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Stress Analysis of Multilayer Pressure Vessel

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Abstract

Multilayer pressure vessel is designed to work under high pressure condition. In this paper, the stress analysis of multi-layer pressure vessel made of a homogeneous and isotropic material and subjected to internal pressure is considered. The hoop stresses for 1, 2 and 3-layer pressure vessel is calculated theoretically. The modeling of pressure vessel is carried out in CATIA V5 and this model is imported in ANSYS Workbench where stress analysis is carried out. The shrink fit is applied during the CAD modeling of multilayer pressure vessel. Both theoretical and FE results are compared and effect of multi-layering on stresses induced and the volume requirement to sustain the given pressure is calculated. Also optimization of number of layers is carried out for multi-layering the pressure vessel. From calculations it is observed that as the numbers of layers increases, the hoop stresses decreases.

Keywords: Residual stress; Contact pressure; Multi-layer cylinder; Optimization

Introduction

The pressure vessels are used to store fluids under pressure. Pressure vessels find wide applications in thermal and nuclear power plants, process and chemical industries, in space and ocean depths and in water, steam, gas and in air supply system, in process Industries like chemical and petroleum industries [1]. Design of shrunk-fit multilayer pressure vessel is much simplified if each layer is considered as a separate layer subjected to the shrinkage pressure and the working pressure.

To increase the pressure capacity of thick-walled layers, two or more layers (multi-layer) are shrunk into each other with different diametric differences to form compound pressure vessel. In three-layer compound pressure vessel the usual practice is to shrink the outer layer 3 on to the intermediate layer 2 and then shrink the resulting compound pressure vessel on to the inner layer 1. When the outer layer contracts on cooling the inner layer is brought into a state of compression. The outer layer will conversely be brought into a state of tension. If this compound pressure vessel is subjected to internal pressure the resultant hoop stresses will be the algebraic sum of those resulting from internal pressure and those resulting from shrinkage.

Introduction to Problem, Scope and Methodology

Pressure vessels often have a combination of high pressures together with high temperatures, and in some cases flammable fluids or highly radioactive materials. Because of such hazards it is imperative that the design be such that no leakage can occur. In addition these vessels have to be designed carefully to cope up with the operating temperature and pressure. To fulfill these requirements the multilayer pressure vessel is designed [2]. For designing the multilayer pressure vessel, first analytical analysis is carried out by using different mathematical equations. In analytical calculations hoop stresses developed in the pressure vessel is carried out. Also the total interference and contact pressure is calculated at the junction of inner and outer layer.

To compare and analyze the results the analysis is carried out on 1. 2 and 3-layer pressure vessel. The modeling of pressure vessel is carried out in CATIA V5 and the same model is imported in ANSYS Workbench where stress analysis is carried out on CAD model of pressure vessel. To optimize the number of layers in multilayer pressure vessel, the hoop stress calculation is carried out up to 10-layer pressure vessel and the percentage reduction in hoop stresses with increase in number of layers is determined [3].

For FE and theoretical analysis of pressure vessel, hoop stresses are calculated by varying the number of layers. For analysis, the internal pressure is assumed as 60 Mpa, inner diameter is assumed as 100 mm. The layers of multilayer pressure vessel are brought in contact by applying shrink fit between the layers of pressure vessel. The thickness is calculated by using the analytical formula and to increase the number of layers, thickness is divided equally. Material for all the three layers is assumed as same i. e. steel with yield strength of 250 MPa.

Analytical Estimation of Hoop Stresses in 1-layer Pressure Vessel

Thickness (t) and outer diameter (d_2) of 1-layer pressure vessel is given by equation 1 and 2,

$$Thickness = t = r_i\left[\sqrt{\frac{\sigma_t + p_i}{\sigma_t - p_i}} - 1\right] \tag{1}$$

$$Outer\ diameter\left(d_2\right) = d_i + \left(t \times 2\right) \tag{2}$$

Hoop stress in 1-layer pressure vessel is given by equation (3),

$$\sigma_\theta = \frac{p_i\left(r_2^{\,2} + r_1^{\,2}\right)}{\left(r_2^{\,2} - r_1^{\,2}\right)} \tag{3}$$

Analytical Estimation of Hoop Stresses in 2-Layer Pressure Vessel

The Hoop stresses induced at outer and inner radius of layer 1 only due to contact pressure (p_{s12}) at the Junction is given by equation (4) and (5),

