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Strategic Joining and Pricing Policies in a Retrial Queue With Orbital Search and Its Application to Call Centers

SHAN GAO AND JIE ZHANG
School of Mathematics and Statistics, Fuyang Normal University, Fuyang 236037, China

Corresponding author: Shan Gao (sgao_09@yeah.net)

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ABSTRACT This paper treats strategic joining and pricing policies in an M/M/1 retrial queue with orbital search which is motivated by the application in call centers, where the server will make orbital search or remain idle whenever he completes a service, the orbital search time follows exponential distribution. Given a natural reward-cost structure and imposed on an admission fee, all arriving customers decide to whether to join the orbit or balk when they find the server busy. Using queueing theory and game theory, we first analyze the Nash equilibrium mixed joining strategy for individual customer. Further we investigate the optimal joining probabilities and corresponding admission pricing problems that maximize the administrator's revenue and social profit, respectively. Finally, we present some numerical examples to demonstrate the effect of some system parameters on the sensitivity of the solutions of the individual maximization, administrator's maximization and social optimization.

INDEX TERMS Retrial queue, orbital search, pricing, joining probability, Nash equilibrium.

I. INTRODUCTION

In daily-life situations, customers are often forced to leave the service area when their services can't immediately begin at their arrival epochs. However, these customers repeat their request after staying some time in a virtual waiting room (called as orbit). Queues with such characters are called as retrial queues. Generally, customers in the orbit can access to the server according to four different retial policies when the server becomes idle: the first is the classical retrial policy, in which the intervals between successive repeated attempts are exponentially distributed with rate $n\nu$, when the number of customers in the orbit is n, see Yang and Templeton [1] and Falin [2]; the second is the constant retrial policy, in which repeated customers in orbit form a queue according to first-come-first served (FCFS) and only the customers at the head of the orbit queue can request a service after an exponentially distributed retrial time with rate $\alpha(1-\delta_{n,0})$, when the orbit size is n and $\delta_{n,0}$ denotes Kronecker's delta function, see Fayolle [3], and Farahmand [4]; the third is the linear retrial policy, in which time intervals between successive repeated attempts are exponentially distributed with parameter $\alpha(1-\delta_{n,0})+n\nu$, when the orbit size is n, see Artalejo and Gomez-Corral [5]; the forth is the general retrial time policy, which is the generalization of the second by assuming that the retrial time is governed by an arbitrary distribution and that the customer at the head of the orbit queue is allowed access to the server, see Falin [2]. In recent decades, retrial queueing systems have been extensively investigated due to their applications in practical situations, such as call centers, communication systems, computer networks, neural networks and inventory-production systems *et al.*, see the books [6], [7], recent literatures [8]–[16] and references therein.

Most existing literatures assume that upon the completion of a service, the server does nothing but stays idle in the system until the next customer from the outside or the orbit arrives. This assumption ignores the initiative of the service provider and is not applicable for some real situations, especially in systems with human servers such as call centers, mobile phone, information and communication network, etc. For example, in a call center of a credit card

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company with an interactive voice response units (IVRUs) (see [17]) and without a waiting buffer, an incoming call can be served immediately when the operator (server) is idle at his arrival epoch, otherwise, he may hang up and retry to access to the server later with probability q or leaves forever with probability $1 - q$. Such a retrying caller is said to be in orbit. In practice, the callers in the orbit can be regarded as a waitlisted calls and the operator will search for a customer from the wait list to inform the customer to pay or to offer some different proposals during his idle period. The main goal of the operator is to reduce waiting times of the calls in the orbit. This motivates us to consider such a retrial model with interaction between the server and the orbit. About recent work on retrial queues with server-orbit interaction, readers are referred to Arivudainambi and Dodhandaraman [13], Dragieva and Phung-Duc [18], Krishnamoorthy *et al.* [19], Chakravarthy *et al.* [20], Deepak *et al.* [21], Gao and Wang [22] and references therein. It should be remarked that in [19], [21] and [22], the orbital search time is neglected and authors mainly focused on the system performance measures for the considered retrial queues. However, to the best of our knowledge, few papers consider the economic analysis for the retrial queue with server-orbit interaction. Hence, in this work our objective is to study the strategic joining and pricing policy for a retrial queue with server-orbit interaction from the economic view point.

The economic analysis of queueing systems is concerned with customers' decisions to join or balk based on a given reward-cost structure, which incorporates the customers' desire for service as well as their unwillingness to wait. More specifically, on one hand, there is a reward of R units for each customer that completes a service in the system, which quantifies his/her satisfaction and/or the added value of being served. On the other hand, there also exists a waiting cost of C units per time unit as long as a customer remains in the system (in queue or in service), which quantifies his/her unwillingness to wait and/or lost benefits. In such queueing system, customers are selfish and want to maximize their own net benefit. Therefore, this system can be modeled as a symmetric noncooperative game among customers. The economic analysis of queueing systems can be traced back to the pioneering works of Naor [23] and Edelson and Hildebrand [24], where an observable and an unobservable single-server queue were, respectively, studied. for a detailed introduction to this area and reviews on the economic analysis of various queueing systems, readers are referred to the books of Hassin [25], Hassin and Haviv [26]. Since the works of Naor [23] and Edelson and Hildebrand [24], the economic analysis of variants of the M/M/1 queue has been extensively studied. Economou and Kanta [27] discussed the equilibrium balking strategies of an observable M/M/1 queue with breakdowns and repairs. Li *et al.* [28] extended the results in Economou and Kanta [27] to the unobservable case and mixed balking strategies were presented. Wang and Zhang [29] and Yu *et al.* [30] studied the M/M/1 queue with balking and delayed repair in observable and unobservable cases, respectively. Boudali and Economou [31], [32] studied the strategic behavior of customers in M/M/1 queueing systems with catastrophes, in which all customers are removed whenever a catastrophe occurs. Boundali and Economou [33] considered equilibrium joining strategies of a batch service queueing systems in observable and unobservable cases.

Recently, economic analysis of the retrial queueing systems has increasingly attracted attention of scholars. Retrial queueing systems are more appropriate in various fields, such as telephone switching systems and call centers, telecommunication networks with retransmission, etc. The strategic analysis of retrial queueing systems was initiated by Economou and Kanta [34], where they considered equilibrium customer strategies and social-profit maximization in the M/M/1 constant retrial queue. Afterwards, these results were further extended by many authors. Wang and Zhang [35] investigated the equilibrium and socially optimal balking strategies in the observable and unobservable M/M/1 classical retrial queue. Wang and Zhang [36] studied the customers' Nash equilibrium strategies and socially optimal strategies in an M/M/1 retrial queue with constant retrial policy and delayed vacations, in which the socially optimal pricing strategy was also determined. Wang and Li [37] studied the joining strategies of the SUs in cognitive radio networks with random access in the noncooperative and cooperative cases. Wang *et al.* [38] considered the customers' strategic behavior and social maximization problem in an M/M/1 retrial queue with constant retrials under N-policy. Gao *et al.* [39] treated the economic analysis for an M/M/1 retrial queue with an unreliable server, in which limited idle period and single vacation were introduced into the retrial queue. Recently, Zhang *et al.* [40] studied the optimal pricing strategies for a retrial queue with service interruptions and general service times, which is used to model a cognitive radio system. To the best of our knowledge, there are no papers dealing with the economic analysis of the retrial queue with server-orbit interaction. Our contributions of this work are as follows:

- We introduce the server-orbit interaction schedule into an M/M/1 retrial queue, which can effectively reduce the sojourn time of the customer in the orbit.
- We extensively investigate the optimal joining strategies, respectively, from the aspects of the individual, the administrator and the social planner. More specially, on one hand, we identify the Nash equilibrium joining probability for the customers, see [34], on the other hand, we obtain the optimal joining probabilities that maximize the administrator's profit and the social net benefit per unit time, respectively.
- We consider the revenue-optimal pricing problem (i.e., the optimal price which is selected to induce the customers to behave in the joining policy that maximizes the administrator's profit per unit time), and the social-optimal pricing problem (i.e., the optimal price which is selected to eliminate differences between the optimal joining probabilities of the individual and the social).

