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Strange-Quark Condensate in Nucleon Based on an Effective Lagrangian Incorporating $U_A(1)$ Anomaly*)

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Strange-quark condensate in the nucleon (proton) is examined on the basis of the $SU(3)$-Nambu-Jona-Lasinio model incorporating the $U_A(1)$ anomaly, which gives rise to flavour mixings. It is shown that the flavour mixing due to the anomaly solely gives small values for $\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u|N\rangle = 0.07 \sim 0.10$.

Recently, much attention has been paid to the strange-quark (s quark) condensate in the nucleon.[2),3)] It is related to πN and KN-Σ terms, the latter of which turns out to be the strength of the driving force of the kaon condensation.[4)] Theoretical values of the πN-Σ term obtained by assuming the OZI-rule $\langle N|\bar{s}s|N\rangle_c \ll \langle N|\bar{u}u|N\rangle_c$ are about a half of the experimental value $\sim 57\,\mathrm{MeV}$. Donoghue and Nappi[2)] have directly estimated the condensates in the nucleon and obtained

$$z = \langle N|\bar{s}s|N\rangle_c / \langle N|\bar{u}u|N\rangle_c \sim 0.6\,, \tag{1}$$

where the subscript "c" stands for "connected diagram". Thus, they have argued that the OZI-rule is considerably violated in the nucleon. This is certainly an interesting possibility.

Very recently, Hatsuda and the present author[5)] have analysed the quark condensates and meson spectra using a generalised Nambu-Jona-Lasinio model incorporating the $U_A(1)$ anomaly,[6)] which causes a flavour mixing i.e., an OZI-violating effect; they have also determined the strength of the determinantial interaction representing the anomaly. Therefore, it is intriguing to examine the s quark condensate in the nucleon using the model Lagrangian. In fact, we have already evaluated the quantity and the results have been reported in Ref. 5) (see also Ref. 1)). In this short note, we will give calculational details such as the models and/or the ansats employed, as well as the numerical results.

The Lagrangian is expressed as follows:

$$L = \bar{q}(i\gamma\cdot\partial - \boldsymbol{m})q + g_s/2\sum_{a=0}^{8}((\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_5\lambda_a q)^2) + g_D[\det(\bar{q}(1-\gamma_5)q) + \mathrm{h.c.}]\,, \tag{2}$$

where $\bar{q}q \equiv \sum_{a=1}^{N_c}\sum_{i=1}^{3}\bar{q}_i^a q_i^a$ with a and i being the colour and flavour indices, respectively. Here, $\boldsymbol{m} = \mathrm{diag}(m_u, m_d, m_s)$ and the λ_a's are the Gell-Mann matrices with $\lambda_0 = \sqrt{2/3}\cdot\mathbf{1}$. We will assume the $SU_f(2)$ symmetry, i.e., $m_u = m_d$, throughout the paper. The relevance of the NJL-model to QCD has often been explained, so we do not refrain it here but only give some references.[7),8)] Here, it should be noted that the

*) Preliminary results have been reported in the meeting on "Pions and Quarks in Nuclei" held at the RIFP on 3-5 Novem., 1987.[1)]

effects of the confinement are not explicitly included in the model, nevertheless the effects can be nicely simulated.[8] The QCD sum-rule approach gives $\hat{m}=(m_u+m_d)/2=5.5$ MeV.[9] The remaining parameters to be specified in the model are the current s quark mass m_s, the coupling constants g_s and g_D and the cutoff Λ. We take the three momentum cutoff-scheme so that the application of this model to the finite temperature case is easy. The parameters were determined so as to reproduce the masses of pion (138 MeV), kaon (500 MeV) and η'-meson (960 MeV) and the pion decay constant (93 MeV). The results are

$$m_s=138\ \mathrm{MeV},\quad \Lambda=631.38\ \mathrm{MeV},\quad g_s=0.33\ \mathrm{fm}^2 \text{ and } g_D=0.00235\ \mathrm{fm}^5 .$$

With the parameters, we get for the vacuum condensates of the $u(d)$ and s quarks,

$$\langle \bar{u}u\rangle=(-246.8\ \mathrm{MeV})^3 \quad\text{and}\quad \langle \bar{s}s\rangle=(-267.2\ \mathrm{MeV})^3 .$$

