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STRESS AND SETTLEMENT OF FOOTINGS IN SAND

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ABSTRACT

In current engineering practice, the magnitude of the settlement of a footing in sand, as compared to the settlement of a different size footing in the same sand, is considered to be a non-linear function of the footing width. Further, the settlement is considered to be proportional to the density of the sand. Results of finite element analysis of settlement for footings of three sizes placed in two different sand types show that the settlement in sand is a direct function of neither footing size nor soil density. Instead, the settlement should be related to the steady state line of the sand and to the upsilon distance of the sand, that is, the initial void ratio distance to the steady state line at equal mean stress and at homologous points. This requirement imposes scaling-rules for model tests and it limits the range of application of a small-scale test to a prototype behavior. Moreover, it imposes boundaries on the geometric scale, because a model test cannot realistically be carried out in a sand that is looser than the maximum void ratio, and it is meaningless if performed in a sand close to the minimum void ratio, because it would then not be representative for any prototype.

INTRODUCTION

In current foundation engineering practice, when assessing settlement of footings in sand, conditions are normally so favorable that it is obvious that the settlement will not exceed the usual one-inch limit. Sometimes, however, existence of a favorable situation is not that obvious and a closer look is required. The closer look, invariably, involves calculation and analysis.

Most calculation of settlement of footings in sand involves empirical methods whereby the settlement is determined in relation to an average N-value, cone penetrometer data, or other indirect method based on in-situ testing. Sometimes, a modulus of elasticity of the sand is estimated and combined with the Boussinesq stress distribution below the center of the footing (or below some point between the center and a side, such as the so-called characteristic point). In both cases, the settlement of the footing is the accumulated compression calculated for a series of sub-layers.

Generally, agreement between calculated settlement and reality has little correlation to whether an empirical method or a sophisticated method is used, or to the degree of complexity of the approach. The governing aspect is the experience data base of the person making the settlement estimate.

For qualitative assessment of settlement, it is generally considered that the denser the sand, the smaller the settlement for a given applied stress and footing size. The density of the sand is usually expressed in terms of its density index, I_D (formerly called ‘relative density’). Unfortunately, many reports omit to mention the values of actual density, void ratio, or porosity, forgetting that the I_D-value alone has little meaning. Sometimes, of course, the reason for omitting the actual soil density may be because reliable values of the in-situ conditions of a sand are difficult to determine directly. However, there is no good reason for not including the maximum and minimum boundaries used in determining the density index values. (When also, inexplicably, the vital pore pressure information is missing, such as whether the test is performed in dry, wet, or saturated sand, and where the groundwater table is located, the value of the test data is seriously impaired).

It is also generally recognized that sands of different degree of uniformity, angularity of the grains, etc., will behave differently under otherwise equal conditions. Of particular importance are the geographical/geological/mineralogical aspects of the sand; micaceous sand is much more compressible than silica sand (Gilboy 1928), and a footing in a calcareous sand can hardly be expected to settle similarly to one placed in a silica sand, be density indices, density values, coefficients of uniformity, etc. ever so equal. Fortunately, foundations on dense sands only rarely entail major concerns about settlement and loose sands can be densified, lessening the severity of the consequence of an inaccurate prediction of settlement. However, ground improvement treatment costs money and before recommending this, or some other solution, to a settlement problem, the need for it must be shown. Often, a small-scale test is performed and the results are extrapolated to the prototype (full-scale) condition.

The most relevant experience base consists of results from observations of the behavior of existing footings under known loads and/or of results from loading-tests on footings. Because one rarely has the means to perform a full-scale test, a footing test is usually performed at some ratio of scale to the actual footing considered. Moreover, few engineers are fortunate enough to be able to support a settlement assessment by means of a project-specific footing test at any scale. Instead, they have to rely on experience of tests from other projects or on more or less applicable information found in the literature.

LITERATURE

As to extrapolation of results from a small-scale footing test to the behavior of a prototype footing, the Terzaghi and Peck (1967) relation between the settlement of a footing in terms of the settlement for a one-foot reference footing is probably the best known such relation. This relation has been used to state the conclusion that however large a footing, its settlement will never be larger than four times that of a one-foot diameter footing. This conclusion is incorrect. The relation is only intended to apply to relatively small footings. Another well-known relation is that proposed by Bond (1961), who indicated that for footings in dense sands, the ratio of settlement is equal to the square root of the ratio of the footing width and that for loose sand the width ratio exponent is smaller than 0.5.

Of course, extrapolating results from a small-scale test to a full-scale (prototype) test must be with the small-scale footing test performed in the same type of sand as the prototype footing. It has also been taken as self evident that the test should be performed in a sand of the same density as the sand at the prototype footing. The latter postulation is a fallacy, however, which will be addressed in this paper.