The remainder of this paper is organized as follows. Section 2 provides a detailed description of the model and its application. Sections 3 and 4 present individual equilibrium joining strategies and the administrator's maximization problem, respectively. Section 5 considers socially optimal joining strategies and socially optimal pricing problem. Section 6 gives some numerical experiments. Conclusions are given in Section 7.

II. MODEL DESCRIPTION AND ITS APPLICATION

A. MODEL DESCRIPTION

In this section, we consider an M/M/1 retrial queue with orbital search with following characters:

- Customers arrive at the system according to a Poisson process with rate λ. The service times of the customers are independent and follow exponential distribution with rate μ.
- An arriving customer immediately begins his service if he finds the server available at his arrival epoch, otherwise, he joins the retrial orbit with probability q or leaves the system with probability $\bar{q}(=1-q)$. The customers in the orbit are permitted to access to the server under the FCFS discipline when the server becomes idle. The retrial time is exponentially distributed with rate α, i.e., the constant retrial policy is adopted in this paper, see [34] for details.
- After serving a customer and finding that there are customers in the orbit, the server begins searching for the customer from the ahead of the orbit with probability β or remains idle in the system with probability $\bar{\beta}(=1-\beta)$. And assume that the server always remains idle in the system when no customers are in the orbit at the service completion epochs. The search time is assumed to be exponentially distributed with rate θ. If there are customers arriving during the search period, the server immediately stops the search and begins serving the newly arriving customer.
- All the random variables defined above are mutually independent.

For the above queueing system, we are interested in the strategic joining policy of the customers who find a busy server upon their arrivals and pricing policy from the insights of the individual, the administrator and the social, respectively, under a reward-cost structure as follows:

- Each customer receives a reward of R units after completing his service, and bears a sojourn cost of C units per unit time which is incurred for the delay in the system.
- To maximize his revenue per unit time, the administrator of the system charges a price p on each customer who enters the system, where $R-p>\frac{C}{\mu}$, which ensures that the customers who find the sever idle or in the search period do enter in the system and receive their service immediately.
- All customers are risk neutral and aim to maximize their own expected benefit. Decisions made by customers are irrevocable, in the sense that neither reneging of entering customers nor retrials of balking customers is allowed.

B. PRACTICAL APPLICATION

Consider a system of bank call center which is based on communication network, in which there is a single operator (server) who is responsible for product marketing service. Typically, a primary incoming call (customer) is immediately assigned to the operator by an Automatic Call Distributor (ACD) if the operator is idle upon its arrival epoch; otherwise, the incoming call either leaves forever and seeks service elsewhere with some probability $\bar{q}$, or is queued at the ACD (called as "retrial orbit" in queueing terminology) according to FCFS with probability q. When the operator becomes idle, the call at the head of the queue is initiated by the ACD (automatically) to receive its service after an exponentially distributed time (called as "constant retrial" in queueing terminology). To reduce sojourn times of the calls in the queue, the operator may actively search for the customer in the orbit to initiate the service with some probability β and the search time is exponentially distributed (called as "orbital search" in queueing terminology). Assume that primary incoming calls arrive at system according to a Poisson process and the service times of incoming calls are i.i.d., exponentially distributed. Then such a bank call center system can be modelled as an M/M/1 retrial queue with orbital search.

In the bank call center system, on one hand, customers hope to accept their services in time and can obtain a reward R after completing their service (for example, obtain certain products at a discount price), however, under a certain cost structure, customers may suffer a heavy loss if their sojourn time in the system is longer (assuming that there exists a sojourning cost of C units per unit time). On the other hand, according to the benefit principle, which is made in the business requirements of the bank call center, bank call center imposes a service fee, say p, on each customer to make a profit. Under the above reward-cost structure, each customer is allowed to take his own decision to join or balk to maximize his own benefit. Therefore, the optimization problem becomes a game among customers and the first objective is to find the Nash equilibrium points. The second objective is that the administrator of the system and the social planner have to solve the profit maximization problems by taking into account the joining behavior of the customers.

In the next section, we aim to study the individual equilibrium joining strategy.

III. INDIVIDUAL EQUILIBRIUM JOINING STRATEGY

To study the individual's Nash equilibrium strategy, we first discuss the steady-state distribution of the system.

The state of the system at time t can be described by a pair $\{(N(t), J(t)), t \geq 0\}$, where $N(t)$ is the number of customers in the orbit, $J(t)$ is 0, 1 or 2 according as the server is idle, in search period or busy at time t. Then the stochastic process $\{(N(t), J(t)), t \geq 0\}$ is a continuous time Markov chain with

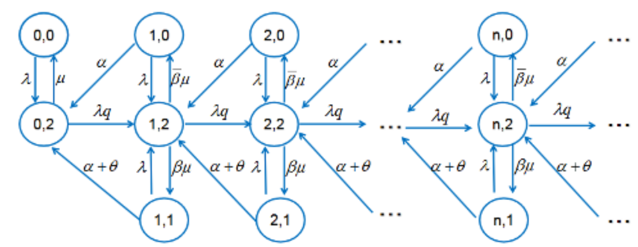

FIGURE 1. Dynamic diagram of the retrial queue with orbital search.

state space $\Omega = \{(0,0)\} \cup \{(0,2)\} \cup \{(n,j) : n \geq 1, j = 0, 1, 2\}$. Transition rate diagram of $\{(N(t), J(t)), t \geq 0\}$ is given in Fig.1.

Let $\rho = \frac{\lambda(\lambda+\alpha)(\lambda+\alpha+\theta)}{\mu[\beta\lambda\theta+\alpha(\lambda+\alpha+\theta)]}$, we have the following results.

Theorem 1: For the M/M/1 retrial queue with orbital search, we have that

(1) The stationary condition of the system is $q\rho < 1$.

(2) Under the steady-state condition $q\rho < 1$, define the steady-state distribution of $\{(N(t), J(t)), t \geq 0\}$ as follows: $\pi_{n,j} = \lim_{t\to\infty} P(N(t) = n, J(t) = j), j = 0, 2, n \geq 0$; $j = 1, n \geq 1$. Then the steady-state distribution are given as:

$$\pi_{0,0} = \frac{\mu}{\lambda}B,$$
$$\pi_{n,0} = \frac{\bar{\beta}\mu}{\lambda+\alpha}B\xi^n, \quad n \geq 1,$$
$$\pi_{n,1} = \frac{\beta\mu}{\lambda+\alpha+\theta}B\xi^n, \quad n \geq 1,$$
$$\pi_{n,2} = B\xi^n, \quad n \geq 0.$$

where

$$B = \left[\frac{\mu}{\lambda} + \frac{1}{1-\xi} + \frac{\mu\xi}{1-\xi}\left(\frac{\bar{\beta}}{\lambda+\alpha} + \frac{\beta}{\lambda+\alpha+\theta}\right)\right]^{-1},$$
$$\xi = q\rho.$$

