Strategic Asset Allocation*

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Abstract

To implement strategic asset allocation, we must determine risk and return expectations for the various asset classes. Starting from the paradigm that long-run asset returns are determined by the long-run fundamentals of the economy, a fair value approach to building expectations is crucial. This paper proposes to formalize a quantitative and systematic methodology for optimizing portfolios, from the determination of long-run fundamental pillars through the modeling of asset returns and the assessment of market risks. We apply forecasting models and build in the specific of the main asset classes (equities, bonds and alternative investments) depending on the uncertainties they represent for the risk-averse investor. Our resulting allocations within the equity asset class, and with regard to the place of alternative investments, question the choices of long-term institutional investors such as pension funds that have shifted their long-run allocations in response to the recent financial crisis.

Keywords: Long-term investment policy, strategic asset allocation, tactical asset allocation, risk premium, long-run economic growth, Solow model, Phillips curve.

JEL classification: E20, E50, G11.

1 Introduction

The recent crisis caused significant damages, with job losses and lower output for the real economy, and sell-offs in the equities and credit markets in the financial world. Monetary and government authorities prevented an outright collapse, by implementing far-reaching, unconventional support policies. Unfortunately, the crisis spread geographically and also over time with negative inter-temporal impacts. Pension funds posted major losses which may affect pension pay-outs in the future. In particular, their performances was hit as a

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result of their exposure to underperforming asset classes, such as equities. The roots of these significant losses lie in the investment policies of pension funds.

This issue reveals confusion regarding the nature of investors' decisions. On the one hand, decisions that are made with respect to structural changes in the economy should relate to the strategic asset allocation. On the other hand, choices that alter the asset mix in the portfolio to reflect the impact of business cycles over the short to medium term are defined as being part of tactical asset allocation. Logically, changes of strategic allocation should be occasional, because structural shocks are less frequent than changes in the business cycle. However, a number of events during the past twenty years have defied this logic. Investors over-weighted equities in the boom years leading up to the dot.com crisis or during the 2003-2007 equity bull market, and completely revised their equity/bond asset mix after the technological bubble burst, and the subprime financial crisis. Following the financial crisis, most strategic asset allocations have been revised by decreasing equity exposure. There is clearly confusion between tactical and strategic allocation, because investors' long-run assumptions appear to have varied too much during this period.

In recent years, solutions have been developed to limit risks and to guarantee the pension funds. Liability-driven investments (LDI) and asset and liability management (ALM) refer to those investment policies in which investors focus on monitoring the difference between their assets and their liabilities. By way of illustration, the idea of the LDI solution is to compare the fund's asset value to the fund's liabilities and to compare their respective sensitivities to market factors (Bruder *et al.*, 2010). This leads to the design of portfolio strategies that will match future pension scheme payments as closely as possible. However, the origin of the problem still remains because the choice of a strategic portfolio is being derived from long-run assumptions on asset returns.

In academic literature, various approaches have been proposed for forecasting long-run asset returns, including reference to historical values or to economic models. Over the past decade, econometric models have been widely used to study the inter-temporal behavior of asset returns in the presence of macro-economic factors (Campbell and Viciera, 2002). In this paper, we propose a comprehensive and systematic approach for determining long-run asset returns. First, we make extensive reference to economic and asset pricing theories for formalizing the long-term behavior of asset returns. In particular, we present the notion of fair value of assets, which states that long-run asset returns could be derived from the long-run path of the real economy. Second, we use limited statistical tools to calibrate long-term relationships between variables, in order to determine long-run asset return forecasts in a systematic manner.

The paper is organized as follows. The second section sets out the theoretical framework of our approach for determining long-run asset returns, the role of output and inflation as economic fundamentals, and the derivations of risk premiums for key asset classes. In the third section, we present the empirical results obtained using macro-econometric regression models. These results are presented with respect to regional variations, and feature forecasts on asset returns from horizons varying from 2020 to 2050. Section four derives long-term portfolios from our risk and return expectations. In particular, we describe a number of possible strategic allocations by considering a pure equity portfolio, then adding bonds, and finally alternative investments. Section five concludes by opening the door to a comprehensive analysis framework that encompasses both strategic and tactical asset allocations.

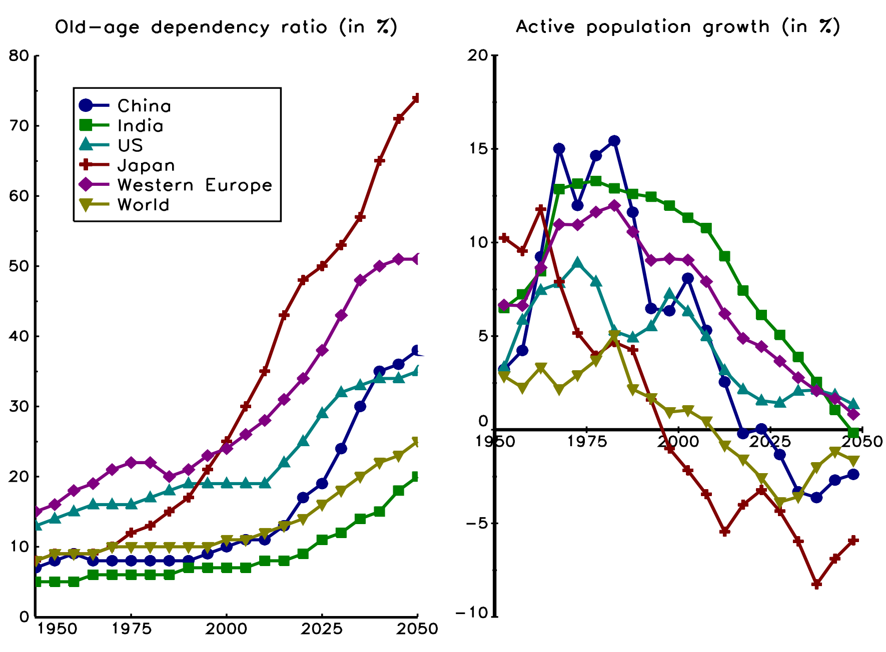
2 Economic modeling of risk premiums

The decision-making process for a strategic allocation requires long-term expectations of asset class returns, volatilities, and correlations as inputs. One popular method consists in using historical figures as a guide for the future. This method is said to be unconditional, meaning that the expected returns are based solely on historical returns, and so do not take into account any global shocks or structural economic changes that could arise. Here are some examples:

- The Bond returns observed during the 80s and 90s may not be repeated in the future, because disinflation is unlikely to re-occur to the same extent;
- The stock/bond correlation depends on the inflation environment;
- The commodity returns depend on population growth and demand from emerging countries.

The strong assumption behind the use of historical figures is not only to model the long-run path of the economies as stationary, which seems reasonable, but to consider it as constant – meaning, repeating itself. However, the world is changing. One of the most structural change is the demographic evolution. In Figure 1, we have represented the dynamic of the old-age dependency ratio[1] and the active population[2] growth until 2050. The world

Figure 1: Old-age dependency ratio and active population growth (in %)



Source: `data.un.org`.

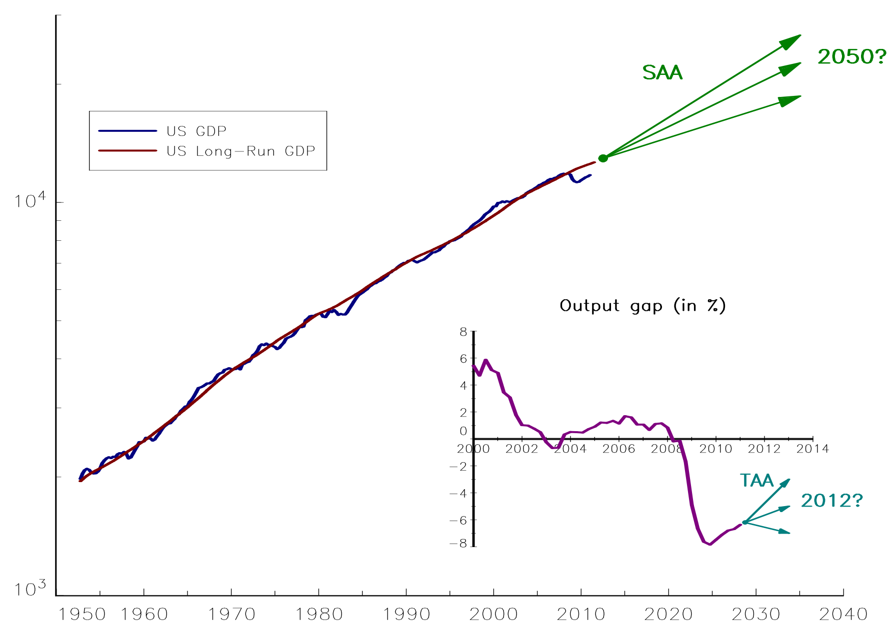
[1]The old-age dependency ratio is the ratio of the population aged 65 years or over to the population aged 15-64.

[2]The working-age population measure is used to give an estimate of the total number of potential workers within an economy. Each region may have a different range of ages, but typically the ages of 20 to 65 are used.

population is expected to decelerate across the regions, with particularly a growing share of old-age people. We note that these trends should be more pronounced in Europe and Japan. These demographic changes should be associated with changes of saving rates, public sector expenditures and economic growth.

Thus, the other way to build expected returns is said to be conditional, meaning that expected asset returns should relate to a global economic scenario. The rationale here is that expected returns should be regarded as the returns required by economic agents, in order to be consistent with this scenario. From an economic perspective, asset prices should converge to a fundamental value (also called *intrinsic* value). A consequence of this fair value methodology is that returns on financial assets must reflect returns on physical assets over the long-run. So, to build asset returns expectations using a fair value methodology, we must determine the long-run fundamentals and how asset returns should depend on these fundamentals values. In this case, long-run risk premiums are stationary but not necessarily constant. In Figure 2, we have reported the dynamic of the US GDP and its breakdown into the long-run GDP and the output gap. The underlying idea is to decompose the dynamic of economic fundamentals by a long-term component and a short-term disequilibrium. This long-term component may be a trend-stationary process. This is the key difference with the methodology using historical values which assumes that the steady-state is constant.

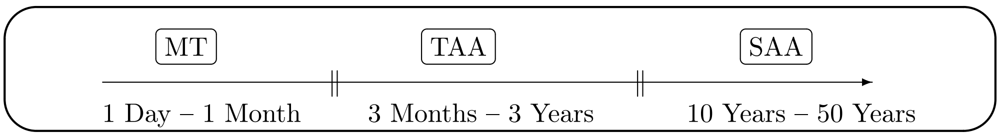
Figure 2: Illustration of long-term component and short-term disequilibrium of output



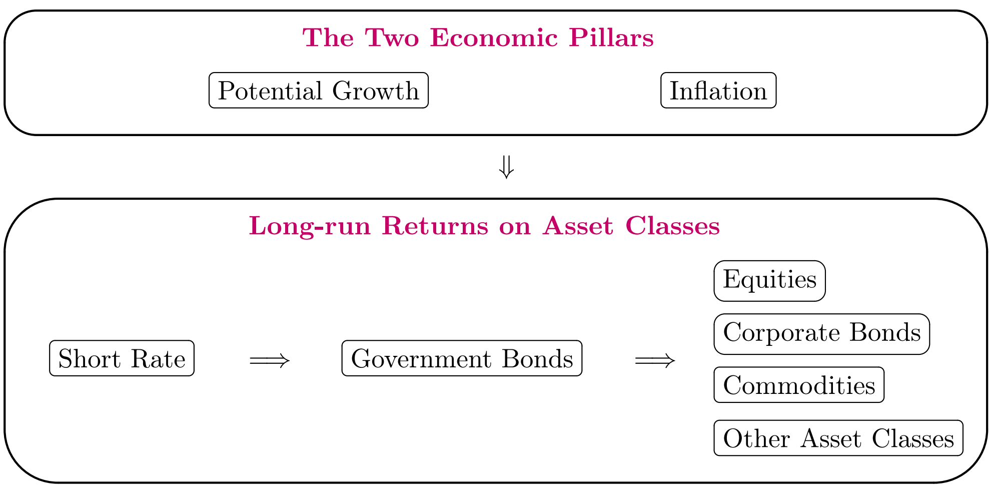
In this context, strategic asset allocation and tactical asset allocation should not be opposed or confused. On the contrary, they should be considered as complementary bodies of a solid long-term investment policy[3]. Thus, SAA should embody the long-run decisions based upon the assumption of stationary risk premiums given an economic scenario, while

[3]It is interesting to note that some people make a confusion between SAA and long-term investment policy.

TAA should allow for adjusting to the business cycle within which risk premiums are time-varying. Lastly, very short-term investment decisions (like for example achieving a volatility target) should be considered as market timing (MT) and are not relied to risk premium (but to the market sentiment).



In conclusion, our approach of SAA could be illustrated by the following scheme. First, we identify two representative economic fundamentals (output and inflation) and we build a long-run economic scenario for these two variables. Second, we derive long-run asset returns from this economic scenario. The long-run short rate is directly obtained by adding together output and inflation. To obtain the long-run government bond return, we add a bond risk premium to the long-run short rate. Finally, the expected returns of the other asset classes are derived from the specific risk premium associated with the nature of the asset class.



2.1 The two economic pillars

The long-term outlook for the economy has been and will continue to be debated extensively. Central banks and governments pay close attention to the economy's long-term prospects in order to optimize their monetary and budgetary policies[4].

2.1.1 Potential output

Economic output, also known as gross domestic product (or GDP), represents the country's nominal aggregated value of all final goods and services. Typically, output grows strongly during periods of economic expansion, and shrinks during a recession. But this cyclical effect vanishes when one look at longer horizons of 20, 30 or 50 years, and we are left with the complex task of describing the long-term outlook for the economy. Does output still behave like a longer cycle, reverting to a somewhat long-term average level? Or does it follow a

[4]For illustration, the US Central bank operates under a dual mandate to achieve both price stability and sustainable growth.

steady path in a given direction with quantifiable intensity? To achieve this, economists generally define so-called potential output as a measure of the economy's productive capacity[5]. In practice, the *full employment* output and the maximum sustainable output consistent with a stable rate of inflation are suitable measures for estimating this capacity. A broad theoretical consensus has been obtained, stating that modeling potential output relies on focusing on the drivers of growth on the supply side of the economy.

Inspired by the neo-classical theory of production, the Solow model presented in Appendix B.1.1 has achieved broad recognition for its ability to formalize the conception of a long-term trend followed by the economy. It assumes that output growth is driven by growth in labor input (or demography) and the accumulation of physical capital. The model also implicitly assumes the existence of factor productivity, a residual part of the output growth that cannot be attributed to the two previous production factors[6]. Basically, Solow (1956) predicts that output growth should converge towards equilibrium[7]. It states that the economy tends to a steady-state, where output growth is only due to long-run constant and exogenous production factors, namely growth in the labor force and growth in productivity[8]:

- Growth in the labor force
  The rationale behind the concept of stable long-run employment growth makes reference to a "natural" unemployment rate. Over the medium term, the unemployment rate could be above or below its natural level. Typically, periods of faster economic growth trigger stronger employment growth, and so lower the unemployment rate. This situation occurs when output exceeds its potential level (positive output gap). Indeed, a positive output gap triggers tensions on resources of production, and the supply of workers becomes scarcer, as explained by Okun's law, as presented in Appendix B.2.3. However, the output gap becomes null over the long term, so the unemployment rate converges to its natural level.

