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Research Article

Stress and Strain Analysis of Functionally Graded Rectangular Plate with Exponentially Varying Properties

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The bending of rectangular plate made of functionally graded material (FGM) is investigated by using three-dimensional elasticity theory. The governing equations obtained here are solved with static analysis considering the types of plates, which properties varying exponentially along z direction. The value of Poisson's ratio has been taken as a constant. The influence of different functionally graded variation on the stress and displacement fields was studied through a numerical example. The exact solution shows that the graded material properties have significant effects on the mechanical behavior of the plate.

1. Introduction

Recently, a new category of composite materials known as functionally graded materials (FGMs) has attracted the interest of many researchers. The FGMs are heterogeneous composite materials in which the mechanical properties vary continuously in certain direction. FGMs are used in many engineering applications such as aviation, rocketry, missiles, chemical, aerospace, and mechanical industries. Therefore, composites that are made of FGMs were considerably attractive in recent years.

Several studies have been performed to analyze the behavior of functionally graded beam, plates, and shells. Hadi et al. [1, 2] studied an Euler-Bernoulli and Timoshenko beam made of functionally graded material subjected to a transverse loading at which Young's modulus of the beam varies by specific function. Reddy [3] has analyzed the static behavior of functionally graded rectangular plates based on his third-order shear deformation plate theory. Cheng and Batra [4] have related the deflections of a simple supported functionally graded polygonal plate given by the first-order shear deformation theory and a third-order shear deformation theory to an equivalent homogeneous Kirchhoff plate. Cheng and Batra [5] also presented results for the buckling and steady state vibrations of a simple supported functionally graded polygonal plate based on Reddy's plate theory. Loy et al. [6] studied the vibration of functionally graded cylindrical shells by using Love's shell theory.

Analytical 3D solutions for plates are useful because provided benchmark results to assess the accuracy of various 2D plate theories and finite element formulations. Cheng and Batra [7] used the method of asymptotic expansion to study the 3D thermoelastic deformations of a functionally graded elliptic plate. Recently, Vel and Batra [8] have presented an exact 3D solution for the thermoelastic deformation of functionally graded simple supported plates of finite dimensions. Reiter et al. [9] performed detailed finite element studies of discrete models containing simulated particulate and skeletal microstructures and compared the results with those that are computed from homogenized models in which effective properties were derived by the Mori-Tanaka and the self-consistent methods.

Tanigawa [10] used a layerwise model to solve a one-dimensional transient heat conduction problem and the associated thermal stress problem of an inhomogeneous plate. He further formulated the optimization problem of the material composition to reduce the thermal stress distribution. Tanaka et al. [11, 12] have designed FGM property profiles by using sensitivity and optimization methods based on the reduction of thermal stresses. Jin and Noda [13] used the minimization of thermal stress intensity for a crack in a metal-ceramic functionally gradient material as a criterion for optimizing material property variation. In the same context, also were studied both the steady state [14] and the transient [15] heat conduction problems by them, but neglected the thermomechanical coupling (see also [16, 17]).

The response of functionally graded ceramic-metal plates has been investigated by Praveen and Reddy [18] with using a plate finite element that accounts for the transversal shear strains, rotatory inertia, and moderately large rotations in von Kármán sense. Reddy and Chin [19] have studied the dynamic thermoelastic response of functionally graded cylinders and plates. Najafizadeh and Eslami [20] presented the buckling analysis of radially loaded solid circular plate that is made of functionally graded material.

In this paper, an exact solution bending of rectangular plate made of functionally graded material (exponential form along z direction) subjected to top and bottom pressures P_T and P_B, respectively, is investigated by using three-dimensional elasticity theory.

2. Analysis

The equilibrium of a weightless homogeneous transversally isotropic elastic FGM plate is considered. The geometry of the elastic FGM plate in relation to the coordinate axes is shown in Figure 1.

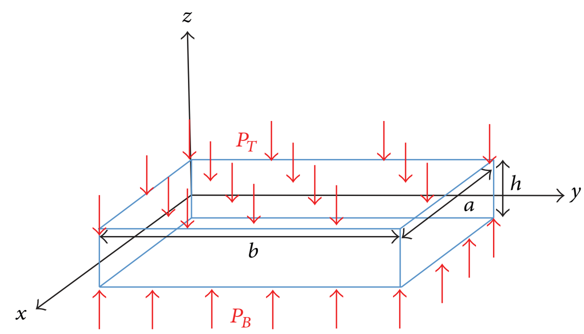

Figure 1: Geometry and boundary conditions of the plate.

