

Numerical SOLUTIONS
*to Odd-Numbered Problems**
of Chapters 1 – 10 of:

Strength and Stiffness
of Engineering Systems

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**Note: Chapters 3 and 8* include several multi-part problems – both odd- and even-numbered – for which each part repeats the question of Part (a), but with different numerical values. For such problems (including even-numbered problems), the numerical solutions are given for Parts (a), (c), (e), etc. The problems are: 3.20, 8.1, 8.3, 8.4, 8.13-8.16 and 8.20.

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However, students, instructors and others using the text, *Strength and Stiffness of Engineering Systems* (Leckie and Dal Bello, Springer, 2009), for academic and learning purposes may print a copy of these numerical solutions for use in learning the material.

Chapter 1

- 1.1 Ultimate, Proof, Working
- 1.3 79.0 kips
- 1.5 $W_{min} = 4710$ lb, 3-ton (6000-lb) crane
- 1.7 2.5; 64%
- 1.9 6400 lb; 12.8 kips, 11.2 kips, 3200 lb; 18.4 kips; 38.4 kips
- 1.11 two 8 kN loads, R_{Ax} , R_{Ay} , R_{By}

Chapter 2

- 2.1 $P_{CD} = 2.5$ kips; $P_{BC} = 1.5$ kips; $P_{EF} = -3.0$ kips; $P_{BE} = 2.50$ kips
- 2.3 $T_2:T_1 = 2:1$
- 2.5 300 lb; 2400 lb-ft
- 2.7 $M_{max} = 41.4$ kN·m for $a = 4$ m or 6 m
- 2.9 2160 lb; 180 lb/ft, 90 lb/ft; 3240 lb-ft
- 2.11 $P_{AB} = -3.23$ kips; $T = 16.79$ kip-in. CCW; $N = 1.120$ kips up

Chapter 3

- 3.1 0.1042%, 31.3 ksi; 0.128 in²; 1.92
- 3.3 -4.04 ksi
- 3.5 Plot; $S_p \sim 225$ MPa; $E = 200$ GPa; $S_y = 255$ MPa; $S_u = 450$ MPa
- 3.7 127.3 MPa; 1.88; 1.455 mm; 9.994 mm
- 3.9 -720×10^{-6} in.; 0.029 in³
- 3.11 156.3 kN·m/m³; 312.5 N·m
- 3.13 30,000 cycles; 40%; 40.8 kN
- 3.15 158 MPa; 131 MPa
- 3.17 75.8 ksi
- 3.19 188.5 kN; 94.3 kN, 3.62 kN·m, 1.810 kN·m
- 3.20 Strain ($\times 10^{-3}$)

Part	ϵ_x	ϵ_y	ϵ_z	γ_{xy}
(a)	+2.86	-0.94	-0.94	+2.68
(c)	+2.39	+0.49	-1.41	+2.68
(e)	+2.14	-2.61	+0.24	+3.57
(g)	-2.39	-0.49	+1.41	+3.57

- 3.21 Strain ($\times 10^{-6}$)

