35.5938	3.3817
				Normality tests				
Jarque-Bera	952	24	97,496	33	1751	19	120,219	55
[p-value]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.0000]	[0.0000]
Lilliefors	0.0493	0.0176	0.2189	0.0213	0.0517	0.0153	0.2264	0.0299
[p-value]	[0.0000]	[0.0697]	[0.0000]	[0.0106]	[0.0000]	[>0.1]	[0.0000]	[0.0000]
Anderson-Darling	14.5295	1.7548	256.8931	2.3783	15.0759	1.4858	269.2609	3.2387
[p-value]	[0.0000]	[0.0002]	[0.0000]	[0.0000]	[0.0000]	[0.0008]	[0.0000]	[0.0000]

123456789...60 (line numbers in margin)

Table 2: The HAR based realized volatility models estimation results for the S&P 500 index

	HAR	HAR-GARCH	AdHAR	AHAR	AHAR-GARCH	AHAR-FIGARCH
a_0	-0.006	-0.008	-0.074*	-0.174*	-0.177*	-0.178*
	(0.013)	(0.013)	(0.022)	(0.032)	(0.031)	(0.031)
$a_{(d)}$	0.301*	0.293*	0.213*	0.140*	0.139*	0.144*
	(0.030)	(0.035)	(0.032)	(0.032)	(0.031)	(0.032)
$a_{(w)}$	0.422*	0.454*	0.507*	0.363*	0.381*	0.375*
	(0.050)	(0.052)	(0.050)	(0.053)	(0.053)	(0.052)
$a_{(m)}$	0.170*	0.146*	0.171*	0.352*	0.344*	0.345*
	(0.044)	(0.044)	(0.042)	(0.046)	(0.045)	(0.045)
$\vartheta_{(d)}$	---	---	-0.137*	-0.089*	-0.083*	-0.083*
			(0.011)	(0.012)	(0.012)	(0.012)
$\vartheta_{(w)}$	---	---	---	-0.106*	-0.104*	-0.103*
				(0.015)	(0.015)	(0.015)
$\vartheta_{(m)}$	---	---	---	-0.056*	-0.057*	-0.058*
				(0.016)	(0.016)	(0.015)
$\gamma_{(d)}$	---	---	0.081*	0.060*	0.059*	0.059*
			(0.020)	(0.019)	(0.019)	(0.018)
$\gamma_{(w)}$	---	---	---	0.055*	0.055*	0.055*
				(0.020)	(0.020)	(0.020)
$\gamma_{(m)}$	---	---	---	0.092*	0.096*	0.096*
				(0.022)	(0.022)	(0.021)
ω	0.280*	0.030	0.255*	0.240*	0.005	0.017
	(0.010)	(0.023)	(0.010)	(0.010)	(0.004)	(0.010)
d_u	---	---	---	---	---	0.222*
						(0.077)
α_1	---	0.057**	---	---	0.021**	---
		(0.025)			(0.009)	
β_1	---	0.832*	---	---	0.955*	0.736*
		(0.104)			(0.022)	(0.070)
φ_1	---	---	---	---	---	0.570*
						(0.085)
$LogL$	-1,163	-1,153	-1,095	-1,049	-1,042	-1,040
AIC	1.573	1.563	1.483	1.427	1.420	1.419
SIC	1.591	1.588	1.508	1.466	1.467	1.469
$Skewness$	0.333	0.309	0.307	0.308	0.306	0.294
$Excess\ Kurt.$	0.529	0.559	0.610	0.766	0.828	0.784
JB	45 [0.000]	43 [0.000]	51 [0.000]	59 [0.000]	65 [0.000]	59 [0.000]
$Q(5)$	[0.560]	[0.657]	[0.095]	[0.748]	[0.829]	[0.837]
$Q(50)$	[0.244]	[0.277]	[0.182]	[0.424]	[0.229]	[0.507]
$Q^2(5)$	[0.001]	[0.259]	[0.031]	[0.319]	[0.392]	[0.794]
$Q^2(50)$	[0.212]	[0.800]	[0.657]	[0.860]	[0.976]	[0.978]
$ARCH-LM(2)$	[0.0007]	[0.4438]	[0.005]	[0.098]	[0.257]	[0.800]
$ARCH-LM(5)$	[0.0055]	[0.6079]	[0.057]	[0.416]	[0.752]	[0.961]

Notes: Standard errors are presented in parentheses. * and ** indicate statistical significance at 1%, and 5% significance levels respectively. *LogL* is the optimized value of the log likelihood. A(S)IC is the Akaike (Schwartz) Information Criterion. $Q(h)$ and $Q^2(h)$ are the Ljung-Box statistics for h^{th} order serial correlation for the standardized and squared standardized residuals respectively. The *p*-values for the Jarque Bera (JB), $Q(\cdot)$, $Q^2(\cdot)$ and the *ALCH-LM*($\cdot$) test statistics are depicted in brackets.