These values are in very good agreement with those obtained through other sources.[9] The total masses M_u and M_s, which can be identified with the constituent quark masses, become

$$M_u=335\ \mathrm{MeV} \qquad\text{and}\qquad M_s=551\ \mathrm{MeV} .$$

It is remarkable that these values are certainly in excellent agreement with those of the constituent quark masses derived by several authors.[9),10] It implies that the constituent quark masses, say, in the nonrelativistic quark models are given by our model through the chiral symmetry breaking.*)

Here, we recapitulate how the vacuum condensates and hence the total quark masses are determined in our model for later convenience. They are determined by the self-consistency condition given as follows:

$$\begin{aligned} M_u&=m_u-2g_s\alpha-2g_D\beta\gamma ,\\ M_d&=m_d-2g_s\beta-2g_D\alpha\gamma ,\\ M_s&=m_s-2g_s\gamma-2g_D\alpha\beta , \end{aligned} \tag{3}$$

where $\alpha=\langle \bar{u}u\rangle=\beta=\langle \bar{d}d\rangle$ and $\gamma=\langle \bar{s}s\rangle$ are expressed as

$$\alpha=N_cM_u\Lambda^2/2\pi^2\cdot\{x^2\ln((1+\sqrt{1+x^2})/x)-\sqrt{1+x^2}\}_{x=M_u/\Lambda} \tag{4}$$

and $\gamma=\langle \bar{s}s\rangle$ with $M_u\to M_s$. From the equation, one can see that the condensates are coupled with each other, hence a change of the one gives rise to that of the other. The strength of the coupling is given by that of the anomaly term.

The s quark condensate in the nucleon: We are now in a position to discuss the s quark condensate in the nucleon. To calculate $\langle N|\bar{s}s|N\rangle$ a model or an ansatz for the nucleon state is needed. In this report, we will take two typical models to

*) The fact that the theory which the confinement is not built in could describe the low energy phenomena related with the chiral symmetry and its breaking seems to support the picture[7),8),11] that the quark condensates and the Nambu-Goldstone (NG) bosons and the would-be NG-boson (η') are determined almost solely by the dynamics in the intermediate scale contained in our Lagrangian or the relatives, and the confinement, which is relevant in the long range scale, hardly affects these observables.

calculate the condensate; the cleansing-out model by Brown, Kubodera and Rho[12] and the Bernard-Jaffe-Meißner (BJM) ansatz using Feynman's theorem.[13]

The cleansing-out model[12] The model is based on a bag picture and assume that chiral symmetry is 'restored' in the nucleon bag; the u and d quark take their current masses $M_u = M_d = m_u$. Thus, our model gives the following:

$$\langle \bar{u}u \rangle_N \equiv \int d^3x \langle N | \bar{u}u | N \rangle_c = (\langle \bar{u}^{(m_u)} u^{(m_u)} \rangle - \langle \bar{u}^{(M_u)} u^{(M_u)} \rangle) V = (244.9\ \text{MeV})^3 \cdot V\,, \tag{5}$$

where $u^{(m_u)}(u^{(M_u)})$ denotes the quark field with mass of $m_u(M_u)$ and V denotes the volume of the nucleon bag. The subscript c stands for connected; note that the vacuum value is subtracted away. The decrease of the u quark condensate causes a change in the s quark condensate as is seen from Eq. (3). Inserting the equation $\beta = \langle \bar{u}^{(m_u)} u^{(m_u)} \rangle = (-69.316)^3$ into Eq. (3) we get for the s quark condensate in the nucleon

$$\langle \bar{s}s \rangle_N \equiv \int d^3x \langle N | \bar{s}s | N \rangle_c = [(-262.5)^3 - (-267.2)^3] \cdot V = (99.6\ \text{MeV})^3 \cdot V\,. \tag{6}$$

Here, we note that the condensate is positive. Thus, the cleansing-out model gives

$$z = \langle \bar{s}s \rangle_N / \langle \bar{u}u \rangle_N (99.6/244.9)^3 = 0.07\,. \tag{7}$$

Feynman-Bernard-Jaffe-Meißner ansatz A method to calculate the quark condensates has been developed by Bernard, Jaffe and Meißner[13] as an application of Feynman's theorem. Here, we first give a somewhat more detailed account of the method than the original paper, putting an emphasis on the presence of a normal ordering in the Hamiltonian to be used, thereby a relation with the cleansing-out model discussed above is clarified.