The plate is assumed under the action of top and bottom pressures P_T and P_B, respectively,

$$P_T = P_{\text{To}} \operatorname{Sin} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}, \quad P_B = P_{\text{Bo}} \operatorname{Sin} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}, \tag{1}$$

where P_{To}, and P_{Bo} are top and bottom pressures in $x = a/2$, $y = b/2$.

In order to account for the changing material properties along the z direction, an exponential relationship is used as follows:

$$E(z) = E_i e^{n(z/h)}. \tag{2}$$

Here E_i is the module of elasticity at $z = 0$, and n is the inhomogeneity constants determined empirically. Equilibrium equations in three dimensions are defined as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0, \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0, \tag{3}$$

where σ_x, σ_y, and σ_z are normal stress components in x, y, and z direction, respectively. τ_{xy}, τ_{yz}, and τ_{xz} are shear stresses components.

The displacement in the x, y, and z directions is denoted by u, v, and w, respectively, six strain components can be expressed as

$$\begin{bmatrix} \varepsilon_x & \lambda_{xy} & \lambda_{xz} \\ \lambda_{xy} & \varepsilon_y & \lambda_{yz} \\ \lambda_{xz} & \lambda_{yz} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} \frac{du}{dx} & \frac{1}{2}\left(\frac{du}{dy} + \frac{dv}{dx}\right) & \frac{1}{2}\left(\frac{du}{dz} + \frac{dw}{dx}\right) \\ \frac{1}{2}\left(\frac{du}{dy} + \frac{dv}{dx}\right) & \frac{dv}{dy} & \frac{1}{2}\left(\frac{dv}{dz} + \frac{dw}{dy}\right) \\ \frac{1}{2}\left(\frac{du}{dz} + \frac{dw}{dx}\right) & \frac{1}{2}\left(\frac{dv}{dz} + \frac{dw}{dy}\right) & \frac{dw}{dz} \end{bmatrix}, \tag{4}$$

where ε_x, ε_y, and ε_z are normal strain components in x, y, and z directions, respectively. λ_{xy}, λ_{yz}, and λ_{xz} are shear strain components.

The stress-strain relations are

$$\sigma_x = \frac{E(z)}{(1-2v)(1+v)}\left[(1-v)\frac{\partial u}{\partial x} + v\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right], \quad \sigma_y = \frac{E(z)}{(1-2v)(1+v)}\left[(1-v)\frac{\partial v}{\partial y} + v\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right], \quad \sigma_z = \frac{E(z)}{(1-2v)(1+v)}\left[(1-v)\frac{\partial w}{\partial z} + v\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right], \tag{5}$$

$$\tau_{xy} = G(z)\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right], \quad \tau_{xz} = G(z)\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right], \quad \tau_{yz} = G(z)\left[\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right], \tag{6}$$

where $G(z) = E(z)/2(1 + v)$ is shear modulus, with $E(z)$ the modulus of elasticity and v Poisson's ratio. The value of Poisson's ratio has been taken as constant.

The boundary conditions of the problem are the following. Along the sides of the plate, we have

$$\begin{aligned} &x = 0, \quad a \longrightarrow w = 0, \quad \sigma_x = 0, \\ &x = 0, \quad b \longrightarrow w = 0, \quad \sigma_y = 0 \end{aligned} \tag{7}$$

the boundary conditions on the plate faces are as below:

$$\begin{aligned} &z = \frac{h}{2} \longrightarrow \sigma_z = P_T, \quad \tau_{xz} = \tau_{yz} = 0, \\ &z = -\frac{h}{2} \longrightarrow \sigma_z = P_B, \quad \tau_{xz} = \tau_{yz} = 0. \end{aligned} \tag{8}$$

By the displacement field below, the boundary conditions (7) are satisfied:

$$\begin{aligned} u &= u(x, y, z) = U(z) \cos \frac{\pi x}{a} \sin \frac{\pi y}{b}, \\ v &= v(x, y, z) = V(z) \sin \frac{\pi x}{a} \cos \frac{\pi y}{b}, \\ w &= w(x, y, z) = W(z) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}. \end{aligned} \tag{9}$$

