Stratified Map Spaces: A Formal Basis for Multi-resolution Spatial Databases

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Abstract

Precision is a key component of spatial data quality and in this era of globally distributed spatial data it is essential to be able to integrate multiple distributed data sets with heterogeneous levels of precision. Imprecision arises through limitations on semantic and geometric resolution of data representations. Generalization, and in particular model-oriented generalization, is an important process in this context, because it enables translation between different levels of precision. This paper provides a formal approach to multi-resolution in spatial data handling. It begins by motivating the work and pointing to some of the background research, and then introduces the basic concepts underlying the approach, focusing on the new concept of a stratified map space. The approach is quite general, and to show its application, the paper uses it to provide a formal foundation for generalization and vague regions.

Keywords: resolution, generalization, vagueness, theory

1. Introduction

With the distributed mode of spatial data handling increasingly becoming the norm, there is a pressing need for a well-founded model of spatial multi-datasets that tackles the issue of heterogeneity amongst component databases. Heterogeneity has several dimensions, and in the context of spatial data these include semantic and geometric differences, as well as heterogeneity of metadata and data quality. A major impediment to achieving data integration is semantic and geometric heterogeneity among the various data sources, both in terms of the data themselves and the precision to which they are represented. This paper proposes a general model that helps to provide a formal basis for processing and reasoning with spatial datasets that are heterogeneous with regard to semantic and geometric precision. The model will be applied particularly to the case of model-oriented generalization, as this is a well-researched subset of the more general area of multi-precision representation, which transforms representations from higher to lower precisions. However, the model is actually quite general, and resolutions of component spatial datasets need not be placed in a strict linear ordering of increasing imprecision. When the word 'generalization' is used in this paper it usually refers to model-oriented generalization, in the sense of Müller *et al.* (1995). We should also emphasize that at this stage of the research there is no attempt to formalise the intricate and detailed processes of cartographic generalization. However, the framework presented here is capable of extension to handle more sophisticated generalization practice.

Resolution or granularity is concerned with the level of discernibility between elements of a phenomenon that is being represented by the dataset. Higher resolutions and finer granularities allow more detail to be observed in the components of the phenomenon, and multi-resolution implies a collection of such levels of details. The issue of multi-resolution spatial datasets has been taken up by several authors. Puppo and Dettori (1995) provide a formal model of some of the topological and metric aspects of multi-resolution using abstract cell complexes and homotopy theory - both topics within algebraic topology. Parts of this work can be seen as extending formal models of single spatial datasets (for example, Egenhofer *et al.* 1989, Pigot 1992, Worboys 1992) to the case of multiple spatial datasets. Rigaux and Scholl (1995) discuss the impact of scale and resolution on spatial data modelling and querying. They develop a theory with spatial and semantic components and apply the ideas to a partial implementation in the object-oriented DBMS O_2.

The major application of the model presented in this paper is developed in the context of generalization, which allows movement from higher to lower resolution representations. Cartographic generalization has been the subject of a great deal of research by the cartographic and GIS communities, particularly on the geometric aspects of the generalization process (see for example Buttenfield and McMaster 1991, Müller *et al.* 1995). Dettori and Puppo (1996) state that geospatial data generalization can be 'considered a non-algorithmic task since it involves both exact rules and as yet unquantified considerations', which the authors later relate to the move from metric to symbolic information at smaller scales. Even though it is the case that an all-embracing automatic approach has not yet been found, there is a large literature on computational aspects of components of the generalization process. The geometric transformations that an entity may undergo as part of cartographic generalization have been categorized by Puppo and Dettori (1995).

To give examples of the kinds of approaches to parts of the generalization process that are amenable to computation, Delaunay triangulations and the dual construct, Voronoi diagrams have been successfully applied to the detection and resolution of graphical conflicts arising from visually appropriate representations of spatial object at smaller scales (Ware and Jones 1996). These triangulations and their duals have been used for more knowledge-oriented tasks associated with generalization, such as the recognition of clusters of buildings (Regnauld 1996). Other examples of automated generalization processes are given in (Robinson and Lee, 1994).