The technical literature abounds with reports on footing tests. Most available references reporting results from tests on footings do not isolate one parameter at a time, making the results difficult to use as support for generalized conclusions. One exception to this is Vesic (1967**;** 1975) who presented results from a comprehensive series of tests on model footings tested in sands of different densities. Fig. 1 compiles load-settlement curves from Vesic's tests on 150 mm diameter circular model footings tested on the surface of a sand. The influence of the sand density is clearly evident. (The dry density of the sand ranged from 1,360 kg/m^3 through 1,540 kg/m^3, corresponding to void ratios ranging from 0.96 through 0.73. The maximum and minimum void ratios were 1.10 and 0.62, respectively).

Another exception is Ismael (1985), who published results from a useful and conclusive series of field tests in Kuwait on rigid footings on "compact fine to medium non plastic cohesionless windblown sand with little silt". The silt content ranged from 5 percent through 12 percent and the tests were performed above the groundwater table. The tests consisted of measuring the settlement induced by incremental loading of four square footings placed at a depth of 1.0 m. The footing diameters were 0.25 m, 0.50 m, 0.75 m, and 1.00 m. The results of the tests are shown in Fig. 2 as contact stress versus settlement and indicate that the larger the footing diameter, the larger the settlement for a certain contact stress. It is of interest to note that despite the large relative deformation, 16 percent for B = 0.25 m, the applied load is well below the capacity of the footing. Additional insight into the results can be obtained if the results are normalized to show the contact stress versus the settlement divided by the footing diameter, as is shown in Fig. 3. The normalization appears to suggest that, for a given contact stress, the settlement is proportional to the footing width.

Ismael (1985) also tested 0.5 m and 1.0 m footings at depths of 0.50 m, 1.00 m, 1.50 m, and 2.00 m below the ground surface at the site, that is, at depth ratios ranging from 0.5 through 4.0. The results are shown in Fig. 4 and indicate that the influence of footing depth is very small. Note, that if the settlement values would be normalized to footing diameter, the two groups of curves would plot within a common band.

STEADY STATE SOIL MECHANICS APPROACH

When extrapolating results from small-scale model tests to the behavior of prototype footings, the scale requirement is not limited to the geometric scale, there is also a stress-scale to consider. This requirement can be addressed by performing the small-scale test in the centrifuge keeping the stress-scale equal to unity (equal stress at homologous points between model and prototype). The centrifuge test is performed using the same sand density and stress field for test and prototype. Small-scale tests outside the centrifuge, however, are performed at normal gravity and the stress-scale is not unity. For test result to represent the prototype behavior requires recognition that the density (void ratio), geometric scale, and stress-scale are related and must be considered together. A detail explanation and discussion of this statement is presented by Altaee and Fellenius (1994) and only major points are presented in this paper.

A first step toward understanding the behavior of sands was taken by Casagrande (1936), who showed that the behavior in shear of a sand can be either contractant, dilatant, or neither. Casagrande established the term “critical void ratio” or “critical density”, which denotes the void ratio or density of a soil subjected to continuous shear under neither dilatant nor contractant behavior, i.e., no volume change. A next step was by Roscoe et al. (1958) who introduced the Critical State Soil Mechanics for clays, which explains the fundamental behavior of a clay as a function of the void ratio and the mean stress. Later, Roscoe and Poorooshasb (1963) suggested that this principle could be extended to the behavior of non-cohesive soil, i.e., sands.

Been and Jefferies (1985) indicated that, in a void ratio vs. mean stress plot, the distance between the actual void ratio and the void ratio at the critical (or steady) state is an important parameter. They demonstrated that a similarity of behavior would occur between samples of the same sand tested at different void ratio and mean stress as long as the states are at equal void ratio difference. Altaee and Fellenius (1994) developed scaling relations for small-scale model testing and analysis and presented a soil model based on steady state behavior as developed by Bardet (1986). The principle of the critical state, or steady state, for sands is illustrated in Fig. 5, stating that the critical void ratio at the critical or steady state of the sand (when it shears with no further volume change) is a linear function of the logarithm of mean stress. The line is defined by its critical void ratio value, Γ (at the reference mean stress of 100 KPa) and its slope, λ. The compression of the sand follows a line with slope κ. When stress is introduced to a sand, the behavior of the

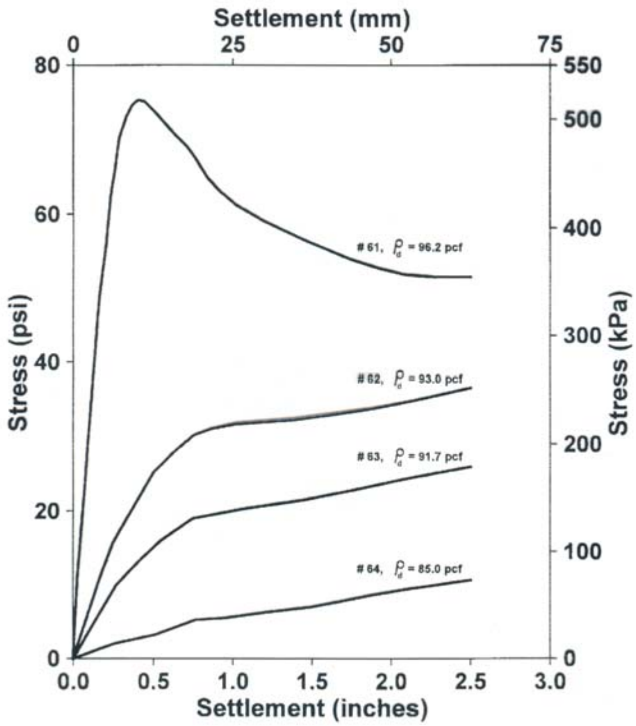

Fig. 1 Contact stress versus settlement of 150 mm footings. (Vesic 1967)