- Productivity growth
  Productivity is a measure of output per unit of input (worker, machine, etc.). Economists have attributed productivity growth to many causes besides technological progress, for example changes in the rate at which capital is used, changes in the quality of labor or the amount of human capital, changes in businesses' organization or culture, or productivity shocks. Meanwhile, as a residual in Solow model, it remains technically difficult to model or predict. Nevertheless, over the long term, it is possible to use a variant of Okun's law to define a long-run productivity growth rate, meaning cyclically adjusted.

[5] An other common approach consists in estimating a long-run stochastic trend to quantify the steady path of long-run output (Beveridge and Nelson, 1981).

[6] The role of productivity has been validated empirically by former academic works that found that a significant part of output changes are explained by the Solow residual (Kydland and Prescott 1991, King and Rebelo 1999, Tang 2002). Today, this assumption is not completely accepted (Francis and Ramey, 2005).

[7] There also exist statistical approaches that aim at estimating the potential output. For instance, Hodrick-Prescott and Kalman filters are often used to extract directly the trend from the output. These methods do require analysts to make assumptions about how the filters are structured, including the values of one or more parameters. However, they could not be used to perform forecast of the output.

[8] Solow specifically models a closed economy where output growth is attributed to the dynamics of capital accumulation and to some effective labor force growth that is assumed to be constant and exogenous. The main result is that, given an initial amount of capital and fixed exogenous savings rate, a steady-state is reached when investment equals depreciation of capital. Capital growth and therefore potential output growth are then constant and equal to the sum of employment growth and productivity growth, after the stock of capital reached its steady-state level.

Remark 1 *The macro-economic framework based on the Solow model is particularly relevant for strategic asset allocation. The other growth economic models (Ramsey model, endogenous growth models or DSGE) are less suitable because they address both tactical and strategic asset allocations.*

2.1.2 Long-run inflation

Inflation is traditionally defined as the rise in the general level of prices of goods and services in an economy. Over the medium term, variations of inflation are usually attributed to business cycles. Typically, inflation accelerates (resp. decelerates) when output exceeds (resp. falls below) its natural level. Indeed, excessive output requires intensive use of inputs which eventually puts pressure on prices. We generally refer to the well-known Phillips curve to illustrate these stylized facts (Phillips 1958, Cahuc and Zylberberg 2004). In this theory, high inflation is associated with low unemployment (Appendix B.2.1). When the unemployment rate falls (resp. rises) below (resp. above) a specific level, then inflation accelerates (resp. decelerates). The specific unemployment level that remains neutral with respect to inflation is called the "non-accelerating inflation rate of unemployment" or NAIRU (Appendix B.2.2).

Over the long term, when the economy converges to equilibrium, output and the unemployment rate should converge respectively to potential output and to a natural unemployment rate, commonly referred as the NAIRU (Blanchard and Katz, 1997). In other words, output and unemployment reach levels that are defined as sustainable and consistent with stable inflation (Bruno and Easterly, 1998). Is there any truth behind this hypothesis? Let us consider rational economic agents. In order to stabilize their purchasing power, they demand higher wages when they expect inflation to rise. This is called the long-run neutrality of inflation: variations of inflation are fully passed on to the nominal value of economic variables, keeping their real value constant. Therefore, rational expectations imply that long-run variations in inflation should have no impact on the equilibrium level of actual output.

However, we have still not addressed the crucial question of the choice of a long-run inflation rate. There is broad agreement among economists that long-run inflation is only a monetary phenomenon, and could be related to monetary policy (quantitative theory of money). Simply stated, the quantity of money in the economy should be conditional on the level of inflation that the Central banks assume to be consistent with sustainable output. A consequence is that Central banks must establish their credibility in fighting inflation. If not, agents would bet on the fact that Central banks will expand (resp. decrease) the money supply rapidly enough to prevent recessions, even at the expense of exacerbating inflation. In fact, the main Central banks declare an official (medium-term) objective for inflation, also called an inflation target. These targets are therefore reliable candidates for long-run inflation (Bernanke *et al.* 1999, Friedman 2003, Woodford 2003).

2.2 Modeling asset returns

According to the economic theory, the return on capital is closely tied with potential growth, and capital stock in particular. From a micro-economic point of view, the level of return on capital should result from the opposing forces of supply and demand for capital. A well-known optimal condition derived from neo-classical theory is achieved when the real return on capital is equal to the marginal productivity of physical capital (Appendix B.1.3). In other words, supply and demand reach equilibrium when adding one dollar of physical capital is equivalent to buying one dollar of the firm's asset. From a macro-economic point of view and according to the Solow model, this equality could be derived in a normative rule, also called the Golden rule which states that the real return on capital consistent with the maximization of output per worker is equal to real potential output growth (Phelps, 1961). What then is the relationship between this synthetic real return of capital derived from the economic theory, and the returns obtained from equities, bonds, and other asset classes? As suggested by Baker *et al.* (2005), the mapping between the real return of capital and asset class returns is completely straightforward only under the assumption of constant risk premiums. According to the asset pricing theory, risk premiums should reflect investors' assessments of the fundamental risks given by the economy and investors' sensitivity to these inherent risks, namely their risk aversion. From an investor's point of view, risk premiums are interpreted as excess returns required in order to buy riskier assets rather than risk-free assets. Lucas (1978) shows that, over the medium term, risk premiums and asset returns vary with the business cycle. More precisely, they are linked through consumption-based models (Campbell, 1999) and production-based models (Cochrane, 1991). Over the long term, risk premiums should stabilize as the influence of the business cycle disappears. The challenge then consists in estimating the long-run values of asset returns.

2.2.1 Short-term interest rates

Short-term interest rates are the principal instrument used by Central banks to implement their monetary policies. They use the short rate in order to attain a set of objectives oriented towards price stability and sustainability of growth. Let us consider a situation where inflation increases to a level that is not consistent with a sustainable output growth. Typically, Central banks could react by hiking interest rates in the hope of stifling demand (consumption and investment), and thereby reducing output to a more sustainable level. The Taylor rule formalizes this idea by quantifying the intensity of change in the short rate that Central banks should apply in response to observed divergences of actual from target inflation rates and of output from potential output[9]. One key result of this rule stipulates that when inflation exceeds its target, not only should the short rate be increased, but it should also overreact to inflation so that the real short rate increases.

Over the long term, the Taylor rule simply states that the short rate converges to the sum of long-run inflation and a long-run real short rate, because divergences in output and inflation from their equilibrium levels vanish over this horizon (Appendix C.1). On the one hand, the Fisher parity tells us that these two variables are not related, meaning that any change in long-run inflation should have no impact on the long-run real short rate. This is the case if, and only if, the long-run nominal interest rate adjusts in step with the rise of inflation. On the other hand, as previously mentioned, the long-run real short rate is generally viewed as an equilibrium level alongside the steady-state path of the economy. According to the Golden rule, it can be set equal to real potential output growth. It is therefore assumed that, at equilibrium, the long-run short rate is the sum of long-run inflation and real potential output growth.

2.2.2 Bond returns

Sovereign bonds The key notion is that bond prices react conversely to shifts in interest rates, and that these shifts typically occur in tandem with business cycles. For example, bonds prices rise as a consequence of falling bond yields, when the economy is slowing

[9] The Taylor rule does not verify the Tinbergen rule, which states that with only one policy instrument, policy can only achieve one goal.

and/or inflation is decelerating. Typically, during difficult economic times Central banks lower their short-term rates in order to restore the conditions required for an economic expansion, and this cut partially spreads to long-term rates. Financial theory formalizes the relation between short-term rates and long-term rates through the rational expectations model of the term structure: if long-term rates are higher than short-term rates, it means that investors anticipate higher short rates (Lutz, 1940). A key result is that for an investor, holding a long-term bond would be the same as rolling a short-term rate over the same period (Appendix C.1). However, there is empirical evidence[10] that the difference between long-term rates and short-term rates (the term spread) is not only related to future short-term rates expectations, but may also be influenced by other factors. Following the economic theory, long-term rates should be especially sensitive to the government policy, while short-term rates are more sensitive to monetary policy[11]. From the investor's point of view, the slope of the yield curve is effectively a bond risk premium required in order to compensate for higher uncertainty over a longer period. Moreover, following Lucas (1978), this term structure spread could logically vary alongside business cycles. The intuition is that investors prefer a smooth consumption stream rather than very high consumption at one stage of the business cycle and very low consumption at another stage. A logical hedging method is to substitute bonds of different maturities. A key result of this is that the slope of the yield curve should rise (resp. fall) during economic slowdowns (resp. expansions), meaning that the bond risk premium required by the investor to buy a long-term bond rather than a short-term bond should rise (resp. fall).

Over the long term, the bond risk premium is supposed to stabilize because the influence of business cycles should vanish. However, this bond risk premium should remain positive as the investor still requires an excess return over the short-term rate to compensate for higher uncertainty over longer time horizons. How then does this uncertainty embody long-run economic risks that characterize the long-run path of the economy? According to the economic theory, these risks could be related to both monetary and budgetary policies. For Central banks, a failure to control inflation would lead to the erosion of the real bond return at maturity. In practice, long-run inflation volatility is a measure of this uncertainty. For governments, a failure to control their budget deficits could trigger higher bond yields, undermining the value of previous bond issues. Here again, the uncertainty surrounding this issue is clearly indicated by the long-run budget balance (or debt) over GDP ratio. The long-run bond risk premium can thus be seen as an indicator of the long-term credibility of monetary and budgetary policies. It follows that a long-run bond yield should therefore be linked to a long-run short rate and a bond risk premium that is positively related to long-run inflation volatility and the equilibrium deficit/GDP ratio. Finally, it is possible to mechanically derive a long-run bond return from the long-run path of the 10-year bond yield.

Corporate bond returns Corporate bonds provide the same payoff structure as sovereign bonds, but are more risky because a company's capacity to service its debt is uncertain whereas sovereign bonds have the status of safe assets[12]. It is therefore only to be expected

[10] Note that the hypothesis that long-term yields equal an average of future expected short-term yields is a statement about the unpredictability of bonds' excess returns. However, Fama and Bliss (1987) find that the spread between the m-year forward rate and the 1-year yield predicts the 1-year excess return of the m-year bond, with a R^2 of about 18%. Campbell and Shiller (1991) find similar results while forecasting yield changes using yield spreads. Cochrane and Piazzesi (2005) extend these results for long-term bonds.

[11] For example, in the IS-LM model of Clarida and Friedman (1983), long-term rates affect the IS curve whereas short-term rates influence the LM curve.

[12] This status of safe assets is especially pronounced during periods of high uncertainty on the economy, with investors fleeing from risky assets to take refuge in government bonds. However, since 2010 this status

that corporate bond should offer a higher expected return compared to sovereign bonds, to compensate for their higher risk. In yield terms, this means that the difference (also called the credit spread) between the corporate bond yield and the sovereign bond yield should be positive. The fundamentals of this theory were formally propounded by Merton (1974), who demonstrated that the credit spread is closely linked to a company's leverage and its assets volatility, as these two variables provide information on the probability of default (Jones *et al.*, 1984). During economic recessions, this uncertainty regarding a company's capacity to service its debt is exacerbated due to the increasing probability of default.

Over the long term, credit risk premiums are not supposed to vary because business cycles are eliminated. In particular, the default rate should converge towards a long-run level, which should be related to long-run output growth, and the volatility on equities should also stabilize close to a long-run average. Long-run corporate bond yields can therefore be broken down into the long-run sovereign bond yield plus a credit risk premium derived from the long-run default rate and volatility. As for sovereign bonds, the long-run corporate bond return can be derived from the long-run path of corporate bond yields.

Remark 2 *Emerging bonds combine the characteristics of both sovereign and corporate bonds. The difference between emerging bond yields and US government bond yields (also called the emerging bond spread) is commonly regarded as a measure of an emerging economy's creditworthiness. Typically, a widening of the emerging bond spread is associated with greater uncertainty regarding a country's economic and/or political situation. Investors focus in particular on the balance of payments structure*[13] *when assessing a country's financial health. In fact, uncertainty about the capacity of emerging economies to finance their economic growth is higher when trade balance and/or budget deficits are large. Also, emerging spreads may depend on currency risk as well as liquidity conditions. An emerging economy crisis is generally characterized by large foreign capital outflows and the depreciation of the local currency. Using the same methodology as for corporate bonds, we then find that long-run emerging bond yields can be broken down into US government long-run bond yields plus an emerging bond risk premium derived from the long-run current account position and the volatility of emerging equities. Lastly, long-run emerging bond returns can be derived from the long-run path of emerging bond yields.*

2.2.3 Equity returns

Equities differ substantially from bonds in that they do not have a defined maturity and their future cash flows are unknown. However, as with bonds, the assumption of no arbitrage requires that equities should be priced at their discounted *intrinsic* values, meaning the present value of their expected cash flows. The discount rate is either interpreted as the cost of capital from the firm's point of view, or as the expected equity return from the investor's point of view. Considering the firm's point of view, equity prices are then inversely related to the cost of capital (also positively correlated to the expected cash flows). In particular, a lower (resp. higher) cost of capital should be associated with higher (resp. lower) equity prices (Campbell and Shiller, 1988). But does this means that equities outperform (resp. underperform) sovereign bonds? No, because this cost of capital makes no distinction between the cost related to the level of the risk-free interest rate and the one related to the

has been undermined due to increasing doubts about European countries' capacity to service their debt.

[13] In economics, the current account is one of the two primary components of the balance of payments, the other being the capital account. The current account is the sum of the balance of trades (exports minus imports of goods and services), net factor income (such as interest and dividends) and net transfer payments (such as foreign aid). A current account surplus increases a country's net foreign assets by the corresponding amount, and a current account deficit does the reverse.

underlying characteristics of the firm. Considering the investor's point of view thus helps us to assess the behavior of this equity risk premium. Cochrane (2001) revisits Lucas' model in order to demonstrate a formal link between equity risk premiums and business cycles[14]. Simply stated, he explains that economic recessions trigger higher equity risk premiums, because the investor becomes particularly averse to buying assets that fail to hedge him against unwelcome changes in his consumption.

However, knowing that over the long-run, business cycles vanish and no longer influence expected returns, where do we stand regarding the behavior of equity risk premium over long horizons? Equity prices should reflect only the long-run expectations of the rational investor. Any deviation of the market price of equities from their fair value should be interpreted as an evidence of market inefficiency and should disappear. For a long-term analysis, a very well suited approach is proposed by Gordon (1959), which assumes that future dividends should grow at a constant rate and that the expected return on equity (cost of capital) required remains constant as well (Appendix C.2). In particular, the model proposes a breakdown of the equity expected return in the sum of a risk-free return and a pure equity risk premium. It is then possible to resolve the model for the equity risk premium which equals the sum of three elements.