Substituting (9) into (5) and the resulting expressions for stress components into equilibrium equations (3), we see that the equilibrium equation is expressed as system of second-order ordinary differential equations:

$$\begin{aligned} &\frac{d^2U}{dz^2} + \frac{n}{h}\frac{dU}{dz} + J_1 U + J_3 V - \frac{b}{\pi} J_3 \frac{dW}{dz} + \frac{\pi n}{ah} W = 0, \\ &\frac{d^2V}{dz^2} + \frac{n}{h}\frac{dV}{dz} + J_2 V + J_3 U - \frac{a}{\pi} J_3 \frac{dW}{dz} + \frac{\pi n}{bh} W = 0, \\ &\frac{b}{\pi} J_3 \frac{dU}{dz} + J_4 b U + \frac{a}{\pi} J_3 \frac{dV}{dz} \\ &\qquad + J_4 a V + J_5 \frac{d^2W}{dz^2} + \frac{J_5 n}{h}\frac{dW}{dz} + J_6 W = 0, \end{aligned} \tag{10}$$

where

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} = \begin{bmatrix} -\pi^2 \left(\frac{1}{b^2} + \frac{2(1-v)}{a^2(1-2v)} \right) & -\pi^2 \left(\frac{1}{a^2} + \frac{2(1-v)}{b^2(1-2v)} \right) \\ \frac{-\pi^2}{ab} \left(\frac{1}{1-2v} \right) & -\frac{2nv\pi}{abh(1-2v)} \\ \frac{2(1-v)}{1-2v} & -\frac{\pi^2}{a^2b^2} \left(a^2 + b^2 \right) \end{bmatrix}. \tag{11}$$

Therefore

$$\begin{aligned} &\left(S^2 + \frac{n}{h} S + J_1 \right) U + J_3 V + \left(-\frac{b}{\pi} J_3 S + \frac{\pi n}{ah} \right) W = 0, \\ &J_3 U + \left(S^2 + \frac{n}{h} S + J_2 \right) V + \left(-\frac{a}{\pi} J_3 S + \frac{\pi n}{bh} \right) W = 0, \\ &\left(\frac{b}{\pi} J_3 S + J_4 b \right) U + \left(\frac{a}{\pi} J_3 S + J_4 a \right) V \\ &\qquad + \left(J_5 S^2 + \frac{J_5 n}{h} S + J_6 \right) W = 0, \\ &\qquad\qquad S = \frac{d}{dz}. \end{aligned} \tag{12}$$

The general solution of (10) is as follows:

$$\begin{aligned} U &= C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} \\ &\quad + C_4 e^{s_4 z} + C_5 e^{s_5 z} + C_6 e^{s_6 z}, \\ V &= C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} \\ &\quad + C_{10} e^{s_4 z} + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}, \\ W &= C_{13} e^{s_1 z} + C_{14} e^{s_2 z} + C_{15} e^{s_3 z} \\ &\quad + C_{16} e^{s_4 z} + C_{17} e^{s_5 z} + C_{18} e^{s_6 z}, \end{aligned} \tag{13}$$

where $s_1, s_2, \ldots, s_6$ are the roots of the equation below:

$$\det \begin{bmatrix} S^2 + \frac{n}{h} S + J_1 & J_3 & -\frac{b}{\pi} J_3 S + \frac{\pi n}{ah} \\ J_3 & S^2 + \frac{n}{h} S + J_2 & -\frac{a}{\pi} J_3 S + \frac{\pi n}{bh} \\ \frac{b}{\pi} J_3 S + J_4 b & \frac{a}{\pi} J_3 S + J_4 a & J_5 S^2 + \frac{J_5 n}{h} S + J_6 \end{bmatrix} = 0, \tag{14}$$

where $C_1, C_2, \ldots, C_{18}$ are arbitrary integration constants.