This paper is concerned with both semantic and geometric aspects of spatial imprecision. Semantic heterogeneity occurs where there is lack of uniformity in the 'meaning, interpretation or intended use' of the data in the collection of datasets (Sheth and Larsen, 1990) and occurs particularly when the datasets are in distinct databases. The motivation for the paper is to provide a generic, well-founded theory of generalization and multi-resolution, in which topics such as the correctness of generalization processes and integration of semantically and geometrically heterogeneous geospatial datasets can be formally studied. A general formal model of multi-resolution spatial data will aid in placing these disparate processes within a unified framework. The framework presented in this paper has the characteristics listed below, and it is the ability to be able to cater for all these aspects which is the unique contribution of this work.

- It covers uniformly both the semantic and geometric components of spatial data.
- It is formal, thus providing a firm foundation for multi-resolution data modelling, including general criteria for assessing correctness of generalization algorithms without prescribing the precise nature of the algorithms.
- In includes a treatment of vagueness and imprecision, which are important aspects of spatial data quality.

2. Overview of the framework

2.1 Extent and granularity

This section begins by introducing two constructs that are fundamental to the framework subsequently developed. Assume we are given a spatial dataset providing information about some geospatial phenomenon and structured according to a prescribed representation. This representation will possess both extent and granularity.

The *extent* of a representation is the totality of the elements that make up the representation. For example, in the case of the semantic component, its extent would include the collection of taxonomic constructs in the model, maybe expressed as a collection of object types. The semantic extent provides the 'vocabulary' in which the domain being modelled may be expressed. For the spatial component, the spatial extent will be the total area spanned by the representation, maybe a country, road network or the domain of a remotely sensed image. The spatial extent specifies the global spatial 'world' (usually a region) that is represented. Aspects of the phenomenon outside the boundaries of that world will not be visible.

The *granularity* of a representation specifies the levels of detail with respect to which the data are registered. For the semantic component, the level of detail may be expressed as the fineness of the object classes available in the object inheritance hierarchy. In a spatial context, the level of detail might be the resolution or granularity of the spatial framework, below which spatial elements become indistinguishable, one from another.

In summary, the extent of a representation is the range of visibility of objects in the dataset, while the granularity is the level of detail within the extent. Limitations in the extent and granularity of a representation lead to incompleteness and imprecision respectively in computations made with respect to the representation.

2.2 Maps, granularities and stratified map spaces

This section outlines the main features of the framework, which is built using five principal concepts *map*, *map space*, *granularity lattice*, *stratified map space*, and *sheaf of stratified map spaces*.

Map

We use the term 'map' to denote an arbitrary finite collection of data. The general framework makes no assumptions about the nature of maps, but a useful example to bear in mind is a set of objects having both semantic and geometric attributes. The term is chosen so that a conventional paper map is one example, but we are thinking of the data conveyed by the paper map, rather than any specific visual realization of it.

Map space

A map space is a set of all possible maps described using some fixed representation vocabulary. In database terms, a map corresponds to a database state i.e. a particular collection of data held at a particular time, whereas a map space corresponds to the set of all possible databases states that are instances of some fixed schema. Map spaces are partially ordered by precision. So if m_1 and m_2 are maps in the same map space, we may have $m_1 \leq m_2$, which means that m_1 is a less vague, or more precise, version of the map m_2.

Granularity lattice

In a map space, the maps can vary in how vague they are, but there is a fixed maximum level of detail which they can contain. The idea of a granularity lattice is that of a set of levels of detail. We take $g_1 \leq g_2$ to mean that the granularity level g_1 is less coarse (or provides for more detail) than the granularity g_2.