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$$\sigma_{\theta 1}=\frac{-P_{s12}\left(r_2^{\,2}+r_1^{\,2}\right)}{r_2^{\,2}-r_1^{\,2}} \tag{4}$$

$$\sigma_{\theta 2}=\frac{-2P_{s12}\left(r_2^{\,2}\right)}{r_2^{\,2}-r_1^{\,2}} \tag{5}$$

The Hoop stresses induced at outer and inner radius of layer 2 only due to contact pressure (p_{s12}) at the junction is given by equations (6) and (7),

$$\sigma_{\theta 3}=\frac{2P_{s12}\left(r_2^{\,2}\right)}{r_3^{\,2}-r_2^{\,2}} \tag{6}$$

$$\sigma_{\theta 4}=\frac{P_{s12}\left(r_3^{\,2}+r_2^{\,2}\right)}{r_3^{\,2}-r_2^{\,2}} \tag{7}$$

The Hoop stresses induced due to internal pressure (P_i) at inner radius of layer 1 is given by equation (8),

$$\sigma_{\theta 5}=\frac{P_i\left(r_3^{\,2}+r_1^{\,2}\right)}{r_3^{\,2}-r_1^{\,2}} \tag{8}$$

The Hoop stresses induced due to internal pressure (P_i) at outer radius of layer 1 and inner radius of layer 2 is given by equation (9),

$$\sigma_{\theta 6}=\frac{P_i.r_1^{\,2}}{r_2^{\,2}}\left[\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_1^{\,2}}\right] \tag{9}$$

The Hoop stresses induced due to internal pressure (P_i) at outer radius of layer 2 is given by equation (10),

$$\sigma_{\theta 7}=\frac{2.P_i.r_1^{\,2}}{r_3^{\,2}-r_1^{\,2}} \tag{10}$$

The resultant hoop stresses at inner radius of layer 1 & 2 is given by the equation (11) and (12),

$$\sigma_{\theta r1}=\frac{P_i\left(r_3^{\,2}+r_1^{\,2}\right)}{r_3^{\,2}-r_1^{\,2}}-\frac{2.P_{s12}\left(r_2^{\,2}\right)}{r_2^{\,2}-r_1^{\,2}} \tag{11}$$

$$\sigma_{\theta r2}=\frac{P_i.r_1^{\,2}}{r_2^{\,2}}\left[\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_1^{\,2}}\right]+\frac{P_{s12}\left(r_3^{\,2}+r_2^{\,2}\right)}{r_3^{\,2}-r_2^{\,2}} \tag{12}$$

Estimation of contact pressure (p_{s12}) between layer 1 and 2

To obtain optimum values of the contact (shrinkage) pressures P_{s12} which will produce equal hoop (tensile) stresses in both the layers, maximum hoop stresses given by the equations (11) and (12) have been equated [4-6].

Equating equations (11) and (12). i.e. $\sigma_{\theta r1}=\sigma_{\theta r2}$ and rearranging,

$$P_i\left[\frac{r_3^{\,2}+r_1^{\,2}}{r_3^{\,2}-r_1^{\,2}}-\frac{r_1^{\,2}}{r_2^{\,2}}\left(\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_1^{\,2}}\right)\right]=P_{s12}\left[\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_2^{\,2}}+\frac{2\left(r_2^{\,2}\right)}{r_2^{\,2}-r_1^{\,2}}\right]$$

$$\text{Let, } K_1=\left[\frac{r_3^{\,2}+r_1^{\,2}}{r_3^{\,2}-r_1^{\,2}}-\frac{r_1^{\,2}}{r_2^{\,2}}\left(\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_1^{\,2}}\right)\right] \tag{13}$$

$$K_2=\left[\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_2^{\,2}}+\frac{2\left(r_2^{\,2}\right)}{r_2^{\,2}-r_1^{\,2}}\right] \tag{14}$$

Thus,

$$P_{s12}=P_i\left[\frac{K_1}{K_2}\right] \tag{15}$$

The total interference between layer 1 and 2 is given by,

$$\delta_{12}=\frac{P_{s12}.r_2}{E}\left[\frac{r_3^{\,2}+r_2^{\,2}}{r_3^{\,2}-r_2^{\,2}}+\frac{r_2^{\,2}+r_1^{\,2}}{r_2^{\,2}-r_1^{\,2}}\right] \tag{16}$$

Analytical Estimation of Hoop Stresses in 3-Layer Pressure Vessel

Figure 1 shows the Radial and Hoop stress distribution in three separate layers of 3-layer pressure vessel.