The resulting displacement field is defined as follows:

$$\begin{aligned} u(x, y, z) &= [C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} + C_4 e^{s_4 z} \\ &\quad + C_5 e^{s_5 z} + C_6 e^{s_6 z}] \operatorname{Cos} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}, \\ v(x, y, z) &= [C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} + C_{10} e^{s_4 z} \\ &\quad + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}] \operatorname{Sin} \frac{\pi x}{a} \operatorname{Cos} \frac{\pi y}{b}, \\ w(x, y, z) &= [C_{13} e^{s_1 z} + C_{14} e^{s_2 z} + C_{15} e^{s_3 z} + C_{16} e^{s_4 z} \\ &\quad + C_{17} e^{s_5 z} + C_{18} e^{s_6 z}] \operatorname{Sin} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}. \end{aligned} \tag{15}$$

Substituting (15) into (5), (6) stress component is calculated as follows:

$$\sigma_x = \frac{E_i e^{nz/h} \operatorname{Sin}(\pi x/a) \operatorname{Sin}(\pi y/b)}{(1-2\upsilon)(1+\upsilon)} \times \left(\begin{array}{c} -\dfrac{(1-\upsilon)\pi\left(C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} + C_4 e^{s_4 z} + C_5 e^{s_5 z} + C_6 e^{s_6 z}\right)}{a} \\ -\dfrac{\upsilon\left(C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} + C_{10} e^{s_4 z} + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}\right)\pi}{b} \\ +\upsilon\left(C_{13} e^{s_1 z} s_1 + C_{14} e^{s_2 z} s_2 + C_{15} e^{s_3 z} s_3 + C_{16} e^{s_4 z} s_4 + C_{17} e^{s_5 z} s_5 + C_{18} e^{s_6 z} s_6\right) \end{array} \right), \tag{16a}$$

$$\sigma_y = \frac{E_i e^{nz/h} \operatorname{Sin}(\pi x/a) \operatorname{Sin}(\pi y/b)}{(1-2\upsilon)(1+\upsilon)} \times \left(\begin{array}{c} -\dfrac{(1-\upsilon)\pi\left(C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} + C_{10} e^{s_4 z} + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}\right)}{b} \\ -\dfrac{\upsilon\left(C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} + C_4 e^{s_4 z} + C_5 e^{s_5 z} + C_6 e^{s_6 z}\right)\pi}{a} \\ +\upsilon\left(C_{13} e^{s_1 z} s_1 + C_{14} e^{s_2 z} s_2 + C_{15} e^{s_3 z} s_3 + C_{16} e^{s_4 z} s_4 + C_{17} e^{s_5 z} s_5 + C_{18} e^{s_6 z} s_6\right) \end{array} \right), \tag{16b}$$

$$\sigma_z = \frac{E_i e^{nz/h} \operatorname{Sin}(\pi x/a) \operatorname{Sin}(\pi y/b)}{(1-2\upsilon)(1+\upsilon)} \times \left(\begin{array}{c} -\dfrac{\upsilon\pi\left(C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} + C_{10} e^{s_4 z} + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}\right)}{b} \\ -\dfrac{\upsilon\pi\left(C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} + C_4 e^{s_4 z} + C_5 e^{s_5 z} + C_6 e^{s_6 z}\right)}{a} \\ +(1-\upsilon)\left(\begin{array}{c} C_{13} e^{s_1 z} s_1 + C_{14} e^{s_2 z} s_2 + C_{15} e^{s_3 z} s_3 \\ +C_{16} e^{s_4 z} s_4 + C_{17} e^{s_5 z} s_5 + C_{18} e^{s_6 z} s_6 \end{array} \right) \end{array} \right), \tag{16c}$$

$$\tau_{xy} = \frac{E_i e^{nz/h} \pi \operatorname{Cos}(\pi x/a) \operatorname{Cos}(\pi y/b)}{2(1+\upsilon)} \left(\begin{array}{c} \dfrac{C_7 e^{s_1 z} + C_8 e^{s_2 z} + C_9 e^{s_3 z} + C_{10} e^{s_4 z} + C_{11} e^{s_5 z} + C_{12} e^{s_6 z}}{a} \\ +\dfrac{C_1 e^{s_1 z} + C_2 e^{s_2 z} + C_3 e^{s_3 z} + C_4 e^{s_4 z} + C_5 e^{s_5 z} + C_6 e^{s_6 z}}{b} \end{array} \right), \tag{16d}$$

