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Stress corrosion constitutive laws as a possible mechanism of intermediate-term and short-term seismic quiescence

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SUMMARY

Three types of seismic quiescence are recognized in the earthquake cycle. *Post-seismic quiescence* occurs due to the stress drop caused by a previous major earthquake, and results in long-term seismic gaps; *intermediate-term quiescence* occurs due to a small stress relaxation in the volume around the next mainshock area; and *short-term quiescence* during foreshock sequences occurs due to slip-weakening or dilatancy hardening concentrated on the nucleation point. In this paper a single quantitative model for intermediate-term and short-term quiescence is developed from (a) observation of subcritical crack growth due to stress corrosion, and (b) a general model for subcritical damage development where a fractal population of fractures results, irrespective of the underlying mechanism. In the former the stress intensity K of a single dominant macrocrack is the appropriate constitutive variable, while the latter more general formulation relies on a mean potential strain energy release rate $\langle G \rangle$ proportional to (a) the square of the applied effective stress and (b) the mean fracture length. Stress corrosion provides an important concrete example as well as a useful analogy for interpreting the general theory.

We then consider the effect of a stress decrease in the intermediate-term on seismic event rates using the approporiate constitutive laws for both variables. Simple calculations for a material of stress corrosion index $n = 30$ show that a 45 per cent reduction in event rates is consistent with only a 2 per cent reduction in K, and a 90 per cent reduction results from only a 7 per cent decrease in K. A similar order of magnitude of quiescence can be predicted theoretically by considering the effect of similar small changes in $\langle G \rangle$ for a fractal pupulation of faults or cracks averaged over a range of length scales. Such intermediate-term quiescence occurs in the model when K or $\langle G \rangle$ decreases because of the reduction in applied stress, during a phase of strain softening late in the earthquake cycle. Such a decrease in K or $\langle G \rangle$ moves the system temporarily further from the failure condition $K = K_c$ (or $\langle G \rangle = \langle G \rangle_c$) and is therefore stable. This temporary stability is consistent with the relatively long duration of intermediate-term quiescence (months or years). Observed intermediate-term seismic quiescences are relatively easily explained in the model by stable, regional decreases in stress in a volume much larger than the mainshock area, but prior to the period when concentrated accelerating crack growth in the nucleation zone dominates the strain softening. The general mechanism is compared to the Kaiser effect as a possible alternative explanation for intermediate-term seismic quiescence. The two models are not necessarily mutually exclusive, though the Kaiser effect involves *local* stress and strain relaxation, and predicts more abrupt changes in event rate when the stress decreases.

Short-term quiescence is also a feature of the general model, and is predicted when a concomitant decrease in seismic b-value occurs, as the larger events near the nucleation zone begin to dominate the stress relaxation, thereby inhibiting fracture on a smaller scale. Thus K (or $\langle G \rangle$) are increasing in this phase despite the stress reduction, because the length of the dominant crack (or the mean crack length of the ensemble) is increasing at a rate fast enough to more than offset the stress decrease. This increase in K or $\langle G \rangle$ means that the system is inherently unstable, consistent with the very short duration (hours or days) of short-term quiescence.

Key words: Kaiser effect, seismic quiescence, stress corrosion.

INTRODUCTION

Seismic quiescences, or a significant drop in the occurrence rate of earthquakes above an arbitrary threshold magnitude, are recognized in three of the important phases of the earthquake cycle (Scholz 1988). *Post-seismic quiescence* occurs in the region of the rupture zone of a major earthquake, and persists for a substantial fraction of the interevent time. *Intermediate-term quiescences* appear in a zone equal to or greater than the rupture area, and their durations appear to scale with the mainshock size (Mogi 1987; Rikitake 1987). Finally *short-term quiescences* associated with the immediate hypocentral area are sometimes observed hours or days prior to a mainshock.