- The first is dividend yield (calculated as the dividend over price ratio). This variable is commonly assumed to be constant. Campbell and Shiller (1998) state that over long time horizons, the dividend yield forms a stationary time-series. Specifically, dividend yields are highly persistent, and any deviation from a long-term average should be regarded as temporary. However, the last decade has produced growing evidence of slow time-varying dividend yields related to demographic variables (Favero *et al.*, 2009). In brief, these studies show that dividend yield is inversely related to the ratio of middle-aged to young population. A long-run dividend yield should therefore be consistent with demographic assumptions.

- The second is constant dividend growth rate, which is assumed to be consistent with earnings growth. Dividends represent a share of the company's earnings which is empirically stable over the long run, despite a long period of structural decline during the 90s[15]. Also, earnings growth is supposed to be consistent with long-run output growth. Indeed, the output produced is the overall income which is distributed between wages and earnings. Earnings may therefore grow faster than overall income for a time, but not indefinitely.

- The third is more commonly embodied by long-run government bond yields rather than long-run short rates. The equity risk premium is therefore defined by reference to the yield on a government bond.

Remark 3 *An interesting case is that of small caps, which are defined as equities with low market capitalizations. Traditionally, their equity risk premiums are higher than for large caps, because they present a liquidity risk and greater uncertainty regarding their future cash flows. Small caps usually underperform large caps during a recession. The natural way of assessing long-run returns on small caps is to sum the long-run equity return and the historical excess return of small caps over large caps.*

[14] Bansal and Yaron (2004) find encouraging empirical results, especially by using specific forms of utility functions.

[15] During this period, we observe a structural decline of the payout ratio (the share of a company's earnings out in dividends) due to changes in corporate management policies. The low cost of capital prompted companies to finance their corporate investments through debt rather than equity.

2.2.4 Returns on other asset classes

Commodities Unlike equities and bonds, the existence of a commodity risk premium – excess return of commodities over the risk-free rate – has been the object of much debate[16]. The risk premium theory advanced by Keynes (1923) relates futures prices to anticipated future spot prices, arguing that speculators bear risks and must be compensated for their risk-bearing services in the form of a discount, known as normal backwardation[17]. On the other hand, the theory of storage as proposed by Kaldor (1939), postulates that the return from purchasing a commodity at time t and selling it forward for delivery at time T, should be equal to a cost of storage minus a convenience yield. The empirical evidences for the existence of time-varying commodity risk premiums related to business cycles are strong (Deaton and Laroque, 1992). Typically, periods of strong global demand are associated with low inventories, implying a surge of commodities prices. However, the evidence for a long-run commodity risk premium is patchy. One could therefore argue that over long time horizons, commodity returns should be close to a risk-free rate[18]. However, from an economic point of view, the necessity of leaving a share of natural resources for future generations to use, and the growing role of emerging economies could structurally drive prices higher, justifying a risk premium over the long term. Long-run commodity returns could thus be defined as the sum of the long-run short-term rate and a risk premium. This latter should relate to the long-run balance between resources and consumption, potentially proxied by the combination of global output growth and the emerging economies' share of global output (USDA, 2010).

Other alternative asset classes Since the early 90s, institutional investors' interest in "alternative" asset classes has grown significantly. Higher volatility in the equity markets and record low bond yields have led investors to demand alternative sources of return. The major categories include real estate, private equity and hedge funds. Over the long term, the empirical evidence for return on alternative assets confirms certain key characteristics (Roxburgh *et al.* 2009, Fugazza *et al.* 2007, Phalippou 2007). First of all, there is the potential for additional diversification. Alternative assets generate different returns characteristics than traditional asset classes. Their returns are to a certain degree less correlated with traditional equity and fixed income, thereby mitigating portfolio risk. Also, alternative investments are often required to provide superior risk-adjusted performances compared to traditional investment, mainly to offset their higher degree of illiquidity and prevailing suspicions regarding their transparency. Because of this relative illiquidity, they are therefore best suited for long-term horizons. Think of institutional investors that are willing to take a longer-term view when investing in these alternatives, which sometimes impose lock-ups. However, unlike equities and bonds, in the absence of a clearly identifiable body of theoretical work, it is not possible to propose a consistent fair value framework. We therefore

[16] The return from holding physical commodities derives from both their price appreciation and the economic benefit of physical commodity storage. In contrast, the return on a commodity futures contract takes into account the price appreciation of the physical commodity, a collateral yield that is the interest earned on the US Treasury bills used as collateral for the futures position, and a yield realized based on the futures term structure. This last source of return on commodities futures is called the roll yield and represents the difference between the change in futures prices and the change in the spot price, at any time. These changes are due to the convergence of futures contracts with the spot price as the contracts approach expiry.

[17] The market is said to be in backwardation when futures prices are lower than spot prices. This situation allows investors to earn money from buying the discounted futures contracts, which continuously roll up to the higher spot price. In this case investors capture a positive roll yield. When the opposite occurs, the market is said to be in contango.

[18] It is worth mentioning that commodities are traditionally considered to be a good hedge against inflation because they represent a cost for consumers and producers and are therefore an underlying source of inflation.

propose to define the long-run returns on alternative asset classes in terms of their historical relative performances with traditional asset classes.

Currencies When assessing the sources of return on foreign currencies, financial theory emphasizes equilibrium relationships between interest rates differentials across various economic regions, as well as differences in purchasing power between countries. In particular, uncovered interest rate parity postulates that, for *domestic* investors, an attractive interest rate differential in favor of a *foreign* country, should be reflected in the corresponding exchange rate so that investors willing to trade this spread see their profits offset once they convert the invested currency back to the currency of the domestic country (Frenkel and Levich, 1975). However, some empirical evidence effectively contradicts this no-arbitrage condition, which is exploited using the so-called carry trade. From an investor's point of view, these facts suggest the existence of a time-varying currency risk premium. Alternatively, economic theory introduces the notion of a real exchange rate, defined as the nominal exchange rate adjusted by the differential between the price of goods of two countries. Over the medium term, real exchange rates vary alongside changes of business conditions. Countries with low real short rates, large trade deficits, and negative output gaps tend to be associated with lower real exchange rates. But over the long term, the real exchange rate should stabilize. This theory is expressed through purchasing power parity, which states that a basket of goods should have the same price worldwide after nominal exchange rates are taken into account. This implies that long-run exchange rates should be determined by long-run inflation differentials between economies. Relative stabilization of long-run inflation differentials should generally be consistent with negligible currency returns. It is worth noting that structurally, these mechanisms should favor emerging currencies over developed ones. Indeed, as long as an economy is categorized as *emerging*, it should exhibit higher inflation as a result of the Balassa-Samuelson effect[19], which leads to the appreciation of its real exchange rate. On the other hand, convergence of long-run inflation differentials between developed countries should be consistent with limited currency returns.

2.3 Assessing market risks

2.3.1 Volatility

In its generic formulation, volatility is traditionally related to the quantity of risk embodied in the real and financial worlds. Low volatility is usually interpreted as limited uncertainty regarding the future growth of the economy and/or assets, and therefore as low risk. The US stock market exhibits historical volatility of around 15% when calculated over the two last decades, 14% if we consider long-term data. Bonds exhibit significantly lower risk when measured by their volatility, at 9%. Traditionally, volatility is considered to be stationary with mean-reverting properties, and can therfore be forecast using econometric tools. Typically, these methods postulate that periods of high (resp. low) volatility are followed by periods of low (resp. high) volatility. However, from an economic point of view, these periods of high (resp. low) volatility could be related to fundamental volatility, meaning the volatility of output or/and inflation (Tang 2002, Diebold and Yilmaz 2010). By way of illustration, the "Great Moderation" in the volatility of economic activity is often dated to the mid-eighties, and has contributed to a structural decline in equity, credit and bond volatility. The case of commodities is especially interesting, with the oil price shocks of

[19] The least developed countries have a manufacturing sector in which productivity gains increase rapidly. The resulting salary increases spread to all other sectors of the economy – notably services where the productivity gains are weaker – and result in structurally stronger inflation. At a more or less rapid pace, price levels in developing countries converge with those of advanced countries.

the seventies showing a positive relationship between financial and fundamental volatilities. Regarding other alternative assets, their volatility is allegedly related less to fundamentals and more to liquidity risk. However, there are differences between alternative assets. For example, hedge funds volatility is historically half that of equities, whereas private equity volatility is twice as high.

Over the long term, the reference to historical figures is more commonly admitted than for returns, due to the statistical properties of volatility. However, a long-run volatility forecast will have to be consistent with the one of the economic pillars. More specifically, if one expects deviations between output and its potential levels to be limited in the future, and/or inflation to be kept under control by Central banks, then fundamental volatility will be lower, resulting in a low level of volatility in the markets. Bond volatility will be closely linked to inflation volatility, whereas the volatility of equities and credit markets will be linked to both output and bond yield volatility. Also, expected stock and bond market volatility will depend upon the degree of diversification. For example, it would seem logical to forecast a decline in emerging market volatility. Volatility on small caps should remain higher than the one on large caps, as these stocks exhibit a specific liquidity risk and higher uncertainty. Regarding commodities, dwindling resources and the growing role of emerging countries suggests that volatility should not diminish. For alternative assets, it would make sense to forecast future volatility consistently with past volatility, especially for hedge funds whereas real estate and private equity may be influenced more by equity volatility.

2.3.2 Correlation

The correlation between stock and bond returns is generally positive over time, meaning that rising (resp. falling) equity returns are associated with rising (resp. falling) bond returns. An interpretation of this positive correlation is derived from the present value model, which prices equities using the discounted value of future dividends: falling bond yields (positive bond returns) lift the fair value of equities (positive equity returns). However, this correlation can be influenced by stock market uncertainty and economic considerations. First, periods of "flight to quality" show a negative stock/bond correlation, with large outflows from equity markets that find refuge in "safe" government bonds (Ilmanen, 2003). Second, periods of low inflation are also associated with negative stock/bond correlation, when investors become much less sensitive to changes in bond yield levels. Credit markets tend to be positively correlated with equity markets as their risk premiums exhibit the same kind of correlation with business conditions, while their correlation with sovereign bonds is unclear. The correlation of commodities with other asset classes is ambiguous, especially due to their close relationship with inflation. The case of other alternative asset classes intuitively suggests that they should exhibit a low level of correlation with traditional asset classes, as they are supposed to offer a high degree of diversification.

As for volatility, over the long term, the reference to historical figures is more commonly admitted than for returns. However, this long-run forecast of correlation must be consistent with the long-run outlook on economic pillars, especially inflation (Li, 2002). The correlation between traditional asset classes and alternative asset classes should remain constant, and should therefore be derived from historical value. The correlation between distinct markets within a specific asset class should depend on the degree of globalization that is expected over the long term. The intuition is that globalization should reinforce the correlation between equity markets, and also between bond markets[20].

[20] The decline of home bias is well documented by economic studies (see e.g. Sørensen *et al.*, 2007).

3 Empirical estimation of risk premiums

The objective of this empirical section is to set out forecasts for different long-term horizons (2020, 2030 and 2050). First we provide forecasts for economic pillars, and then we describe our methodology for determining long-term views on returns from different asset classes. These will be used as inputs for a strategic asset allocation. We distinguish between the following regions: US, Euro, Japan, Pacific ex Japan (Australia) and emerging countries. We use a large body of annual data from January 1970 to September 2010 provided by the IMF, OECD, the World Bank and Datastream[21].

3.1 Potential output and inflation

We use forecasts provided by various organisms as inputs, provided they adopt the same approach as we do. More specifically, they must make reference to the theory derived from the Solow growth model. They should also use economic tools such as the Phillips curve and the NAIRU for determining long-run inflation. Finally, they should consider the long-run structural changes expected for the coming years, such as demographic changes[22] and globalization. Using forecasts of different organisms, we observe some differences which may be explained by different approaches regarding the long-run determinants of economic fundamentals. We build a synthetic forecast by averaging the cited organisms' forecasts, taking into account differences in terms of horizons.

For the 2050 horizon, potential GDP growth is close to 2% for developed countries, but higher for the US (2.6%) than for the eurozone (1.7%) and Japan (1.4%). These forecasts are lower than values obtained during the past three decades, mainly due to a lower working population growth forecast. Emerging potential GDP growth converges with that of developed countries, but still remains higher at 4.4%. Inflation stabilizes between 2.1% and 2.2% in the US and the eurozone, at 1.2% in Japan, and at 4.7% in emerging countries. These levels are lower than those obtained during the past three decades which were characterized by structural disinflation. Indeed, inflation figures reached a peak at the end of the 90s, on the back of oil price shocks. Regarding our long-run forecast, the implicit assumption is that inflation should not be associated with rising inflation risks.

[21] In particular, we use IMF and OECD sources for historical economic data such as GDP, inflation, budget balances, population, productivity and short rates. We use Datastream for historical financial data such as equity prices, bond yields, and commodity prices. When we have to build long-run forecasts on key exogenous variables, we take into account a large set of sources. For example, for long-run GDP and long-run inflation, we compare forecasts by the IMF, OECD, World Bank, CEPII, CBO, US Federal Reserve, etc. Our data set ranges from 1970 to 2010, depending on the underlying economy. It is possible to obtain more historical data for US, but our objective is to build estimates covering the same historical sample of data.

[22] The world population is ageing quickly due to declining fertility rates and longer life expectancies. This process should be associated with lower potential output growth and higher inflation. Indeed, the Solow growth model teaches us that lower working active population growth implies lower potential output growth. Also, while the theoretical impact on inflation remains unclear, empirically there is a positive relationship between an ageing population and higher inflation. The intuition is that ageing people decrease their savings and so increase demand, which then triggers higher inflation. On the other hand, globalization, and notably the opening up of China and India, modify the long-run outlook of the global share of output and inflation. First, a key prediction of the Solow model is that the income levels of poor countries will tend to converge towards the income levels of rich countries. Second, the growing role of emerging economies suggests upward pressures on commodity prices, while the impact on other prices should abate progressively as emerging economies converge to the developed economies.

Table 1: Economic forecast for GDP

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	3.7%	3.1%	3.1%	3.9%	2.4%	2.5%	2.6%
EURO	2.7%	2.1%	2.2%	1.8%	1.6%	1.7%	1.7%
JAPAN	4.4%	3.7%	1.7%	1.5%	1.3%	1.4%	1.4%
PACIFIC		3.4%	3.3%	2.9%	3.1%	2.8%	2.7%
EM		3.5%	3.3%	5.9%	5.7%	4.8%	4.4%

Table 2: Economic forecast for inflation

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	8.1%	5.6%	3.0%	2.5%	2.1%	2.2%	2.2%
EURO	7.7%	5.9%	2.6%	2.2%	2.0%	2.1%	2.1%
JAPAN	7.8%	2.7%	1.2%	−0.6%	1.1%	1.2%	1.2%
PACIFIC		8.4%	2.5%	3.2%	2.5%	2.5%	2.5%
EM		10.0%	8.0%	6.8%	4.1%	4.5%	4.7%

3.2 Asset returns

Our objective is to build long-run return forecasts for a large range of asset classes: short rates, governments bonds, corporate and high yield bonds, developed and emerging equities, currencies, commodities and alternative investments. In section three, we have detailed the theoretical framework proposed for each asset class based on a fair value approach. Then, our long-run asset returns are interpreted as expected returns required by investors. In a perfectly rational world, required returns should not differ largely from historical returns, as investors rationally price the relevant information over time. However, the academic literature emphasizes differences between realized and required returns[23]. The interpretation is that required risk premiums might not be good predictors of realized risk premiums, or that realized values might not be justified from a fair value perspective. A pragmatic solution to circumvent this problem is to consider that the true equity risk premium is somewhere between its realized and required values. Therefore, we estimate the relationships between the realized risk premiums and the theoretical determinants of the required risk premiums.