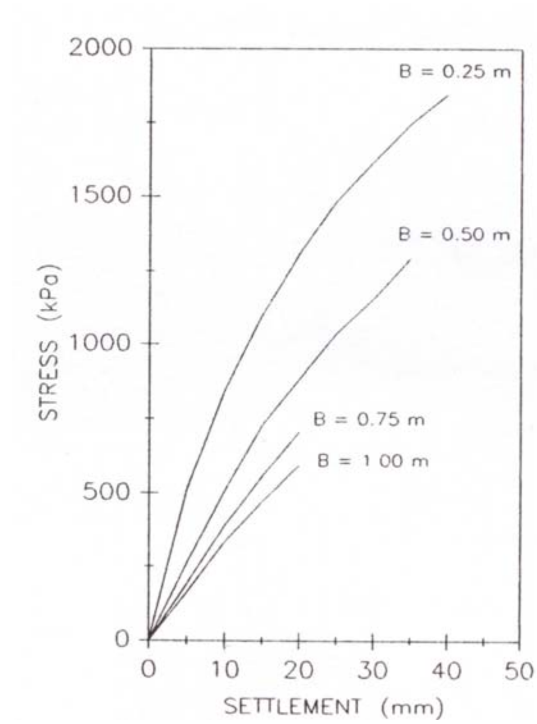

Fig. 2 Contact stress versus versus settlement of 0.25 m - 1.00 m footings (Ismael 1985)

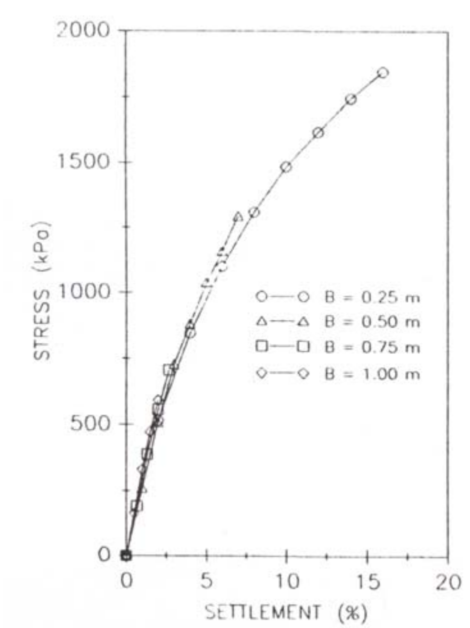

Fig. 3 Stress vs. normalized settlement of the data shown in Fig. 2.

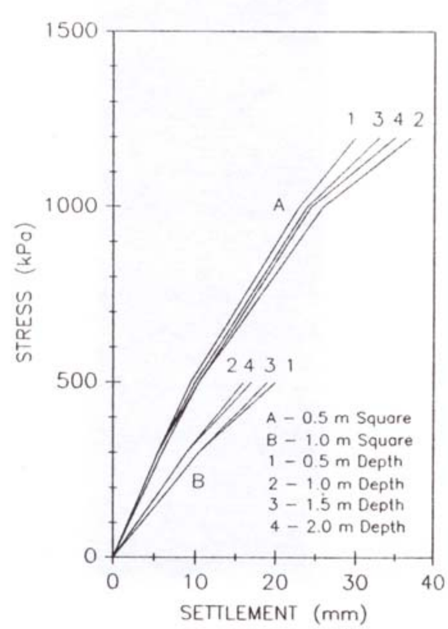

Fig. 4 Influence of footing depth on stress-settlement (Ismael 1985)

sand is a function of its state location in this void-ratio versus mean-stress diagram and the void ratio distance (the upsilon value, Υ) from the initial void ratio to the void ratio along the steady state line at the same mean initial pressure. Positive upsilon values (upsilon distance above the line) indicate contractant behavior and negative values dilatant behavior.

Fig. 6 shows several steady-state lines from various sources as compiled by the authors (Altaee and Fellenius, 1994). The compilation indicates a vast variety of slope and critical void ratio values, which demonstrate the very variable behavior exhibited by different sands. No two sands can *a priori* be assumed similar in behavior.