Post-seismic quiescence is a consequence of the unloading of the Earth's brittle crust due to the sudden stress drop associated with a major seismic event, and typically lasts for upwards of 50 per cent of the interevent time. For example, Thatcher (1990) has recently reported a marked tendency of larger earthquakes in the circum-Pacific seismic belt to occur later rather than earlier in the recurrence cycle, leading to long-term seismic gaps.

Intermediate-term quiescence is thought to be due to a quasi-static decrease in stress caused by stress relaxation in the volume around the mainshock area, and has been widely reported as an earthquake precursor, summarized in Habermann (1988) and Wyss & Habermann (1988). The characteristic of such intermediate-term quiescence is a relatively large and abrupt decrease in seismic event rate by between 45 and 90 per cent, lasting for 15–17 months prior to the mainshock. Wyss & Habermann (1988) claim to have retrospectively 'predicted' 17 earthquakes by applying a statistical test to a large numer of data sets after the mainshock has occurred, with about a 50 per cent false alarm rate (i.e., where a 'prediction' was not followed by an event). Although the particular statistical test they used has been critized by Matthews & Reasenberg (1987), intermediate-term seismic quiescence remains one of the most statistically significant of reported earthquake precursors. Real-time predictions based on intermediate-term quiescence were made by Ohtake, Matumoto & Latham (1977), Wyss & Burford (1987) and Kisslinger (1988). More recently Wyss, Bodin & Habermann (1990a) have predicted the reccurrence of a magnitude $M_L = 5.7$ earthquake on the Parkfield segment of the San Andreas fault by 1992, based on similar observations of seismic quiescence. This prediction is also based on a coincident decrease in deformation rate as measured by a 5 km aperture geodimeter network straddling the fault at Parkfield (Wyss, Slater & Burford 1990b), though the significance and timing of this strain anomaly has been questioned by Langbein (1991). A further characteristic of intermediate-term quiescence is the large volume affected. For the Parkfield quiescence anomaly this comprises a zone 40 km long by 5 km wide around a predicted rupture segment approximately 35 km long. In this case the quiescence is closely associated with the predicted fault plane, but in other cases from the Aleutians (Kisslinger 1988); Hawaii (Wyss & Habermannn 1988); and Mexico (Ohtake *et al.* 1977) the quiescence is distributed about a volume 10–20 km in depth, and 10–100 km in length and width.

In contrast, short-term quiescence is usually associated with a small zone concentrated around the nucleation point of the mainshock, and is thought to result from slip-weakening or dilatancy hardening in the local area around a nucleation patch in the final quasi-static phase of earthquake preparation (Scholz 1988). The most well-known example of short-term quiescence within a localized foreshock sequence is the 1975 Haicheng earthquake ($M = 7.5$; Raleigh *et al.* 1977).

There are two possible explantions described in the literature for intermediate-term quiescence: the stress memory effect known as the 'Kaiser effect' (Scholz 1988) and a qualitative model based on time-dependent stress corrosion (Ohnaka 1985). The purpose of this paper is to show, by extrapolation from laboratory results based on acoustic emission monitoring, and by an appropriate theoretical model, that stress corrosion is quantitatively a plausible mechanism for both intermediate-term and short-term quiescence. The appropriate constitutive variable for the laboratory experiments is the stress intensity K, a measure of the intensity of the stress field concentrated at the tip of a pre-existing macrocrack. The appropriate variable for the more general theory is the mean potential strain energy release rate $\langle G \rangle$ (per unit surface area) for a fractal population of fractures produced by *any* mechanism, including stress corrosion. The conclusion of theory and experiment alike is that only a small concomitant reduction in stress and stress intensity is required to produce a large decrease in event rate in the intermediate term. Such intermediate-term quiescence is associated with an increasing or constant seismic b-value, and is predicted to be relatively stable because of the reduction in stress intensity. In contrast short-term quiescence results when the stress intensity is increasing despite a reduction in stress, accompanied by a dramatic decrease in seismic b-value. The increase in stress intensity is consistent with the short-term, unstable nature of short-term quiescence. These conditions are most closely associated with the final phase of earthquake nucleation.