In the previous section, the risk premium is defined as the difference between the expected return and a risk-free return. The first important choice is to decide if the short rate, being a natural candidate for the risk-free rate, or another variable should be the right anchor to model the risk premium for a given asset class. Therefore we would find the anchor variable that implies a risk premium displaying the most stationary relationship in the long term. That's why we use the co-integration theory to select the risk premium variable which presents the more co-integrated pricing equation and the relevant components of the corresponding co-integrated vector (Engle and Granger, 1987). Specifically, the choice of the anchor variable depends on the asset class. For example, our empirical results suggest that the right anchor variable for commodities is the long-run short interest rate whereas the long-run inflation rate is more relevant for sovereign bonds.

[23]Fama and French (2002), using a discounted dividend model, estimate the implied equity risk premium to be between 2.55% and 4.32% for the period 1951-2000, far below the historical risk premium measured at 7.43%.

3.2.1 Short rates

We propose to derive long-run short rates r_∞ from the lower bound of the normative Golden rule:

$$\boxed{r_\infty = g_\infty + \pi_\infty}$$

where g_∞ is the long-run real potential output growth and π_∞ is the long-run inflation. This simple methodology provides several key results, in particular with regard to developed countries[24]. US expected short rates come to 4.8% while those of the eurozone and Japan ones come respectively to 3.8% and 2.6%, this hierarchy being fully consistent with that derived from long-run output and inflation forecasts. These long-run short rates are low compared to historical standards of the last quarter-century, because short rates have progressively decreased in tandem with inflation since 1980. For the years to come, our long-run inflation forecast implies that long-run short rates should remain low. If we consider short rates in real terms, we also find lower long-run forecasts than historical standards. This last result is explained by lower potential output growth forecasts. It is also interesting to note that these forecasts imply significant increases of short rates from current historical low levels in developed economies. Indeed, following the subprime crisis, Central banks have promoted very accommodating monetary policies in order to restore the conditions for growth.

Table 3: Economic forecast for short rates

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	11.8%	8.7%	6.1%	6.4%	4.5%	4.7%	4.8%
EURO	10.4%	8.0%	4.8%	4.0%	3.6%	3.7%	3.8%
JAPAN	12.2%	6.4%	2.9%	0.9%	2.4%	2.5%	2.6%
PACIFIC		14.5%	7.2%	5.5%	5.6%	5.3%	5.2%
EM					9.8%	9.3%	9.1%

3.2.2 Bonds

Sovereign bonds From a fair value point of view, the bond risk premium required by an investor should increase with higher inflation risk and/or government debt risk. Then, we consider the inflation volatility σ_t^π as a good proxy for inflation risk, and use a 10-year moving average of inflation volatility[25]. Also, we consider the government balance on output ratio $(B/Y)_t$ as a good proxy for debt risk[26]. Specifically, we estimate the long-term dependence of the 10-year real bond yield $\mathcal{R}_t^{\mathrm{b}}$ with respect to the real short rate $\mathfrak{r}_t$ and these two explanatory variables, by means of regression:

$$\mathcal{R}_t^{\mathrm{b}} = \beta_0 + \beta_1 \mathfrak{r}_t + \beta_2 \sigma_t^\pi + \beta_3 (B/Y)_t + \varepsilon_t$$

Then, we obtain the long-run value of the nominal bond yield R_∞^b:

$$\boxed{R_\infty^b = \mathcal{R}_\infty^{\mathrm{b}} + \pi_\infty}$$

[24] Emerging long-run short rates seem very attractive. However, investors should take into account two factors: increased uncertainty regarding the currency outlook and economic improvements.

[25] We regress the long-run inflation volatility on the long-run inflation, then we use our long-run inflation forecast to derive a long-run inflation volatility forecast.

[26] Over a very long-run horizon, official organisms generally consider a progressive normalization of deficits. However, we mix this assumption with the implicit forecast derived from a regression between government balance on output ratio on the one hand, and the output growth and real short rates on the other hand.

where the long-run real bond yield $\mathcal{R}^{\mathrm{b}}_{\infty}$ is obtained using the estimated regression model:

$$\boxed{\mathcal{R}^{\mathrm{b}}_{\infty} = \hat{\beta}_0 + \hat{\beta}_1 \mathfrak{r}_{\infty} + \hat{\beta}_2 \sigma^{\pi}_{\infty} + \hat{\beta}_3 \left(B/Y\right)_{\infty}}$$

Remark 4 *In order to facilitate the reading of this section, the numerical results of the regression models have been included in Appendix D.*

Among the key conclusions derived from regression estimates, we pay attention to the economic interpretation of the results. Coefficients of real short rates are consistent with the economic intuition: real bond yields underreact to real short rate shifts. For illustration, a 100 bps movement in the real short rate is associated with a 50 bps movement in the real bond yield on average, regardless of the geographic zone. Also, lower inflation volatility and a higher government balance on output ratio are associated with a lower real bond yield. Finally, we find a positive coefficient β_0, which is consistent with the existence of a liquidity risk premium. Our long-run forecasts exhibit interesting features. Indeed, we find that US and eurozone bond yields stabilize between 4.8% and 5.1% at the 2050 horizon, which should be associated with an annualized return of 4.3% for the US and 4% for the eurozone, taking into account the trajectory of bond yields. These figures contrast with the 9% annualized average performance posted since 1980[27], a period which was associated with structural disinflation and decreasing inflation risk. Regarding contemporaneous low bond yields across the regions, our bond yield forecasts imply mixed performances for the horizon 2010-2020.

Investment grade, high yield and emerging bonds We have chosen to regroup these three asset classes because their long-run returns are defined as the sum of the government bond return and a bond risk premium. As for government bonds, we propose to work with a bond yield, and then derive an expected return. We propose to analyze the difference s^{cr}_t between the credit bond yield and the government bond yield, and to estimate the relationship with proxies for risk premium. For the investment grade and high yield spreads, we consider the following regression model:

$$s^{\mathrm{cr}}_t = \beta_0 + \beta_1 \sigma^e_t + \beta_2 g_t + \varepsilon_t$$

where σ^e_t denotes the equity volatility and g_t is the output growth. For the emerging bond spread, the regression model becomes:

$$s^{\mathrm{cr}}_t = \beta_0 + \beta_1 \sigma^e_t + \beta_2 \left(CA/Y\right)_t + \varepsilon_t$$

where $\left(CA/Y\right)_t$ is the current account on output ratio. Then again, we deduce the long-run spread s^{cr}_{∞} and the implied long-run bond yield R^{cr}_{∞} using the relationship[28]:

$$\boxed{R^{\mathrm{cr}}_{\infty} = R^{\mathrm{b}}_{\infty} + s^{\mathrm{cr}}_{\infty}}$$

[27] Over a very long term, Dimson *et al.* (2010) find that bond excess returns over cash is about 1% on average, which is higher than our long-run forecasts, but lower than the 2% observed during the last quarter-century.

[28] For the current account, we assume a null value at the horizon 2050, knowing that emerging economies typically exhibit a positive current account as long as they converge to developed economies. For the volatility, we consider long-run historical values, which are lower than those obtained during the two last decades, with the exception of the volatility of the emerging equity markets.

Our coefficient results are consistent with the economic intuition. In particular, we find that high volatility is associated with a wider credit spread for investment grade, high yield and emerging bonds. Also, we observe that lower output growth is associated with a wider credit spread for investment grade and high yield. Lastly, for emerging bonds, the observed negative current account position (typically negative trade balance) is associated with a widening of the emerging bond spread. Results show that US and eurozone high yield returns, as well as emerging returns, are close to 10% at the 2050 horizon. These performances are quite similar to those observed over the last two decades. Also, consistent with the economic intuition, investment grade returns are lower than high yield returns, and are similar to the performances observed over the last two decades.

Table 4: Economic forecast for 10-year bond yields

	2010	2020	2030	2050
Sovereign bonds				
US	2.8%	4.9%	5.1%	5.1%
EURO	2.6%	4.5%	4.7%	4.8%
JAPAN	1.1%	3.3%	3.5%	3.6%
PACIFIC	5.5%	6.5%	6.3%	6.2%
EM	5.5%	9.4%	10.1%	10.7%
Corporate bonds				
IG US	6.5%	6.3%	6.4%	6.5%
IG EURO	3.5%	4.8%	5.0%	5.1%
HY US	7.8%	10.2%	10.3%	10.3%
HY EURO	7.8%	10.1%	10.2%	10.2%

Table 5: Expected returns of bonds

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
Sovereign bonds							
US	6.4%	11.5%	9.3%	6.1%	1.9%	3.5%	4.3%
EURO		8.4%	8.2%	5.5%	1.8%	3.2%	4.0%
JAPAN			7.3%	2.5%	0.0%	1.7%	2.6%
PACIFIC			12.5%	6.8%	5.5%	6.1%	6.2%
EM			14.2%	10.0%	5.6%	7.6%	9.0%
Corporate bonds							
IG US			8.0%	6.8%	6.1%	6.2%	6.3%
IG EURO				4.0%	3.7%	4.3%	4.6%
HY US			11.0%	7.0%	8.9%	9.6%	9.9%
HY EURO				4.0%	8.6%	9.4%	9.8%

3.2.3 Equities

We previously defined the required equity risk premium as the sum of a long-run dividend yield and the dividend growth minus the long-run bond yield. We build a 10-year rolling equity excess return by establishing the difference between the geometric mean of equity total return and the bond total return. As suggested in the previous section, we replace the dividend yield by the price earnings ratio, and also consider long-run values of price earnings ratio derived from a demographic scenario. Lastly, it is possible to replace the long-run dividend growth by the sum of the potential output growth and the long-run inflation. However, it is common to consider that this variable could be proxied by a constant when using a long-run statistical approach. Then, we estimate the equity risk premium $\mathcal{R}_t^{\mathrm{e}}$ using the following regression model:

$$\mathcal{R}_{t+10}^{\mathrm{e}} = \beta_0 + \beta_1 \,\mathrm{PE}_t + \beta_2 R_t^{\mathrm{b}} + \varepsilon_t$$

where $\mathcal{R}_{t+10}^{\mathrm{e}}$ is the 10-year forward equity excess return, PE_t is the price earning ratio and R_t^{b} is the 10-year bond yield. Then, we define the long-run equity returns as follows:

$$\boxed{R_{\infty}^{\mathrm{e}} = R_{\infty}^{\mathrm{b}} + \mathcal{R}_{\infty}^{\mathrm{e}}}$$

where $\mathcal{R}_{\infty}^{\mathrm{e}}$ is defined by the long-run relationship[29]:

$$\boxed{\mathcal{R}_{\infty}^{\mathrm{e}} = \hat{\beta}_0 + \hat{\beta}_1 \,\mathrm{PE}_{\infty} + \hat{\beta}_2 R_{\infty}^{\mathrm{b}}}$$

The results of the estimation are consistent with the economic intuition[30]. Indeed, higher price earnings ratios and higher bond yields are associated with lower equity risk premiums, and we also mention that the positive constant is interpreted as the positive impact of long-run output growth. Our long-run regression forecasts show that long-run equity risk premium forecasts for 2050 are close to long-run historical standards, with the exception of Japan. Indeed, we obtain an equity risk premium of 4.8% for the US and the eurozone, and 3% for Japan[31], while the historical values stand between 4% and 5%, depending on the methodology used and the period. However, our results are higher than the values obtained during the last three decades, which saw a fall of equity risk premiums on the back of the dot.com and subprime crises. We also compare our forecasts with those derived from a non-statistical approach, and find quite similar values. Another result is that emerging risk premiums are higher than those in developed countries, which is consistent with the economic intuition. However, we notice that the emerging equity risk premium collapses if we replace the US bond yield with the local sovereign bond yield. In terms of equity returns, the addition of long-run bond returns lifts the US and European returns to 9% and only 5.6% for Japan. Regarding small caps, we obtain a long-run return of 12%, assuming a 3% excess return over large caps, consistent with historical standards.

3.2.4 Other asset classes returns

Commodities We consider the performances of the commodity price index rather than the total return index, in order to extract the contribution of the cash return. Following our definition of commodity returns, these commodity prices variations are assimilated to the variations in a commodity risk premium. We proxy the long-run commodity risk premium

[29] The long-run price earnings ratio PE_{∞} is calculated on the basis of the average of the price earnings ratio PE_t for the last thirty years.

[30] The case of the emerging equity market is more ambiguous due to the lack of historical data. We therefore propose to use the pure required equity risk premium instead of a regression. This equity risk premium is defined as the sum of the potential output growth, the long-run inflation and the long-run dividend yield, minus the US bond yield.

[31] Regarding the 2020 horizon, our forecast shows higher equity risk premiums across the regions in comparison to 2030 and 2050, these results being explained by contemporaneously low bond yields and price earnings ratios.

Table 6: Expected returns of equities

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
US	2.8%	15.2%	18.3%	−1.2%	9.2%	8.4%	9.1%
EURO	8.1%	12.8%	16.8%	0.4%	9.7%	8.2%	8.7%
JAPAN	2.6%	20.1%	−0.5%	−3.4%	8.8%	4.9%	5.6%
PACIFIC			14.0%	8.7%	14.7%	9.1%	9.5%
EM			8.4%	14.0%	10.7%	10.4%	10.8%
Small cap			9.3%	5.9%	12.2%	11.4%	12.1%

by the share of emerging countries in the world output and world output growth. The intuition is that the commodity risk premium $\mathcal{R}_t^{\mathrm{co}}$ should be positively correlated to these two variables. The regression model is :

$$\mathcal{R}_t^{\mathrm{co}} = \beta_0 + \beta_1 \Delta \left(Y^{\mathrm{EM}} / Y^{\mathrm{W}} \right)_t + \beta_2 g_t^{\mathrm{W}} + \varepsilon_t$$

where $\Delta \left(Y^{\mathrm{EM}} / Y^{\mathrm{W}} \right)_t$ represents the change of emerging output over world output ratio, and g_t^{W} represents the world output growth. We deduce that the long-run commodity total return is:

$$\boxed{R_\infty^{\mathrm{co}} = r_\infty + \mathcal{R}_\infty^{\mathrm{co}}}$$

where the long-run commodity risk premium $\mathcal{R}_\infty^{\mathrm{co}}$ is obtained using the relationship:

$$\boxed{\mathcal{R}_\infty^{\mathrm{co}} = \hat{\beta}_0 + \hat{\beta}_1 \Delta \left(Y^{\mathrm{EM}} / Y^{\mathrm{W}} \right)_\infty + \hat{\beta}_2 g_\infty^{\mathrm{W}}}$$

Our regression results show that the sign of coefficients are consistent with the economic intuition. Then, we derive long-run commodity price returns by considering our long-run output growth forecasts, and a forecast derived from the IMF and Goldman Sachs for the share of emerging economies in the world output. Over the long-run, we obtain a commodity risk premium of 4.2%, this result being justified by the assumption of an increase in the share of emerging economies in world output from roughly 40% to 70% at the 2050 horizon. Finally, we derive long-run commodity total returns by adding the long-run cash return, and obtain roughly 9%.