Proof: (1) From Fig.1, we see that $\{(N(t), J(t)), t \geq 0\}$ is a continuous time quasi-birth-death (QBD) process. Using the lexicographical sequence for the states, the infinitesimal generator $\mathbb{Q}$ is given by

$$\mathbb{Q} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{C}_0 & & & & \\ \mathbf{A}_1 & \mathbf{B} & \mathbf{C} & & & \\ & \mathbf{A} & \mathbf{B} & \mathbf{C} & & \\ & & \mathbf{A} & \mathbf{B} & \mathbf{C} & \\ & & & \mathbf{A} & \mathbf{B} & \mathbf{C} \\ & & & & \ddots & \ddots & \ddots \end{bmatrix},$$

where

$$\mathbf{A}_0 = \begin{bmatrix} -\lambda & \lambda \\ \mu & -(\lambda q+\mu) \end{bmatrix}, \quad \mathbf{C}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda q \end{bmatrix},$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & \alpha \\ 0 & \alpha+\theta \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & \alpha+\theta \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -(\lambda+\alpha) & 0 & \lambda \\ 0 & -(\lambda+\alpha+\theta) & \lambda \\ \bar{\beta}\mu & \beta\mu & -(\lambda q+\mu) \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda q \end{bmatrix}.$$

Let $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$, then

$$\mathbf{D} = \begin{bmatrix} -(\lambda+\alpha) & 0 & \lambda+\alpha \\ 0 & -(\lambda+\alpha+\theta) & \lambda+\alpha+\theta \\ \bar{\beta}\mu & \beta\mu & -\mu, \end{bmatrix}$$

and $\mathbf{D}$ is obviously a generator matrix, its associated stationary probability vector $\mathbf{d} = (d_0, d_1, d_2)$ can be derived by $\mathbf{dD} = \mathbf{O}_2$ and $\mathbf{de}_3 = 1$, where $\mathbf{O}_2 = (0, 0, 0)$, $\mathbf{e}_3 = (1, 1, 1)^T$.

Thus we have $\mathbf{d} = (\frac{\bar{\beta}\mu(\lambda+\alpha+\theta)}{\omega}, \frac{\beta\mu(\lambda+\alpha)}{\omega}, \frac{(\lambda+\alpha)(\lambda+\alpha+\theta)}{\omega})$, where $\omega = \bar{\beta}\mu\theta + (\lambda+\alpha)(\lambda+\alpha+\mu+\theta)$. From the Theorem 3.1.1 in [41] which states that the necessary and sufficient condition for stability of QBD process is $\mathbf{dCe}_3 < \mathbf{dAe}_3$, then we obtain that $q\rho < 1$ is the stationary condition for our retrial system.

(2) The balance equations for the stationary distribution are given as follows:

$$\lambda\pi_{0,0} = \mu\pi_{0,2}, \tag{1}$$
$$(\lambda+\alpha)\pi_{n,0} = \bar{\beta}\mu\pi_{n,2} \quad n \geq 1, \tag{2}$$
$$(\lambda+\alpha+\theta)\pi_{n,1} = \beta\mu\pi_{n,2} \quad n \geq 1, \tag{3}$$
$$(\lambda q+\mu)\pi_{0,2} = \lambda\pi_{0,0} + (\alpha+\theta)\pi_{1,1} + \alpha\pi_{1,0}, \tag{4}$$
$$\begin{aligned}(\lambda q+\mu)\pi_{n,2} = {} & \lambda\pi_{n,0} + \alpha\pi_{n+1,0} + \lambda\pi_{n,1} \\ & + (\alpha+\theta)\pi_{n+1,1} + \lambda q\pi_{n-1,2}, \quad n \geq 1.\end{aligned} \tag{5}$$

Using (1),(2),(3), we obtain that

$$\pi_{0,0} = \frac{\mu}{\lambda}\pi_{0,2}, \tag{6}$$
$$\pi_{n,0} = \frac{\bar{\beta}\mu}{\lambda+\alpha}\pi_{n,2}, \quad n \geq 1, \tag{7}$$
$$\pi_{n,1} = \frac{\beta\mu}{\lambda+\alpha+\theta}\pi_{n,2} \quad n \geq 1. \tag{8}$$

Inserting (7) and (8) into (5), we find that $\{\pi_{n,2}, n = 0, 1, 2, \cdots\}$ satisfies the following homogeneous linear difference equation with constant coefficients

$$\begin{aligned}&\mu[\beta\lambda\theta + \alpha(\lambda+\alpha+\theta)]\pi_{n+1,2} \\ &\quad - [\beta\lambda\theta\mu + (\lambda+\alpha+\theta)(\alpha\mu + \lambda q(\lambda+\mu))]\pi_{n,2} \\ &\quad + \lambda q(\lambda+\alpha)(\lambda+\alpha+\theta)\pi_{n-1,2} = 0, \quad n \geq 1.\end{aligned} \tag{9}$$

The characteristic equation corresponding to (9) has two roots 1 and ξ, then we obtain that

$$\pi_{n,2} = A + B\xi^n, \quad n \geq 0, \tag{10}$$

where $\xi = q\frac{\lambda(\lambda+\alpha)(\lambda+\alpha+\theta)}{\mu[\beta\lambda\theta+\alpha(\lambda+\alpha+\theta)]} = q\rho$.

Inserting (10) into (4), we obtain that $A = 0$. So

$$\pi_{n,2} = B\xi^n, \quad n \geq 0. \tag{11}$$

From (6), (7), (8) and (11), and using normalization condition $\sum_{n=0}^{\infty}(\pi_{n,0} + \pi_{n,2}) + \sum_{n=1}^{\infty}\pi_{n,1} = 1$, we can calculate that $B = \left[\frac{\mu}{\lambda} + \frac{1}{1-\xi} + \frac{\mu\xi}{1-\xi}\left(\frac{\overline{\beta}}{\lambda+\alpha} + \frac{\beta}{\lambda+\alpha+\theta}\right)\right]^{-1}$, which completes the proof of Theorem 1.

Let Π_2 be the stationary probability that the server is busy, then $\Pi_2 = B/(1-\xi)$. Due to the PASTA property, we can get the effective arrival rate in the retrial orbit as follows

$$\lambda_{eff} = \lambda q \Pi_2 = \frac{\lambda q B}{1-\xi}. \tag{12}$$

On the other hand, from Theorem 1, the expected number of customer in the orbit is given by

$$\begin{aligned} E[N] &= \sum_{n=1}^{\infty} n\pi_{n,0} + \sum_{n=1}^{\infty} n\pi_{n,1} + \sum_{n=1}^{\infty} n\pi_{n,2} \\ &= \left(1 + \frac{\overline{\beta}\mu}{\lambda+\alpha} + \frac{\beta\mu}{\lambda+\alpha+\theta}\right) B \frac{\xi}{(1-\xi)^2}. \end{aligned} \tag{13}$$

Applying Little's law and using (12), (13), we obtain the mean waiting time of a customer in the orbit as follows

$$E[W_o] = \frac{E[N]}{\lambda_{eff}} = \frac{C_1}{1-\xi},$$

where $C_1 = \frac{(\lambda+\alpha+\mu)(\lambda+\alpha+\theta)-\beta\mu\theta}{\mu[\beta\lambda\theta+\alpha(\lambda+\alpha+\theta)]}$.

Denote by $E[W_s|J=2]$ the expected sojourn time of a customer in the system given that he finds the server busy upon his arrival, then we have

$$E[W_s|J=2] = E[W_o] + \frac{1}{\mu} = \frac{C_1}{1-\xi} + \frac{1}{\mu}. \tag{14}$$

In the sequel, we first assume that $\rho < 1$. Based on (14), we can derive the equilibrium joining strategy for the customer.