MECHANISMS OF INTERMEDIATE-TERM QUIESCENCE

(a) The Kaiser effect

The Kaiser effect is a well-known stress memory effect where a material which has been subjected to a load at

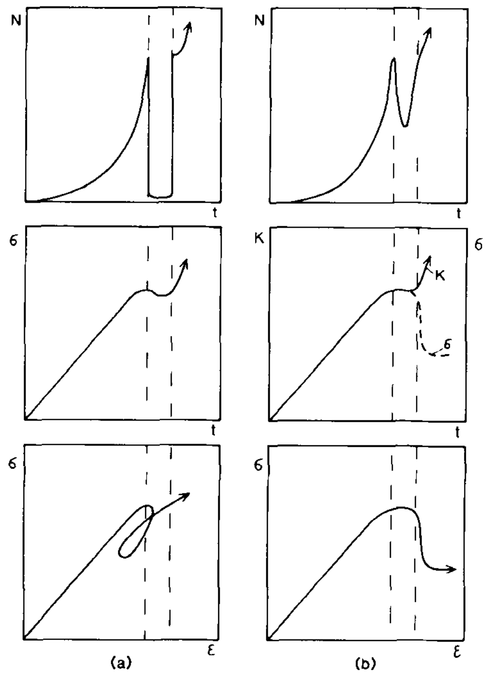

Figure 1. Two proposed models of seismic quiescence: (a) the Kaiser effect, after Scholz (1988), and (b) from constitutive laws derived from stress corrosion experiments and a model of fractal damage evolution (this paper). (a) involves local stress (σ) and strain (ε) relaxation, whereas (b) occurs during strain softening (increasing remote strain). The Kaiser effect is characterized by a more abrupt decrease in event rate (N) during a period of decreasing stress.

which acoustic emissions (AE) are produced, ceases to radiate AE when the load is relaxed. After reloading further AE are not generated until the previous peak stress has been exceeded. A characteristic of the Kaiser effect is the sudden switch-off in AE event rate when the stress drops even slightly. However this effect was first defined (Kaiser 1953) for the case of cyclical loading, where both stress *and* strain are relaxed before being reapplied, as illustrated in Fig. 1(a). Thus it may not be appropriate to apply the Kaiser effect to the interpretation of laboratory experiments or earthquake precursors unless such concomitant stress and strain relaxation can be independently demonstrated. For the usual boundary condition of constant strain rate appropriate to plate tectonic loading (Stuart 1979; Main 1988), the remote strain increases at a constant rate, and only *local* strain relaxation may take place. It has also been suggested that intermediate-term quiescence might occur when a relatively small increase in rock strength occurs due to the dilatancy hardening mechanism (Scholz 1988). In this case the Kaiser effect mechanism relies on tracing the history of the *effective* stress.

(b) Stress corrosion

Stress corrosion cracking results from the quasi-static, physico-chemical interaction between the stress applied to a flawed medium and the active chemical weakening of atomic bonds in the stress concentration at the tip of an existing crack or flaw. The significance of this is that cracks can propagate at accelerating rates even if no extra stress is applied, at stress intensities K much lower than the critical stress intensity or fracture toughness, K_c, of a material. This anelastic mechanism contrasts with the usual elastic Griffith energy balance criterion of crack growth when $K \geq K_c$, and can allow stable, subcritical crack growth to occur in the presence of an active chemical environment even when the stress is constant or decreasing. The extremely non-linear nature of the governing equations between crack growth, stress intensity and event rates (Meredith & Atkinson 1983) also introduces an inherently sensitive time dependence into the process, making accurate predictions difficult. The importance of stress corrosion in a geological context has recently been reviewed by Atkinson & Meredith (1987).