$$\tau_{xz} = \frac{E_i e^{nz/h} \pi \operatorname{Cos}(\pi x/a) \operatorname{Sin}(\pi y/b)}{2(1+\upsilon)} \times \left(\begin{array}{c} \dfrac{C_{13} e^{s_1 z} + C_{14} e^{s_2 z} + C_{15} e^{s_3 z} + C_{16} e^{s_4 z} + C_{17} e^{s_5 z} + C_{18} e^{s_6 z}}{a} \\ +\dfrac{C_1 s_1 e^{s_1 z} + C_2 s_2 e^{s_2 z} + C_3 s_3 e^{s_3 z} + C_4 s_4 e^{s_4 z} + C_5 s_5 e^{s_5 z} + C_6 s_6 e^{s_6 z}}{\pi} \end{array} \right), \tag{16e}$$

$$\tau_{yz} = \frac{E_i e^{nz/h} \pi \operatorname{Sin}(\pi x/a) \operatorname{Cos}(\pi y/b)}{2(1+\upsilon)} \times \left(\begin{array}{c} \dfrac{C_{13} e^{s_1 z} + C_{14} e^{s_2 z} + C_{15} e^{s_3 z} + C_{16} e^{s_4 z} + C_{17} e^{s_5 z} + C_{18} e^{s_6 z}}{b} \\ +\dfrac{C_7 s_1 e^{s_1 z} + C_8 s_2 e^{s_2 z} + C_9 s_3 e^{s_3 z} + C_{10} s_4 e^{s_4 z} + C_{11} s_5 e^{s_5 z} + C_{12} s_6 e^{s_6 z}}{\pi} \end{array} \right). \tag{16f}$$

The resultant moments on a unit of length are obtained by using relations (16a)–(16f):

$$M_x = \frac{E_i \operatorname{Sin}(\pi x/a)\operatorname{Sin}(\pi y/b)}{(1-2v)(1+v)} \times \left[\sum_{i=1}^{6}\left(ab(n+hs_i)^2\right)\left[\begin{array}{l} e^{(h/2)(n/h+s_i)} h\left(\frac{hn}{2}+h\left(-1+\frac{hs_i}{2}\right)\right)\left(\begin{array}{c}-aC_{i+6}v\pi \\ +b\left(\begin{array}{c}C_i(-1+v)\pi \\ +aC_{i+12}vs_i\end{array}\right)\end{array}\right) \\ -e^{-(h/2)(n/h+s_i)} h\left(-\frac{hn}{2}+h\left(-1-\frac{hs_i}{2}\right)\right)\left(\begin{array}{c}-aC_{i+6}v\pi \\ +b\left(\begin{array}{c}C_i(-1+v)\pi \\ +aC_{i+12}vs_i\end{array}\right)\end{array}\right)\end{array}\right]\right], \quad (17a)$$

$$M_y = \frac{E_i \operatorname{Sin}(\pi x/a)\operatorname{Sin}(\pi y/b)}{(1-2v)(1+v)} \times \left[\sum_{i=1}^{6}\frac{1}{ab(n+hs_i)^2}\left[\begin{array}{l} e^{(h/2)(n/h+s_i)} h\left(\frac{hn}{2}+h\left(-1+\frac{hs_i}{2}\right)\right)\left(\begin{array}{c}-bC_i v\pi \\ +a\left(\begin{array}{c}C_{i+6}(-1+v)\pi \\ +bC_{i+12}vs_i\end{array}\right)\end{array}\right) \\ -e^{-(h/2)(n/h+s_i)} h\left(-\frac{hn}{2}+h\left(-1-\frac{hs_i}{2}\right)\right)\left(\begin{array}{c}-bC_i v\pi \\ +a\left(\begin{array}{c}C_{i+6}(-1+v)\pi \\ +bC_{i+12}vs_i\end{array}\right)\end{array}\right)\end{array}\right]\right], \quad (17b)$$

$$M_{xy} = \frac{E_i\pi \operatorname{Cos}(\pi x/a)\operatorname{Cos}(\pi y/b)}{2(1+v)} \times \left[\sum_{i=1}^{6}\frac{h(aC_i+bC_{i+6})}{(n+hs_i)^2}\left[\begin{array}{l} e^{(h/2)(n/h+s_i)}\left(\frac{hn}{2}+h\left(-1+\frac{hs_i}{2}\right)\right) \\ -e^{-(h/2)(n/h+s_i)}\left(-\frac{hn}{2}+h\left(-1+\frac{hs_i}{2}\right)\right)\end{array}\right]\right]. \quad (17c)$$