Figures 2 and 3 shows the interference between layer 1 and 2 and 2 and 3. Interference is the difference between the outer radius of inner layer and inner radius of outer layer. By using this interference the layers are bring in contact with each other [7]. Hoop stresses induced in outer and inner radius of layer 1 only due to contact pressure (P_{s12}) is given by equations (17) and (18),

$$\sigma_{\theta 1}=-P_{s12}\left[\frac{r_2^{\,2}+r_1^{\,2}}{r_2^{\,2}-r_1^{\,2}}\right] \tag{17}$$

$$\sigma_{\theta 2}=-\left[\frac{2P_{s12}r_2^{\,2}}{r_2^{\,2}-r_1^{\,2}}\right] \tag{18}$$

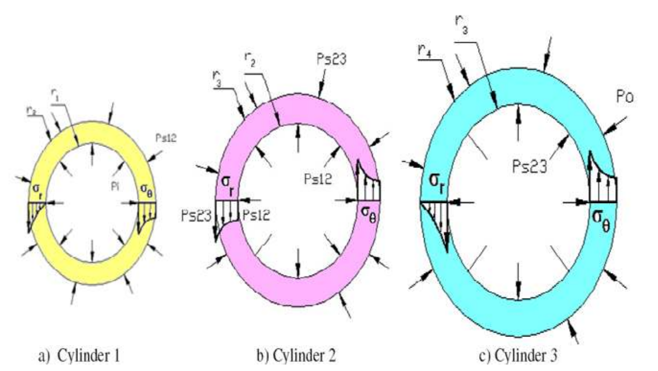

Figure 1: Radial and hoop stress distribution.

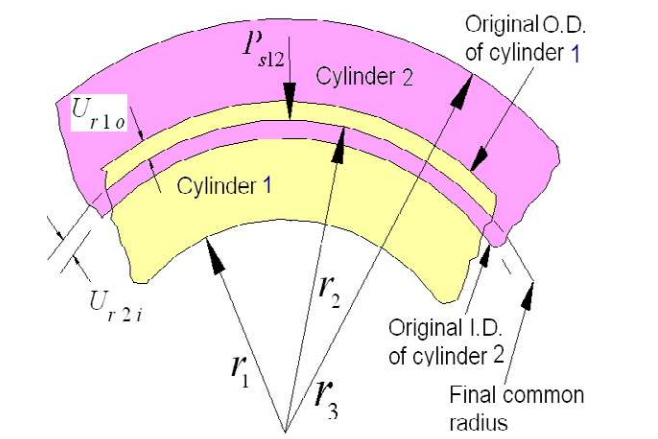

Figure 2: Interference between layer 1 and 2.

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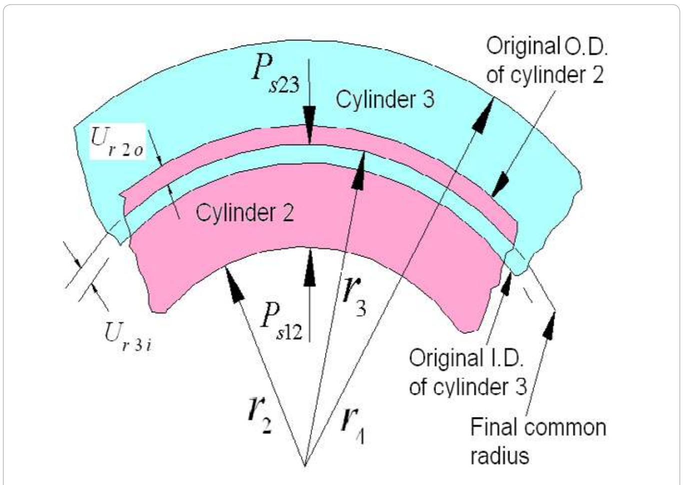

Figure 3: Interference between layer 2 and 3.

Radial displacement U_{rlo} due to contact pressure at outer radius of layer 1 is given by equation (19),

$$U_{rlo} = \frac{-P_{s12} r_2^2 (1+V)}{E}\left[(1-V)\left(\frac{r_2^2+r_1^2}{r_2^2-r_1^2}\right)-V\right] \tag{19}$$

Hoop stresses induced in outer and inner radius of layer 2 only due to contact pressure (P_{s12}) is given by equations (20) and (21),

$$\sigma_{\theta 3} = \frac{2P_{s12} r_2^2}{r_3^2-r_2^2} - \frac{P_{s23}\left(r_3^2+r_2^2\right)}{r_3^2-r_2^2} \tag{20}$$

$$= \frac{P_{12}\left(r_3^2 \quad r_2^2\right)}{r_3^2-r_2^2} - \frac{P_{23} r_3}{r_3^2-r_2^2} \tag{21}$$