Theorem 2: In the M/M/1 retrial queue with orbital search and $\rho < 1$, there exists a unique mixed equilibrium joining strategy 'enter the orbit with probability q_e while finding the server busy', where

$$q_e = \begin{cases} 0, & \frac{1}{\mu} < \frac{R-p}{C} < L_e, \\ q_e^*, & L_e \le \frac{R-p}{C} \le U_e, \\ 1, & \frac{R-p}{C} > U_e, \end{cases} \tag{15}$$

where

$$L_e = \frac{1}{\mu} + C_1, \quad U_e = \frac{1}{\mu} + \frac{C_1}{1-\rho},$$

$$q_e^* = \frac{1}{\rho}\left[1 - C_1\left(\frac{R-p}{C} - \frac{1}{\mu}\right)^{-1}\right]. \tag{16}$$

Proof: Assume that all customers follow a given entering strategy q. Then from (14), we can obtain the expected net benefit of a tagged arriving customer who finds the server busy and decides to enter the orbit as

$$\begin{aligned} S_e(q) &= R - p - CE[W_s|J=2] \\ &= R - p - C\left(\frac{C_1}{1-q\rho} + \frac{1}{\mu}\right), \end{aligned} \tag{17}$$

which indicates that $S_e(q)$ strictly decreases for $q \in [0,1]$, its unique maximum and unique minimum are, respectively, given by $S_e(0) = R - p - CL_e$ and $S_e(1) = R - p - CU_e$.

To determine the equilibrium point q_e, we consider three cases:

- Case I. when $\frac{1}{\mu} < \frac{R-p}{C} < L_e$, we have $S_e(q) < 0$ for all $q \in [0,1]$. In this case, the best response for the arriving tagged customer who finds the server busy is balking. That is, $q_e = 0$ is the unique equilibrium point and we have the first branch of (15).
- Case II. when $L_e \le \frac{R-p}{C} \le U_e$, which yields that $S_e(0) \ge 0$ and $S_e(1) \le 0$. Then the equation $Se(q) = 0$ has a unique root in [0, 1], denoted by q_e^*, which is given by (16). In this case, $q_e = q_e^*$ is the best response. The reason is as follows:
From (17), the expected net benefit of a tagged customer that enters the orbit with probability q' given that the server is found busy at his arrival epoch, when the others follow a strategy q is given by $F(q', q) = q'S_e(q)$. Following from the definition of q_e, we can see that $F(q_e, q_e) \ge F(q', q_e)$, for every $q' \in [0,1]$, then q_e is a (symmetric Nash) equilibrium (see [35]), which means that q_e is a best response against itself, that is, if all customers follow it, no one can benefit by changing it. Then we have the second branch of (15)
- Case III. when $\frac{R-p}{C} > U_e$, we have $S_e(q) > 0$ for all $q \in [0,1]$. In this case, the best response for the arriving tagged customer who finds the server busy is entering. That is, $q_e = 1$ is the unique equilibrium point and we have the third branch of (15).

By Theorem 2, we see that $S_e(q)$ is a strictly decreasing function. Whenever the entering probability q adopted by other customers is smaller than q_e, the expected net benefit of a tagged arriving customer is positive if he choose to enter the orbit when he finds the server busy at his arrival epoch, then the best response is 1. If $q = q_e$, any strategy is the best response because the expected net benefit of the tagged customer is always 0. If $q > q_e$, the best response of the tagged customer is 0 because the expected net benefit of the tagged customer is negative. This illustrates that the best response is a non-increasing function of the strategy selected by other customers, which shows that an "Avoid the crowd" (ATC) situation exists.

Remark 1: In the M/M/1 retrial queue with orbital search and $\rho \ge 1$, following the familiar analogue, when $\rho \ge 1$ and $q\rho < 1$, there exists a unique mixed equilibrium joining strategy 'enter the orbit with probability q_e while finding the server busy', where

$$q_e = \begin{cases} 0, & \frac{1}{\mu} < \frac{R-p}{C} < L_e, \\ q_e^*, & \frac{R-p}{C} \ge L_e. \end{cases}$$

IV. ADMINISTRATOR'S PROBLEM

In this section, we discuss the profit maximization problem of the administrator, who charges an entrance fee p on the customers for completing their services. We are interested in the joining strategy q_{prof} which maximizes the administrator's profit per unit time.

Theorem 3: In the M/M/1 retrial queue with orbital search and $\rho < 1$, there exists a unique mixed equilibrium joining strategy 'enter the orbit with probability q_{prof} while finding the server busy', where

$$q_{prof} = \begin{cases} 0, & \frac{R}{C} < L_{prof}, \\ q^*_{prof}, & L_{prof} \le \frac{R}{C} \le U_{prof}, \\ 1, & \frac{R}{C} > U_{prof}, \end{cases} \tag{18}$$

where

$$L_{prof} = \frac{1}{\mu} - \frac{1}{\lambda} + \frac{(\lambda+\mu)^2(\lambda+\alpha)^2(\lambda+\alpha+\theta)^2}{\lambda\mu^2(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))^2}, \tag{19}$$

$$U_{prof} = \frac{1}{\mu} - \frac{1}{\lambda} + \frac{\mu^2(\lambda+\alpha)^2(\lambda+\alpha+\theta)^2}{\lambda(\beta\mu\lambda\theta+(\lambda+\alpha+\theta)(\alpha\mu-\lambda(\lambda+\alpha)))^2}, \tag{20}$$

$$\sigma = \frac{1}{(\lambda+\alpha)(\lambda+\alpha+\theta)}\sqrt{1+\lambda\Big(\frac{R}{C}-\frac{1}{\mu}\Big)}, \tag{21}$$

$$q^*_{prof} = \frac{\mu+\lambda-\sigma\mu(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))}{\lambda(1-\sigma(\lambda+\alpha)(\lambda+\alpha+\theta))}. \tag{22}$$

Proof: For a given joining strategy q, denote by $S_{prof}(q)$ the administrator's net profit per unit time, $\lambda^*(q)$ the mean effective arrival rate, $p(q)$ the entrance fee levied by the administrator to induce the joining strategy q. Then

$$S_{prof}(q) = \lambda^*(q)p(q), \tag{23}$$

where $\lambda^*(q)$ is given by

$$\lambda^*(q) = \lambda\,[1-\bar{q}\Pi_2] = \frac{\lambda\mu}{\mu+\lambda\bar{q}}. \tag{24}$$

The objective of the administrator is to maximize his own net profit, which means that the expected net benefit of a tagged arriving customer finding the server busy and deciding to enter the orbit can't arrive to positive value. Then by (17), we have that

$$R - p - C\left[\frac{C_1}{1-q\rho} + \frac{1}{\mu}\right] = 0,$$

which leads to

$$p(q) = R - C\left(\frac{C_1}{1-q\rho} + \frac{1}{\mu}\right). \tag{25}$$

Inserting (24) and (25) into (23) yields

$$\begin{aligned} S_{prof}(q) &= \frac{\lambda\mu}{\mu+\lambda\bar{q}}\left(R - C\left(\frac{C_1}{1-q\rho}+\frac{1}{\mu}\right)\right) \\ &= \left(R - \frac{C}{\mu} + \frac{C}{\lambda}\right)\frac{\lambda\mu}{\mu+\lambda\bar{q}} \\ &\quad - \frac{C\mu(\lambda+\alpha)(\lambda+\alpha+\theta)}{\beta\mu\lambda\theta+(\lambda+\alpha+\theta)[\alpha\mu-\lambda q(\lambda+\alpha)]}. \end{aligned} \tag{26}$$