Table 3: The HAR based realized volatility models estimation results for the DJIA index

	HAR	HAR-GARCH	AdHAR	AHAR	AHAR-GARCH	AHAR-FIGARCH
a_0	0.002	-0.006	-0.070*	-0.168*	-0.168*	-0.170*
	(0.013)	(0.013)	(0.022)	(0.031)	(0.030)	(0.030)
$a_{(d)}$	0.291*	0.291*	0.224*	0.167*	0.170*	0.169*
	(0.030)	(0.034)	(0.032)	(0.032)	(0.035)	(0.032)
$a_{(w)}$	0.450*	0.450*	0.510*	0.389*	0.400*	0.403*
	(0.050)	(0.053)	(0.051)	(0.054)	(0.056)	(0.053)
$a_{(m)}$	0.144*	0.146*	0.154*	0.301*	0.291*	0.291*
	(0.044)	(0.044)	(0.044)	(0.045)	(0.045)	(0.046)
$\vartheta_{(d)}$	---	---	-0.114*	-0.073*	-0.070*	-0.070*
			(0.012)	(0.013)	(0.013)	(0.013)
$\vartheta_{(w)}$	---	---	---	-0.092*	-0.088*	-0.089*
				(0.015)	(0.016)	(0.016)
$\vartheta_{(m)}$	---	---	---	-0.050*	-0.050*	-0.050*
				(0.016)	(0.016)	(0.016)
$\gamma_{(d)}$	---	---	0.091*	0.067*	0.067*	0.067*
			(0.021)	(0.020)	(0.020)	(0.020)
$\gamma_{(w)}$	---	---	---	0.068*	0.065*	0.066*
				(0.021)	(0.021)	(0.021)
$\gamma_{(m)}$	---	---	---	0.089*	0.087*	0.088*
				(0.022)	(0.022)	(0.022)
ω	0.265*	0.086**	0.249*	0.237*	0.078	0.162*
	(0.009)	(0.042)	0.010	(0.010)	(0.074)	(0.029)
d_u	---	---	---	---	---	0.050**
						(0.024)
α_1	---	0.087*	---	---	0.046	---
		(0.031)			(0.038)	
β_1	---	0.585*	---	---	0.621	---
		(0.178)			(0.331)	
φ_1	---	---	---	---	---	---
$LogL$	-1,121	-1,113	-1,076	-1,040	-1,037	-1,037
AIC	1.517	1.508	1.458	1.414	1.414	1.412
SIC	1.535	1.533	1.483	1.454	1.460	1.455
$Skewness$	0.300	0.265	0.254	0.236	0.237	0.244
$Excess\ Kurt.$	0.767	0.653	0.796	1.019	0.921	0.911
JB	58 [0.000]	43 [0.000]	55 [0.000]	78 [0.000]	[0.000]	66 [0.000]
$Q(5)$	[0.255]	[0.422]	[0.136]	[0.651]	[0.715]	[0.721]
$Q(50)$	[0.074]	[0.078]	[0.049]	[0.051]	[0.050]	[0.056]
$Q^2(5)$	[0.005]	[0.207]	[0.088]	[0.534]	[0.669]	[0.953]
$Q^2(50)$	[0.201]	[0.639]	[0.035]	[0.512]	[0.631]	[0.781]
$ARCH-LM(2)$	[0.005]	[0.832]	[0.057]	[0.342]	[0.632]	[0.724]
$ARCH-LM(5)$	[0.009]	[0.485]	[0.131]	[0.556]	[0.908]	[0.953]

Notes: Standard errors are presented in parentheses. * and ** indicate statistical significance at 1%, and 5% significance levels respectively. *LogL* is the optimized value of the log likelihood. A(S)IC is the Akaike (Schwartz) Information Criterion. $Q(h)$ and $Q^2(h)$ are the Ljung-Box statistics for h^{th} order serial correlation for the standardized and squared standardized residuals respectively. The *p*-values for the Jarque Bera (JB), $Q(\cdot)$, $Q^2(\cdot)$ and the *ALCH-LM*(·) test statistics are depicted in brackets.

Table 4: The ARFIMA realized volatility models estimation results for the S&P 500 and DJIA indices