Stratified map space

The notion of a stratified map space lies at the heart of our approach to multi-resolution data handling, and allows translation between maps representing the same extent at different levels of detail. A stratified map space (see figure 1) consists of a granularity lattice, G, and for each granularity $g \in G$, a map space **Maps**(g). There are two special transfer functions, namely **Gen** that transfers by some coarsening process a map from a lower to a higher level of imprecision, and **Lift** that transfers a map from a higher to a lower level of imprecision. **Lift** may be thought of as an enlargement operation that adds no new level of detail. In formal terms, whenever $g_1 \leq g_2$ in G, there are functions

$$\mathbf{Gen}[g_1, g_2] : \mathbf{Maps}(g_1) \rightarrow \mathbf{Maps}(g_2)$$

$$\mathbf{Lift}[g_1, g_2] : \mathbf{Maps}(g_2) \rightarrow \mathbf{Maps}(g_1)$$

between the map spaces **Maps**(g_1) and **Maps**(g_2).

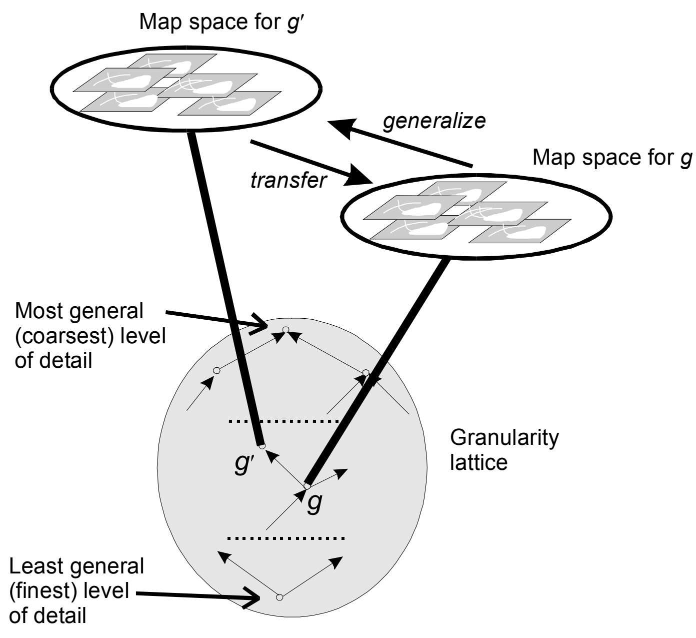

Figure 1: A stratified map space

Criteria for the correctness of generalization algorithms arise from the interaction between lifting and generalization, and development of constraints on **Gen** and **Lift** help us to specify particular aspects of generalization. It seems reasonable to require that for all $m \in$ **Maps**(g_1), we have $m \leq$ **Lift**(**Gen**(m)), where we have omitted the parameters g_1, and g_2 of **Lift** and **Gen** to aid readability. Also important is that for all $m \in$ **Maps**(g_2), we have $m \leq$ **Gen**(**Lift**(m)).

A particular model of generalization might impose additional constraints, such as making generalization functorial. Certainly, we would expect $\mathbf{Gen}[g_1, g_1]$ to be the identity on $\mathbf{Maps}(g_1)$, and we might also feel that the composition equation

$$\mathbf{Gen}[g_1, g_3] = \mathbf{Gen}[g_2, g_3]\mathbf{Gen}[g_1, g_2]$$

should hold, as it might be reasonable to assume that if we generalized a map from granularity g_1 to g_2 , and then performed another generalization from g_2 to g_3, then this is the same as generalizing directly from g_1 to g_3. However, we might weaken this to

$$\text{For all } m \in \mathbf{Maps}(g_1),\ \mathbf{Gen}[g_1, g_3](m) \leq \mathbf{Gen}[g_2, g_3](\mathbf{Gen}[g_1, g_2](m)),$$

which allows information loss under repeated generalization.