From (26), we can obtain that

$$S'_{prof}(q) = f_1(q)f_2(q),$$

where

$$\begin{aligned} f_1(q) &= \frac{C\lambda\mu(\lambda+\alpha)^2(\lambda+\alpha+\theta)^2}{(\mu+\lambda\bar{q})^2(\beta\mu\lambda\theta+(\lambda+\alpha+\theta)(\alpha\mu-\lambda q(\lambda+\alpha)))^2} \\ &\quad \times \Big(\sigma(\beta\mu\lambda\theta+(\lambda+\alpha+\theta)(\alpha\mu-\lambda q(\lambda+\alpha))) \\ &\quad + (\mu+\lambda\bar{q})\Big), \\ f_2(q) &= \sigma(\beta\mu\lambda\theta+(\lambda+\alpha+\theta)(\alpha\mu-\lambda q(\lambda+\alpha))) \\ &\quad - (\mu+\lambda\bar{q}). \end{aligned}$$

Obviously, $f_1(q) > 0$ holds for any $q \in [0,1]$. Because $f_2(q)$ can be rewritten as

$$\begin{aligned} f_2(q) &= \lambda q(1-\sigma(\lambda+\alpha)(\lambda+\alpha+\theta)) \\ &\quad - [(\mu+\lambda) - \sigma(\beta\lambda\mu\theta+\alpha\mu(\lambda+\alpha+\theta))], \end{aligned}$$

which shows that $f_2(q)$ is a decreasing function. The root of $f_2(q) = 0$ is $q = q^*_{prof}$, which is given by (22). We can easily testify that $0 \le q^*_{prof} \le 1 \Leftrightarrow L_{prof} \le R/C \le U_{prof}$, where L_{prof} and U_{prof} are given, respectively, by (19) and (20). From $S'_{prof}(q) = 0 \Leftrightarrow f_2(q) = 0$, we have the following three cases:

- Case I. If $R/C < L_{prof}$, which is equivalent to $q^*_{prof} < 0$. In this case, $f_2(q) < 0$ for any $q \in [0,1]$, which leads to $S'_{prof}(q) < 0$, $q \in [0,1]$. Therefore, $S_{prof}(q)$ is a non-increasing function, which indicates that the best joining policy is $q_{prof} = 0$.
- Case II. If $L_{prof} \le R/C \le U_{prof}$, which is equivalent to $0 \le q^*_{prof} \le 1$. In this case, $f_2(q) \ge 0$ for $q \in [0, q^*_{prof}]$ and $f_2(q) \le 0$ for $q \in [q^*_{prof}, 1]$, which yields $S'_{prof}(q) \ge 0$ for $q \in [0, q^*_{prof}]$ and $S'_{prof}(q) \le 0$ for $q \in [q^*_{prof}, 1]$. Therefore, $S_{prof}(q)$ attains its maximum at q^*_{prof}, i.e., the best joining policy is $q_{prof} = q^*_{prof}$.
- Case III. If $R/C > U_{prof}$, which is equivalent to $q^*_{prof} > 1$. In this case, $f_2(q) > 0$ for any $q \in [0,1]$, then $S'_{prof}(q) > 0$ holds for any $q \in [0,1]$, which shows that $S_{prof}(q)$ is an increasing function, so the best joining policy is $q_{prof} = 1$.

Remark 2: In the M/M/1 retrial queue with orbital search and $\rho \ge 1$. In this case, $0 \le q^*_{prof}\rho < 1$ always holds for $R/C \ge L_{prof}$. The unique mixed equilibrium joining strategy 'enter the orbit with probability q_{prof} while finding the server busy', where

$$q_{prof} = \begin{cases} 0, & \frac{R}{C} < L_{prof}, \\ q^*_{prof}, & \frac{R}{C} \ge L_{prof}. \end{cases}$$

In the next, we give the administrator's optimal price, which is selected to induce the customers to behave in the joining policy q_{prof}. First we present the result when $\rho < 1$.

Corollary 1: In the M/M/1 retrial queue with orbital search and $\rho < 1$, the optimal pricing that induces the customers to behave in the optimal joining policy q_{prof} to maximize the administrator's net profit is given by

$$p_{prof} = \begin{cases} R - CL_e, & \frac{R}{C} < L_{prof}, \\ p^*_{prof}, & L_{prof} \le \frac{R}{C} \le U_{prof}, \\ R - CU_e, & \frac{R}{C} > U_{prof}, \end{cases}$$

where $p^*_{prof} = R - C\left(\frac{C_1}{1-q^*_{prof}\rho} + \frac{1}{\mu}\right)$.

Proof: From Theorem 3 and (25), we can readily obtain the expected result.

Remark 3: In the M/M/1 retrial queue with orbital search and $\rho \ge 1$, the optimal pricing that induces the customers to behave in the optimal joining policy q_{prof} to maximize the administrator's net profit is given by

$$p_{prof} = \begin{cases} R - CL_e, & \frac{R}{C} < L_{prof}, \\ p^*_{prof}, & \frac{R}{C} \ge L_{prof}. \end{cases}$$

V. SOCIALLY OPTIMAL PROBLEM

Now we can proceed to discuss the problem of the social optimization, that is, we focus on the joining strategy q_{soc} which maximizes the social net welfare per unit time. we have the following result.

Theorem 4: In the M/M/1 retrial queue with orbital search and $\rho < 1$, there exists a unique mixed joining strategy 'enter the orbit with probability q_{soc} while finding the server busy' which maximizes the social net welfare per unit time, where

$$q_{soc} = \begin{cases} 0, & \frac{R}{C} < L_{soc}, \\ q^*_{soc}, & L_{soc} \le \frac{R}{C} \le U_{soc}, \\ 1, & \frac{R}{C} > U_{soc}, \end{cases} \tag{27}$$

where

$$L_{soc} = \frac{(\lambda+\mu)^2(\lambda+\alpha+\theta)(\lambda+\alpha)}{\lambda\mu^2(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))} - \frac{1}{\lambda},$$

$$U_{soc} = \frac{\mu^2(\lambda+\mu)(\lambda+\alpha+\theta)(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))}{\lambda[\beta\lambda\mu\theta+(\alpha+\lambda+\theta)(\alpha\mu-\lambda(\lambda+\alpha))]^2} - \frac{1}{\lambda},$$

$$\tau = \frac{1}{(\lambda+\alpha)(\lambda+\alpha+\theta)}\sqrt{\left(1+\frac{\lambda R}{C}\right)\frac{(\lambda+\alpha)(\lambda+\alpha+\theta)}{\beta\lambda\theta+\alpha(\lambda+\alpha+\theta)}},$$

$$q^*_{soc} = \frac{\lambda+\mu[1-\tau(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))]}{\lambda[1-\tau(\lambda+\alpha)(\lambda+\alpha+\theta)]}.$$

Proof: For a given joining strategy q, the social net welfare per unit time, denoted as $S_{soc}(q)$, is the sum of the net profit of the administrator per unit time and the net benefit of a customer per unit time, then

$$\begin{aligned} S_{soc}(q) &= \lambda[1-\bar{q}\Pi_2]p + \lambda(1-\Pi_2)(R-p-C/\mu) \\ &\quad + \lambda q\Pi_2\left(R-p-C\left(\frac{C_1}{1-q\rho}+\frac{1}{\mu}\right)\right) \\ &= \frac{\mu(\lambda R+C)}{\mu+\lambda(1-q)} \\ &\quad - \frac{C\mu(\beta\lambda\theta+\alpha(\lambda+\alpha+\theta))}{\beta\mu\lambda\theta+(\alpha+\lambda+\theta)(\alpha\mu-\lambda q(\alpha+\lambda))}. \end{aligned}$$

Following the same line with the proof of Theorem 3, we can obtain the result (27).