Figure 2 shows the non-linear dependence of the crack growth velocity V and the AE event rate N during double-torsion tensile crack propagation experiments on Whin Sill dolerite. The data show a power-law relationship of the form

$$V/V_0 = (K/K_0)^n, \tag{1}$$

$$N/N_0 = (K/K_0)^{n'}, \tag{2}$$

where K_0, V_0, N_0, n and n' are all empirical constants. The most striking feature of Fig. 2 is the parallel nature of the two curves, which accords with the very similar values for the power-law exponent, determined by regression: $n = 29.0$ (0.990), $n' = 29.1$ (0.977), with correlation coefficients in brackets. Equation (1) is known as Charles' law (Charles

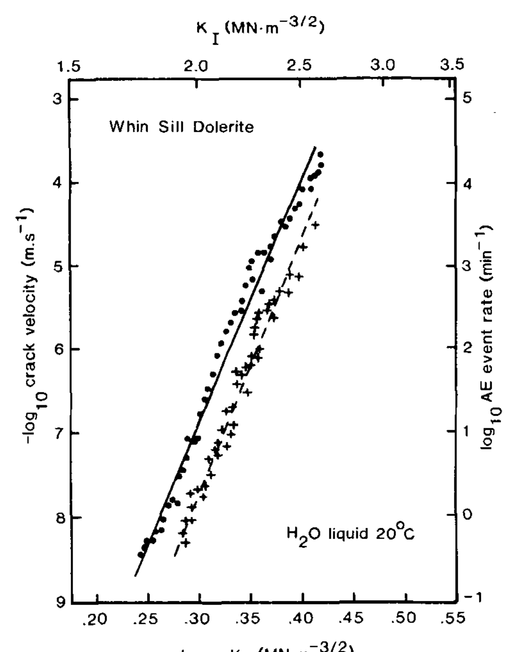

Figure 2. Dependence of crack velocity and event rate on measured (mode I) stress intensity K_I from the double torsion tensile failure experiments of Meredith & Atkinson (1983). The stress corrosion index is the slop of the log–log plot of crack extension velocity v. stress intensity (solid line), and a power law of similar exponent applies to the event rate dependence (dashed line), as evidenced by the parallel nature of the two lines. Tests were carried out in water at room temperature.

1958), and the exponent n is usually called the stress corrosion index. Alternatively, an exponential relationship of the form

$$V/V_0 = \exp[(-E^* + \alpha K)/RT] \quad (3)$$

has been proposed by Weiderhorn & Bolz (1970), based on reaction rate theory. However, when n is large it is difficult in practice to distinguish between the power-law and exponential forms, and laboratory results are often interpreted using (1) rather than the more physically sound equation (3). Here E^* is the stress-free activation energy, and α is an empirical constant. The empricial parameters V_0, K_0 and N_0 in equations (1) and (2) implicitly include the effect of temperature T.

The most important implication of the experimental results is that the AE event rate is very sensitive to small changes in the stress intensity because of the large values of n' found for heterogeneous materials like rocks. This sensitivity of the AE event rate during subcritical crack growth by the mechanism of stress corrosion, represented by equation (2), is the justification for the event rate model shown in Fig. 1(b). Here the strain is monotonically increasing at some remote boundary, and the rock responds at first elastically, then strain hardens and strain softens quasi-statically before producing dynamic rupture and maximum rate of stress drop. In order to produce intermediate-term seismic quiescence, the stress intensity must also decrease before increasing again in the final stages of mainshock nucleation, most likely as foreshocks. This model is a variant of the general model proposed to explain precursory variations in seismic b-value, also based on empirical observation of subcritical crack growth due to stress corrosion (Meredith, Main & Jones 1990; model C).