Sheaf of stratified map spaces

In a stratified map space we have variation of data quality (vagueness) and variation of level of detail (granularity), but the extent is fixed. For instance, we might have in a stratified map space a framework which allows us to deal with maps of a particular country showing both road and rail networks. We can talk about maps at different scales (geometric granularity), and levels of semantic detail (whether only main roads or all roads are displayed for example). However a single stratified map space cannot deal with maps which cover only a region within the country (limitation of geometric extent) or which show railways only and ignore roads altogether (limitation of semantic extent).

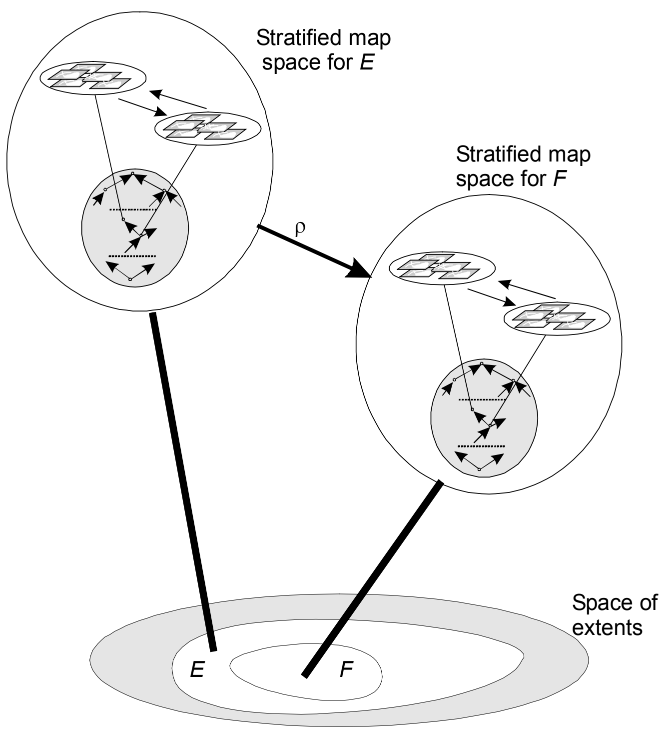

Figure 2: A sheaf of stratified map spaces

The collection of extents can be thought of as the open sets of a topological space, which we call the *extent space*. A *sheaf* over this extent space assigns to each extent, E, a stratified map space $\Sigma(E)$ and whenever we have a sub-extent $F \subseteq E$, there is a restriction function $\rho : \Sigma(E) \rightarrow \Sigma(F)$ (see figure 2). To fill in the details of this construction we would need to define ρ to be a morphism of stratified map spaces, that is a morphism of the granularity lattices with a family of functions between the map spaces which all interact in an appropriate manner with lifting, generalization and the vagueness ordering.

The sheaf construction is very helpful in formalizing map spaces, as combining (glueing together) consistent map spaces with overlapping extents into a seamless whole is handled quite naturally in the sheaf environment. This allows the framework to model integration of heterogeneous data sets.

3. Case Study of Vague Regions and Generalization

The aim of this section is to present a specific model which is an instance of the general framework outlined in the previous section. Although the model is relatively simple it demonstrates the way in which the framework is sufficiently rich to deal with several issues and the relationships between them. The model deals with maps combining both spatial and semantic information. This information exists at various levels of detail, and definitions are given of functions which translate between the various levels. We reemphasize that we do not intend a formalization of the complex processes of cartographic generalization, but a framework in which reasoning can begin to take place about spaces of generalized maps and their resulting imprecisions.

We now outline the main features of the model before going into the technical details. The granularity lattice is the product of two lattices, one handling spatial granularity, and one handling taxonomic granularity. Thus each level of granularity is a pair $\langle\sigma,\tau\rangle$ where σ is a spatial, and τ a taxonomic level of detail. Associated to each such pair there are partially ordered sets **VRegions**(σ) and **Classes**(τ) These are respectively sets of vague regions and of taxonomic classifications which are available as semantic attributes of objects at the stated granularity.