	S&P 500			DJIA		
	ARFIMA	*ARFIMA-FIGARCH*	*ARFIMAX-FIGARCH*	*ARFIMA*	*ARFIMA-FIGARCH*	*ARFIMAX-FIGARCH*
d_{RV}	0.497*	0.502*	0.584*	0.511*	0.509*	0.582*
	(0.028)	(0.029)	(0.047)	(0.028)	(0.029)	(0.046)
μ	-0.565	-0.551	-0.669**	-0.356	-0.370	-0.442
	(0.268)	(0.272)	(0.314)	(0.264)	(0.254)	(0.291)
ψ_1	-0.107*	-0.109*	---	-0.129*	-0.121*	---
	(0.037)	(0.038)	---	(0.036)	(0.038)	---
δ_1	---	---	-0.255*	---		-0.244*
	---	---	(0.060)	---		(0.059)
$\vartheta_{(d)}$	---	---	-0.091*	---		-0.076*
	---	---	(0.010)	---		(0.011)
$\gamma_{(d)}$	---	---	0.051*	---		0.063*
	---	---	(0.017)	---		(0.018)
d_u	---	0.093*	0.077*	---	0.078*	0.069*
	---	(0.025)	(0.024)	---	(0.025)	(0.025)
ω	0.281*	0.138*	0.148*	0.265*	0.147*	0.152*
	(0.010)	(0.025)	(0.027)	(0.009)	(0.027)	(0.028)
LogL	-1,166	-1,156	-1,118	-1,123	-1,117	-1,091
AIC	1.575	1.562	1.516	1.518	1.510	1.479
SIC	1.590	1.580	1.541	1.532	1.527	1.504
Skewness	0.328	0.305	0.330	0.279	0.261	0.253
Excess Kurt.	0.516	0.564	0.565	0.767	0.703	0.66
JB	43 [0.000]	42.8 [0.000]	46 [0.000]	55 [0.000]	47 [0.000]	43 [0.000]
$Q(5)$	[0.240]	[0.140]	[0.275]	[0.295]	[0.193]	[0.483]
$Q(50)$	[0.131]	[0.079]	[0.128]	[0.034]	[0.018]	[0.039]
$Q^2(5)$	[0.002]	[0.800]	[0.970]	[0.012]	[0.619]	[0.977]
$Q^2(50)$	[0.060]	[0.899]	[0.975]	[0.255]	[0.672]	[0.232]
ARCH-LM(2)	[0.001]	[0.964]	[0.723]	[0.010]	[0.997]	[0.980]
ARCH-LM(5)	[0.007]	[0.810]	[0.975]	[0.018]	[0.626]	[0.978]

Notes: Standard errors are presented in parentheses. * and ** indicate statistical significance at 1%, and 5% significance levels respectively. *LogL* is the optimized value of the log likelihood. A(S)IC is the Akaike (Schwartz) Information Criterion. $Q(h)$ and $Q^2(h)$ are the Ljung-Box statistics for h^{th} order serial correlation for the standardized and squared standardized residuals respectively. The *p*-values for the Jarque Bera (JB), $Q(\cdot)$, $Q^2(\cdot)$ and the *ALCH-LM(·)* test statistics are depicted in brackets.

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Table 5: The forecast loss functions

$$MSE = (1/n) \sum_{t=1}^{n} \left(\sigma_{RV,t} - \hat{\sigma}_{RV,t/t-1}^{j} \right)^2$$

$$MAE = (1/n) \sum_{t=1}^{n} \left| \sigma_{RV,t} - \hat{\sigma}_{RV,t/t-1}^{j} \right|$$

$$MAPE = (1/n) \sum_{t=1}^{n} \left(\left| \sigma_{RV,t} - \hat{\sigma}_{RV,t/t-1}^{j} \right| / \sigma_{RV,t} \right)$$

$$MLAE = (1/n) \sum_{t=1}^{n} \left(\log \left| \sigma_{RV,t} - \hat{\sigma}_{RV,t/t-1}^{j} \right| \right)$$

$$QLIKE = (1/n) \sum_{t=1}^{n} \left(\log \left(\hat{\sigma}_{RV,t/t-1}^{j} \right) + \sigma_{RV,t} / \hat{\sigma}_{RV,t/t-1}^{j} \right)$$

$$R2LOG = (1/n) \sum_{t=1}^{n} \left(\log \left(\sigma_{RV,t} / \hat{\sigma}_{RV,t/t-1}^{j} \right) \right)^2$$

$$HMSE = (1/n) \sum_{t=1}^{n} \left(\left(\sigma_{RV,t} / \hat{\sigma}_{RV,t/t-1}^{j} \right) - 1 \right)^2$$

Table 6: Forecast loss functions for the S&P 500 index

				1 day ahead			
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.02754 (11)	0.13093 (11)	0.22473 (11)	-2.44812 (10)	0.54104 (10)	0.06521 (11)	0.05721 (9)
HAR-GARCH	0.02698 (8)	0.12915 (9)	0.22145 (10)	-2.47833 (9)	0.54056 (8)	0.06400 (9)	0.05664 (7)
A*d*HAR	0.02567 (6)	0.12470 (6)	0.21038 (6)	-2.49526 (7)	0.53752 (5)	0.05749 (6)	0.05047 (6)
AHAR	0.02471 (5)	0.12124 (5)	0.20292 (5)	-2.53066 (5)	0.53625 (4)	0.05446 (5)	**0.04885 (1)**
AHAR-GARCH	0.02436 (4)	0.12035 (4)	0.20129 (4)	-2.53859 (4)	0.53611 (3)	0.05397 (4)	0.04897 (4)
AHAR-FIGARCH	0.02429 (3)	0.12011 (3)	0.20081 (3)	-2.55167 (2)	0.53608 (2)	0.05387 (3)	0.04903 (5)
AHAR-RPV-GARCH	0.02422 (2)	0.12000 (2)	0.20053 (2)	-2.54340 (3)	**0.53599 (1)**	0.05366(2)	0.04886 (2)
AHAR-RPV-FIGARCH	**0.02418 (1)**	**0.11979 (1)**	**0.20016 (1)**	**-2.55954 (1)**	**0.53599 (1)**	**0.05362(1)**	0.04895 (3)
ARFIMA	0.02741 (10)	0.13019 (10)	0.22192 (9)	-2.44282 (11)	0.54080 (9)	0.06428 (10)	0.05765 (11)
ARFIMA-FIGARCH	0.02679 (7)	0.12813 (8)	0.21818 (8)	-2.48507 (8)	0.54030 (7)	0.06302 (8)	0.05712 (8)
ARFIMAX-FIGARCH	0.02710 (9)	0.12676 (7)	0.21337 (7)	-2.50304 (6)	0.53999 (6)	0.06179 (7)	0.05761 (10)