Radial displacement U_{r2i} due to contact pressure at inner radius of layer 2 is given by equation (22),

$$U_{r2i} = \frac{r_2(1+V)}{E}\left[P_{s12}\left((1-V)\frac{\left(r_3^2+r_2^2\right)}{r_3^2-r_2^2}+V\right)-(1-V)\left(\frac{2P_{s23} r_3^2}{r_3^2-r_2^2}\right)\right] \tag{22}$$

Using equations (19) and (22), total interference δ_{12} at the contact between layer 1 and 2 is given by equation (23),

$$\delta_{12} = U_{r2i} - U_{rlo}$$

$$\delta_{12} = \frac{r_2\left(1-V^2\right)}{E}\left[P_{s12}\left(\frac{r_3^2+r_2^2}{r_3^2-r_2^2}+\frac{r_2^2+r_1^2}{r_2^2-r_1^2}\right) - 2P_{s23}\left(\frac{r_3^2}{r_3^2-r_2^2}\right)\right] \tag{23}$$

Radial displacement U_{r20} due to contact pressure at outer radius of layer 2 is given by equation (24),

$$U_{r2o} = \frac{r_3}{E}\left[\frac{2P_{s12}\left(1-V^2\right) r_2^2}{r_3^2-r_2^2} - P_{s23}(1+V)\left((1-V)\frac{r_3^2+r_2^2}{r_3^2-r_2^2}-V\right)\right] \tag{24}$$

Hoop stresses induced in outer and inner radius of layer 3 only due to contact pressure (P_{s12}) is given by equations (25) and (26),

$$\sigma_{\theta 5} = \frac{2P_{s23} r_3^2}{r_4^2-r_3^2} \tag{25}$$

$$\sigma_{\theta 6} = \frac{P_{s23}\left(r_4^2+r_3^2\right)}{r_4^2-r_3^2} \tag{26}$$

Radial displacement U_{r3i} due to contact pressure at inner radius of layer 3 is given by equation (27),

$$U_{r3i} = \frac{P_{s23} r_3 (1+V)}{E}\left[(1-V)\left(\frac{r_4^2+r_3^2}{r_4^2-r_3^2}\right)+V\right] \tag{27}$$

Using equations (24) and (27), total interference δ_{23} at the contact between layer 2 and 3 is given by equation (28),

$$\delta_{23} = U_{r3i} - U_{r20}$$

$$\delta_{23} = \frac{r_3\left(1-V^2\right)}{E}\left[P_{s23}\left(\frac{r_4^2+r_3^2}{r_4^2-r_3^2}+\frac{r_3^2+r_2^2}{r_3^2-r_2^2}\right) - \frac{2P_{s12} r_2^2}{r_3^2-r_2^2}\right] \tag{28}$$

Hoop stress at any radius r in pressure vessel due to internal pressure only is given by,

$$\sigma_\theta = \frac{P_i r_1^2}{r_4^2-r_1^2}\left[\frac{r_4^2}{r^2}+1\right] \tag{29}$$

After finding hoop stresses at all the radii, the principle of superposition is applied (Figures 4-9), i.e. the various stresses are then combined algebraically to produce the resultant hoop stresses in the compound pressure vessel subjected to both shrinkage pressures and internal pressure P_i.

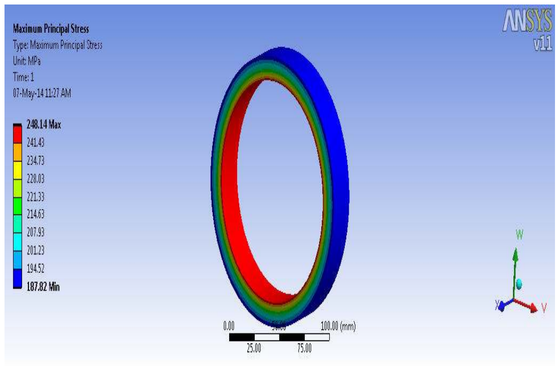

Figure 4: Hoop stresses.

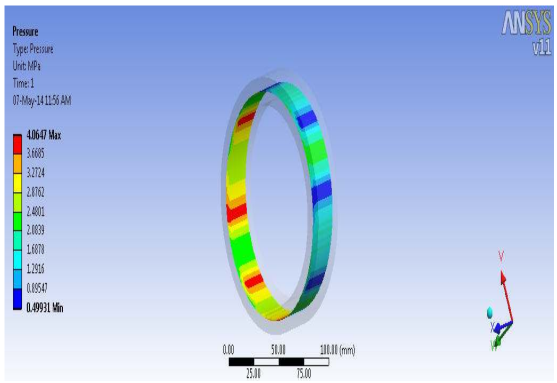

Figure 5: Contact pressure (Ps12).