Using (16a), (16b), (16c), (16d), (16e), and (16f) the transverse shearing forces on a unit of length are by definition

$$Q_x = \frac{E_i \operatorname{Cos}(\pi x/a)\operatorname{Sin}(\pi y/b)}{2a(1+v)}\left[\sum_{i=1}^{6}\frac{h(C_{i+12}\pi + aC_i s_i)}{n+hs_i}\left[e^{(h/2)(n/h+s_i)} - e^{-(h/2)(n/h+s_i)}\right]\right], \quad (18a)$$

$$Q_y = \frac{E_i \operatorname{Sin}(\pi x/a)\operatorname{Cos}(\pi y/b)}{2b(1+v)}\left[\sum_{i=1}^{6}\frac{h(C_{i+12}\pi + bC_{i+6} s_i)}{n+hs_i}\left[e^{(h/2)(n/h+s_i)} - e^{-(h/2)(n/h+s_i)}\right]\right]. \quad (18b)$$

3. Results and Discussion

In the following, the obtained solution will be employed to analyze the effect of material inhomogeneity on the elastic field in the rectangular plate. Consider a rectangular plate with length $a = 1.5$ m, width $b = 1$ m, and thickness $h = 0.2$ m, with material property $E_i = 70$ GPa, subjected to top and bottom pressures $P_{\mathrm{To}} = -3 \times 10^6$ Pa and $P_{\mathrm{Bo}} = -1.5 \times 10^6$ Pa, respectively. It is assumed the Poisson's ratio v has a constant value of 0.3. Dimensionless and normalized variables are used. For different values of n, dimensionless modulus of elasticity along the z direction is plotted in Figure 2. According to this figure, at the same position $-0.5 \le z/h \le 0$, dimensionless modulus of elasticity is increasing as the parameter n *is* decreasing, while for $0 \le z/h \le 0.5$, dimensionless modulus of elasticity is increasing as the parameter n *is* increasing.

Figure 3 displays the nondimensional displacement of the plate in z direction for different values of parameter n. This plot displays that the magnitude of changing w by z is low, so for, $n < 0$ assuming plane strain is true and reasonable, but, for $n > 0$, the amount of changing w by z is high.

In Figure 4 is shown the nondimensional displacement in the x direction versus x/a for $z = h/3$, $y = b/2$. In this plot

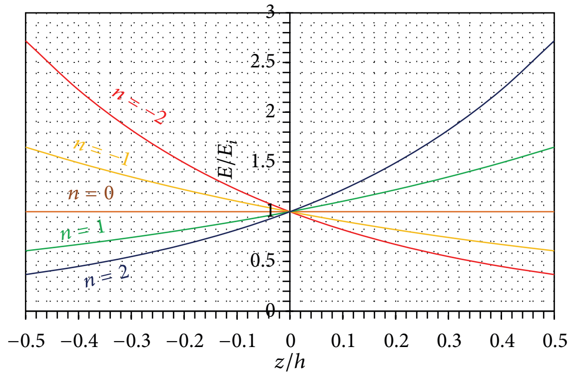

FIGURE 2: Distribution of modulus of elasticity.

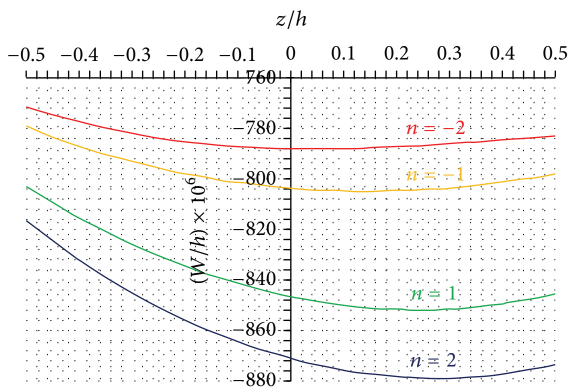

FIGURE 3: Distribution of displacement in z direction of the plate versus z/h at $x = a/2$, $y = b/2$.