				5 days ahead			
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.07903 (11)	0.22149 (11)	0.15750 (11)	-1.92672 (11)	1.36566 (11)	0.03447 (11)	0.03276 (11)
HAR-GARCH	0.07678 (10)	0.21836 (10)	0.15511 (10)	-1.93484 (10)	1.36525 (10)	0.03360 (10)	0.03198 (9)
A*d*HAR	0.06925 (6)	0.20407 (6)	0.14334 (6)	-2.03282 (6)	1.36317 (6)	0.02922 (6)	0.02798 (6)
AHAR	0.06815 (5)	0.19887 (5)	0.13702 (5)	-2.08784 (5)	1.36222 (5)	0.02720 (5)	0.02624 (5)
AHAR-GARCH	0.06629 (4)	0.19707 (4)	0.13570 (4)	-2.10483 (2)	1.36204 (4)	0.02673 (4)	0.02604 (2)
AHAR-FIGARCH	0.06598 (3)	0.19666 (3)	0.13536 (3)	**-2.10584 (1)**	1.36202 (3)	0.02667 (3)	0.02607 (4)
AHAR-RPV-GARCH	0.06589 (2)	0.19658 (2)	0.13529 (2)	-2.09471 (3)	1.36200 (2)	0.02660 (2)	0.02605 (3)
AHAR-RPV-FIGARCH	**0.06549 (1)**	**0.19608 (1)**	**0.13492 (1)**	-2.09350 (4)	**1.36197 (1)**	**0.02651 (1)**	**0.02603 (1)**
ARFIMA	0.07558 (9)	0.21737 (9)	0.15370 (9)	-1.96463 (8)	1.36513 (9)	0.03309 (9)	0.03240 (10)
ARFIMA-FIGARCH	0.07301 (8)	0.21339 (8)	0.15086 (8)	-1.96396 (9)	1.36472 (8)	0.03219 (8)	0.03163 (8)
ARFIMAX-FIGARCH	0.07260 (7)	0.21084 (7)	0.14761 (7)	-1.98752 (7)	1.36433 (7)	0.03125 (7)	0.03108 (7)

				22 days ahead			
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.36614 (11)	0.46121 (11)	0.15018 (11)	-1.26200 (9)	2.12040 (11)	0.03282 (11)	0.03122 (10)
HAR-GARCH	0.35599 (10)	0.45441 (9)	0.14793 (10)	-1.26179 (8)	2.12003 (9)	0.03204 (10)	0.03046 (8)
A*d*HAR	0.35048 (8)	0.44981 (8)	0.14572 (8)	-1.27375 (7)	2.11969 (7)	0.03115 (8)	0.03015 (6)
AHAR	0.34198 (7)	0.43575 (5)	0.14064 (5)	**-1.33178 (1)**	2.11905 (5)	0.02995 (5)	0.02868 (5)
AHAR-GARCH	0.33365 (4)	0.43166 (3)	0.13927 (4)	-1.31668 (5)	2.11888 (2)	0.02953 (3)	0.02849 (2)
AHAR-FIGARCH	0.33209 (2)	0.43068 (2)	0.13894 (2)	-1.32441 (2)	**2.11886 (1)**	0.02947 (2)	**0.02848 (1)**
AHAR-RPV-GARCH	0.33285 (3)	0.43173 (4)	0.13925 (3)	-1.32093 (3)	2.11893 (4)	0.02957 (4)	0.02872 (4)
AHAR-RPV-FIGARCH	**0.33087 (1)**	**0.43052 (1)**	**0.13882 (1)**	-1.31868 (4)	2.11889 (3)	**0.02946 (1)**	0.02866 (3)
ARFIMA	0.35210 (9)	0.45681 (10)	0.14790 (9)	-1.22333 (11)	2.12004 (10)	0.03170 (9)	0.03134 (11)
ARFIMA-FIGARCH	0.34099 (5)	0.44845 (7)	0.14533 (7)	-1.24961 (10)	2.11970 (8)	0.03098 (7)	0.03065 (9)
ARFIMAX-FIGARCH	0.34112 (6)	0.44562 (6)	0.14346 (6)	-1.28563 (6)	2.11953 (6)	0.03054 (6)	0.03042 (7)