The stratified map space has a set of maps **Maps**$\langle\sigma,\tau\rangle$ for each granularity. The elements of **Maps**$\langle\sigma,\tau\rangle$. are structures consisting of a set of object ids, say *X*, and two functions *loc* : $X \rightarrow$ **VRegions**(σ) and *class* : $X \rightarrow$ **Classes**(τ). The functions assign to each object its location and its taxonomic class, or semantic attribute. An example of a map is illustrated in figure 3

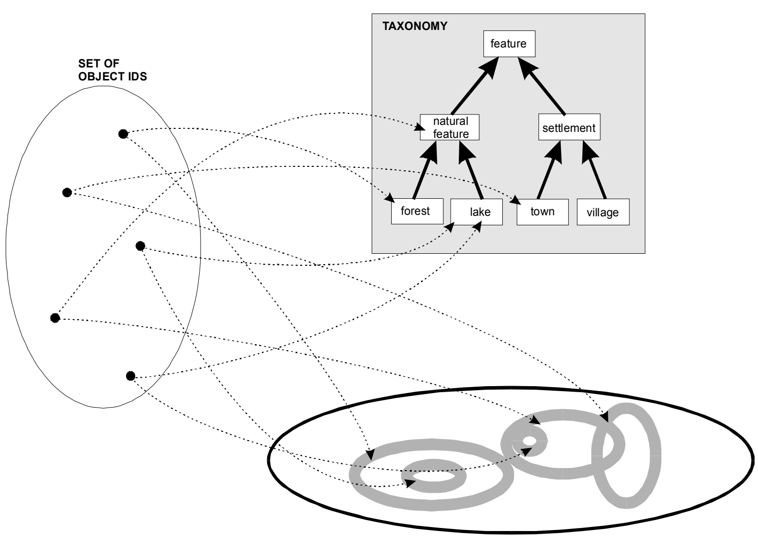

Figure 3: An example of a map

3.1 The Structure of Regions

Vague Regions

We assume a set, R, the elements of which are basic regions in some sense. These basic regions might be cells in some subdivision of physical space, or possibly pixels. By taking unions of elements of R we can generate other regions, the set of all such regions is denoted **R.**

A partition of R gives rise to a function $\sigma: R \rightarrow A$, where A is the set of equivalence classes of the partition. The set of all partitions of R is partially ordered by putting $\sigma_1 \leq \sigma_2$ where $\sigma_i: R \rightarrow A_i$, whenever there is a function $\varphi: A_1 \rightarrow A_2$ such that $\sigma_2 = \varphi\, \sigma_1$. If $\sigma_1 \leq \sigma_2$ this means that σ_1 is less general, i.e. more detailed than σ_2.

Each partition of R is a level of granularity with respect to which we can define vague regions over **R**. We can define a vague region over **R**, with respect to a partition $\sigma: R \rightarrow A$ to be a function $v: A \rightarrow \{in, maybe, out\}$. Such a function gives rise to a pair of subsets of R. The set of basic regions definitely included in the vague region is $lower(v) = \{r \in R \mid v\, \sigma\, r = in\}$. The set of basic regions possibly included in the vague region is $upper(v) = \{r \in R \mid v\, \sigma\, r \neq out\}$. Each vague region, v, can be thought of as standing for any region $\mathbf{r} \in \mathbf{R}$ such that $\mathbf{r}$ lies between the union of all elements of $lower(v)$ and the union of all elements of $upper(v)$. Further details of this type of vague region, which are closely related to the theory of rough sets, can be found in Worboys (1997).

The set of all vague regions of **R** with respect to $\sigma: R \rightarrow A$ will be denoted by **VRegions**(σ). This set is partially ordered by setting $v_1 \leq v_2$ if for all $a \in A$, $v_1 a \leq v_2 a$ where $in \leq maybe \geq out$, and *in* and *out* are incomparable. The ordering on **VRegions**(σ) models relative precision between vague regions. Knowing that $v_1 \leq v_2$ tells us that the set of crisp regions which v_1 might denote is a subset of the set of crisp regions which v_2 might denote.