Part	ϵ_x	ϵ_y	ϵ_z
(a)	+350	-83.3	+133.3
(b)	-650	+433	+217
(c)	+200	+200	+200

Chapter 4

- 4.1 -40.0 MPa; -571×10^{-6} ; -686 μ m
- 4.3 57.6 MPa, -40.7 MPa; 1.16 mm, -0.582 mm = -582 μ m
- 4.5 9.00 ksi, 3.00 ksi; +0.0540 in., +0.0120 in.; 0.0540 in. down, 0.0660 in. down
- 4.7 -957 psi; -6.12×10^{-3} in.
- 4.9 6.43 kips left; 8.57 kips left; 0.00343 in. right
- 4.11 $-FL/(2AE)$; plot; $-FL/(3AE)$
- 4.13 $\frac{-2}{\pi(0.4+x)^2}$ MPa, x in meters; -34.1 μ m
- 4.15 582 μ m right, 582 μ m up
- 4.17 56.6 MPa, -56.6 MPa; 4.2; +1.143 mm, -1.143 mm; 1.616 mm right, 0 mm
- 4.19 80 MPa, -80 MPa; 3.0; +2.29 mm, -2.29 mm; 4.58 mm right
- 4.21 $\sigma_{AB} = 56.6$ MPa, $\sigma_{BC} = -56.6$ MPa, $\sigma_{BD} = 0$ MPa; 4.2; $\Delta_{AB} = +1.143$ mm; $\Delta_{BC} = -1.143$ mm; $\Delta_{BD} = 0$ mm; 1.616 mm right
- 4.23 $\sigma_s = -4.80$ ksi; $\sigma_c = -0.640$ ksi; -0.0230 in.; 8330 ksi
- 4.25 0.214 in²; 0.0672 in.
- 4.27 -40.3 MPa
- 4.29 11.43 ksi, 8.57 ksi; 317°F
- 4.31 392 MPa (matrix), -727 MPa
- 4.33 40.0 MPa; 5.00 MPa; 140×10^{-6}
- 4.35 7.5 mm minimum; 18.3 mm
- 4.37 589 lb; -82.5×10^{-6} ; 250×10^{-6}
- 4.39 3.38 MPa; 62.8 kN/m
- 4.41 120 kips
- 4.43 $3\sigma_{ave}$; 2.67a ($n = 2.67$ radii)
- 4.45 64.6 N·m, 22.9 N·m; 8.75 mm down; 2.29 MN/m
- 4.47 8.0 N·m, 22.6 N·m; 3.06 mm right; 6.53 MN/m
- 4.49 $3FL/(AE)$ down; $EA/(3L)$; $m = 9K(\rho/E)L^2$; minimize ρ/E or maximize E/ρ

P.4

- 4.51 $U_{AB} = U_{BC} = (3EA\upsilon^2)/(16L)$,
 $U_{BD} = (EA\upsilon^2)/(2\sqrt{3}L)$;
 4.19×10^{-3} in.; 1195 kips/in.
- 4.53 4.40 N·m; 0.220 mm down
- 4.55 25.4 lb-in.; 0.0254 in. down
- 4.57 $P_{AD} = 100.7$ kN, $P_{BD} = 112.0$ kN,
 $P_{CD} = -28.0$ kN; 107.0 kN right,
142.6 kN down; 168.2 kN

Chapter 5

- 5.1 25.1 in⁴; 3.98 ksi; 5.0; 0.622°
- 5.3 5.796×10^{-6} m⁴; 5.726×10^{-6} m⁴; -1.2%
- 5.5 45.7 mm; 11.44 mm, 45.8 mm;
62.0 mm, 6.20 mm; 6.0%, 63.2%
savings
- 5.7 10.2 ksi; 2.95°
- 5.9 2.03 ksi, 1.510 ksi; 0.716°
- 5.11 15.71 kip-in.; 2.74 in., 3.0 in.; 63%
savings
- 5.13 174.3 N·m; 0.130°
- 5.15 1.273 ksi; 1.12° (approx.)
- 5.17 1.48 in.
- 5.19 14.3 kN·m; 73.0 MPa; 3.34°; 128 mm
- 5.21 3183 N·m; 32.3 mm; 41.9 mm; 41.9 mm

Chapter 6

- 6.1 (a) $0 < x < 4$ m: $V(x) = -6.0$ kN,
 $M(x) = 6x$ kN·m;
 $4 < x < 10$ m: $V(x) = +4.0$ kN;
 $M(x) = -4x + 40$ kN·m;
(b) $V_{max} = 6$ kN; $M_{max} = 24$ kN·m at
 $x = 4$ m
- 6.3 22.9 in.; 2.18%
- 6.5 S12×31, W14×26; W14×26
- 6.7 72 kip-in.; 6.00 ksi; 10.39 in.
- 6.9 480 MPa
- 6.11 4.10 ksi at $x = 21$ ft
- 6.13 794 psi
- 6.15 105.6 MPa; 2.27
- 6.17 $M = \frac{-w_0}{6L}(L-x)^3$; 29.4 ksi