Table 7: Forecast loss functions for the DJIA index

	1 day ahead						
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.03400 (10)	0.14391 (11)	0.23456 (11)	-2.37560 (8)	0.59615 (9)	0.07068 (11)	0.06238 (8)
HAR-GARCH	0.03375 (9)	0.14310 (9)	0.23303 (10)	-2.37477 (9)	0.59587 (7)	0.07002 (10)	0.06197 (7)
A*d*HAR	0.03229 (5)	0.13783 (6)	0.22141 (6)	-2.41244 (6)	0.59306 (4)	0.06393 (6)	0.05646 (6)
AHAR	0.03103 (4)	0.13411 (5)	0.21412 (5)	-2.43164 (5)	0.59171 (3)	0.06059 (5)	**0.05498 (1)**
AHAR-GARCH	0.03088 (3)	0.13357 (4)	0.21320 (4)	-2.43635 (3)	0.59166 (2)	0.06039 (4)	0.05509 (4)
AHAR-FIGARCH	0.03088 (3)	0.13354 (3)	0.21302 (3)	-2.43243 (4)	0.59166 (2)	0.06036 (3)	0.05516 (5)
AHAR-RPV-GARCH	**0.03060 (1)**	0.13277 (2)	0.21157 (2)	-2.45302 (2)	**0.59146 (1)**	0.05980 (2)	0.05500 (2)
AHAR-RPV-FIGARCH	0.03061 (2)	**0.13276 (1)**	**0.21144 (1)**	**-2.45835 (1)**	**0.59146 (1)**	**0.05978 (1)**	0.05507 (3)
ARFIMA	0.03395 (8)	0.14331 (10)	0.23197 (9)	-2.35751 (11)	0.59588 (8)	0.06967 (9)	0.06289 (11)
ARFIMA-FIGARCH	0.03336 (6)	0.14143 (8)	0.22761 (8)	-2.36889 (10)	0.59529 (6)	0.06804 (8)	0.06244 (9)
ARFIMAX-FIGARCH	0.03338 (7)	0.14036 (7)	0.22474 (7)	-2.38481 (7)	0.59507 (5)	0.06717 (7)	0.06281 (10)

	5 days ahead						
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.08536 (11)	0.22905 (11)	0.15484 (11)	-1.92239 (9)	1.42230 (10)	0.03360 (11)	0.03119 (11)
HAR-GARCH	0.08487 (10)	0.22799 (10)	0.15400 (10)	-1.90838 (11)	1.42211 (9)	0.03325 (10)	0.03068 (10)
A*d*HAR	0.07425 (6)	0.21252 (6)	0.14220 (6)	-1.96637 (6)	1.41989 (5)	0.02844 (6)	0.02667 (6)
AHAR	0.07128 (5)	0.20555 (5)	0.13502 (5)	-2.03602 (5)	1.41884 (4)	0.02843 (5)	0.02481 (5)
AHAR-GARCH	0.07051 (4)	0.20430 (4)	0.13421 (4)	-2.04570 (2)	1.41873 (3)	0.02596 (4)	0.02462 (3)
AHAR-FIGARCH	0.07043 (3)	0.20422 (3)	0.13408 (3)	-2.04446 (3)	1.41873 (3)	0.02593 (3)	0.02465 (4)
AHAR-RPV-GARCH	0.06956 (2)	0.20315 (2)	0.13328 (2)	**-2.04698 (1)**	1.41863 (2)	0.02567 (2)	**0.02457 (1)**
AHAR-RPV-FIGARCH	**0.06949 (1)**	**0.20304 (1)**	**0.13314 (1)**	-2.04294 (4)	**1.41862 (1)**	**0.02564 (1)**	0.02458 (2)
ARFIMA	0.08172 (9)	0.22492 (9)	0.15122 (9)	-1.91136 (10)	1.42174 (8)	0.03221 (9)	0.03054 (9)
ARFIMA-FIGARCH	0.08011 (8)	0.22157 (8)	0.14824 (8)	-1.93425 (8)	1.42140 (7)	0.03210 (8)	0.03010 (8)
ARFIMAX-FIGARCH	0.07835 (7)	0.21790 (7)	0.14540 (7)	-1.94435 (7)	1.42095 (6)	0.03037 (7)	0.02939 (7)

	22 days ahead						
	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.38902 (11)	0.47317 (11)	0.14591 (11)	-1.24074 (11)	2.17779 (11)	0.03141 (11)	0.02962 (11)
HAR-GARCH	0.38482 (10)	0.46746 (10)	0.14407 (10)	-1.26285 (9)	2.17750 (10)	0.03088 (10)	0.02883 (8)
A*d*HAR	0.35515 (6)	0.45124 (6)	0.13897 (7)	-1.29824 (6)	2.17668 (6)	0.02888 (6)	0.02792 (6)
AHAR	0.34309 (5)	0.43626 (5)	0.13439 (5)	-1.32586 (5)	2.17602 (5)	0.02767 (5)	0.02628 (5)
AHAR-GARCH	0.33838 (3)	0.43309 (3)	0.13333 (3)	-1.33379 (4)	2.17581 (2)	0.02728 (3)	**0.02579 (1)**
AHAR-FIGARCH	0.33882 (4)	0.43344 (4)	0.13337 (4)	-1.33834 (3)	2.17585 (4)	0.02732 (4)	0.02591 (2)
AHAR-RPV-GARCH	**0.33481 (1)**	**0.43158 (1)**	**0.13277 (1)**	-1.34850 (2)	**2.17580 (1)**	**0.02716 (1)**	0.02597 (3)
AHAR-RPV-FIGARCH	0.33519 (2)	0.43182 (2)	0.13279 (2)	**-1.35292 (1)**	2.17583 (3)	0.02719 (2)	0.02608 (4)
ARFIMA	0.37422 (9)	0.46482 (9)	0.14261 (9)	-1.24730 (10)	2.17738 (9)	0.03024 (9)	0.02948 (10)
ARFIMA-FIGARCH	0.36799 (8)	0.45784 (8)	0.14004 (8)	-1.27711 (8)	2.17713 (8)	0.02968 (8)	0.02901 (9)
ARFIMAX-FIGARCH	0.36250 (7)	0.45334 (7)	0.13848 (6)	-1.27851 (7)	2.17685 (7)	0.02903 (7)	0.02859 (7)