Generalizing and Lifting Vague Regions

If $\sigma_1 \leq \sigma_2$ we define the region generalization function **RegGen**[σ_1,σ_2] by:

$$\mathbf{RegGen}[\sigma_1,\sigma_2] : \mathbf{VRegions}(\sigma_1) \rightarrow \mathbf{VRegions}(\sigma_2)$$

$$\mathbf{RegGen}[\sigma_1,\sigma_2]\; v\; a = max\; v\varphi^{-1}a$$

where *max* denotes the maximum with respect to the ordering of {*in*, *maybe*, *out*} defined above. It is easy to see that **VRegions**(σ_2) $\subseteq$ **VRegions**(σ_1) so we have an inclusion function **RegLift**[σ_1, σ_2]: **VRegions**(σ_2) $\rightarrow$ **VRegions**(σ_1). It can be shown that for any region $v \in$ **VRegions**(σ_1), the region **RegGen**[σ_1,σ_2]v is the best possible representation of v at the σ_2 level of detail.

3.2 The Structure of Taxonomies

Vague Classifications

We assume a semantic hierarchy is a tree, T (see figure 3 for an example with **feature** as the root). A level of granularity for T is a *cut* in the tree in the sense of Rigaux and Scholl (1995). A cut, τ, is a subtree of T having the same root and such that for every node, n of τ, either τ includes all the immediate descendants of n or none of them. Each element of a cut τ is a vague semantic attribute, the least detailed ones being those furthest up the tree i.e. nearest the root.

The set of all classifications with respect to a cut τ is denoted **Classes**(τ). In this basic model, τ and **Classes**(τ) are equal as sets, but it is useful to keep the conceptual distinction between a level of detail and the vocabulary available at this level of detail.

Generalizing and Lifting Classifications

If τ_1 and τ_2 are cuts, and $\tau_1 \leq \tau_2$ there is a taxonomic generalization function

$$\textbf{ClassGen} : \textbf{Classes}(\tau_1) \rightarrow \textbf{Classes}(\tau_2).$$

This function takes $k \in$ **Classes**(τ_1) to the least element of **Classes**(τ_2) which is greater than or equal to k. The fact that we are dealing with trees here means that such a least element will exist. There is also an inclusion function **ClassLift** : **Classes**(τ_2) $\rightarrow$ **Classes**(τ_1).

3.3 The Stratified Map Space

The granularity lattice consists of pairs of the form $\langle\sigma, \tau\rangle$ where σ is a spatial level of detail, and τ is a taxonomic one. The order on these granularities is defined in terms of the orderings on the spatial and the taxonomic parts defined above. Thus $\langle\sigma_1, \tau_1\rangle \leq \langle\sigma_2, \tau_2\rangle$ if and only if $\sigma_1 \leq \sigma_2$ and $\tau_1 \leq \tau_2$.

Associated with a level of detail $\langle\sigma, \tau\rangle$, we have the set of maps **Maps**$\langle\sigma, \tau\rangle$. An element of **Maps**$\langle\sigma, \tau\rangle$ is a set, X, equipped with two functions: $loc : X \rightarrow$ **VRegions**(σ), and $class : X \rightarrow$ **Classes**(τ). The maps are partially ordered in the following way. Suppose that m and m' are maps in **Maps**$\langle\sigma, \tau\rangle$ with m consisting of functions $loc : X \rightarrow$ **VRegions**(σ), and $class : X \rightarrow$ **Classes**(τ), and m' consisting of functions $loc' : X' \rightarrow$ **VRegions**(σ), and $class' : X' \rightarrow$ **Classes**(τ). Then $m \leq m'$ if $X = X'$ and there are functions φ : **VRegions**(σ) $\rightarrow$ **VRegions** $s(\sigma)$, and ψ : **Classes**(τ) $\rightarrow$ **Classes**(τ) such that $loc = \varphi\, loc'$, and $class = \psi\, class'$, and for every $r \in$ **VRegions**(σ), $r \leq \varphi r$ and for every $c \in$ **Classes**(τ), $c \leq \psi c$.