- 6.19 24.9 mm
- 6.21 21.2 MPa; 2.47 mm difference
- 6.23 $v = \frac{P}{6EI}(x^3 - 3Lx^2)$; beam hits structure
- 6.25 $L = a + b$; answers will vary based on
length variables (a, b, L) used.
 $v_1 = \frac{-Pbx}{6EIL}[L^2 - b^2 - x^2]$; $0 < x < a$
 $v_2 = \frac{-P}{6EIL}[ax^3 - 3aLx^2$
 $+ (3a^2L - bL^2 + b^3)x - a^3L]$;
 $a < x < a + b = L$

If $a < b$:

$$\delta_{max} = \frac{-Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL} \text{ at } x = \sqrt{\frac{L^2 - b^2}{3}}$$

6.27

$$v_1 = \frac{P}{6EI}(x^3 - 3Lx^2); \quad 0 < x < L$$

$$v_2 = \frac{-PL^2}{6EI}(3x - 2L); \quad L < x < 3L/2$$

6.29

$$v_1 = \frac{P}{6EI}[x^3 - 3a(a+s)x]; \quad 0 < x < a$$

$$v_2 = \frac{P}{6EI}[3ax^2 - 3a(2a+s)x + a^3]; \quad a < x < a + s$$

6.31 $R_A = 3wL/8$, $R_B = 5wL/8$,

$$M_B = wL^2/8 \text{ CW};$$

$$v = \frac{-w}{48EI}[2x^4 - 3Lx^3 + L^3x];$$

$$v(L/2) = \frac{-wL^4}{192EI}$$

6.33 $R_A = R_B = wL/2$ up,

$$M_A = wL^2/12 \text{ CCW},$$

$$M_B = wL^2/12 \text{ CW};$$

$$\delta_{max} = -wL^4/(384EI)$$

6.35 $\sigma = \pm 12$ MPa at $(2, \mp 0.05)$ m

$$\tau = \pm 0.3 \text{ MPa at } (2 \pm 2, 0)\text{m}$$

$$\delta_{max} = 2.00 \text{ mm down}$$

6.37 15.2 kN; 59.2 MPa

6.39 333 kPa; 533 kPa; 416 kPa

- 6.41 274 MPa; 2.39 MPa
 6.43 6.1 in.
 6.45 42.7 in^3 ; 21.3 in^3 ; 6.28 in^3
 6.47 $AD/2$
 6.49 13.8
 6.51 $80 \times 240 \text{ mm}$; $S_{12} \times 31.8$; 68.6% or
 69.4 lb/ft savings
 6.53 333 lb, deflection
 6.55 Silicon Carbide (SiC), but it is brittle;
 Wood is next best
 6.57

$$m = \left(\frac{5\pi}{96}\right)^{1/2} \left(\frac{wL}{\delta}\right)^{1/2} \left(\frac{\rho^2}{E}\right)^{1/2} \left(\frac{1}{\phi}\right)^{1/2} L^{5/2};$$