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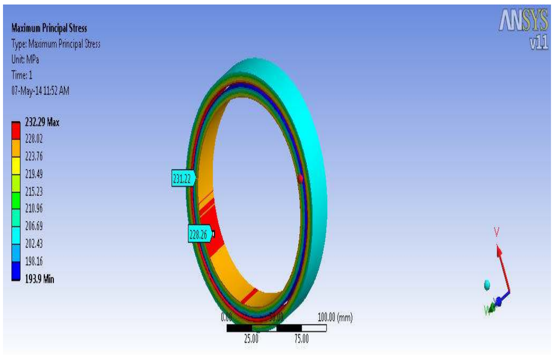

Figure 6: Resultant hoop stresses.

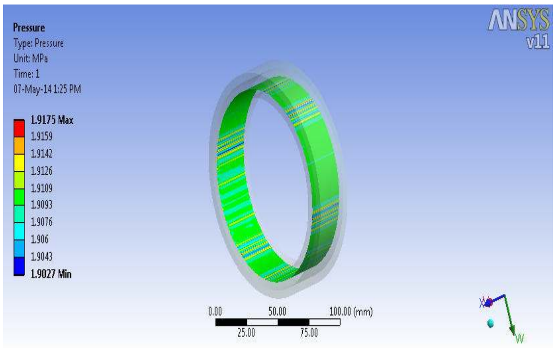

Figure 7: Contact pressure (Ps12).

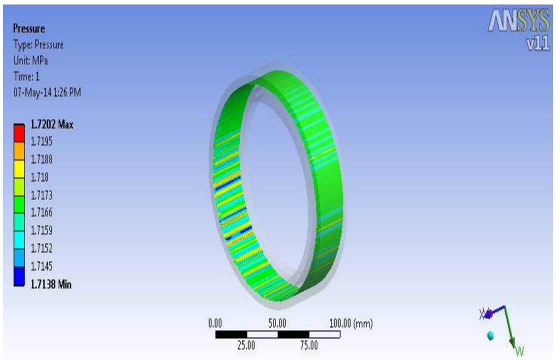

Figure 8: Contact pressure (Ps23).

Using equations (29) and (18), maximum hoop stress at the inner surfaces of layer 1 at r_1 is given by equation (30),

$$\sigma_{\theta r1} = P_i\left[\frac{r_4^2+r_1^2}{r_4^2-r_1^2}\right] - 2P_{s12}\left[\frac{r_2^2}{r_2^2-r_1^2}\right] \tag{30}$$

Using equations (29) and (21), maximum hoop stress at the inner surfaces of layer 2 at r_2 is given by equation (31),

$$\sigma_{\theta r2} = \frac{P_i r_1^2}{r_2^2}\left[\frac{r_4^2+r_2^2}{r_4^2-r_1^2}\right] + \frac{P_{s12}\left(r_3^2+r_2^2\right)-2P_{s23}r_3^2}{r_3^2-r_2^2} \tag{31}$$

Using equations (29) and (26), maximum hoop stress at the inner surfaces of layer 3 at r_3 is given by equation (32),

$$\sigma_{\theta r3} = \frac{P_i r_1^2}{r_3^2}\left[\frac{r_4^2+r_3^2}{r_4^2-r_1^2}\right] + P_{s23}\left[\frac{r_4^2+r_3^2}{r_4^2-r_3^2}\right] \tag{32}$$

Estimation of contact pressure p_{s12} and p_{s23}

To obtain optimum values of the contact (shrinkage) pressures P_{s12} and P_{s23} which will produce equal hoop (tensile) stresses in all the three layers, maximum hoop stresses given by the equations (30), (31) and (32) have been equated [8].

Equating equations (30) and (31) i.e. $\sigma_{\theta r1} = \sigma_{\theta r2}$ and rearranging,

$$P_{s12}\left[\frac{2r_2^2}{r_2^2-r_1^2} + \frac{r_3^2+r_2^2}{r_3^2-r_2^2}\right] p_i\left[\frac{r_4^2+r_1^2}{r_4^2-r_1^2} - \frac{r_1^2}{r_2^2}\left[\frac{r_4^2+r_2^2}{r_4^2-r_1^2}\right]\right] + P_{s23}\frac{2r_3^2}{r_3^2-r_2^2} \tag{33}$$