Remark 4: In the M/M/1 retrial queue with orbital search and $\rho \ge 1$. In this case, $0 \le q^*_{soc}\rho < 1$ always holds for $R/C \ge L_{soc}$. The unique mixed equilibrium joining strategy 'enter the orbit with probability q_{prof} while finding the server busy', where

$$q_{soc} = \begin{cases} 0, & \frac{R}{C} < L_{prof}, \\ q^*_{soc}, & \frac{R}{C} \ge L_{soc}. \end{cases}$$

In the next, we give the socially optimal price which is selected to induce the customers to behave in the socially optimal way, i.e., the social planner aim to eliminate differences between the optimal joining probabilities of the individual and the social. For the scenario $\rho < 1$, we have the following Corollary.

Corollary 2: In the M/M/1 retrial queue with orbital search and $\rho < 1$, the optimal pricing that induces the customers to behave in the optimal joining policy q_{soc} is given by

$$p_{soc} = \begin{cases} C(L_{soc} - L_e), & \frac{R}{C} < L_{soc}, \\ p^*_{soc}, & L_{soc} \le \frac{R}{C} \le U_{soc}, \\ C(U_{soc} - U_e), & \frac{R}{C} > U_{soc}, \end{cases} \tag{28}$$

where

$$p^*_{soc} = R - C\left(\frac{C_1}{1-q^*_{soc}\rho} + \frac{1}{\mu}\right). \tag{29}$$

Proof: Because any customer is selfish to maximize his own net benefit. Then the social optimal price should be selected to eliminate the differences between the optimal joining probabilities of the individual and the social. From Theorem 4, when $\frac{R}{C} < L_{soc}$, the socially optimal joining probability $q_{soc} = 0$. In this case, let $p = C(L_{soc} - L_e)$, then $\frac{R-p}{C} < L_e$, and $q_e = 0$. This indicates that $p_{soc} = C(L_{soc}-L_e)$ is the optimal price that induces the customers to behave in the socially optimal joining policy, and we obtain the first branch of (28). Along the same line, we can obtain the third branch of (28).

When $L_{soc} \le \frac{R}{C} \le U_{soc}$, the socially optimal joining probability is q^*_{soc}. Solving the equation $q^*_{soc} = q^*_e$ with respect to p yields $p = p^*_{soc}$, which is given by (29). If such price is levied on customers, then the inequality $L_e \le \frac{R-p}{C} \le \frac{1}{\mu} + \frac{C_1}{1-\rho}$ holds. In this case, $p_{soc} = p^*_{soc}$ is the optimal price that induces the customers to behave in the socially optimal joining policy, and we obtain the second branch of (28).

Remark 5: In the M/M/1 retrial queue with orbital search and $\rho \geq 1$, the optimal pricing that induces the customers to behave in the optimal joining policy q_{prof} is given by

$$p_{soc} = \begin{cases} C(L_{soc} - L_e), & \frac{R}{C} < L_{soc}, \\ p^*_{soc}, & \frac{R}{C} \geq L_{soc}. \end{cases}$$

VI. NUMERICAL EXAMPLES

In this section, we present some numerical examples to show the tendency of the joining probabilities q_e, q_{prof}, q_{soc} and optimal prices p_{prof}, p_{soc} with some parameters.

Fig.2 gives the curve of q_e vs the admission fee p, which shows that the individual joining probability q_e is non-increasing as function of p. The reason is that for a larger admission fee p levied on the customer, a smaller proportion of customers are willing to join the orbit for their service.

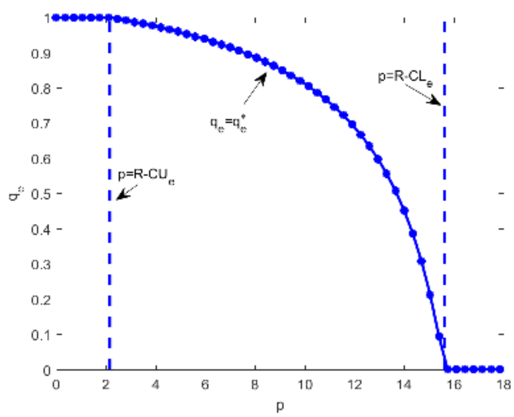

FIGURE 2. q_e **vs.** p $(\lambda, \mu, \beta, \theta, \alpha, R, C) = (2, 3, 0.55, 1, 7.5, 20, 5)$.

For $(\lambda, \mu, \alpha, p) = (2, 3, 7.5, 2.5)$, we plot the curves q_e, q_{soc}, q_{prof} vs. β, θ, R, and C, respectively, in Fig.3(a) $(\theta, R, C) = (1, 18, 5)$, Fig.3(b) $(\beta, R, C) = (0.55, 18, 5)$, Fig.3(c) $(\beta, \theta, C) = (0.55, 1, 5)$ and Fig.3(d) $(\beta, \theta, R) = (0.55, 1, 18)$. It is noted that q_e, q_{soc}, q_{prof} are increasing in Fig.3(a)-(c), and decreasing in Fig.3(d). On one hand, that is because as β and θ increase, the sojourn times of customers in the system will decrease and then reduce the total cost, which encourages the customers to enter the orbit. On the other hand, that is because larger value of R attracts more customers to join the orbit and large cost per unit time C easily discourages customers to join the orbit. Lastly, Fig.3(a)-(d) also shows that $q_e \geq q_{soc} \geq q_{prof}$ generally holds for relatively large R, however, due to selfish individual always want to maximize his own benefit, when R is relatively small, $q_e \leq q_{soc}$ might happen, as shown in Fig.3(c).

For $(\lambda, \mu, \alpha) = (2, 3, 7.5)$, we plot the curves p_{soc}, p_{prof} vs. β, θ, R, and C, respectively, in Fig.4(a) $(\theta, R, C) = (1, 6.5, 5)$, Fig.4(b) $(\beta, R, C) = (0.55, 6.5, 5)$, Fig.4(c) $(\beta, \theta, C) = (0.55, 1, 5)$ and Fig.4(d)

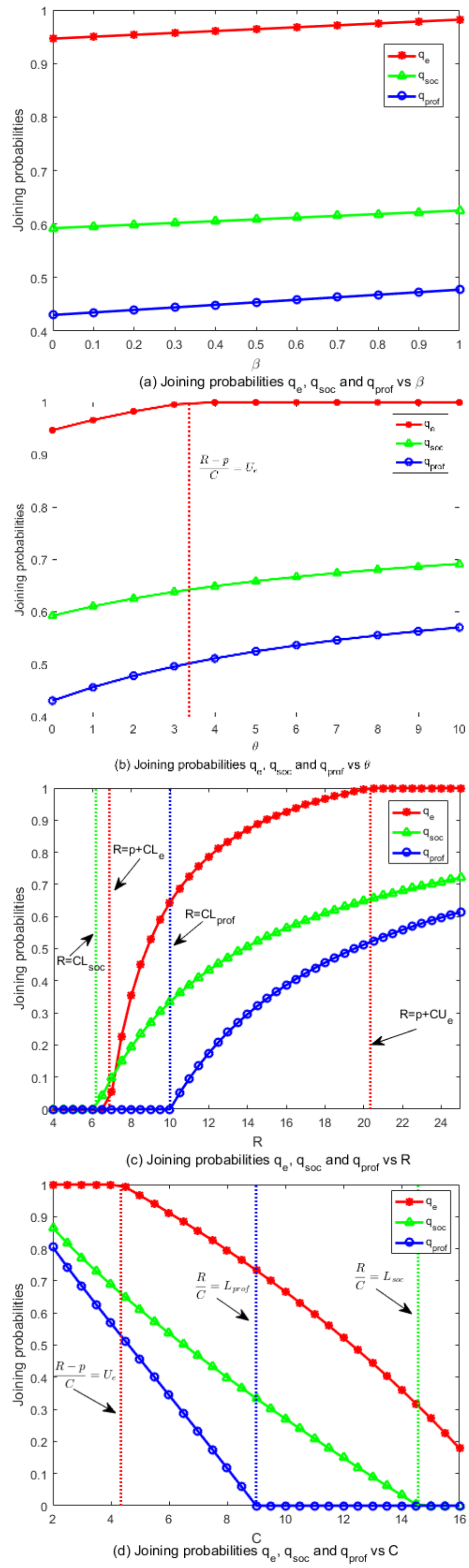

FIGURE 3. q_e, q_{soc}, q_{prof} **vs.** β, θ, R, **and** C**, respectively.**

$(\beta, \theta, R) = (0.55, 1, 18)$. Under the given values of these parameters and in given intervals, we examine that $p_{prof} = R - CL_e$ and $p_{soc} = p^*_{soc}$ in Fig.4(a) and in Fig.4(b).