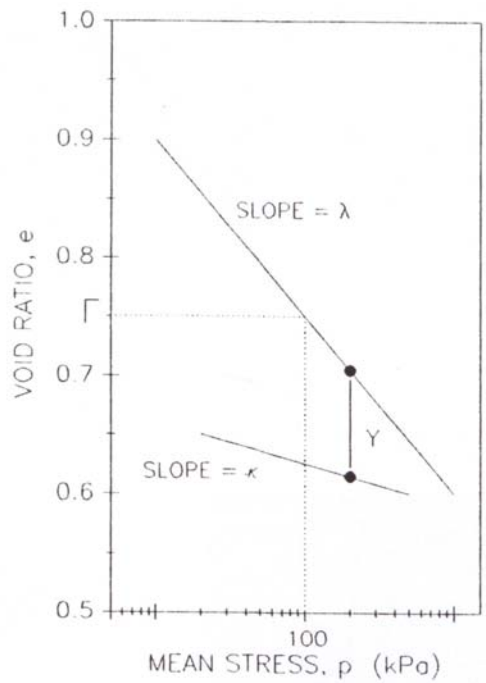

Fig. 5 Definition of steady-state line in the e-ln(p) plane.

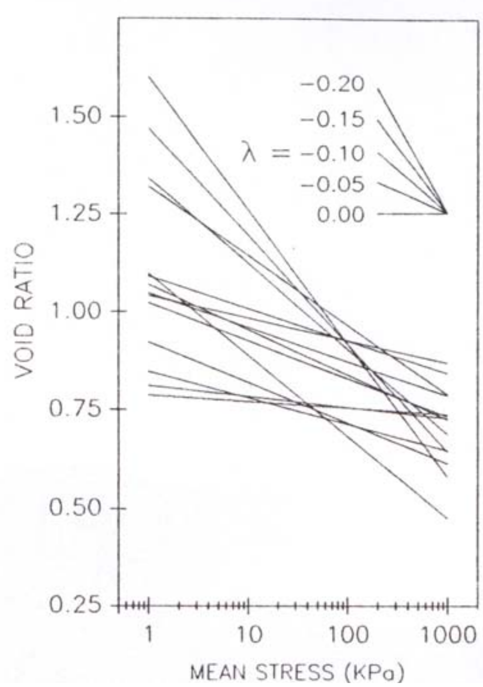

Fig. 6 Compilation of steady state lines (Altaee and Fellenius, 1994)

To demonstrate the importance of the steady-state approach in analyzing the behavior of a sand, we will discuss the load-settlement behavior of footings in two of the sands whose steady state lines**:** Fuji River and Kogyuk sands.

A summary of the soil parameters pertaining to the two sand types is presented in Table 1. The two sands are very different. The steeply sloping line (as indicated by $\lambda = 0.120$) of the Fuji River Sand, as opposed to the flat slope ($\lambda = 0.029$) of the Kogyuk sand sloping line, indicates that the former sand is much more compressible than the latter. Also the peak strengths, as indicated by the peak friction angles in Table 1, differ considerably. Detailed data on these sands are presented by Tatsuoka and Ishihara (1974) and Ishihara et al. (1991), and Been and Jefferies (1985), respectively.

Agreement (not here documented) has been established between computed behavior (simulated tests) and the reported behavior in laboratory tests, which confirms the adequacy and relevance of the soil model and analysis method (Altaee, 1991; Altaee and Fellenius, 1993).

TABLE 1. Comparison of sand parameters

Parameter	Soil Type Fuji River	 Kogyuk
Mean particle size (mm)	0.22	0.35
Uniformity coefficient, C_U	2.21	1.80
Maximum void ratio, e_{max}	1.08	0.83
Minimum void ratio, e_{min}	0.53	0.47
Effective angle of friction (°) from triaxial testing		
Ultimate Compression	36.9	30.5
Ultimate Extension	32.0	30.5
Peak	38.0	35.0
Critical void ratio at 100 KPa, Γ	0.920	0.713
Slope of critical line in e - ln(p) plane, λ	0.120	0.029
Slope of unloading-reloading line in e-ln(p) plane, κ	0.010	0.006
Poisson's ratio, ν	0.30	0.10
Aspect ratio of bounding surface, ρ	2.2	2.0
Hardening parameter, h_o	1.0	1.0

NUMERICAL ANALYSIS

The settlement behavior of footings placed on the two sand types is investigated numerically in a three-part analysis series. The analysis is a plane strain (two-dimensional) finite element analysis incorporating the bounding surface plasticity model for sand. The finite element mesh consists of 300 nodal points selected by means of a parametric study to determine the size of the soil mass to include in the analysis as well as the geometric boundaries. The footings are rigid and continuous with rough contact surface.

Fig. 7 presents a diagram of the data for the Fuji River Sand plotted as void ratio versus mean stress with the initial void ratio and initial mean stress below the footing base for each analysis. The similar plot for the Kogyuk data is not shown. (Note, for reasons of achieving clarity, Fig. 7 shows the conditions at a depth of 3B below the footing**;** the initial void ratio distance—Upsilon value—to the steady state line is different at different depths).

In a first series on each sand type, footings of three sizes (B = 0.5 m, B = 1.0 m, and B = 2.0 m) are placed at the ground surface, at a depth equal to the footing size, and a depth equal to twice the footing size. For all these nine footings (numbered from 1 through 9), the initial void ratio of the soil is essentially the same (note, as the mean stress increases, the void ratio decreases slightly due to compression of the soil). The density indices are also essentially equal. However, the upsilon values (void ratio distance to the steady state line) vary somewhat and more so for the Fuji River Sand than the Kogyuk Sand, because of the different slopes of the steady state lines.