Alternative asset classes Our long-run return forecasts for alternative asset classes combine academic works and historical figures. For private equity, Wilshire (2009) estimates the risk premium for private equity over stocks at 3%, which then produces a long-run return of 12.1%. A literature review by Norman *et al.* (1995) summarizes all findings regarding real estate. More than half of the consulted literature in their paper reports a lower return for real estate compared to stocks. Our long-run forecast considers a long-run underperformance of 1%, which then implies a long-run real estate return of 8.1%. Finally, the literature reports extensive information on biases in hedge fund indices (survivorship bias, self-reporting bias, side pockets, etc.). It is also not possible to use their performance to estimate a long-run historical return. In what follows, we consider a simple rule. We assume that the average portfolio of the hedge fund industry is a well-diversified portfolio of traditional asset classes (or an alternative beta portfolio). We therefore deduce the expected return from the volatility of this diversified portfolio and the assumption that the long-run Sharpe ratio of hedge funds is 0.5. Finally, we add an alpha of 1.2% (Roncalli and Teïletche, 2008).

Table 7: Expected returns of alternative investments

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
Commodity	3.9%	1.1%	4.0%	5.5%	8.4%	8.6%	9.0%
Hedge funds			10.9%	4.0%	7.1%	7.3%	7.4%
Real estate			−0.9%	3.8%	8.2%	7.4%	8.1%
Private equity				2.5%	12.5%	11.7%	12.1%

3.3 Volatility and correlation

We use an Ornstein-Uhlenbeck model to estimate the long-run volatility and then adjust the figures to take into account macro-economic volatility and the inflation level. Our long-run forecasts are much lower than the values obtained during the last decade, which has been marked by two severe recessions. For the long term, we expect low levels of potential output growth and inflation in comparison to historical standards. In this context, US and eurozone long-run equity volatility comes to 15% against 19% during the last decade. For emerging economies, we expect a convergence of equity volatility towards the developed one, due to increasing diversification inside these markets. Regarding bonds, long-run volatility comes to 5% for US and eurozone sovereign bonds, also in the low range of historical standards. The case of emerging bonds is particularly interesting because we expect a progressive decrease of the volatility from 15% in 2020 to 10% in 2050. Indeed, the credit quality of these countries is expected to increase as they converge to the developed economies. Lastly, long-run volatility on alternative asset classes is expected to remain in the range of historical values, as for equities, with the exception of commodities. Indeed, the long-run volatility on commodities increases from 25% during the last decade to 30%. The growing role of emerging economies combined with the growing scarcity of resources should be associated with larger swings in commodity prices.

Our long-run correlation forecasts are similar to the historical values obtained during the last decade. Specifically, key assumptions are related to the low level of long-run inflation and the reinforcement of the globalization. The forecast of low long-run inflation is associated with a negative long-run correlation between equity and sovereign bonds, which is slightly negative at −20% on average for developed countries. In that scenario, rising (resp. falling) bond yields should be associated with rising (resp. falling) equities, as investors flight (resp. rally) from the bond market to rally (resp. flight) the equity market. Regarding bonds, investment grade bonds remain more positively correlated to sovereign bonds, while high yields are more positively correlated to equities. Lastly, the alternative asset classes also exhibit a long-run correlation with traditional asset classes, in spite of their status as uncorrelated assets. In particular, the long-run correlation between commodities and equities stands at 20% on average, but is null with sovereign bonds. Regarding the correlation between geographic zones for a specific asset class, the globalization of financial markets and the world economy is associated with a structural reinforcement of correlations. For example, the long-run correlation between US and eurozone equity markets is 80%, having doubled over the past two decades. The Japanese and emerging equity markets remain less correlated with the US and European equity markets at 70%, but their correlation has also doubled over the past two decades. Regarding the government bond asset class, the correlation between the US and European markets comes to 80% as for equities, while the Japanese market should remain the less correlated. Regarding the corporate bond markets, correlations remain close to levels over the past decade. For illustration, the correlation between the US and European high yield markets is 80%, and the EM bond market should remain less correlated. Between

Table 8: Expected risks

	1975-1979	1980-1990	1990-2000	2000-2010	2020	2030	2050
Sovereign bonds							
US		8.7%	6.5%	7.7%	5.0%	5.0%	5.0%
EURO		7.7%	5.1%	5.0%	5.0%	5.0%	5.0%
JAPAN		7.2%	5.4%	4.0%	8.0%	8.0%	8.0%
PACIFIC					15.0%	15.0%	15.0%
EM			7.2%	16.8%	15.0%	12.0%	10.0%
Corporate bonds							
IG US					7.0%	7.0%	7.0%
IG EURO					7.0%	7.0%	7.0%
HY US			5.4%	8.9%	8.5%	8.5%	8.5%
HY EURO				17.0%	10.0%	10.0%	10.0%
Equity							
US	11.6%	15.2%	12.6%	19.9%	15.0%	15.0%	15.0%
EURO	8.1%	11.0%	12.1%	18.5%	15.0%	15.0%	15.0%
JAPAN	8.0%	12.6%	18.4%	21.3%	15.0%	15.0%	15.0%
PACIFIC			14.9%	14.8%	17.0%	17.0%	17.0%
EM			18.0%	17.5%	18.0%	18.0%	18.0%
Small cap			13.0%	18.1%	17.0%	17.0%	17.0%
Alternative investments							
Commodity			17.0%	25.0%	30.0%	30.0%	30.0%
Hedge funds			7.0%	6.0%	8.0%	8.0%	8.0%
Real estate			15.5%	19.1%	15.0%	15.0%	15.0%
Private equity				30.4%	25.0%	25.0%	25.0%

alternative asset classes, long-run correlation remains low and close to historical standards. Interestingly, certain alternative assets exhibit more correlation with traditional asset classes than with other alternative assets[32].

Table 9: Correlation matrix of bonds

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	US	100%							
(2)	EURO	80%	100%						
(3)	JAPAN	30%	30%	100%					
(4)	EM	10%			100%				
(5)	IG US	60%	40%	20%	50%	100%			
(6)	IG EURO	20%	30%		30%	60%	100%		
(7)	HY US	−20%	−20%		60%	60%	40%	100%	
(8)	HY EURO	−30%	−20%		50%	40%	40%	80%	100%

[32] For example, we obtain a correlation of 50% between hedge funds and high yield bonds or equities.

Table 10: Correlation matrix of equities

		(9)	(10)	(11)	(12)	(13)	(14)
(9)	US	100%					
(10)	EURO	90%	100%				
(11)	JAPAN	70%	60%	100%			
(12)	PACIFIC	80%	80%	60%	100%		
(13)	EM	70%	70%	70%	80%	100%	
(14)	Small cap	80%	80%	70%	80%	80%	100%

Table 11: Correlation matrix of alternative investments

		(15)	(16)	(17)	(18)
(15)	Commodity	100%			
(16)	Hedge funds	40%	100%		
(17)	Real estate	10%	30%	100%	
(18)	Private equity	10%	40%	10%	100%

4 Strategic asset allocation in practice

In this section, we use our previous estimates of expected returns, volatilities and correlations to illustrate three examples of strategic asset allocation. In what follows, we assume that the time horizon of the investor is 2050 and we consider the framework of the Markowitz mean-variance analysis. Of course, other models may be used as illustrated in the first example[33].

4.1 Building a strategic equity portfolio

Let us consider an institutional investor who wants to build an equity portfolio. The first idea to define this portfolio is to consider the weightings of a global diversified equity index. If we consider the MSCI world index, the portfolio is composed of 47% US stocks, 26.5% European stocks, 8.5% Japanese stocks, etc. If we consider the previous figures in terms of risk premiums, volatilities and correlations, this portfolio has an expected return of 9.0% and an ex-ante volatility of 14.2%. The long-term Sharpe ratio is also close to 0.20. We may compare this portfolio with other solutions given by portfolio allocation methods. Results are reported in Table 12.

If we consider the Markowitz framework of mean-variance (MV) optimization, we have to decide a target for the expected return of the strategic equity portfolio. If the expected return is set to 9.2%, the weight of US stocks declines compared to the corresponding weight in the MSCI index whereas the level of European stocks remains almost the same. The weight of emerging markets stocks increases significantly because of their attractive expected return. One problem with the mean-variance framework is that it produces an optimized portfolio from a mathematical point of view, but one which is generally unsatisfactory from a practical point of view (Michaud, 1989). Indeed, if these portfolios present positive properties in terms of volatility diversification, they generally conduct on concentrated portfolios and present a

[33]However, some of them may require additional parameters than expected returns, volatilities and correlations. In this case, one has to be careful to define these new parameters in a coherent way in terms of economic theory.

Table 12: Strategic equity portfolio

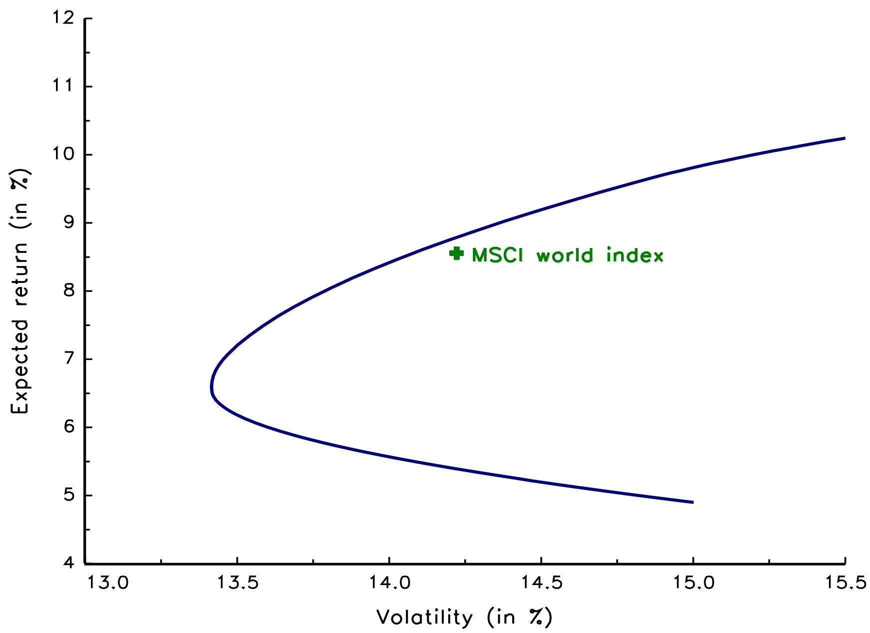
		Without constraints			With constraints		
	MSCI	MV	TE	BL	MV	TE	BL
US	47.0%	40.2%	51.7%	48.5%	39.7%	51.4%	47.3%
EURO	26.5%	26.6%	24.0%	24.4%	26.0%	22.7%	24.1%
JAPAN	8.5%	6.9%	3.5%	3.9%	6.9%	4.3%	4.3%
PACIFIC	3.5%	0.0%	2.4%	4.6%	1.8%	2.2%	5.3%
EM	14.5%	26.3%	18.5%	18.6%	25.7%	19.5%	19.1%
ER	9.0%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%
VOL	14.2%	14.4%	14.4%	14.4%	14.4%	14.4%	14.4%
TR		1.3%	0.6%	0.6%	1.3%	0.6%	0.7%
IR		0.19	0.45	0.41	0.19	0.43	0.37

lack of diversification in terms of weightings. It may be therefore be preferable to use the tracking-error optimization (TE) method. In this case, we define a benchmark and an ex-ante tracking risk (the volatility of the tracking error), and we maximize the expected return of the portfolio under the constraint of a tracking risk level with respect to the benchmark. In our example, the benchmark is the MSCI world index. We consider a level of tracking risk (TR) which matches an expected return of 9.2%. Because the tracking risk is low and equal to 60 bps, we verify that the solution is close to the benchmark portfolio. The third solution to define a strategic allocation is to consider the Black-Litterman model (BL). Compared to the MV solution, the BL solution is more balanced in terms of weightings. This explains why the Black-Litterman model produces better information ratios than the Markowitz model. All these results have been obtained without imposing constraints on the weightings. Generally, one may prefer to impose some bounds on the weightings in order to obtain a solution which does not differ so much from the benchmark. If we consider a constraint of 50% maximum deviation between the weightings of the optimized portfolio and those of the benchmark, we obtain the results given in Table 12. We observe the main differences between results with and without constraints for the mean-variance model. The Black-Litterman model appears to be more robust.

If we consider results based on the Black-Litterman approach, we find an increase of the weight of pacific and emerging markets stocks, a decrease of the weight of Japanese and eurozone stocks and a stability of the weight of US stocks. Roots of these results are in the fundamental assumptions proposed in our scenario on potential growth. Indeed, we expect a deceleration of potential output growth across regions, but less pronounced in the US and emerging markets countries than in eurozone and Japan. As a key determinant of the potential output growth, the active population growth is expected to decelerate in the US and emerging markets countries but decrease in Europe and Japan. Therefore, we could directly derive the differences in weightings from the assumption on the dynamic of the working population. In the case of the Markowitz approach, the results contradict this assumption. Therefore the consistency between the economic scenario and the results of portfolio theory is a key point to choose one approach.

Remark 5 *In this example, the benchmark is already close to the efficient frontier as shown in Figure 3. It explains that optimized portfolios and the MSCI world index have a similar risk/return profile.*

Figure 3: Efficient frontier of the equity portfolio



4.2 The equity/bond allocation policy in a strategic asset allocation

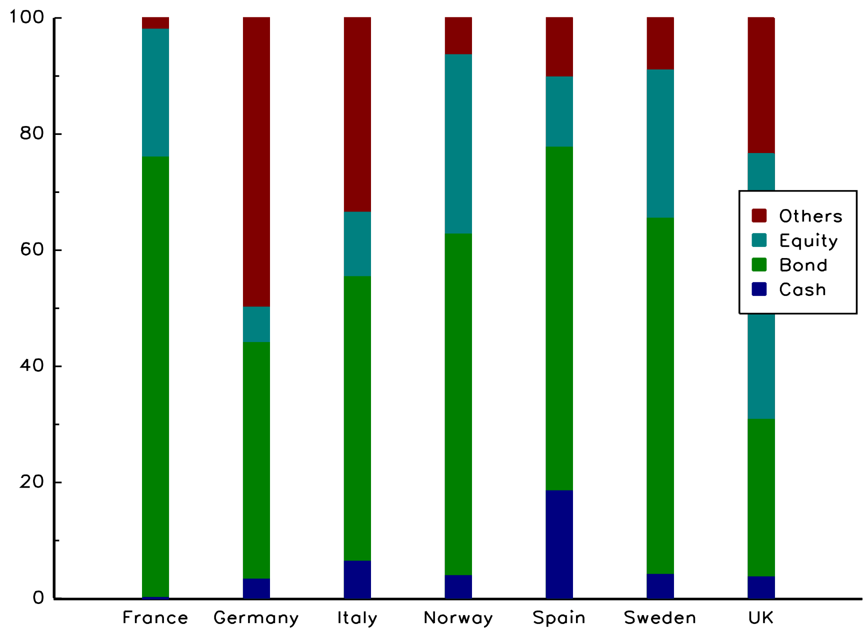
The equity/bond allocation problem is a challenge for pension funds and long-term investors. Until the dot.com crisis, many long-term investors overweighted equities. Thus Ambachtsheer (1987) suggests that the 60/40 equity/bond allocation is a good asset mix. This "cult of the equity" is particular true in the case of UK pension funds (Sutcliffe, 2005). After the turn of the millennium, the Boots Pension Scheme caused surprise by announcing a move to a 100% bond portfolio in July 2001. The question of the equity/bond asset mix policy was partially forgotten during the equity bull market between 2003 and 2007. However, since the financial crisis, it remains a key issue for long-term investors.