Let the ratios $t_1 = \frac{r_2}{r_1} = \frac{d_2}{d_1}$, $t_2 = \frac{r_3}{r_2} = \frac{d_3}{d_2}$, $t_3 = \frac{r_4}{r_3} = \frac{d_4}{d_3}$

Where d_1, d_2, d_3, d_4 are diameters corresponding to radii r_1, r_2, r_3, r_4.

$$\text{Let, } C_1 = \frac{2r_2^2}{r_2^2-r_1^2} + \frac{r_3^2+r_2^2}{r_3^2-r_2^2} \tag{34}$$

$$C_2 = \frac{r_4^2+r_1^2}{r_4^2-r_1^2} - \frac{r_1^2}{r_2^2}\left[\frac{r_4^2+r_2^2}{r_4^2-r_1^2}\right] \tag{35}$$

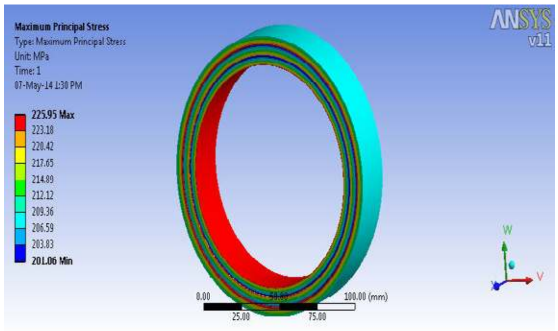

Figure 9: Resultant hoop stresses.

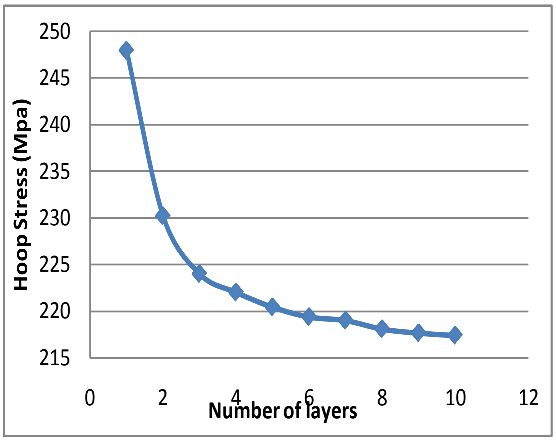

Figure 10: Effect of Number of layers on hoop stress.

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$$C_3 = \frac{2r_3^2}{r_3^2 - r_2^2} \quad (36)$$

Hence equation (33) becomes, $P_{s12} = P_i\left[\frac{C_2}{C_1}\right] + P_{s23}\left[\frac{C_3}{C_1}\right]$ (37)

Equating equation (31) and (32) i. e. $\sigma_{\theta r2} = \sigma_{\theta r3}$ and rearranging,

$$P_{s12}\frac{r_3^2+r_2^2}{r_3^2-r_2^2} = P_i\left[\frac{r_1^2}{r_3^2}\frac{(r_4^2+r_3^2)}{(r_4^2-r_1^2)} - \frac{r_1^2\left(r_4^2+r_2^2\right)}{r_2^2\left(r_4^2-r_1^2\right)}\right] + P_{s23}\left[\frac{r_4^2+r_3^2}{r_4^2-r_3^2} + \frac{2r_3^2}{r_3^2-r_2^2}\right] \quad (38)$$

Let, $C_4 = \dfrac{r_3^2+r_2^2}{r_3^2-r_2^2}$ (39)

$$C_5 = \frac{r_1^2}{r_3^2}\frac{(r_4^2+r_3^2)}{(r_4^2-r_1^2)} - \frac{r_1^2\left(r_4^2+r_2^2\right)}{r_2^2\left(r_4^2-r_1^2\right)} \quad (40)$$

$$C_6 = \frac{r_4^2+r_3^2}{r_4^2-r_3^2} + \frac{2r_3^2}{r_3^2-r_2^2} \quad (41)$$

Hence equation (38) becomes, $P_{s12} = P_i\left[\frac{C_5}{C_4}\right] + P_{s23}\left[\frac{C_6}{C_4}\right]$ (42)

Equation (37) and (42) have been solved to get P_{s12} and P_{s23} in terms of Pi as follows,

$$P_{s12} = P_i\left[\frac{(C_5/C_6)-(C_2/C_3)}{(C_4/C_6)-(C_1/C_6)}\right] \quad (43)$$

$$P_{s23} = P_i\left[\frac{(C_5/C_4)-(C_2/C_1)}{(C_3/C_1)-(C_6/C_4)}\right] \quad (44)$$