In a second series, to demonstrate clearly the effect of varying upsilon value, six footings of equal size (1.0 m) are placed at equal depth (1.0 m), but in sands of different upsilon values, initial void ratios, and initial mean stresses. These analysis cases are numbered 10, 11, 12, 15, and 16. Case 5 of Series 1 fits into this series too.

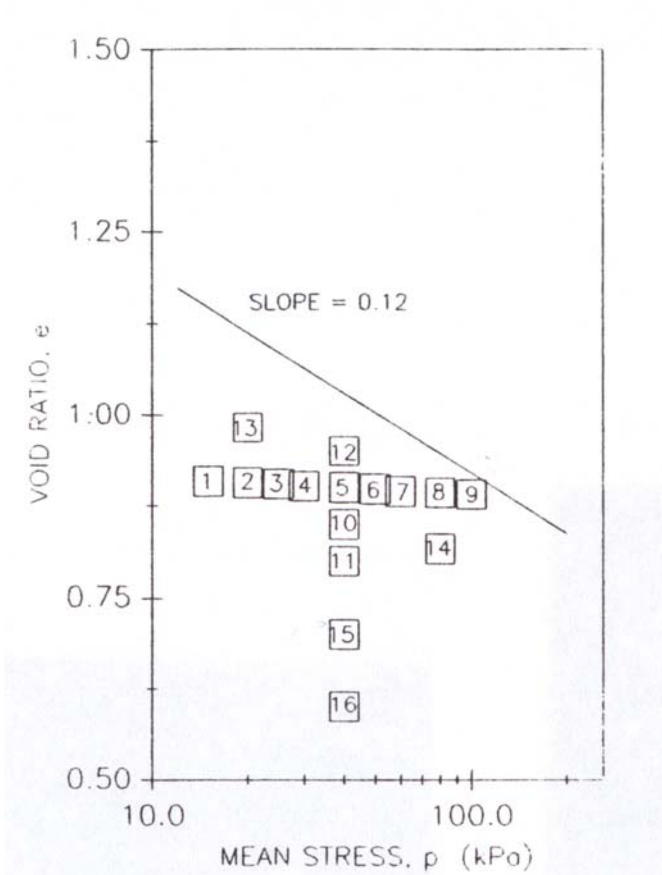

Fig. 7 e-ln(p) diagram for the initial state of homologous points for the analyses.

The third analysis series includes the footing of different sizes placed at a one-diameter depth in the sand having the same upsilon value, but different void ratios. These analyses are numbered 13 and 14. Again, analysis No. 5 of Series 1 fits into this series.

The void ratios, density indices, and upsilon values pertaining to the initial states of the three analysis series at a depth of 1B below the footing base are listed in Table 2. Note that the void ratio values vary with depth and mean stress.

TABLE 2. Void Ratios, Density Indices, and Upsilon Values

Analysis #	Series	B (m)	z (m)	Fuji River Sand e (-)	Fuji River Sand Y (-)	Fuji River Sand I_D (%)	Kogyuk Sand e (-)	Kogyuk Sand Y (-)	Kogyuk Sand I_D (%)
#1	1	0.5	0	0.909	-0.239	31	0.680	-0.088	42
#2	1	0.5	B	0.907	-0.206	31	0.678	-0.079	42
#3	1	0.5	2B	0.905	-0.181	32	0.677	-0.076	42
#4	1	1.0	0	0.902	-0.162	32	0.676	-0.072	43
#5	1, 2, 3	1.0	B	0.900	-0.130	33	0.676	-0.063	43
#6	1	1.0	2B	0.898	-0.104	33	0.675	-0.058	43
#7	1	2.0	0	0.895	-0.086	34	0.671	-0.057	44
#8	1	2.0	B	0.893	-0.054	34	0.671	-0.048	44
#9	1	2.0	2B	0.891	-0.029	34	0.670	-0.043	44
#10	2	1.0	B	0.850	-0.180	42	0.650	-0.089	50
#11	2	1.0	B	0.800	-0.230	51	0.625	-0.114	60
#12	2	1.0	B	0.950	-0.080	24	0.725	-0.014	29
#13	3	0.5	B	0.983	-0.130	18			
#14	3	2.0	B	0.817	-0.130	48			
#15	2	1.0	B	0.700	-0.330	69			
#16	2	1.0	B	0.600	-0.430	97			

ANALYSIS RESULTS

The computations proceeded by calculating the contact stress for imposed values of settlement. Figs. 8 and 9 show the results of the calculations of for Series 1 (different size footings placed at different depths, i.e., different stress at homologous points) for footings in the Fuji River and Kogyuk sands, respectively. The shape of the curves is very similar to that of the field tests presented in Fig. 2 taken from the work of Ismael (1985). Note, that no sign of impeding failure can be seen despite the settlement ratio reaching a value of 10 percent of the footing width.