$$\phi_{st} = 64.7; \phi_{al} = 22.2$$

Chapter 7

- 7.1 -89.1 MPa ; -2.60 MPa
 7.3 at A : $\sigma = +142.9 \text{ psi}$; B : $\sigma = -28.6 \text{ psi}$
 7.5 at A : $\sigma_y = 5728 \text{ psi}$; B : $\sigma_y = -5731 \text{ psi}$;
 C : $\sigma_y = -1.6 \text{ MPa}$, $\tau_{xy} = -32. \text{ psi}$
Note: $\sigma_a = -1.6 \text{ psi}$ (at A , B , C);
 $\sigma_b = 5729.6 \text{ psi}$ (+ at A , - at B)
 7.7 at A : $+6.67 \text{ MPa}$; B : -1.33 MPa ;
 C : -9.33 MPa ; D : -1.33 MPa
 7.9 -2.39 MPa ; 1.90 MPa
 7.11 $R/4$
 7.13 $4Pd/(3L^2)$
 7.15 $\sigma_b = 82.8 \text{ MPa}$ (+ at B);
 $\tau_T = 6.90 \text{ MPa}$ (+ at A , B ; - at C);
 $\tau_V = 0.28 \text{ MPa}$ (- at A , C)
 7.17 $\sigma_a = Pd/A = 67.9 \text{ psi}$ (- at A , B , C);
 $\sigma_{b,x} = FeD/(2I) = 1207 \text{ psi}$ (+ at B);
 $\sigma_{b,y} = PhD/(2I) = 2897 \text{ psi}$ (+ at A ,
 - at C);
 $\tau_T = FhD/(2J) = 966 \text{ psi}$ (+ at A , B ;
 - at C);
 $\tau_V = 4F/(3A) = 60.4 \text{ psi}$ (- at A , C)
 7.19

$$v = \frac{-wx}{24EI} [L_1^3 - 2L_1x^2 + x^3] - \frac{wL_2x}{2AE};$$

$$\delta_{max} = -0.820 \text{ in. down at } x = 121.25 \text{ in.}$$

7.21 $v = \frac{-wx}{48EI} [-2x^3 + 3Lx^2 - L^3],$
 $v' = \frac{-w}{48EI} [-8x^3 + 9Lx^2 - L^3];$
 $A_y = R = \frac{3wL}{8}, B_y = \frac{5wL}{8};$
 $M_B = \frac{wL^2}{8} \text{ CW}$

Chapter 8

- 8.1 (a) 188.3 MPa , 111.7 MPa , -32.1 MPa
 (c) 244.3 MPa , 55.7 MPa , -3.3 MPa
 (e) 176.5 MPa , -76.5 MPa , -113.6 MPa
 (g) 242.3 MPa , 57.7 MPa , 19.6 MPa
 (i) -179.1 MPa , -120.9 MPa , 89.7 MPa
 8.3 (a) $17.1 \text{ ksi@}-19.3^\circ$, $-2.1 \text{ ksi@}70.7^\circ$
 (c) $16.4 \text{ ksi@}12.8^\circ$, $-11.4 \text{ ksi@}102.8^\circ$
 (e) $19.0 \text{ ksi@}-33.7^\circ$, $6.0 \text{ ksi@}56.3^\circ$
 (g) $11.7 \text{ ksi@}-15.5^\circ$, $-11.7 \text{ ksi@}74.5^\circ$
 8.4 (a) $9.6 \text{ ksi@}115.7^\circ$, $-9.6 \text{ ksi@}25.7^\circ$,
 7.5 ksi
 (c) $13.9 \text{ ksi@}-32.3^\circ$, $-13.9 \text{ ksi@}57.8^\circ$,
 2.5 ksi
 (e) $6.5 \text{ ksi@}101.3^\circ$, $-6.5 \text{ ksi@}11.3^\circ$,
 12.5 ksi
 (g) $11.7 \text{ ksi@}119.5^\circ$, $-11.7 \text{ ksi@}29.5^\circ$,
 0 ksi
 8.5 45.0 MPa , -26.0 MPa ; 30.0 MPa
 8.7 $\sigma = -2.60 \text{ MPa}$, $\tau = -89.09 \text{ MPa}$;
 $\sigma_{P,II} = +87.8 \text{ MPa}$, -90.4 MPa ;
 $\tau_{max} = 89.14 \text{ MPa}$
 8.9 46.4 MPa ; 0 MPa ; 0 MPa , -42.8 MPa
 8.11 1.74 ksi @ Pt. B ; 1.14 ksi @ Pt. B .
 8.12 (a) 7.7 ksi , -7.70 ksi , 0.885 ksi
 (b) 15.4 ksi , -15.4 ksi , 4.74 ksi
 (c) 15.0 ksi , -15.8 ksi , 0 ksi
 (d) 16.8 ksi , -17.8 ksi , 0 ksi (normal
 stresses along y -axis, at points not
 on x - or z - axes)
 (e) 3.85 ksi , 8.61 ksi , 7.9 ksi , 8.88 ksi ,
 corresponding to (a)-(d)
 8.13 (a) 14.42 ksi , 10.58 ksi , -1.61 ksi
 (c) 3.55 ksi , 1.45 ksi , 13.83 ksi