Putting the values of t_1, t_2 and t_3, the equation (23) and (28) can be written as,

$$\delta_{12} = \frac{r_2\left(1-V^2\right)}{E}\left[P_{s12}\left(\frac{t_2^2+1}{t_2^2-1} + \frac{t_1^2+1}{t_1^2-1}\right) - 2P_{s23}\left(\frac{t_2^2}{t_2^2-1}\right)\right] \quad (45)$$

$$\delta_{23} = \frac{r_3\left(1-V^2\right)}{E}\left[P_{s23}\left(\frac{t_3^2+1}{t_3^2-1} + \frac{t_2^2+1}{t_2^2-1}\right) - \frac{2P_{s12}}{t_2^2-1}\right] \quad (46)$$

Analytical Results For 1, 2 and 3-Layer Pressure Vessels

Analytical results for 1-layer pressure vessel are shown in Table 1.

Consider, P_i=60 Mpa, d_1=100 mm

Analytical results for 2-layer pressure vessel are shown in Table 2.

Consider, P_i=60 Mpa, d_1=100 mm, d_2=114 mm, d_3=128 mm.

Analytical results for 3-layer pressure vessel are shown in Table 3.

Consider,

Parameters	t (mm)	d_2 (mm)	Volume (mm³)	Hoop stress (σ_θ) (Mpa)
Analytical	14	128	5011.44	247.97

Table 1: Analytical results for 1-layer pressure vessel.

Parameters	P_{s12} (Mpa)	δ_{12} (mm)	$\sigma_{\theta 1}$ (Mpa)	$\sigma_{\theta 2}$ (Mpa)
Analytical	2.046	0.00953	230.21	230.21

Table 2: Analytical results for 2-layer pressure vessel.

Param-eters	P_{s12}	P_{s23}	δ_{12}	δ_{23}	$\sigma_{\theta 1}$	$\sigma_{\theta 2}$	$\sigma_{\theta 3}$
Analyti-cal	1.89	1.602	0.00564	0.0056	224	224	224

Table 3: Analytical results for 3-layer pressure vessel.

Parameters	Hoop stress (σ_θ) (Mpa)
Analytical	247.97
FEA	248.14
Difference (%)	0.068

Table 4: Comparison of analytical and FEA results of 1-layer pressure vessel.

Parameters	P_{s12} (Mpa)	$\sigma_{\theta 1}$ (Mpa)	$\sigma_{\theta 2}$ (Mpa)
Analytical	2.046	230.21	230.21
FEA	2.27	228.26	231.22
Difference (%)	9.8	0.85	0.43

Table 5: Comparison of analytical and FEA results of 2-layer pressure vessel.

Parameters	P_{s12} (Mpa)	P_{s23} (Mpa)	$\sigma_{\theta 1}$ (Mpa)	$\sigma_{\theta 2}$ (Mpa)	$\sigma_{\theta 2}$ (Mpa)
Analytical	1.89	1.602	224	224	224
FEA	1.9	1.7	224.48	223.2	225.7
Difference (%)	0.52	5.76	0.21	0.34	0.78

Table 6: Comparison of analytical and FEA results of 3-layer pressure vessel.

Pi=60 Mpa, $d_{1=}$100 mm, $d_{2=}$109.33 mm, d_3=118.66 mm, d_4=128 mm.

Validation of Analytical Results by Fea

Using the values of δ_{12} and δ_{23}, shrink fit is applied between layer 1 and 2 and between layer 2 and 3 respectively. Contact between layer 1 & 2 as well as between layer 2 and 3 is applied using contact tool in ANSYS Workbench. The surface between layer 1 and 2 and 2 and 3 respectively defined as "ROUGH".

Fe analysis of 1-layer pressure vessel

The data used for CAD modeling is given as follows-

d_1=100 mm,

d_2=128 mm.

Fe analysis of 2-layer pressure vessel

The data used for CAD modeling is given as follows-

d_1=100 mm, d_2=114 mm, d2i=113.981 mm, d_3=128 mm.

Fe analysis of 3-layer pressure vessel

The data used for CAD modeling is given as follows-

d_1=100 mm, d_2=109.33 mm, d_{2i}= 109.319 mm, d_3=118.66 mm, d_{3i}=118.648 mm, d_4=128 mm.

Optimization of Number of Layers in Multilayer Pressure Vessel

From the observations it is concluded that with increase in number of layers, hoop stress reduces. But in detailed study it is observed that reduction in hoop stress is not uniform over each layer.

Thus, it is necessary to know the optimum value of number of layers (n) in designing of pressure vessel to minimize the work in multi-layering the pressure vessel, So that the cost of manufacturing of pressure vessel can be reduced.