To study the effect of the footing size, the Series 1 stress-settlement data are normalized to the footing widths in Fig. 10 and Fig. 11, respectively, for the two sand types (three pairs of data in each diagram). The rather small differences in the normalized behavior appear to suggest that, in conformity with the field tests (Fig. 3), the stress-settlement behavior of footings in sands of equal density might be independent of the footing width and difference in initial stress at homologous points. Note, however, that the behavior of the footing at the ground surface of the Kogyuk sand deviates from its counterpart on the Fuji River sand, which shows that the footing width is not always even approximately useful as a normalizing parameter.

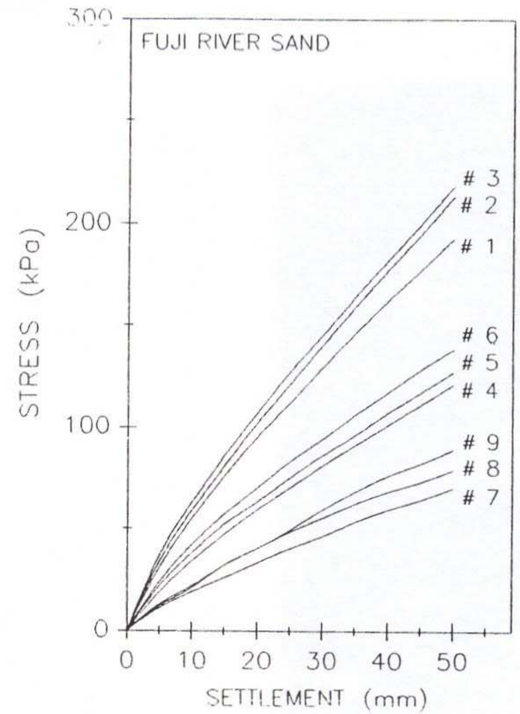

Fig. 8 Contact stress vs. settlement Series 1, Fuji River sand

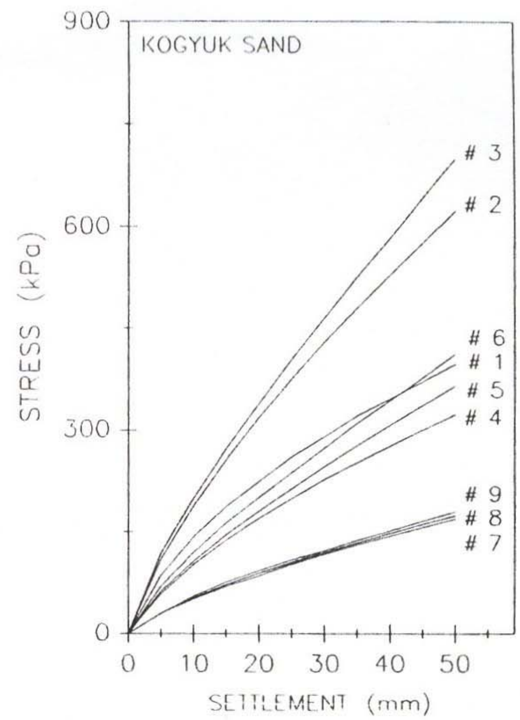

Fig. 9 Contact stress vs. settlement Series 1, Kogyuk sand

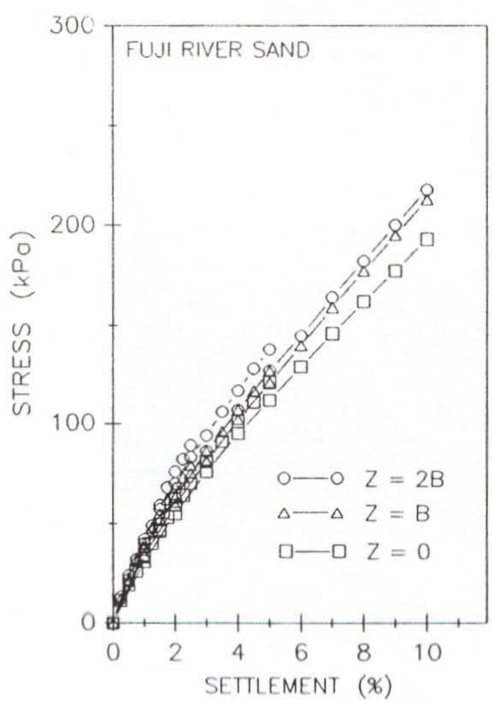

Fig. 10 Stress vs. normalized settlement Series 1, Fuji River sand

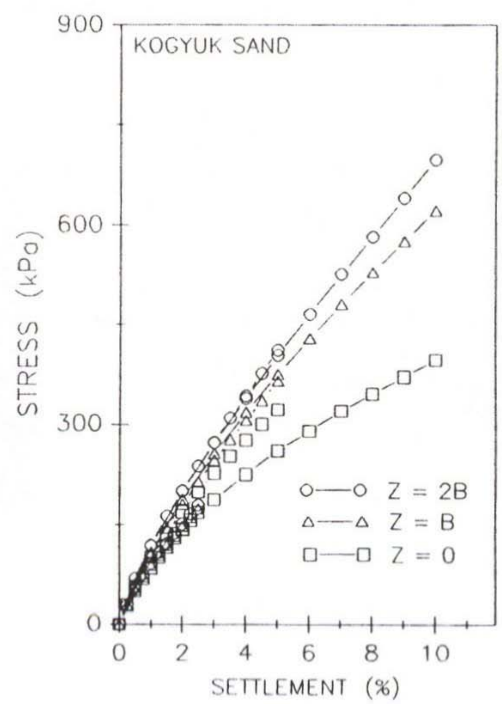

Fig. 11 Stress vs. normalized settlement Series 1, Kogyuk sand

Fig. 12 shows the contact stress versus settlement for footings of equal width (B = 1.0 m) placed at ground surface and at depths of 1.0 m and 2.0 m. The void ratios and the density indices are essentially equal for the sands of each of the two series. The diagrams show that while the influence of depth, that is, of initial mean stress and upsilon values, is small, it