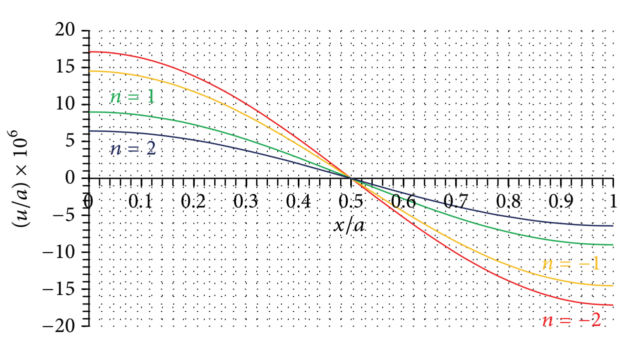

FIGURE 4: Distribution of displacement in x direction of the plate versus x/a at $y = b/2$, $z = h/3$.

displacement is decreasing as the parameter n is increasing. In Figure 5 is shown the nondimensional displacement in the y direction versus y/b for $z = h/3$, $x = a/2$.

In Figure 6 is shown the variation of nondimensional stress in the z direction versus nondimensional thickness at the $x = a/2$, $y = b/2$. This plot shows that the boundary conditions at the up and down surfaces are satisfied. Also at the constant z, by increasing the parameter n, it is observed the stress is decreased.

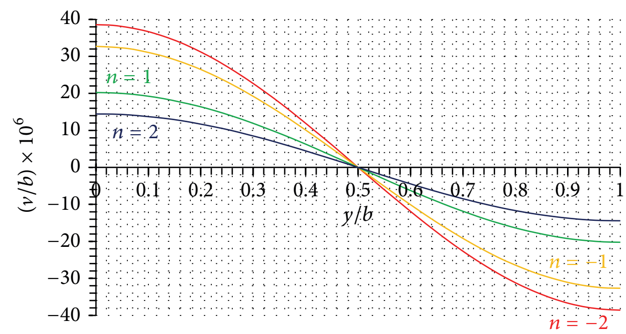

FIGURE 5: Distribution of displacement in x direction of the plate versus y/b at $x = a/2$, $z = h/3$.

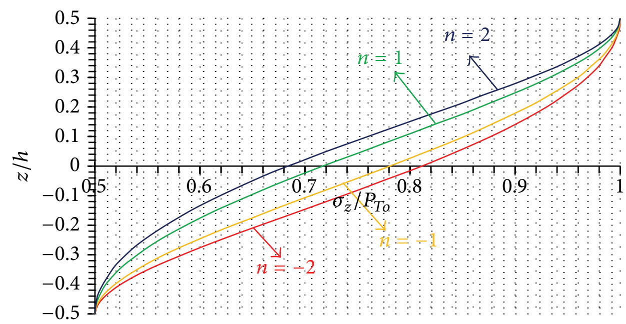

FIGURE 6: Distribution of nondimensional stress in the z direction versus z/h at $x = a/2$, $y = b/2$.

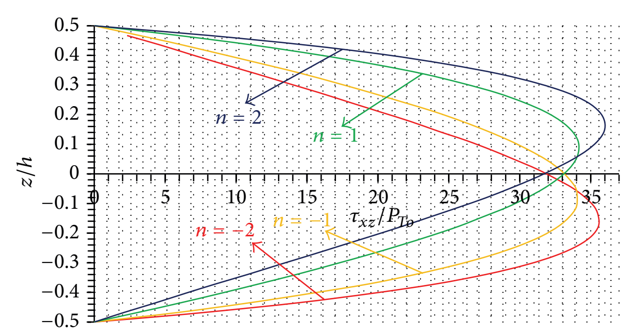

FIGURE 7: Distribution of $\tau_{xz}/P_{\mathrm{To}}$ versus z/h at $x = a/2$, $y = b/2$.

Figure 7 shows the $(\tau_{xz}/P_{\mathrm{To}}) \times 10^{18}$ according to the z/h for the variable amount of n: this figure probes that the stress's component can be deleted proportionally to other components.