Sr.no.	Number of layers (n)	Max. hoop stresses (Mpa)	% Reduction in hoop stresses
1	1-layer	247.97	-
2	2-layers	230.21	7.16
3	3-layers	224.00	2.69
4	4-layers	222.019	0.884
5	5-layers	220.434	0.71
6	6-layers	219.38	0.47
7	7-layers	219.00	0.036
8	8-layers	218.08	0.57
9	9-layers	217.653	0.19
10	10-layers	217.40	0.12

Table 7: Effect of number of layers on hoop stresses.

Analytical estimation of hoop stresses in pressure vessel consists of n-layers

To calculate stresses of pressure vessel having more than 3-layers some different equation have to use. These equations are derived by basic equation to calculate hoop stresses in less time and minimum efforts.

The equation to calculate hoop stresses in pressure vessel is given as-

$$\sigma_\theta = -P_o + \frac{P_i - P_o}{F - 1} \quad (47)$$

The equation given above is to calculate hoop stress due to multi-layering.

Where, $F = C_1 \times C_2 \times C_3 \ldots . \times C_r \times C_{r+1} \ldots . \times C_n$ (48)

C_{r+1} is given by, $C_{r+1} = \frac{2.K^2_{r+1}}{1+K^2_{r+1}}$ and $K_{r+1} = \frac{d_{r+1}}{d_r}$

Discussion

Analytical and FEA (ANSYS) results are summarized in Tables 4-7.

The graph shown in Figure 10 represents the effect of varying number of layers on hoop stresses. It is showing that, as the number of layers increases hoop stresses reduces.

Also as numbers of layers are increasing the variation in rate of change of hoop stress is decreasing. It represents that the effectiveness of layers to reduce the hoop stresses is reducing with increasing the number of layers.

Conclusion

Analytically, the maximum hoop stress for 1-layer pressure vessel is obtained as 247.97 Mpa, for 2-layer pressure vessel 230.21 Mpa and for 3-layer pressure vessel 224 Mpa.

Thus, after observing these results it is concluded that -

- Multi-layering of pressure vessel is very useful in high pressure applications.
- By comparison it is observed that increasing the number of layers reduces the hoop stresses in pressure vessel.
- Multi-layering of vessel decreases the hoop stresses at innermost surface of pressure vessel and it can decreases the difference between maximum and minimum hoop stress as compared to monobloc (1-layer) vessel.
- In optimization of number of layers (n), after observing all the results for hoop stresses on 10 layers it is concluded that the multi-layering of pressure vessel is effective up to only 3-layers. Multi-layering above 3-layers is not advisable as percentage reduction in hoop stresses is very less above 3-layers (n>3). Hence, multi-layering up to 3-layers is recommended in designing of multi-layer pressure vessel.

Nomenclature

P	Internal pressure (Mpa)
P_0	External pressure (Mpa)
σ_θ	Hoop stress in the layers (Mpa)
$\sigma_{\theta r1}$	Resultant hoop stress in layer 1 (Mpa)
$\sigma_{\theta r2}$	Resultant hoop stress in layer 2 (Mpa)
$\sigma_{\theta r3}$	Resultant hoop stress in layer 3 (Mpa)
d_1	Inner diameter of layer 1 (mm)
d_2	Outer diameter of layer 1 and Inner diameter of layer 2 (mm)
d_3	Outer diameter of layer 2 and Inner diameter of layer 3 (mm)
d_4	Outer diameter of layer 3 (mm)
r_1	Inner diameter of layer 1 (mm)
r_2	Outer radius of layer 1 and Inner radius of layer 2 (mm)
r_3	Outer radius of layer 2 and Inner radius of layer 3 (mm)
r_4	Outer radius of layer 3 (mm)
P_{S12}	Contact pressure between layer 1 and 2 (Mpa)
P_{S23}	Contact pressure between layer 2 and 3 (Mpa)
U_{rio}	Radial displacement at outer wall of layer 1 (mm)
U_{r2i}	Radial displacement at inner wall of layer 2 (mm)
δ_{12}	Total interference at the contact between layer 1 and 2 (mm)
U_{r2o}	Radial displacement at outer wall of layer 2 (mm)
U_{r3i}	Radial displacement at inner wall of layer 3 (mm)
δ_{23}	Total interference at the contact between layer 2 and 3 (mm)
V	Poisson's ratio
K_{r+1}	Ratio of outer diameter to inner diameter of $(r+1)^{th}$ shell (dr+1/dr)

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