P.6

- (e) 141.0 MPa, 59.0 MPa, 17.8 MPa
(g) 70.9 MPa, -30.9 MPa, -94.9 MPa
- 8.14** Note: θ_s defines plane where $\tau_{max} > 0$.
- (a) 15 ksi@0°, 10 ksi@90°;
 $\tau_{max} = 2.5$ ksi, $\sigma_{ave} = 12.5$ ksi,
 $\theta_s = -45^\circ$
- (c) 16.4 ksi@12.8°, -11.4 ksi@102.8°;
 $\tau_{max} = 13.9$ ksi, $\sigma_{ave} = 2.5$ ksi,
 $\theta_s = -32.2^\circ$
- (e) 144.7 MPa@31.7°,
55.3 MPa@121.7°;
 $\tau_{max} = 44.7$ MPa, $\sigma_{ave} = 100$ MPa,
 $\theta_s = -13.3^\circ$
- (g) 127.7 MPa@-10.9°,
-87.7 MPa@79.1°;
 $\tau_{max} = 107.7$ MPa, $\sigma_{ave} = 20$ MPa,
 $\theta_s = -55.9^\circ$
- 8.15** (a) 169.9×10^{-6} , 80.1×10^{-6} , 29.1×10^{-6}
(c) 71.4×10^{-6} , 178.6×10^{-6} , -31.9×10^{-6}
- 8.16** (a) 172.2×10^{-6} @29.0°, 77.8×10^{-6}
@119.0°; 94.3×10^{-6} , 7.08 MPa
- 8.17** 121 GPa, 0.28
- 8.18** (a) 208×10^{-6} , 91.7×10^{-6} , 116.6×10^{-6}
- 8.19** $\epsilon_x = \epsilon_A$, $\epsilon_y = \frac{1}{3}[-\epsilon_A + 2\epsilon_B + 2\epsilon_C]$,
$$\gamma_{xy} = \frac{2}{\sqrt{3}}[\epsilon_B - \epsilon_C]$$
- 8.20** (a) 5.0 ksi; 7.5 ksi; I-III (out-of-plane)
(c) 2.5 ksi; 3.5 ksi; I-III (out-of-plane)

Chapter 9

- 9.1** Tresca or von Mises; Maximum Normal Stress
- 9.3** $\sigma_1 = 3.1$ ksi, does not fail
Try: $T = 120$ lb and $M = 160$ lb-ft
($\sigma_1 = 14.4$ ksi)

9.5

$$U_D = \frac{1}{2E} \left[\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - 2\nu(\sigma_I\sigma_{II} + \sigma_{II}\sigma_{III} + \sigma_{III}\sigma_I) \right]$$

$$U_D = \frac{1}{2E} \left[\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - (\sigma_I\sigma_{II} + \sigma_{II}\sigma_{III} + \sigma_{III}\sigma_I) \right]$$

$$\sigma_o = \left[\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}{2} \right]^{1/2}$$

Chapter 10

- 10.1** 140 in. = 11.7 ft
- 10.3** 395 kN, AE and CD
- 10.5** 106.1 in. = 8.84 ft
- 10.7** (a) 708 kips (y -axis buckling); 760 kips (z -axis buckling); buckling about weak-axis;
(b) 1390 kips (y -axis buckling); 760 kips (z -axis buckling); buckling about strong-axis
- 10.9** $\frac{K_T \theta}{L \sin \theta + e \cos \theta}$; plot
- 10.11** 0.36 μm
- 10.13** $m = \frac{2}{\pi^2} P_{cr} \left(\frac{\rho}{E} \right) \frac{L^3}{R^2}$;
1st: Ceramic, 2nd: Composite