is not insignificant. Note that although the difference in density index between the two sands is not large, the settlement (at equal contact stress) in the Fuji River Sand is several times larger than that in the Kogyuk Sand. Note also that the higher friction angle of the Fuji River Sand appears to be of no consequence.

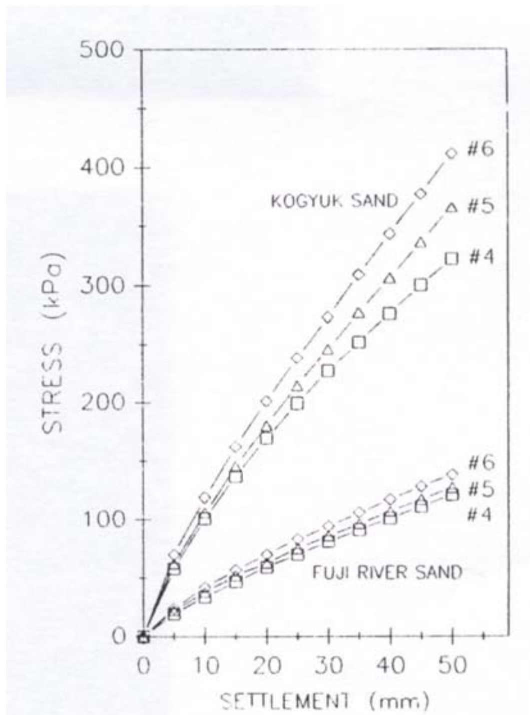

Fig. 12 Stress versus settlement for footings at different depths (z = 0; z = B; z = 2B) Series 1 Fuji River and Kogyuk sands

The results of Series 2 (same size footing placed at equal depth, but in sands of varying density—varying upsilon value) are shown in Figs. 13 and 14. The results indicate that the density of the sand has a significant influence on the stress-settlement behavior—not particularly novel a discovery, of course. Notice also that the upsilon value at homologous points of the sand differs between the different footings.

Fig. 15 shows the results from Series 3 (different footing width, different initial density, different initial stress, but same upsilon value). The stress-settlement behavior of these three analyses are equal, which demonstrates that the important parameter governing the settlement behavior is the upsilon value. In fact, for the behavior of a model test to agree with the behavior of its prototype requires that the test is performed at a density (void ratio) that has an equal distance to the steady state line, that is, the test has to be made at an upsilon distance that for homologous points is equal to that of the prototype.