Table 8: Forecast loss functions for the S&P 500 and DJIA indices using the TTS estimator for the realized variance

	1 day ahead						
S&P 500	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.02545 (5)	0.12797 (5)	0.24574 (5)	-2.42973 (5)	0.43656 (5)	0.07380 (5)	0.06207 (5)
A*d*HAR	0.02462 (4)	0.12427 (4)	0.23622 (4)	-2.47604 (4)	0.43481 (4)	0.06952 (4)	0.05959 (4)
AHAR	0.02367 (3)	0.12034 (3)	0.22804 (3)	-2.51254 (3)	0.43319 (3)	0.06608 (3)	0.05648 (3)
AHAR-GARCH	0.02307 (2)	0.11877 (2)	0.22524 (2)	**-2.54546 (1)**	0.43285 (2)	0.06513 (2)	0.05628 (2)
AHAR-FIGARCH	**0.02295 (1)**	**0.11843 (1)**	**0.22451 (1)**	-2.53538 (2)	**0.43276 (1)**	**0.06487 (1)**	**0.05625 (1)**
	5 days ahead						
S&P 500	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.06819 (5)	0.20515 (5)	0.16177 (5)	-1.98649 (5)	1.26367 (5)	0.03651 (5)	0.03463 (5)
A*d*HAR	0.06942 (4)	0.20185 (4)	0.15736 (4)	-2.03961 (4)	1.26318 (4)	0.03530 (4)	0.03409 (4)
AHAR	0.06682 (3)	0.19411 (3)	0.14901 (3)	-2.10834 (3)	1.26175 (3)	0.03236 (3)	0.03120 (3)
AHAR-GARCH	0.06376 (2)	0.19029 (2)	0.14641 (2)	-2.11952 (2)	1.26135 (2)	0.03149 (2)	0.03049 (2)
AHAR-FIGARCH	**0.06323 (1)**	**0.18939 (1)**	**0.14566 (1)**	**-2.12412 (1)**	**1.26128 (1)**	**0.03129 (1)**	**0.03043 (1)**
	22 days ahead						
S&P 500	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.31042 (3)	0.43054 (4)	0.15572 (4)	-1.28592 (4)	2.02073	0.03489 (4)	0.03455 (4)
A*d*HAR	0.32753 (5)	0.43622 (5)	0.15688 (5)	-1.28370 (5)	2.02132	0.03590 (5)	0.03611 (5)
AHAR	0.31869 (4)	0.42104 (3)	0.15087 (3)	-1.34205 (3)	2.02043	0.03433 (3)	0.03379 (3)
AHAR-GARCH	0.30298 (2)	0.41113 (2)	0.14783 (2)	**-1.37854 (1)**	2.01995	0.03331 (2)	0.03283 (2)
AHAR-FIGARCH	**0.29963 (1)**	**0.40941 (1)**	**0.14718 (1)**	-1.36595 (2)	**2.01985 (1)**	**0.03308 (1)**	**0.03271 (1)**
	1 day ahead						
DJIA	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.02515 (5)	0.12618 (5)	0.24126 (5)	-2.45846 (5)	0.43795 (5)	0.07174 (5)	0.06001 (5)
A*d*HAR	0.02425 (4)	0.12160 (4)	0.23107 (4)	-2.54627 (2)	0.43624 (4)	0.06766 (4)	0.05736 (4)
AHAR	0.02352 (3)	0.11934 (3)	0.22612 (3)	-2.53401 (4)	0.43493 (3)	0.06500 (3)	0.05458 (3)
AHAR-GARCH	0.02308 (2)	0.11803 (2)	0.22339 (2)	-2.54121 (3)	0.43462 (2)	0.06411 (2)	0.05449 (2)
AHAR-FIGARCH	**0.02297 (1)**	**0.11777 (1)**	**0.22292 (1)**	**-2.54734 (1)**	**0.43453 (1)**	**0.06389 (1)**	**0.05437 (1)**
	5 days ahead						
DJIA	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.06746 (5)	0.20329 (5)	0.16041 (5)	-1.99894 (5)	1.26457 (5)	0.03582 (5)	0.03360 (5)
A*d*HAR	0.06599 (4)	0.19758 (4)	0.15460 (4)	-2.06719 (4)	1.26369 (4)	0.03383 (4)	0.03224 (4)
AHAR	0.06433 (3)	0.19111 (3)	0.14775 (3)	-2.13247 (3)	1.26260 (3)	0.03168 (3)	0.02987 (3)
AHAR-GARCH	0.06229 (2)	0.18777 (2)	0.14531 (2)	-2.14638 (2)	1.26231 (2)	0.03103 (2)	0.02944 (2)
AHAR-FIGARCH	**0.06165 (1)**	**0.18655 (1)**	**0.14433 (1)**	**-2.17193 (1)**	**1.26219 (1)**	**0.03074 (1)**	**0.02925 (1)**
	22 days ahead						
DJIA	MSE	MAE	MAPE	MALE	QLIKE	R2LOG	HMSE
HAR	0.30261 (3)	0.42755 (5)	0.15500 (5)	-1.29708 (5)	2.02256 (4)	0.03416 (4)	0.03309 (4)
A*d*HAR	0.30532 (5)	0.42217 (4)	0.15285 (4)	-1.31018 (4)	2.02273 (5)	0.03427 (5)	0.03403 (5)
AHAR	0.30498 (4)	0.41118 (3)	0.14889 (3)	-1.36204 (3)	2.02228 (3)	0.03367 (3)	0.03264 (3)
AHAR-GARCH	0.29477 (2)	0.40346 (2)	0.14646 (2)	-1.40309 (2)	2.02197 (2)	0.03299 (2)	0.03207 (2)
AHAR-FIGARCH	**0.29100 (1)**	**0.40126 (1)**	**0.14557 (1)**	**-1.40438 (1)**	**2.02180 (1)**	**0.03261 (1)**	**0.03177 (1)**