4. Conclusion

It is apparent that close form solutions are important to simplified kinds of real engineering problems. In this paper is studied the rectangular plate that is made of functionally graded material with the variable properties (exponential form) by using 3D elasticity theory. Then some exact solution packages for stresses, displacements are presented.

To show the effect of inhomogeneity on the stress distributions, different values were considered for material inhomogeneity parameter *n*. The presented results show that the material inhomogeneity has a significant influence on the mechanical behaviors of the solid rectangular plate made of exponentially FG.

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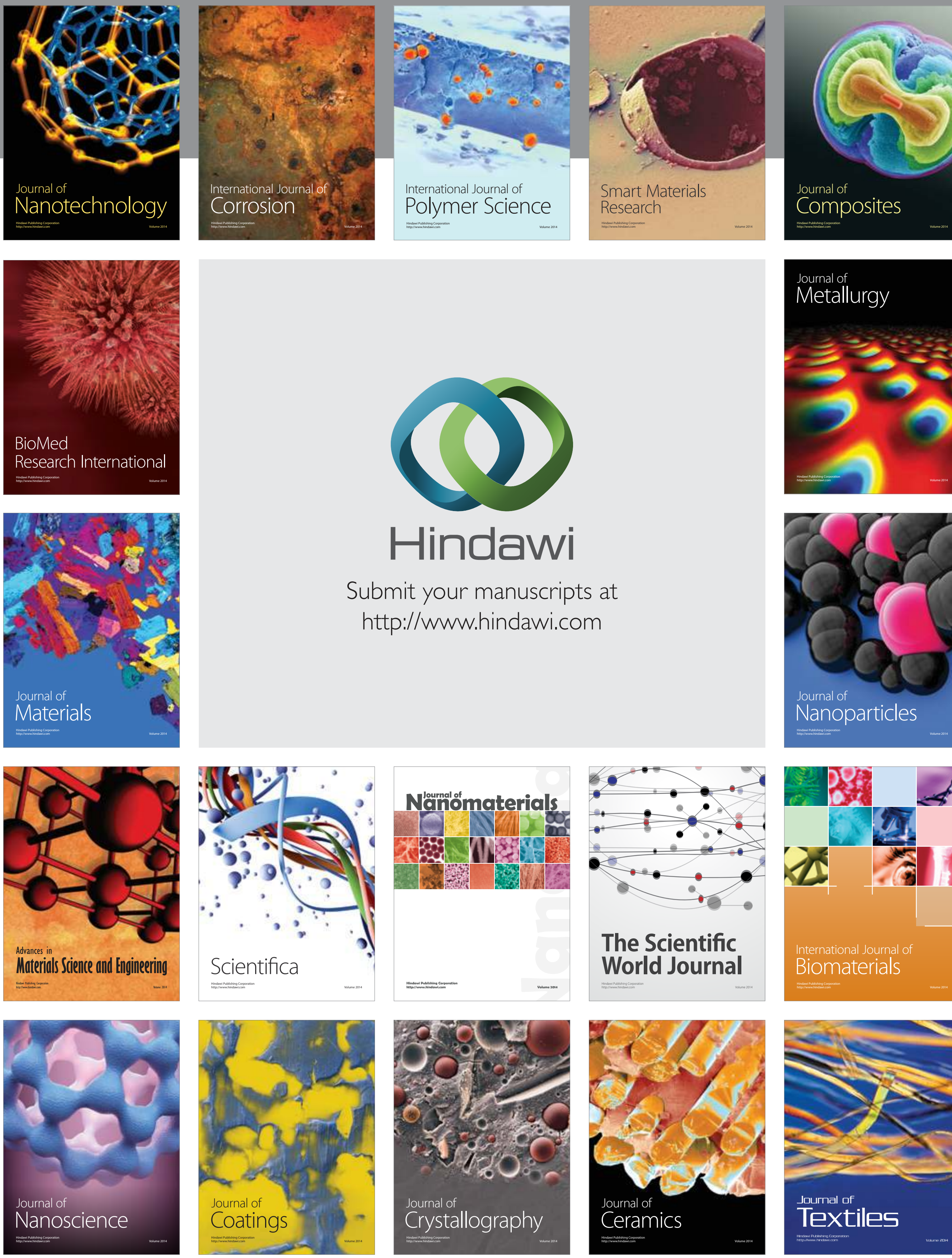
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