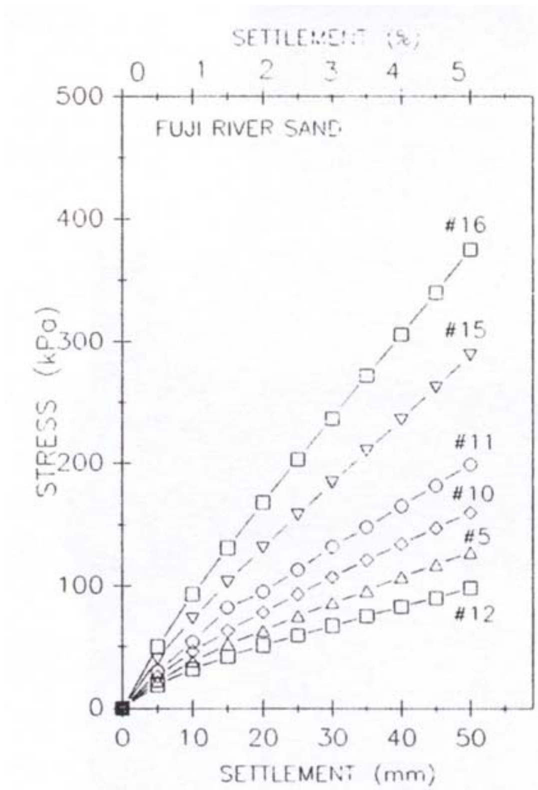

Fig. 13 Stress vs. settlement
Series 2, Fuji River Sand

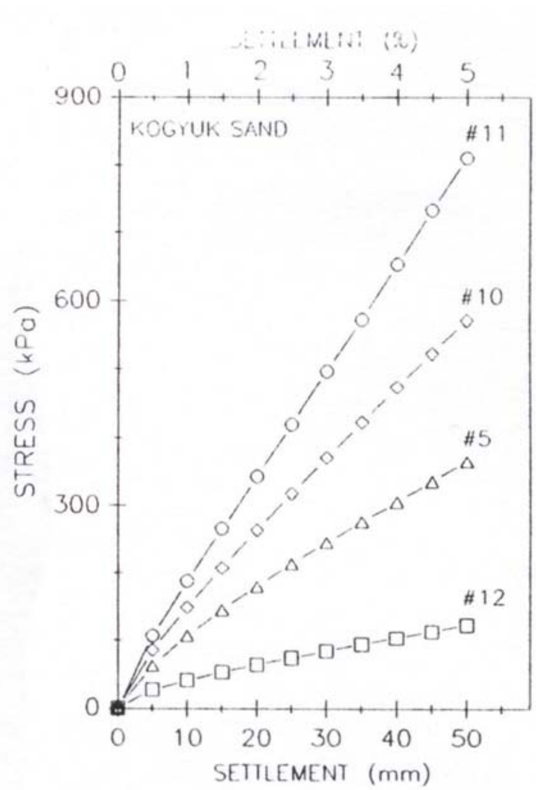

Fig. 14 Stress vs. settlement
Series 2, Kogyuk Sand

Furthermore, the requirement of equal upsilon value means that a model test in a soil of some density emulates the behavior of a prototype (if larger than the model, of course) in a denser soil. Therefore, small-scale model tests in a dense soil have very limited application, because when applied to a prototype of some size, the density of the relevant prototype soil very quickly exceeds the maximum density of the soil. At the same time, when investigating settlement, as well as capacity, in small-scale tests, the emulation of large prototypes must be performed in soils very much looser than that of the prototype soil. The ratio of geometrical scale is determined by the practical limit of how loose the sand can be. Very small footing tests, for example, have limited application to the behavior of full size foundations.

Note that the stress-settlement behavior for the analyses results shown in Fig. 15 is the same for all curves, in contrast to the results shown in Figs. 13 and 14, although the initial void ratios are different. This means that, when comparing footings of different size in the same sand, the settlement is not a function of the density per se, nor is it a function of the density index.

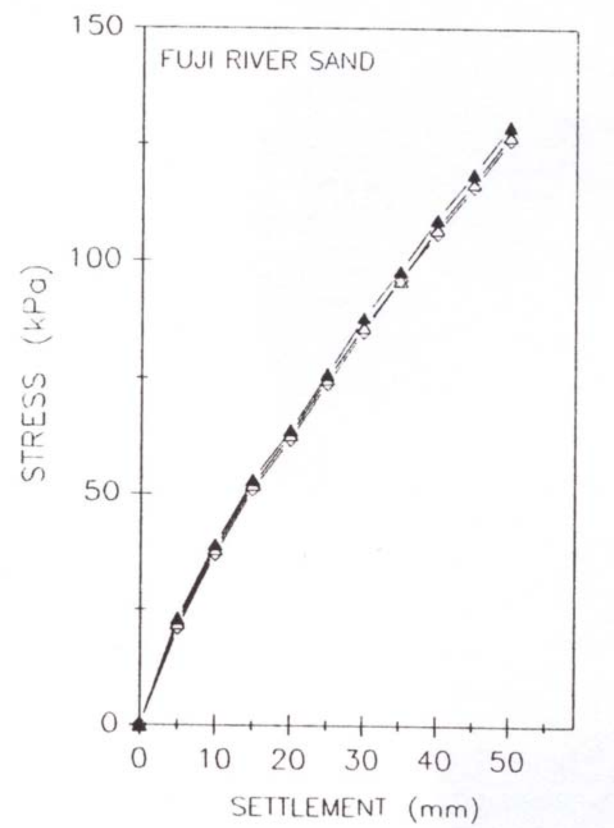

Fig. 15 Stress-settlement Series 3, Cases 5, 13, and 14
Fuji River Sand

CONCLUSIONS

The settlement of footings of different widths placed in sand of different void ratios and at different depths below the ground surface can be directly related if the conditions of steady state are considered and if the tests are performed at equal void ratio distance to the steady state line—equal upsilon value. Small-scale models will only be representative for prototype behavior if this requirement is fulfilled.

When comparing the behavior of footings of different size in the same sand, the settlement is not a function of the density per se, nor is it a function of the density index (the relative density).

The requirement of equal upsilon value means that the small-scale model test must always be performed in a soil that is looser than the prototype soil. This imposes boundaries on the geometric scale, because, first, a model test cannot be carried out in a sand that is looser than the maximum void ratio. Second, a model test must not be performed in a soil that is denser than what corresponds to realistic density of its prototype soil. Therefore, a model test is meaningless—has no corresponding prototype—if it is performed in a sand close to the minimum void ratio, because, if so, the density of the prototype sand would have to approach zero void ratio, that is, cease to be a soil.

Furthermore, although the stress-settlement relation is approximately similar for footings of varying size in sand of uniform density, normalization to footing size does not strictly provide a similitude of results.

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