In Figure 4, we have reported the average pension fund asset allocation for some European countries. We notice some big differences between countries. For example, the weight of bonds is equal to 76% in France whereas it represents only 27.1% in the UK. Another example is the weight of equities, which stands at 12.1% in Spain and 45.8% in the UK. If we calculate the ratio of the weights of cash, bills and bonds in the portfolio, we obtain an average of 55.3% for European countries. However, we observe considerable disparity between countries, with levels ranging between 31% and 78%. If we consider pension funds within a single country, we may also observe some differences, but they are smaller than those between countries. There is therefore no consensus on the asset mix policy between bonds and equities.

In Figure 5, we report the efficient frontier when considering only developed bonds or only developed equities (US, eurozone and Japan). We notice that the combination of bonds and equities allows for the construction of portfolios that are more efficient and more diversified[34].

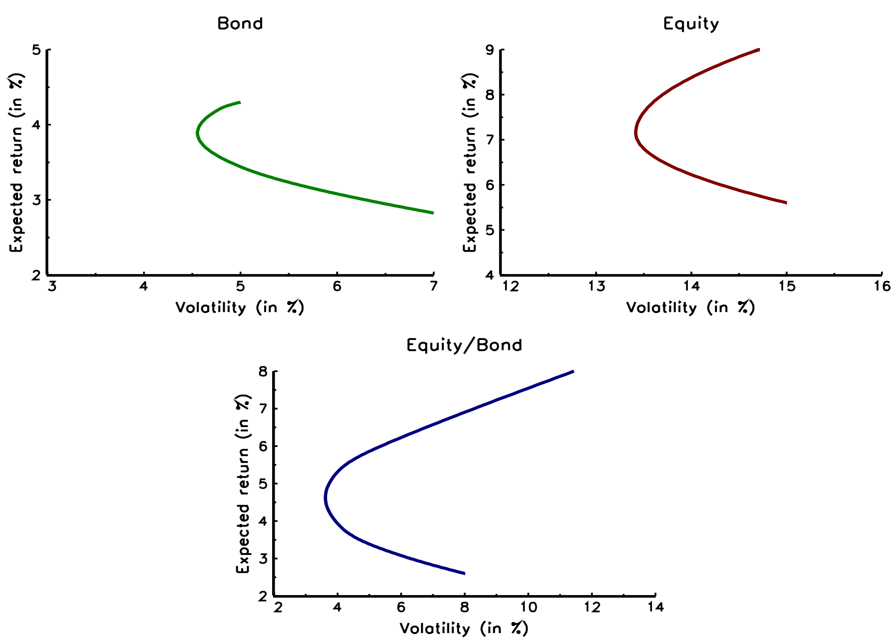
[34]Indeed, we obtain a strong diversification effect. This is consistent with our economic scenario and our

Figure 4: Average allocation of European pension funds



Source: Investment & Pensions Europe, September 2010.

Figure 5: Efficient frontier for the equity/bond allocation policy



In Table 13, we indicate the allocation between bonds and equities with respect to the risk aversion of the investor, which is measured by the ex-ante volatility of the portfolio. For example, if the investor targets a volatility of 5%, the optimal allocation is 65% in bonds and 35% in equities. A balanced portfolio of 50% in bonds and 50% in equities corresponds to an ex-ante volatility which is equal to 7.0%. We have also reported the optimal portfolio in the case of CARA utility and a risk-aversion parameter γ equal to 5, which is considered standard for long-term investors. In this case, the optimal allocation is about 70% in bonds and 30% in equities with a volatility of 4.6%. We also notice that the Sharpe ratio of optimal portfolios is about 0.3, which is not very high.

Table 13: Weights of equity/bond asset mix

VOL	Weights		ER
	Bond	Equity	
3.6%	82.8%	17.2%	4.6%
4.0%	76.2%	23.8%	5.3%
4.5%	69.5%	30.5%	5.6%
($\gamma = 5$) 4.6%	68.1%	31.9%	5.7%
5.0%	64.5%	35.5%	5.9%
5.5%	60.5%	39.5%	6.1%
6.0%	56.9%	43.1%	6.2%
8.0%	43.4%	56.6%	6.9%
10.0%	30.5%	69.5%	7.5%
12.0%	18.0%	82.0%	8.2%
15.0%	0.0%	100.0%	9.1%

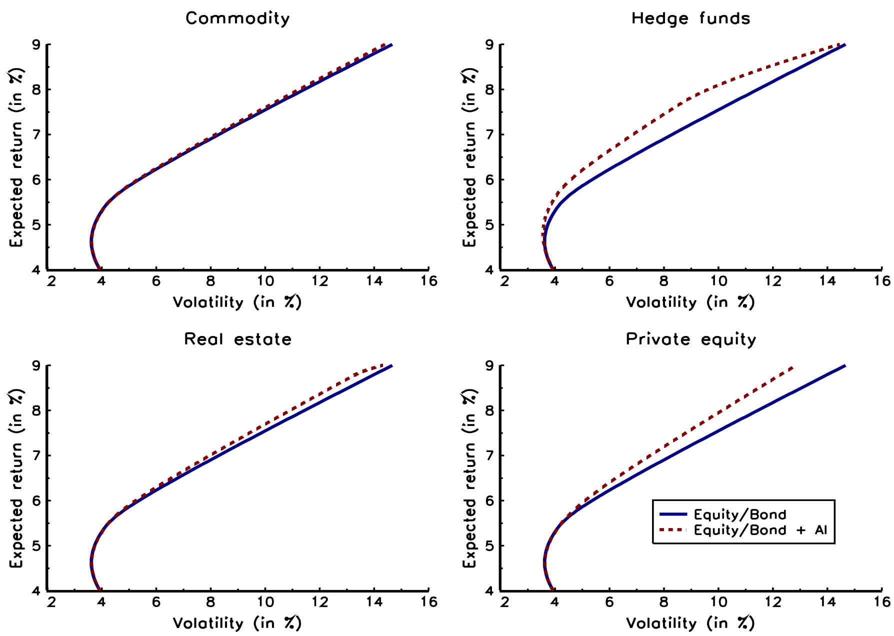
4.3 The place of alternative investments in a long-term portfolio

Another big issue faced by long-term investors is the place of alternative investments in their strategic asset allocation. In Figure 6, we consider the previous mean-variance framework with bonds and equities by adding the alternative asset classes. We assume that alternative investments are not correlated with developed bonds. The correlation with developed equities is set to 30% for commodities, 50% for hedge funds, real estate and private equity. We notice that alternative investments particularly improve the risk/return profile when the risk of the portfolio is high. This is particularly true for private equity and hedge funds and in a less pronounced way for commodities and real estate.

In Table 14, we report the weights of optimized portfolios when the universe corresponds to developed bonds and equities (US, eurozone and Japan) and the four alternative asset classes (commodity, hedge funds, real estate and private equity). In order to be conservative, we assume that the four alternative investments asset classes are perfectly correlated whereas the correlation is 50% with equities. We notice that incorporating alternative investments improved the portfolio return for a given level of risk aversion. This means that one may achieve an expected return for the portfolio with a lower volatility. For example,

assumption on long-run inflation. We forecast that inflation should remain low in comparison to historical standards. In this environment, the correlation between equities and bonds should remain low or negative. In the case where our economic scenario would imply higher inflation rates, less diversification effects would be expected.

Figure 6: Efficient frontier with alternative investments



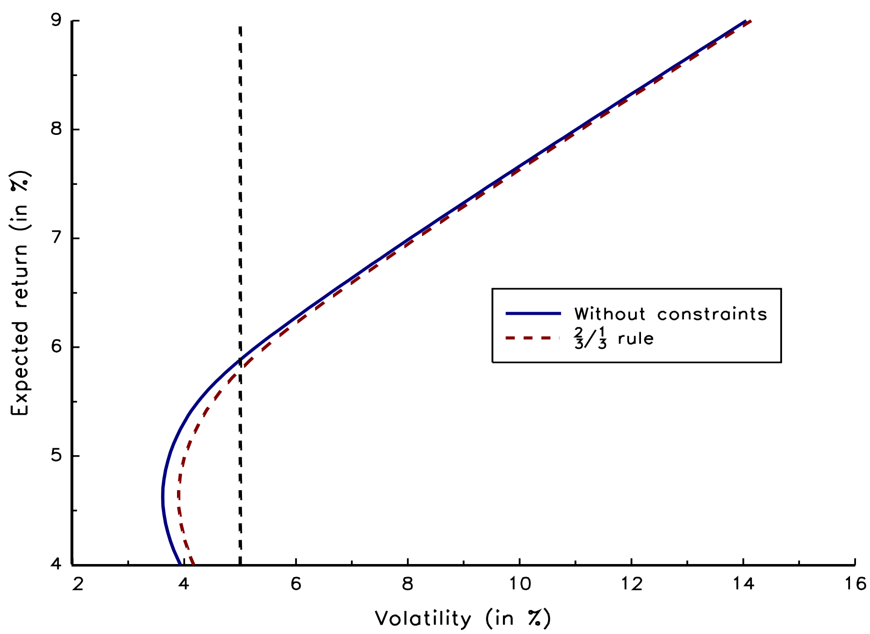
if the expected return is equal to 6.5% (resp. 8%), the gain in volatility amounts to 0.7% (resp. 2.5%). If we compare results in Tables 13 and 14, we notice also that the weights of bonds is relatively the same. Alternative investments can therefore substitute equities. In Figure 7, we compare the risk/return profile of an optimized portfolio in the case without constraints (corresponding to results in Table 14) and in the case where the weight of alternative investments must be half the weight of equities. This means that equities and alternative investments represent respectively $2/3$ and $1/3$ of the risky part of the portfolio. We note that this $2/3 - 1/3$ rule gives similar results to the case without constraints where the volatility of the optimized portfolio is above 5%. This result contradicts market practices. Generally, alternative investments represent a smaller proportion of the risky part of the portfolio, e.g. less than 10%. There are several explanations. Firstly, long-term investors

Table 14: Weights of bond, equity and alternative investments

VOL	Weights			ER
	Bond	Equity	AI	
3.6%	82.8%	17.2%	0.0%	3.9%
4.0%	75.9%	22.3%	1.9%	4.6%
4.5%	69.1%	26.0%	4.9%	5.0%
5.0%	63.7%	28.9%	7.5%	5.3%
5.5%	59.1%	31.3%	9.6%	5.5%
6.0%	54.9%	33.5%	11.6%	5.7%
8.0%	40.0%	42.1%	17.9%	6.5%
10.0%	26.0%	50.4%	23.6%	7.2%
12.0%	12.3%	58.6%	29.1%	7.9%

have a marked preference for equities because they are easier to understand. Another explanation is that alternative investments may present some hidden risks like kurtosis risk (Amin and Kat, 2003). The third explanation concerns liquidity. As alternative investments are less liquid than equities, it is more difficult to implement tactical asset allocation with these asset classes. Nevertheless, even if we take into account these drawbacks, the place of alternative investments in strategic asset allocation is today certainly under-estimated.

Figure 7: Illustration of the $^2/_3 - ^1/_3$ rule



5 Conclusion

This paper proposes a comprehensive methodology for building long-run assumptions on asset returns, in order to derive consistent strategic allocations. This approach presents two key characteristics. The first is an entire reference to the economic and asset pricing theories for formalizing the behavior of asset returns, while the second is the use of limited statistical tools to calibrate the long-run relationships and then forecast long-run asset returns in a systematic way. The study on asset classes covering the main geographic areas provides several key results. Regarding government bonds, forecasts of long-run returns are low in comparison to the last quarter-century. We obtain an annualized return of 4.3% for US bonds and 4.0% for eurozone bonds at the 2050 horizon, a figure contrasting with the 9.0% annualized performance posted since 1980. Regarding equities, long-run risk premiums forecasts for 2050 are close to historical standards at 4.8% for US and eurozone equities, with the exception of Japan which comes in lower at 3.0%. However, these forecasts are higher than those obtained during the last three decades, mainly due to the dot.com and subprime crises. We obtain an annualized return of 9.1% for the US, 8.7% for the eurozone, and 5.6% for Japan.

Then, we illustrate strategic allocations derived from our long-run asset returns forecasts, subject to different optimizing methodologies and different risk targets. In particular, we begin by considering the case of a pure equity portfolio, then we work on an equity/bond portfolio, and finally question the place of alternative investments. We conclude from this analysis that the equity/bond asset mix in a strategic portfolio is still a widely debated topic, with proportions differing greatly between countries, especially in Europe. We show that institutional investors sometimes appear myopic, forgetting this asset mix question during the 2003-2007 equity bull market, and then reconsidering it as a key element after the recent financial crisis. Regarding the place of alternative investments, we also find that allocations probably remain under-estimated, although this can be explained by inherent risks associated with these investment vehicles.

An important question that arises nowadays is the choice of the relevant horizons for the main investment decisions in a strategic portfolio. Indeed, long-term investors are probably confused about the definition of strategic and tactical asset allocations. Most of them, willing to tackle the issue of the large swings on the financial markets during the recent crisis, decided to reduce the horizon for their strategic allocation revisions and to implement short-term money management on a small part of their portfolios, which they call tactical allocations. We think that this framework is not consistent with economic theory. There is still a need for a strategic portfolio that is designed to be consistent with the long-term path of the economy. This should not be revised unless major structural breaks in the economy occur, like productivity shocks or changes in demography or monetary policy. By definition, relevant horizons for such investments are much longer than the business cycle. Nevertheless, there is still room for the implementation of a dynamic asset allocation. Merton (1973) and Lucas (1978) paved the way for this by showing that risk premiums vary over time[35]. Over the past two decades, Barberis (2000) and Campbell and Viceira (2002) among others, showed in which proportions asset weights should change in accordance with time-varying risk premiums. Then, from our point of view, tactical asset allocation is justifiable as long as it is designed to hedge against unwanted variations of the economic cycle. This means that the tactical investment horizons range from a few months to two or three years. All other shorter-term bets in the portfolio should be considered as a way to time the market. Then, we could imagine a hybrid investment policy for long-term investors, where the strategic allocation could be obtained as a reference portfolio, while the tactical allocation would justify adjustments of this reference portfolio within the business cycle. In this framework, confusions between market timing and tactical asset allocation are avoided[36].

[35] See Darolles *et al.* (2010) for a survey on the subject.

[36] This does not exclude market timing from a long-term investment policy. It only means that TAA is fundamentally an economic asset allocation.