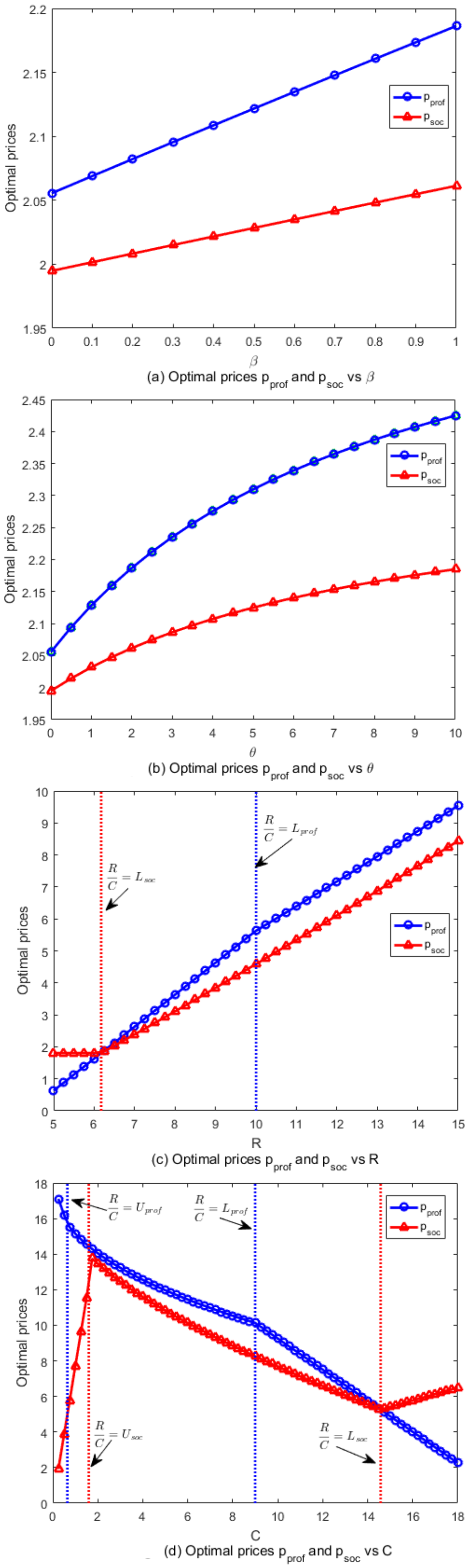

FIGURE 4. p_{soc}, p_{prof} **vs.** β, θ, R, **and** C, **respectively.**

Fig.4(a)-(c) indicate that more customers are encouraged to enter the orbit for their service when β, θ and R increase, so the system administrator and social manager can benefit more from higher entrance fee, then p_{soc}, p_{prof} are non-decreasing in Fig.4(a)-(c) as function of β, θ and R, respectively. However, less customers would like to enter for larger value of C, then the optimal price of the administrator is decreasing as function of C, as shown in Fig.4(d). However the social manager always want to maximize the social net welfare and coordinate the difference between, when C is relatively smaller or larger, the optimal socially price p_{soc} is increasing, as shown by the curve of p_{soc} in Fig.4(d), which is corresponding the first branch and the third branch in (28). When C is relatively moderate, the optimal socially price p_{soc} is decreasing as function of C, the reason is that the social manager can incite the customer to enter the orbit by reducing the admission fee when the cost C increases during a moderate interval, which is corresponding the second branch and the third branch in (28).

VII. CONCLUSION

This paper has developed an extensive discussion of the joining probabilities and optimal pricing problems for an M/M/1 retrial queue with orbital search, where arriving customers have the option to decide whether to enter the obit or not when finding a busy server. When the server becomes idle, the customers in the orbit can either actively access to the server or be searched by the administrator of the system for their service. For this model, we have investigated the individual equilibrium joining probability and presented the optimal joining probabilities and corresponding optimal prices from the perspective of system administrator and social manager. As extensions of this work, one can consider the equilibrium balking strategies and optimal pricing problems for the retrial queue with linear retrial policy and orbital search or the retrial queues with many servers and orbital search. In the so called linear retrial policy, the intervals between successive repeated attempts of the customers in the orbit are exponentially distributed random variable with parameter $\nu_n = \alpha(1 - \delta_{n,0}) + n\nu$ when the orbit size is n, $\delta_{n,0}$ denotes kronecker's delta function. Obviously, ν_n depends on the number of customers in the orbit when $\nu > 0$, however, the retrial policy used in this paper is a special case of the linear retrial policy when $\nu = 0$. Moreover, retrial queues with many servers are more realistic in practice. Further investigation of the equilibrium balking strategies and optimal pricing problems for these retrial queues would be worth studying.

REFERENCES

[1] T. Yang and J. G. C. Templeton, "A survey on retrial queues," *Queueing Syst.*, vol. 2, no. 3, pp. 201–233, Sep. 1987.

[2] G. Falin, "A survey of retrial queues," *Queueing Syst.*, vol. 7, no. 2, pp. 127–168, Jun. 1990.

[3] G. Fayolle, "A simple telephone exchange with delayed feedback," in *Teletraffic Analysis and Computer Performance Evaluation*, O. J. Boxma, J. W. Cohen, and H. C. Tijms, Eds. Amsterdam, The Netherlands: Elsevier, 1986, pp. 245–253.

[4] K. Farahmand, "Single line queue with repeated demands," *Queueing Syst.*, vol. 6, no. 1, pp. 223–228, Dec. 1990.

[5] J. R. Artalejo and A. Gómez-Corral, "Steady state solution of a single-server queue with linear repeated requests," *J. Appl. Probab.*, vol. 34, no. 1, pp. 223–233, 1997.