The argument goes as follows: Let H_S be the Hamiltonian of the strong interactions. H_S contains the current quark masses linearly. Then, the normalized one-nucleon state $|N\rangle$ is an eigenstate of H_S belonging to an eigenvalue M_N, the nucleon mass,

$$H_S |N\rangle = M_N |N\rangle\,, \qquad \langle N | N \rangle = 1\,. \tag{8}$$

Then the following relation holds:

$$\frac{\partial M_N}{\partial m_i} = \langle N | \frac{\partial H_S}{\partial m_i} | N \rangle\,. \tag{9}$$

This is nothing but the Feynman's theorem. And

$$\frac{\partial H_S}{\partial m_i} = \int d^3x \colon \bar{q}_i(x) q_i(x) \colon . \quad (i = u, s) \tag{10}$$

Here, note that the normal ordering with respect to the true vacuum of H_S must be taken; but for the normal ordering, the infinite vacuum energy must be added to M_N in the eigenvalue equation. Thus, one finally get the following formula:

$$\int d^3x \langle N | \bar{q}_i(x) q_i(x) | N \rangle_c \equiv \int d^3x \langle N | \colon \bar{q}_i(x) q_i(x) \colon | N \rangle = \frac{\partial M_N}{\partial m_i}\,. \tag{11}$$

This equation states that the change rate of the nucleon mass due to a change of the current quark mass gives the quark condensate in the nucleon. Up to this, the argument is exact. The problem is to obtain M_N in terms of the current quark masses. Here, we take the valence quark model, i.e., $M_N \simeq 2M_u + M_d$. Then, one get the following nice relation,

$$\langle \bar{q}_i q_i \rangle_P \equiv \int d^3x \langle N | \bar{q}_i(x) q_i(x) | N \rangle_c = 2\frac{\partial M_u}{\partial m_i} + \frac{\partial M_d}{\partial m_i} . \tag{12}$$

The constituent quark mass can be obtained in terms of the current quark masses from Eq. (3) in our model. A simple manipulation gives, for example,

$$\frac{\partial M_u}{\partial m_u} + \frac{\partial M_u}{\partial m_d} = (2g_s R_s + 1)/D ,$$

$$\frac{\partial M_u}{\partial m_s} = -g_D R_s \beta / D , \tag{13}$$

where

$$D = ((2g_s + g_D \gamma) R_u + 1)(2g_s R_s + 1) - (g_D \beta)^2 R_u R_s \tag{14}$$

and

$$R_i = \Lambda \frac{d}{dm_i} \beta . \tag{15}$$

A numerical evaluation gives the following:

$$\langle \bar{u}u + \bar{d}d \rangle_P = 7.74 ,$$

$$\langle \bar{d}d \rangle_P = 3.02$$

and

$$\langle \bar{s}s \rangle_P = .45 ,$$

hence z is small and ~ 0.10. And $y \equiv 2\langle \bar{s}s \rangle_P / \langle \bar{u}u + \bar{d}d \rangle_P = 0.12$.

It is noteworthy that the value of z is very similar to the one obtained in the cleansing-out model; furthermore the positiveness of z, which is also consistent with the picture in the cleansing-out model, is natural in the light of the presence of the normal ordering by which the large and negative vacuum expectation values are subtracted away.

In summary, we have estimated the strange quark condensate in the nucleon on the basis of an effective Lagrangian, which includes the $U_A(1)$ anomaly term which in turn causes a flavour mixing. We get a rather small s quark condensate in the nucleon, both in the cleansing-out model and the Bernard-Jaffe-Meißner ansats. However, it should be noted that we have not taken into account the so-called Casimir effects of the negative energy sea in the bag[14] nor meson clouds surrounding the bag or nucleon.[15] They may contribute considerably to the quark condensate in the nucleon and change the results obtained here.

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Note added in proof: Although the cleansing-out model and the Bernard-Jaffe-Meiβner (BJM) ansatz give similar values for z, they yield different values for the πN-Σ term $\Sigma_{\pi N}$ itself from each other. $\Sigma_{\pi N}$ in the BJM ansatz is 43 MeV. On the other hand, the cleansing-out model yields $\Sigma_{\pi N}$ dependent on the bag radius R ; $\Sigma_{\pi N} = (19 \rightarrow 60)$ MeV for $R=(0.6 \rightarrow 0.88)$ fm.

After completion of the present work, the author got informed that Bernard, Jaffe and Meiβner (MIT preprint, CTP # 1547) and Kohyama, Kubodera and Takizawa (Sophia University preprint) examined the same subject as that in the present paper on the basis of effective Lagrangians similar with but different from ours. They have also obtained small values for $|z| \lesssim 0.1$. It should be noted, however, that the sign of the strange quark condensate in the nucleon is negative solely in the Bernard-Jaffe-Meiβner calculation.