Suppose now that we have granularities $g_1 = \langle\sigma_1, \tau_1\rangle$ and $g_2 = \langle\sigma_2, \tau_2\rangle$ where $g_1 \leq g_2$ We need to define the generalization function **Gen**[g_1, g_2] : **Maps**(g_1) $\rightarrow$ **Maps**(g_2), and the lifting function **Lift**[g_1, g_2] : **Maps**(g_2) $\rightarrow$ **Maps**(g_1). The generalization function assigns to the map $\langle loc_1 : X \rightarrow$ **VRegions**(σ_1), $class_1 : X \rightarrow$ **VRegions**(τ_1)$\rangle$, the map $\langle$**RegGen**[σ_1, σ_2] loc_1, **ClassGen**[τ_1, τ_2] $class_1\rangle$,. The lifting function is constructed in a similar way; it assigns to a map $\langle loc_2 : X \rightarrow$ **VRegions**(σ_2), $class_2 : X \rightarrow$ **VRegions**(τ_2)$\rangle$, the map $\langle$**RegLift**[σ_1, σ_2] loc_2, **ClassLift**[τ_1, τ_2] $class_2\rangle$.

3.4 Variation over Extents

Next we need to consider the extents. It is reasonable to assume that set **R** of all crisp regions is a complete Heyting algebra. The precise details of this structure are not important here, but further details of the relevance of Heyting algebras to spatial information theory and be found in Stell and Worboys (1997). The most important aspect is that we have a notion of one region being a sub-region of another. Associated to each crisp region $\mathbf{r} \in \mathbf{R}$, we have the granularity lattice **RegGran**(**r**), the elements of which are partitions of the set of basic regions in **r**. When $\mathbf{r}_1 \subseteq \mathbf{r}_2$, so $\mathbf{r}_1$ is a sub-region of $\mathbf{r}_2$, any partition of $\mathbf{r}_2$ can be restricted to a partition of $\mathbf{r}_1$. This gives rise to a restriction function **RegRest**[$\mathbf{r}_1,\mathbf{r}_2$] : **RegGran**($\mathbf{r}_2$) $\rightarrow$ **RegGran**($\mathbf{r}_1$). Similarly any map with a granularity in **RegGran**($\mathbf{r}_2$) can be restricted to a map with the restricted granularity. The idea here is simply that of forgetting about all objects whose location has no intersection with the region $\mathbf{r}_1$.

For taxonomic extents, it is sufficient for the purposes of this case study to consider the set of upper sets of the tree *T*. There are arguments for allowing more general subtrees of *T*, which might not have the same root as *T*. These can be dealt with in our framework, but a detailed treatment of this case introduces algebraic complications which there is not space to discuss here. A treatment of semantic structures more general than trees, and of generalization with respect to them can be found in Worboys (1998).The set of all upper sets of any poset, and in particular of any tree, is well known to be the set of open sets for a topology on the poset. Thus the set of upper sets of *T* forms a complete Heyting algebra where the ordering is opposite to that of subset inclusion. Thus, $u_1 \leq u_2$ means that $u_1 \supseteq u_2$. When $u_1 \leq u_2$ any cut in the tree u_2 is already a cut in the tree u_1. This gives a restriction function **ClassRest**$[u_1, u_2]$: **Cuts**$(u_2) \rightarrow$ **Cuts**(u_1). Analogously to the spatial case, maps over u_2 can be restricted to maps over u_1 by forgetting about objects whose semantic attributes are not included in u_1.

Combining both the spatial and semantic aspects we obtain a set **EXT** the elements of which are pairs consisting of a spatial extent and a semantic extent.