A Notations

Table 15 describes the different variables used in this paper. Let X_t be an economic or financial variable. The corresponding long-run steady state or value will be denoted by X_∞. In order to distinguish nominal and real (or excess) returns, we use a calligraphic symbol for the real return. Parameters used in the different models are in lowercase whereas coefficients of regression models are denoted by the Greek letter β and estimates correspond to $\hat{\beta}$.

Table 15: Notation and description of the economic and financial variables

Notation	Description	Formula
π_t	Inflation	
g_t	Output growth	
σ_t^π	Inflation volatility	
Y_t	Output	
C_t	Consumption	
I_t	Investment	$I_t = Y_t - C_t$
K_t	Capital	
L_t	Labor force	
$(B/Y)_t$	Ratio of the government balance on output	
$(CA/Y)_t$	Ratio of the current account on output	
r_t	Short-term interest rate	
$\mathfrak{r}_t$	Real interest rate	$\mathfrak{r}_t = r_t - \pi_t$
$R_t^{(m)}$	Interest rate of maturity m	
S_t	Spot exchange rate	
$F_t^{(m)}$	Forward exchange rate of maturity m	
R_t^{b}	Nominal sovereign bond yield	
$\mathcal{R}_t^{\mathrm{b}}$	Real sovereign bond yield	$\mathcal{R}_t^{\mathrm{b}} = R_t^{\mathrm{b}} - \pi_t$
R_t^{cr}	Nominal credit bond yield	
s_t^{cr}	Credit bond spread	$s_t^{\mathrm{cr}} = R_t^{\mathrm{cr}} - R_t^{\mathrm{b}}$
R_t^{e}	Equity return	
$\mathcal{R}_t^{\mathrm{e}}$	Equity excess return	$\mathcal{R}_t^{\mathrm{e}} = R_t^{\mathrm{e}} - R_t^{\mathrm{b}}$
R_t^{co}	Commodity total return	
$\mathcal{R}_t^{\mathrm{co}}$	Commodity excess return	$\mathcal{R}_t^{\mathrm{co}} = R_t^{\mathrm{co}} - r_t$
σ_t^{e}	Volatility of equity returns	
P_t	Price	
D_t	Dividend	
DY_t	Dividend yield	$\mathrm{DY}_t = D_t/P_t$
PE_t	Price earnings ratio	

B Macro-economics pillars

B.1 Modeling economic growth

B.1.1 The Solow growth model

In its original form, the Solow model is a pure production model of a closed economy where the output Y_t is a production function depending on capital, labor and technology. This function is assumed to exhibit constant returns to scale (CRTS) and decreasing marginal productivity of capital. The model attempts to capture the dynamics of economic growth, stating that it converges to an equilibrium where the determinants that create additions to capital and those that make capital decrease exactly offset each other. The only endogenous variable of the model is the capital K_t. This means that variations of the output are related only to endogenous variations of capital. The working population is represented by the labor force L_t and productivity gains are due to some technological progress A_t. Solow (1956) assumes that these two exogenous variables grow at constant rates η and γ. He does not assume any model for the consumer. All the consumers own all the firms, their income is provided by the output of the firms, and the proportion of output that is not consumed is re-invested in the firms next period.

Thus, we have:

$$Y_t = F(K_t, A_t L_t)$$

where $A_t L_t$ represents the effective labor and F is the production function. Given the CRTS condition, we can write:

$$\frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}, 1\right) = f\left(\frac{K_t}{A_t L_t}\right)$$

We denote $y_t = Y_t / (A_t L_t)$ and $k_t = K_t / (A_t L_t)$ respectively the output per effective worker and capital per effective worker. Then, the model relies on the following equations.

1. The production function implies that, at any time, output per effective worker depends only on capital per effective worker[37]:

   $$y_t = f(k_t)$$

2. We assume that the investment I_t which is defined by the difference between the output Y_t and the consumptions C_t is a constant proportion of the output Y_t:

   $$I_t = Y_t - C_t = sY_t$$

   with s denoting the savings rate. The rule of accumulation of capital then drives the dynamics of the capital:

   $$\frac{\mathrm{d}K_t}{\mathrm{d}t} = I_t - \delta K_t$$

   where δ is the constant rate of capital depreciation.

[37] It is also assumed that $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$. This last condition ensures that the marginal productivity of capital is decreasing. Finally, the function f is defined so that it verifies Inada conditions, that is $\lim_{k \to 0} f'(k) = +\infty$ and $\lim_{k \to +\infty} f'(k) = 0$.

3. The following two equations characterize the evolution of effective labor, stating that population and technological progress grow at exogenous and constant rates:

$$\begin{aligned}\frac{\mathrm{d}L_t}{\mathrm{d}t} &= \eta L_t \\ \frac{\mathrm{d}A_t}{\mathrm{d}t} &= \gamma A_t\end{aligned}$$

Using the previous relationships, the derivation of the dynamics of the capital per effective worker comes easily:

$$\frac{\mathrm{d}k_t}{\mathrm{d}t} = sf(k_t) - (\delta + \eta + \gamma) k_t$$

meaning that the variation of capital per unit of effective labor is equal to the actual investment in physical capital per unit of effective labor minus the effective depreciation of capital per unit of effective labor. Due to Inada conditions and given specific initial conditions for capital, labor and technology, the amount of capital per unit of effective labor converges to a unique $k^\star$, which is the solution to the dynamics equation $\mathrm{d}k_t/\mathrm{d}t = 0$. k^* is a function of the exogenous parameters s, η, γ, and δ and we note $k^\star = k^*(s, \eta, \gamma, \delta)$. This equilibrium level is called the steady-state level of capital, at which capital per effective worker remains constant over time. It is reached when actual investment is equal to effective depreciation. At the steady-state, we have then:

$$sf(k^\star) = (\delta + \eta + \gamma) k^\star$$

We know that effective labor grows at the rate $\eta+\gamma$. We can deduce that, at the steady-state, the quantity of capital also grows at the rate $\eta + \gamma$. So the aggregate output rate of growth is $\eta + \gamma$, as a consequence of the CTRS property of the production function. It is worth noting that capital per effective unit of labor, and thus output per effective unit of labor are constant in the steady-state. Therefore, one important consequence of the Solow model is that the economy should converge towards an equilibrium in which economic growth is equal to the sum of working population growth and technological progress.

B.1.2 Effects of the exogenous savings rate on consumption

At this stage, we note again that the working population and the technological progress grow at constant rates and the savings rate is assumed to be fixed and exogenous. However, this does not mean that we cannot measure the effect of instantaneous changes of these variables on the steady-state of the economy. For example, we know that an increase in the savings rate s implies an increase in actual investment I_t. Consequently, the capital must grow to a new steady-state value that is higher than the previous one, and in transition, output per effective unit of labor growth also increases. But the steady-state of output growth is independent of the level of s. So, even if an increase in the rate of savings permanently raises the steady-state levels of capital and output per effective worker, the increase in output per effective worker growth is only temporary[38].

Yet, we have not addressed the question of welfare. Social welfare certainly depends on consumption rather than output. In the Solow model, the consumption rate is equal to $c_t = (1 - s) f(k_t)$. Therefore, an instantaneous increase in the savings rate will cause consumption to decrease at first, but then it will increase again as a result of the rise in output. At the new equilibrium, it might be greater or smaller than before. We study

[38] In the Solow model, only changes in technological progress have a permanent impact on economic growth.

the effect of changes in the savings rate on consumption at the steady-state, by deriving consumption in the steady-state $c^\star$ with respect to s. We have $c^\star = f(k^\star) - (\delta + \eta + \gamma) k^\star$ and $\partial_s c^\star = (f'(k^\star) - (\delta + \eta + \gamma)) \cdot \partial_s k^\star$. We know that $\partial_s k^\star > 0$. Also, from Inada conditions, the quantity $f'(k^\star) - (\delta + \eta + \gamma)$ is positive for small values of $k^\star$ and negative for large values of $k^\star$. Thus, everything else being equal, steady-state consumption admits a maximum with respect to the savings rate, which is the solution to $\partial_s c^\star = 0$. The highest possible level of consumption in the steady-state is also attained at $f'(k^\star) - \delta = \eta + \gamma$. At equilibrium, consumption per effective unit of labor is at a maximum when the marginal product of capital net of depreciation is equal to the growth rate of of the economy.

B.1.3 The Golden rule of capital accumulation

Historically, the term *Golden rule* is an ethical code that is a base concept of human rights and that can be formulated as follows:

> *"Treat others according to how you would like others to treat you"*.

Applied to our economic context, the terms "you" and "others" refer to the actual and the next generation respectively. The underlying idea is that if we (more precisely, the "government") are concerned about social welfare, how should we act so that ours is identical to the next generations? Probably, by maximizing consumption per capita at the equilibrium of the economy. We introduce the quantity $e_t = \partial_k y_t / (y_t / k_t)$ which measures the elasticity of the output with respect to the capital. In the steady-state, we have:

$$e^\star = \frac{s}{\delta + \eta + \gamma} m^\star$$

where $m^\star = f'(k^\star)$ is the marginal product of capital. Thus, the savings rate that the government should try to attain is the one that maximizes consumption in the steady-state, and hence, is equal to $e^\star$. How does this relate to the financial assets rate of return? In the neo-classical theory of production, the level of production depends only on the supply of goods in the context of pure and perfect competition. Then the producer tries to maximize his profit $\Pi = Py - Rk$ where P and R denote respectively the price of output and the cost of capital, under the constraint $y = f(k)$. The first-order condition is $\partial_k f(k) = R/P$, which means that the marginal product of capital is equal to the real cost of capital (denoted r):

$$m = r$$

From an investor's point of view, it is exactly the minimum expected return for an investment to be worthwhile. As such, it can be thought of as the required risk-free rate for financial assets' risk premium evaluation. At the steady-state, we have then:

$$r - \frac{e}{s} \delta = \frac{e}{s} (\eta + \gamma)$$

Finally, applying the Solow model and according to the Golden rule $e = s$, the financial markets' risk-free rate net of depreciation of capital $\mathfrak{r}^\star = r_c - \delta$ should be equal to the real rate of growth $g^\star$ of the economy at equilibrium:

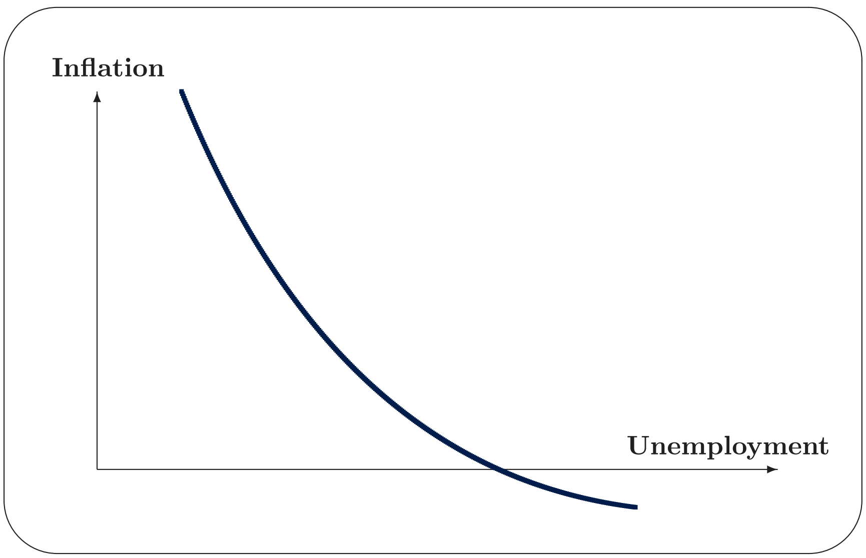
$$\mathfrak{r}^\star = g^\star = \eta + \gamma$$

B.2 Inflation

B.2.1 The Phillips curve

A fundamental concept in inflation analysis is the relationship between inflation and unemployment, called the Phillips curve. This model suggests that there is a trade-off between price stability and employment: the lower the unemployment in an economy, the higher the rate of inflation (see Figure 8). For illustration, an economic recession drives the unemployment level higher. As the economy re-starts, employment increases and wages start to rise. This will increase the firm's cost of production and the high costs are usually passed on to consumers in the form of higher prices. Therefore a decrease in unemployment has led to an increase in inflation and vice versa.

Figure 8: The Phillips curve



The original evidence in favor of the existence of the Phillips curve was graphical. Phillips (1958) describes how he observed an inverse relationship between money wage changes and unemployment in the British economy over the period examined. In the years following his paper, many economists in advanced industrial countries believed that Phillips' results showed that there was a stable relationship between inflation and unemployment (Samuelson and Solow, 1960, Cahuc and Zylberberg, 2004).

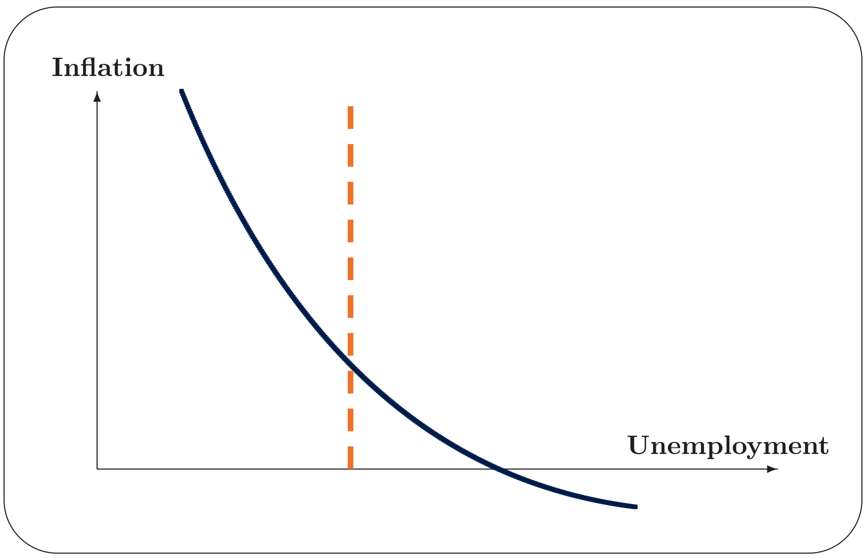
B.2.2 The NAIRU

In the seventies, the concept of a stable Phillips curve showed a breakdown as the economy suffered from both high inflation and high unemployment simultaneously. Economists refer to this kind of situation as stagflation where stagnant economies and rising inflation occur together. New theories, such as rational expectations and the NAIRU ("non-accelerating inflation rate of unemployment"), arose to explain how stagflation could occur, by distinguishing between a short-term and a long-term Phillips curve as illustrated in Figure 9.

- The short-term curve looks more like the original curve: it is also known as the "expectations-augmented Phillips curve" as it shifts up when the expectation of inflation increases as argued by the economists Milton Friedman and Edmund Phelps.

- The long-term curve is a vertical line and is the result of the non-accelerating inflation rate of unemployment theory which is also known as NAIRU. In the long term, the unemployment rate converges to a *natural* level which should be consistent with the potential output. Then, the long-run Phillips curve becomes vertical, meaning that there is no trade-off between inflation and unemployment.