Table 9: Superior Predictive Ability (SPA) test *p*-values

| | S&P 500 | | | | | |
| | 1 day ahead | | 5 days ahead | | 22 days ahead | |
	MSE	QLIKE	MSE	QLIKE	MSE	QLIKE
HAR	0.000	0.000	0.000	0.000	0.000	0.002
HAR-GARCH	0.000	0.008	0.007	0.000	0.028	0.039
A*d*HAR	0.000	0.015	0.071	0.000	0.008	0.047
AHAR	0.000	0.000	0.000	0.001	0.001	0.013
AHAR-GARCH	0.001	0.000	0.010	0.053	0.086	**0.514**
AHAR-FIGARCH	0.002	0.003	0.005	0.043	**0.364**	**0.996**
AHAR-RPV-GARCH	0.071	0.089	**0.206**	**0.508**	0.033	**0.172**
AHAR-RPV-FIGARCH	**0.909**	**0.512**	**0.877**	**0.492**	**0.951**	**0.623**
ARFIMA	0.000	0.000	0.000	0.000	0.000	0.009
ARFIMA-FIGARCH	0.014	0.000	0.052	0.000	**0.549**	**0.300**
ARFIMAX-FIGARCH	0.005	0.000	**0.130**	0.000	**0.595**	**0.407**

| | DJIA | | | | | |
| | 1 day ahead | | 5 days ahead | | 22 days ahead | |
	MSE	QLIKE	MSE	QLIKE	MSE	QLIKE
HAR	0.005	0.000	0.000	0.000	0.007	0.006
HAR-GARCH	0.005	0.000	0.002	0.000	0.021	0.010
A*d*HAR	0.014	0.000	0.067	0.000	0.065	0.023
AHAR	0.000	0.000	0.001	0.000	0.004	0.000
AHAR-GARCH	0.000	0.000	0.001	0.011	0.011	**0.745**
AHAR-FIGARCH	0.000	0.000	0.001	0.014	0.003	**0.110**
AHAR-RPV-GARCH	**0.733**	**0.647**	**0.437**	**0.292**	**0.994**	**0.961**
AHAR-RPV-FIGARCH	**0.291**	**0.353**	**0.932**	**0.711**	**0.518**	**0.152**
ARFIMA	0.000	0.000	0.008	0.001	0.038	0.042
ARFIMA-FIGARCH	0.019	0.000	0.016	0.000	**0.139**	0.079
ARFIMAX-FIGARCH	0.024	0.000	0.064	0.000	**0.241**	**0.116**