[6] J. R. Artalejo and A. Gómez-Corral, *Retrial Queueing Systems: A Computational Approach*. Berlin, Germany: Springer, 2008.
[7] G. Falin and J. G. C. Templeton, *Retrial Queues*. London, U.K.: Chapman & Hall, 1997.
[8] J. R. Artalejo, "A classified bibliography of research on retrial queues: Progress in 1990–1999," *Top*, vol. 7, no. 2, pp. 187–211, Dec. 1999.
[9] J. R. Artalejo, "Accessible bibliography on retrial queues: Progress in 2000–2009," *Math. Comput. Model.*, vol. 5, pp. 1071–1081, May 2010.
[10] J. R. Artalejo, A. N. Dudin, and V. I. Klimenok, "Stationary analysis of a retrial queue with preemptive repeated attempts," *Oper. Res. Lett.*, vol. 28, no. 4, pp. 173–180, May 2001.
[11] A. Krishnamoorthy, D. Shajin, and B. Lakshmy, "On a queueing-inventory with reservation, cancellation, common life time and retrial," *Ann. Oper. Res.*, vol. 247, no. 1, pp. 365–389, Dec. 2016.
[12] A. Gómez-Corral, "Stochastic analysis of a single server retrial queue with general retrial times," *Nav. Res. Logist.*, vol. 46 no. 5, pp. 561–581, Jul. 1999.
[13] D. Arivudainambi and P. Godhandaraman, "Retrial queueing system with balking, optional service and vacation," *Ann. Oper. Res.*, vol. 229, no. 1, pp. 67–84, Jun. 2015.
[14] J. Kim and B. Kim, "A survey of retrial queueing systems," *Ann. Oper. Res.*, vol. 247, no. 1, pp. 3–36, Dec. 2016.
[15] I. Dimitriou, "A mixed priority retrial queue with negative arrivals, unreliable server and multiple vacations," *Appl. Math. Model.*, vol. 37, no. 3, pp. 1295–1309, Feb. 2013.
[16] I. Dimitriou, "A two class retrial system with coupled orbit queues," *Probab. Eng. Inf. Sci.*, vol. 31, no. 2, pp. 139–179, Apr. 2017.
[17] R. Srinivasan, J. Talim, and J. Wang, "Performance analysis of a call center with interactive voice response units," *Top*, vol. 12, pp. 91–110, Jun. 2004.
[18] V. Dragieva and T. Phung-Duc, "Two-way communication $M/M/1$ retrial queue with server-orbit interaction," in *Proc. 11th Int. Conf. Queueing Theory Netw. Appl. (QTNA)*, 2016, Art. no. 11. doi: 10.1145/3016032.3016049.
[19] A. Krishnamoorthy, T. G. Deepak, and V. C. Joshua, "An $M/G/1$ retrial queue with nonpersistent customers and orbital search," *Stochastic Anal. Appl.*, vol. 23, pp. 975–997, Mar. 2005.
[20] S. R. Chakravarthy, A. Krishnamoorthy, and V. C. Joshua, "Analysis of a multi-server retrial queue with search of customers from the orbit," *Perform. Eval.*, vol. 63, no. 8, pp. 776–798, Aug. 2006.
[21] T. G. Deepak, A. N. Dudin, V. C. Joshua, and A. Krishnamoorthy, "On an $M^{(X)}/G/1$ Retrial System with two types of search of customers from the orbit," *Stochastic Anal. Appl.*, vol. 31, no. 1, pp. 92–107, 2013.
[22] S. Gao and J. Wang, "Performance and reliability analysis of an M/G/1-G retrial queue with orbital search and non-persistent customers," *Oper. Res. Lett.*, vol. 236, no. 2, pp. 561–572, Jul. 2014.
[23] P. Naor, "The regulation of queue size by levying tolls," *Econometrica*, vol. 37, no. 1, pp. 15–24, Jan. 1969.
[24] N. M. Edelson and D. K. Hilderbrand, "Congestion tolls for Poisson queuing processes," *Econometrica*, vol. 43, no. 1, pp. 81–92, Jan. 1975.
[25] R. Hassin, *Rational Queueing*. London, U.K.: Chapman & Hall, 2016.
[26] R. Hassin and M. Haviv, *To Queue or Not to Queue: Equilibrium Behavior in Queueing Systems*. Boston, MA, USA: Kluwer, 2003.
[27] A. Economou and S. Kanta, "Equilibrium balking strategies in the observable single-server queue with breakdowns and repairs," *Oper. Res. Lett.*, vol. 36, no. 6, pp. 696–699, Nov. 2008.
[28] X. Li, J. Wang, and F. Zhang, "New results on equilibrium balking strategies in the single-server queue with breakdowns and repairs," *Appl. Math. Comput.*, vol. 241, no. 15, pp. 380–388, Aug. 2014.
[29] J. Wang and F. Zhang, "Equilibrium analysis of the observable queues with balking and delayed repairs," *Appl. Math. Comput.*, vol. 218, no. 6, pp. 2716–2729, Nov. 2011.
[30] S. Yu, Z. Liu, and J. Wu, "Equilibrium strategies of the unobservable $M/M/1$ queue with balking and delayed repairs," *Appl. Math. Comput.*, vol. 290, pp. 56–65, Nov. 2016.
[31] O. Boudali and A. Economou, "Optimal and equilibrium balking strategies in the single server Markovian queue with catastrophes," *Eur. J. Oper. Res.*, vol. 218, pp. 708–715, May 2012.
[32] O. Boudali and A. Economou, "The effect of catastrophes on the strategic customer behavior in queueing systems," *Nav. Res. Logistics*, vol. 60, no. 7, pp. 571–587, Oct. 2013.
[33] O. Boundali and A. Economou, "Equilibrium joining strategies in batch service queueing systems," *Eur. J. Oper. Res.*, vol. 260, no. 3, pp. 1142–1151, Aug. 2017.
[34] A. Economou and S. Kanta, "Equilibrium customer strategies and social–profit maximization in the single-server constant retrial queue," *Nav. Res. Logistics*, vol. 58, no. 2, pp. 107–122, Mar. 2011.
[35] J. Wang and F. Zhang, "Strategic joining in M/M/1 retrial queues," *Eur. J. Oper. Res.*, vol. 230, no. 1, pp. 76–87, Oct. 2013.
[36] J. Wang and F. Zhang, "Monopoly pricing in a retrial queue with delayed vacations for local area network applications," *IMA J. Manage. Math.*, vol. 27, no. 2, pp. 315–334, Apr. 2016.
[37] J. Wang and W. W. Li, "Noncooperative and cooperative joining strategies in cognitive radio networks with random access," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5624–5636, Jul. 2016.
[38] J. Wang, X. Zhang, and P. Huang, "Strategic behavior and social optimization in a constant retrial queue with the N-policy," *Eur. J. Oper. Res.*, vol. 256, pp. 841–849, Feb. 2017.
[39] S. Gao, H. Dong, and X. Wang, "Equilibrium and pricing analysis for an unreliable retrial queue with limited idle period and single vacation," *Oper. Res.*, pp. 1–23, Nov. 2018. doi: 10.1007/s12351-018-0437-7.
[40] Y. Zhang, J. Wang, and W. W. Li, "Optimal pricing strategies in cognitive radio networks with heterogeneous secondary users and retrials," *IEEE Access*, vol. 7, pp. 30937–30950, Mar. 2019.
[41] M. F. Neuts, *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach*. Baltimore, MD, USA: The Johns Hopkins Univ. Press, 1981.

SHAN GAO was born in Shandong, China, in 1975. She received the Ph.D. degree in probability theory and mathematical statistics from Central South University, China, in 2011. Since 2015, she has been a Professor with the School of Mathematics and Statistics, Fuyang Normal College.

She has published over 30 articles in international journals, such as the *European Journal of Operational Research*, *Applied Mathematical Modeling*, *Annals of Operations Research*, the *Journal of Computational and Applied Mathematics*, *Mathematical and Computer Modeling*, the *International Journal of Computer Mathematics*, and *Quality Technology & Quantitative Management*, etc. Her research interests include queueing theory and risk theory. She is currently a member of the Operations Research Society of China (ORSC).

JIE ZHANG was born in Anhui, China, in 1981. She received the master's degree in applied mathematics from Capital Normal University, China, in 2011. Since 2008, she has been a Lecturer with the School of Mathematics and Statistics, Fuyang Normal University. Her research interests include queueing theory and supply chain management. She has coauthored almost seven articles in the areas of her research interests. She is currently a member of the Operations Research Society of China (ORSC).