Conclusions

This paper has developed a formal approach to imprecision and vagueness associated with multi-resolution spatial datasets. The partially ordered granularity lattice induces a stratification of the space of maps. Varying the extents on which the stratified maps are based leads to a structure with sheaf-like characteristics. The sheaf structure seems to the authors to have considerable possibilities for use in the field of geographic data handling and spatial reasoning. For example, spatio-temporal information systems have extents in both spatial and temporal dimensions. It is possible that sheaves will provided a purely algebraic approach to such systems, which have up to now been mainly formalized using logic (e.g. modal and temporal logics), and this is work that the authors are currently undertaking.

References

1. Buttenfield, B. and McMaster, R. (1991). *Map Generalisation: Making Rules for Knowledge Representation*, London: Longman.
2. Dettori, G. and Puppo, E. (1996). How generalization interacts with the topological and metric structure of maps, *Proceedings of 7th International Symposium on Spatial Data Handling, Delft*, Netherlands, Taylor and Francis, pp 9A.27-9A.38, 1996.
3. Egenhofer, M., Frank, A.U. and Jackson, J.P. (1989). A topological model for spatial databases. In *Proceedings of SSD'89*, Lecture Notes in Computer Science, 409, Berlin: Springer, pp. 271-286.
4. Müller, J.C., Lagrange, J.P. and Weibel, R. (eds.) (1995*). GIS and Generalisation: Methodology and Practice*, London: Taylor and Francis.
5. Müller, J.C., R. Weibel, R., J.P. Lagrange, J.P. and Salgé, F. (1995), Generalization - state of the art and issues, in *GIS and Generalization: Methodology and Practice*, Müller, J.C., Lagrange, J.P. and Weibel, R. (eds.), Taylor and Francis, pp. 3-17.
6. Pigot, S. (1992). A topological model of a 3D spatial information system*. Proc. SDH'92, Charleston*, SC, International Geographic Union, pp. 344-360.
7. Puppo, E. and Dettori, G. (1995). Towards a formal model for multiresolution spatial maps. In *Proceedings of SSD'95*, Lecture Notes in Computer Science, 951, Berlin: Springer, pp. 152-169.

8. P. Rigaux and M. Scholl. Multi-scale partitions: Applications to spatial and statistical databases. In *Proceedings of SSD'95*, Lecture Notes in Computer Science, 951, Berlin: Springer, pp. 170-183.

9. Regnauld, N. (1996). How generalization interacts with the topological and metric structure of maps, *Proceedings of 7th International Symposium on Spatial Data Handling*, Delft, Netherlands, Taylor and Francis, pp 9A.27-9A.38.

10. Robinson, G. and Lee, F. (1994). An automatic generalization system for large-scale topographic maps. In *Innovations in GIS* (ed. M. Worboys), London: Taylor and Francis, pp. 53-64.

11. Sheth, A.P. and Larsen, J. (1990). Federated database systems for managing distributed, heterogeneous and autonomous databases. ACM Computing Surveys, 22:183-236.

12. Stell, J.G. and Worboys, M.F. (1997). The algebraic structure of sets of regions, in *Proceedings of the Conference on Spatial Information Theory*, Pittsburgh, Hirtle, S. and Frank, A.U. (eds.), Springer Verlag, Lecture Notes in Computer Science 1329, pp. 163-174.

13. Ware, J.M. and Jones, C.B. (1996). A spatial model for detecting (and resolving) conflict caused by scale reduction, *Proceedings of 7th International Symposium on Spatial Data Handling*, Delft, Netherlands, Taylor and Francis, pp 9A.15-9A.26.

14. Worboys, M.F. (1992). A generic model for planar geometric objects. *International Journal of GIS* 6(5): 353-372.

15. Worboys, M.F. (1997). Imprecision in finite resolution spatial data. Submitted for publication, 1997. In draft form at `http://www.keele.ac.uk/depts/cs/Staff/Homes/Michael/publications.html`.

16. Worboys, M.F.(1998). Computation with imprecise geospatial data. Submitted for publication, 1998. In draft form at `http://www.keele.ac.uk/depts/cs/Staff/Homes/Michael/publications.html`.