Notes: This Table shows the *p*-values of the SPA test statistic. The benchmark model, model 0, in the first column is tested against its competitors, models *k*, with *k = 1…l.* for the specific loss functions denoted in the subsequent columns. The null hypothesis that the benchmark model is not outperformed by any of its competitors for the specific loss function is accepted at a 10% significance level when the *p*-value is greater than 0.10. The bold face fonts indicate acceptance of the null at a 10% significance level.
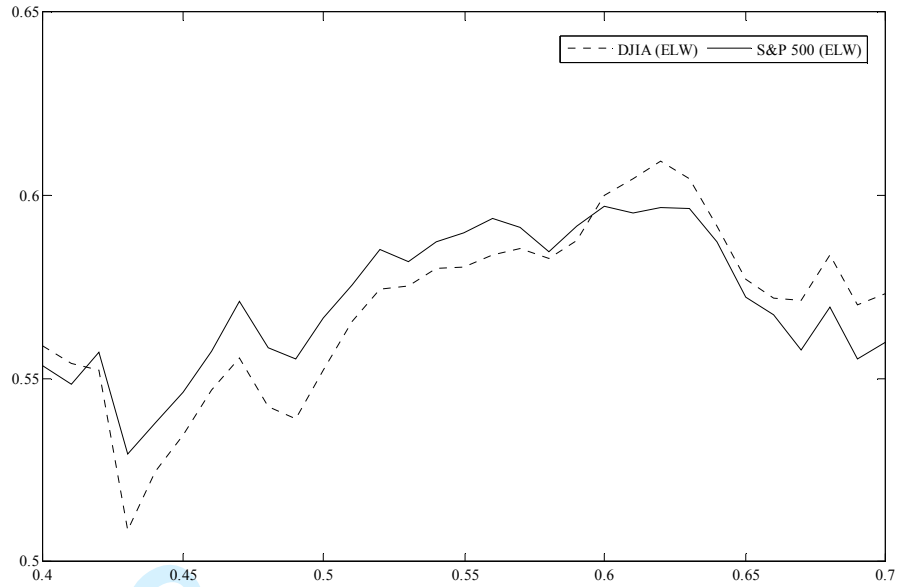
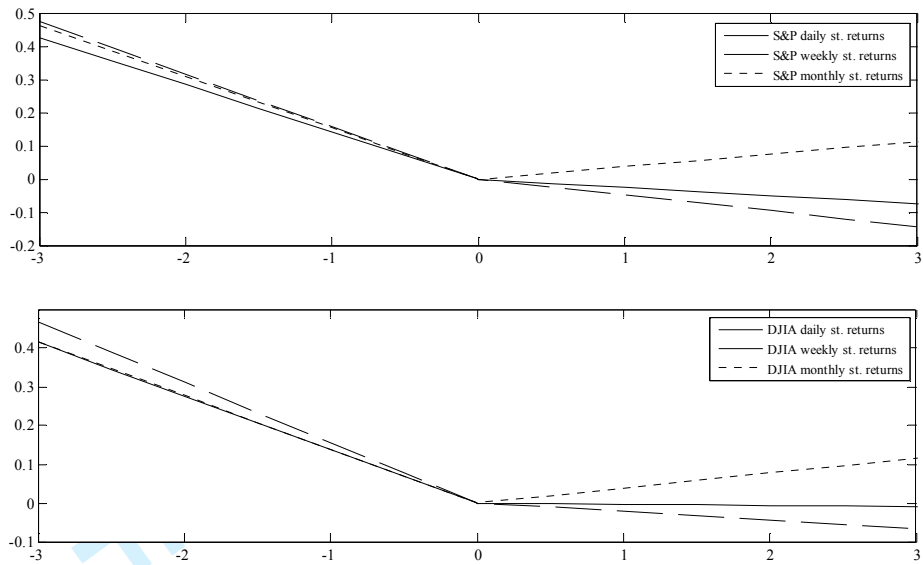**Fig. 1. S&P 500 and DJIA indices log realized variance ELW fractional differencing parameter estimates**

Fig. 2. The impact on volatility from lagged negative and positive daily, weekly and monthly standardized returns for the S&P 500 and DJIA indices
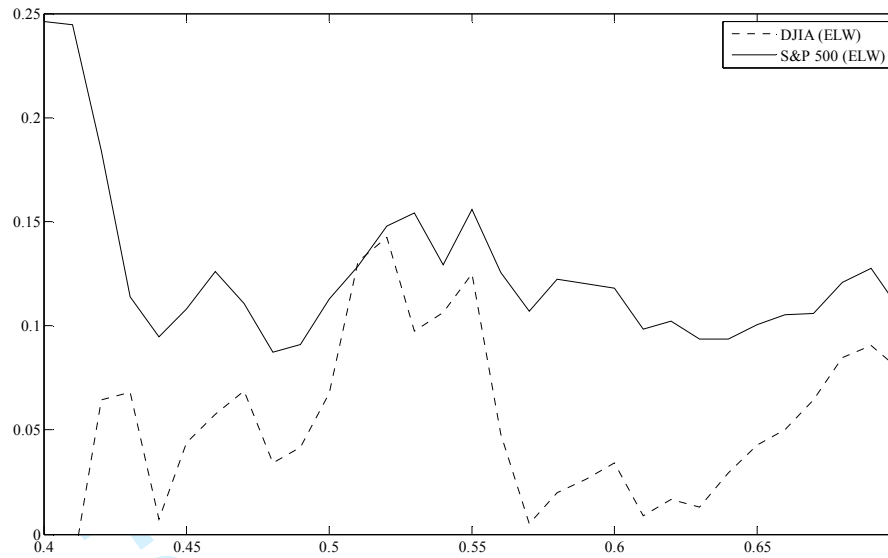
Fig. 3. S&P 500 and DJIA indices HAR squared residuals ELW fractional differencing parameter estimates