Figure 9: The NAIRU curve

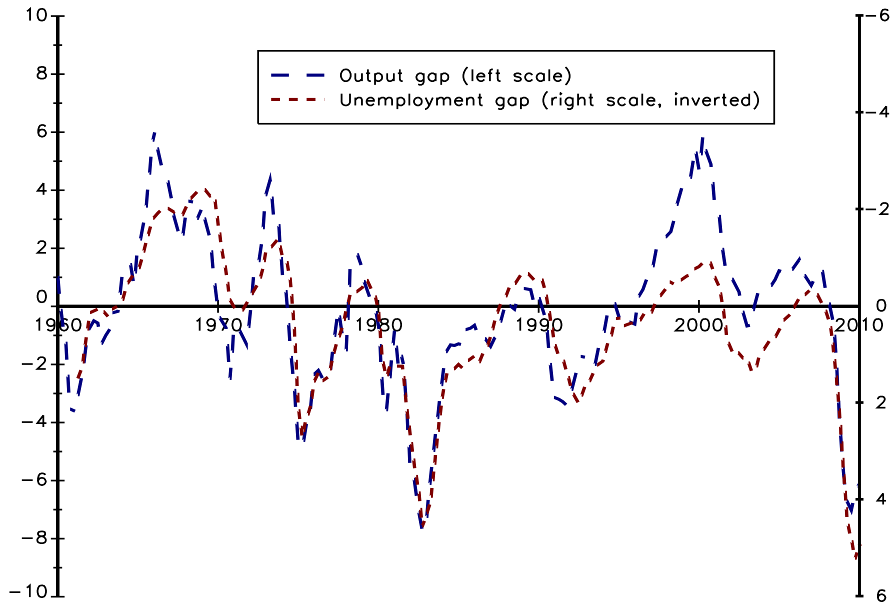


B.2.3 Okun's law

The other common way to understand the links between inflation and the business cycle is Okun's law (Okun, 1971, Prachowny, 1993). Let us define the output gap as the percentage difference between the output and the potential output and the unemployment gap as the difference between the unemployment rate and the natural rate of unemployment. This empirical law shows an inverse relationship between the size of the output gap and the size of the unemployment gap. According to Okun's law, periods when the output exceeds (resp. is below) its potential level are associated with periods when the rate of unemployment is below (resp. exceeds) the natural rate of unemployment (see Figure 10). Indeed, a necessary condition to lift output is to raise the intensity of resources (labor, capital, or productivity) so that the labor force increases and the unemployment rate falls.

Over the long term, the output gap and the unemployment gap are null. The output converges to its potential which is defined as a measure of sustainable output, specifically a level which neither adds or subtracts inflationary pressure derived from the intensity of resources. The unemployment rate converges to a natural rate of unemployment, which is frequently named as the NAIRU, a level which should not contribute to undesirable inflation changes.

Figure 10: Illustration of Okun's law



C Asset returns models

C.1 Interest rates

The theory of rational expectations was formulated by Lutz in 1940. Let $R_t^{(m)}$ be the interest rate with a maturity of m periods at time t and r_t the short-term interest rate with a maturity of one period. In this theory, we have:

$$R_t^{(m)} = \frac{1}{m} \sum_{i=1}^{m} \mathbb{E}_t \left[r_{t+i} \right]$$

This means that the long-term interest rate is an average of the expectation of the short-term interest rate, which is the key variable. As the yield curve is determined by economic agents' forecasts, it reveals their expectations on the dynamic of short-term interest rates. Since the seminal work of Lutz, the theory of rational expectations has been expanded, e.g. to take into account a liquidity premium. Until now, however, it is the most coherent theory for explaining the stylized facts about the term structure of interest rates.

At the beginning of the nineties, the economist John Taylor proposed describing the US monetary policy using an interest-rate feedback rule. This rule fixes the short-term interest rates (more precisely the federal funds rate) in order to achieve a short-term objective of stabilizing economic activity and a long-term objective of controlling inflation. The general form of the Taylor rule is:

$$r_t = \mathfrak{r}^\star + \pi_t + \varpi_\pi \left(\pi_t - \pi^\star \right) + \varpi_g (g_t - g^\star)$$

It also depends on four factors: a real equilibrium interest rate $\mathfrak{r}^\star$, current inflation π_t, the difference between current inflation and the inflation target $\pi^\star$ and the output gap

$(g_t - g^\star)$. Typically, if π_t becomes superior to $\pi^\star$, and/or $(g_t - g^\star)$ is positive, then the Central bank should react by hiking r_t with the hope that it would dampen demand (consumption and investment), and consequently would lower the output to a more sustainable level. In particular, the Taylor rule stipulates that r_t should overreact (i.e. move by more than one for one) to π_t, meaning that the real short rate $\mathfrak{r}_t$ should increase.

To summarize, the economic modeling of interest rates tells us that long-term interest rates are influenced by expectations on short-term interest rates. In particular, an upward sloping yield curve signals that short rates are expected to rise in the future. But in fact, structural differences between long yields and short interest rates also integrate uncertainties on both inflation and government debt. The first factor materializes uncertainty regarding the ability of Central banks to control inflation whereas the second corresponds to fears of failure of governments in terms of controlling their deficits. In other words, the slope of the term structure is related to the credibility of monetary and budgetary policies.

C.2 Equities

In the view of fundamental analysis, stock valuation based on fundamentals aims to give an estimate of the intrinsic value of stocks, based on predictions of the future cash flows and profitability of the business. The Gordon (1959) model is the best known of a class of discounted dividend models. It is based on assumptions that are very suited to a long-term analysis. First, it assumes that the company issues a dividend that grows at a constant rate over time. Second, it also assumes that the required rate of return for the stock remains constant. We start from the definition of the equity return:

$$R^{\mathrm{e}}_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t}$$

where D_t, P_t and R^{e}_t are respectively the dividend, the price and the return at time t.

Then it is possible to re-write this equality to equate the equity price with the expected discounted valuation of the price and the future dividends:

$$P_t = \mathbb{E}_t \left[\frac{P_{t+1} + D_{t+1}}{1 + R^{\mathrm{e}}_{t+1}} \right]$$

where R^{e}_{t+1} becomes a discount rate which corresponds to the expected return or/and the required return of the investor. For a long-term analysis, it is convenient to assume that this discount rate is constant over time, which is a first assumption of the Gordon growth model. Then, by repeating over k periods, we obtain:

$$P_t = \mathbb{E}_t \left[\sum_{i=1}^{k} \frac{1}{(1 + R^{\mathrm{e}})^i} D_{t+i} \right] + \mathbb{E}_t \left[\frac{1}{(1 + R^{\mathrm{e}})^k} P_{t+k} \right]$$

When k tends towards infinity, the first term corresponds to the definition of the equity fundamental value, and the second term tends towards zero. This last condition, namely transversality, prohibits any case where equity prices are expected to grow infinitely and thus the possibility of a bubble. Then, we assume that dividends grow at a constant rate g^{d} over time, which corresponds to the second assumption of the Gordon growth model. Over the long term, this hypothesis is commonly justified for two reasons. Firstly, dividends represent a portion of the company's earnings (the payout ratio) which is stable over the long term. Secondly, earnings growth is supposed to be constant because the distribution of income between wages and earnings is stable over the long term, and income growth converges to the potential output growth. Then, we obtain:

$$P_t = \sum_{i=1}^{\infty} D_t \left(\frac{1 + g^{\mathrm{d}}}{1 + R^{\mathrm{e}}} \right)^i = D_t \left(\frac{1 + g^{\mathrm{d}}}{R^{\mathrm{e}} - g^{\mathrm{d}}} \right)$$

We re-write this relation to equate the expected return on equity R^{e} to the dividend growth rate g^{d} and the so-called dividend yield $\mathrm{DY}_t = D_{t+1}/P_t$:

$$R^{\mathrm{e}} = \mathrm{DY}_t + g^{\mathrm{d}}$$

Then, it is common to replace g^{d} by the sum of the real potential output growth $g^{\star}$ and the long-run inflation $\pi^{\star}$, as suggested by the assumption of constant dividend growth consistent with the potential output growth. Also, the equity return could be broken in two parts, namely a risk free rate $r^{\star}$ which is commonly replaced by the government bond yield for long term analysis, and the equity risk premium:

$$\mathcal{R}^{\mathrm{e}} = \mathrm{DY} + g^{\star} + \pi^{\star} - r^{\star}$$

C.3 Currencies

The covered interest rate parity (CIP) plays an essential role in foreign exchange markets by connecting interest rates, spot and forward exchange rates. Let S_t and $F_t^{(m)}$ be respectively the spot exchange rate and the forward exchange rate of maturity m. We have:

$$F_t^{(m)} = \frac{1 + R_t^{(m)}}{1 + R_t^{\star(m)}} S_t$$

with $R_t^{(m)}$ and $R_t^{\star(m)}$ the domestic and foreign interest rates for the period $[t, t+m]$. This equation is an equilibrium relationship based on arbitrage theory:

- We borrow 1 euro for the period $[t, t+m]$. At time $t+m$, we pay back $1 + R_t^{(m)}$.
- At time t, we convert the 1 euro into S_t dollars and we sell a forward contract with notional $\left(1 + R_t^{\star(m)}\right) \big/ S_t$ at price $F_t^{(m)}$.
- The P&L of this strategy is equal to:

$$\mathrm{PnL} = F_t^{(m)} \left(1 + R_t^{\star(m)}\right) \Big/ S_t - \left(1 + R_t^{(m)}\right)$$

There is no arbitrage when $\mathrm{PnL} = 0$. This relationship is known as the *covered interest rate parity*.

In the *uncovered interest rate parity* (or UIP), we assume that the forward rate is an unbiased estimator of the future spot rate:

$$F_t^{(m)} = \mathbb{E}_t \left[S_{t+m} \right]$$

The underlying idea is that prices reflect all the available information in an efficient market. In this situation, the forward rate is the best forecast of the future spot rate if expectations are rational. Indeed, if differences in interest rates between two distinct geographical economic areas were not accompanied by a change in the corresponding expected money exchange rate, then investors willing to trade on this spread would expect to generate profits once they convert the foreign currency back to the domestic currency.

While CIP and UIP determine exchange rates in the short term, the purchasing power parity (PPP) is useful to explain the level of exchange rates in the long term. It states that the exchange rate between two countries is in equilibrium when their purchasing power is the same in each of these two countries. More precisely, we have in the long term this relationship:

$$P_t = S_t P_t^{\star}$$

where P_t and $P_t^{\star}$ are the domestic and foreign price levels. In other words, this equation states that any good should have the same price worldwide. The important notion here is that an economic agent should be indifferent to acquiring a physical good in one country rather than another, after nominal exchange rates are taken into account.

C.4 Credit

The assessment of corporate debt was introduced in the seminal work of Black and Scholes (1973) on option pricing, and precisely developed by Merton (1974) as a structural credit risk model. In the Merton model, the corporation's value V_t is equal to a single equity issue E_t that pays no dividend and a single debt with nominal K that is represented by a zero-coupon bond B_t:

$$V_t = E_t + B_t$$

At maturity T, if the corporation's value exceeds the debt nominal amount, the bondholders receive K and the equity holders are left with the remaining value $E_T = V_T - K$. If the firm defaults on the debt payment, the bondholders become the only owners of the corporation and $E_T = 0$. So the equity value of the corporation at maturity can be re-written as the payoff of a call option on the corporation's assets, with a strike equal to the maturity value of its debt:

$$E_T = \max\left(V_T - K, 0\right)$$

However, in the context of credit risk, we are particularly interested in the debt value that we can derive using the put-call parity from option pricing theory. It involves then a guaranteed (risk-free) payment of the amount K at maturity plus a short put option displaying the same characteristics of the call mentioned above:

$$B_T = K - \max\left(K - V_T, 0\right)$$

If we assume that V_t is a geometric Brownian motion with drift μ_V and volatility σ_V, and the existence of a risk-free rate r, the risk-neutral probability of default is:

$$P = \Phi\left(-d_2\right)$$

with:

$$d_2 = \frac{1}{\sigma_V\sqrt{T-t}} \ln\left(\frac{V_t}{K} + r\left(T-t\right)\right) - \frac{1}{2}\sigma_V\sqrt{T-t}$$

It is worth noting that the model implies that the probability of default depends on the following observable variables: the corporation's leverage V_t/K, its asset volatility and the time to repayment of its debt. This result makes this model a very interesting tool from a practical point of view. Then, introducing the yield to maturity y_t, defined implicitly so that $B_t = Ke^{-y_t(T-t)}$, it is possible to obtain the expression for the so-called credit spread as an increasing function f of the default probability PD:

$$s_t = y_t - r = f\,(\text{PD})$$

The pricing of default probabilities implying credit spreads is consistent with a risk-neutral world. In practical terms, in a risk averse world, the corporate bond yield is actually comprised of a government bond yield (assuming that governments never default), plus a credit risk premium that is positively correlated to the firm's default probability but that depends also on the uncertainty on the default probability (materialized through the firm's asset volatility).

D Results of regression models

Tables 16 and 17 present the results of the different regression models. For each model, we indicate the period of study, the variables used as explanatory regressors, the estimated values of the coefficients, the associated t-value (in parenthesis) and the R^2 of the model.

Table 16: Coefficient estimates for bond regressions

	Study Period	Constant	$\mathfrak{r}_t$	σ_t^π	$(B/Y)_t$	σ_t^e	g_t	$(CA/Y)_t$	R^2
US	1982–2009	0.008 (2.006)	0.59 (3.63)	0.67 (1.51)	−0.11 (−1.00)				0.82
EURO	1982–2009	0.007 (1.988)	0.47 (4.15)	2.03 (2.07)	−0.10 (−0.84)				0.94
JAPAN	1982–2009	0.011 (3.379)	0.66 (5.57)	0.21 (1.82)	−0.05 (−1.10)				0.85
PACIFIC	1982–2009	0.017 (1.212)	0.47 (3.96)	0.15 (0.19)	−0.32 (−2.42)				0.69
EM	1993–2009	0.046 (4.200)				0.05 (0.78)		−0.51 (−2.64)	0.34
IG US	1983–2009	0.012 (4.878)				0.03 (2.57)	−0.09 (−2.15)		0.44
IG EURO	1999–2009	0.017 (1.212)				0.04 (2.31)	−0.03 (−0.57)		0.50
HY US	1986–2009	0.054 (6.056)				0.12 (3.40)	−0.80 (−4.90)		0.73
HY EURO	1997–2009	0.011 (0.571)				0.38 (4.25)	−0.79 (−2.59)		0.80

Table 17: Coefficient estimates for equity and commodity regressions

	Study Period	Constant	PE_t	R_t^{b}	$\Delta\left(Y^{\mathrm{EM}}/Y^{\mathrm{W}}\right)_t$	g_t^{W}	R^2
US	1983–2009	0.433 (3.587)	−11.16 (−3.52)	−0.89 (−1.73)			0.40
EURO	1983–2009	0.703 (6.409)	−20.64 (−6.16)	−1.14 (−4.30)			0.62
JAPAN	1983–2009	0.760 (4.944)	−18.88 (−5.79)	−1.38 (−1.89)			0.72
PACIFIC	1983–2009	0.509 (9.414)	−12.18 (−7.84)	−1.33 (−7.17)			0.75
COMMODITY	1980–2009	−0.313 (−4.673)			2.98 (1.32)	8.90 (4.45)	0.47

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