Comparison of the two proposed mechanisms for intermediate-term quiescence

The 'stress corrosion' Model (Fig.1b) differs from that of Fig. 1(a) (Scholz 1988), in that the changes in event rate are both smoother and less marked than the abrupt changes predicted by the Kaiser effect. Another major conceptual difference is that the Kaiser effect is due to *locally* relaxing stress and strain, whereas the stress corrosion model is couched in the terms of fracture mechanics, where *remotely* applied stress and strain are the appropriate variables. The strain softening mechanisms of stress relaxation [such as those invoked by Stuart (1979) and Main, Meredith & Jones (1989)] inherently involve an increase in remote strain, but may also produce local strain relaxation, for example by precursory slip on the incipient fault. The two models shown in Fig. 1 are therefore not necessarily mutually exclusive. For example stress intensity is a measure of the intensity of the stress field near the tip of a pre-existing crack, so it is not surprising that the local stress in Scholz's (1988) model (Fig. 1a) behaves in a similar way to the stress intensity in Fig. 1(b). Nevertheless strain softening by mechanisms which do not also involve local stress relaxation (for example due to changes the elastic constants of the stressed, fractured medium surrounding the fault) are by definition inconsistent with the Kaiser effect hypothesis. For example experiments carried out to observe AE during the compressional failure of intact laboratory rock samples (Meredith *et al.* 1990) show no evidence of the distinctive, abrupt Kaiser-type quiescence just after peak stress. Instead the event rate levelled off or lowered very slightly during monotonically increasing strain, consistent with a constant or only slightly decreasing stress intensity. These experiments did show evidence of short-term quiescence associated with incipient dynamic rupture, but this was due to the masking effect of events running into each other in the final stages of nucleation, consistent with accelerating microcrack coalescence. It is therefore unlikely that this is in any way similar to the short-term quiescence reported by Scholz (1988) for earthquakes, where the data base is much more sparse and biassed towards the largest magnitudes which are completely reported.

In the next section we show that the marked non-linearity in event rate observed in Fig. 2 can be modelled quantitatively by extending the concept of stress intensity to an ensemble of cracks in a heterogeneous medium. It is shown that any *any* mechanism of subcritical crack growth or stable damage development which produces a fractal distribution of cracks, faults or flaws can account numerically for the magnitude of the non-linearity between stress intensity and event rate observed in stress corrosion experiments. Thus the behaviour of a fractal population of microcracks or subsidiary faults is analogous to the effect of stress corrosion applied to rock samples with a dominant macrocrack, whatever the underlying mechanism.

THE EFFECT OF FRACTURE DISTRIBUTIONS ON THE MEAN POTENTIAL STRAIN ENERGY RELEASE RATE $\langle G \rangle$

The assumption in linear elastic fracture mechanics (LEFM) is that subsidiary microcracking around a propagating macrocrack is localized near the crack tip, and represents at most only a few per cent of the total energy release. This assumption is not strictly applicable throughout the anelastic phase of the deformation cycle in rock failure. For example Cox & Scholz (1988) argue that a material with as large an elastic stiffness and as heterogeneous a structure as laboratory rock samples, would require samples as large as 10 m before rigorous fracture mechanics determinations in shear fracture could be achieved in the laboratory. This follows from the large zone of brittle damage known to be associated with the fresh fracture of intact laboratory samples. The size of the zone of damage depends mainly on the ratio of grain size to the macrocrack length, and this effect introduces a scaling (i.e. a gradual increase) of apparent fracture toughness with macrocrack length (e.g. see Meredith 1989, fig. 4). The scaling up of such experiments can therefore be achieved only if some normalization relative to a size-dependent fracture toughness is applied. For example the semi-quantitative model of Meredith *et al.* (1990) is based on exactly such a normalization. Indeed all rock types tested so far produce acoustic emissions with indistinguishable b-values at a given environmental humidity and normalized stress intensity K/K_c. It is this scaling which allowed the direct comparison of laboratory AE statistics with earthquake distributions in Meredith *et al.* (1990).

Extensive damage is also consistent with the large volume associated with intermediate-term earthquake precursors, for example the spatially extensive changes in scattering attenuation before the Haicheng, China, earthquake of 1975 February 4 (Jin & Aki 1986, fig. 7). Rundle & Klein (1989) have suggested a modified version of Griffith theory which includes an 'interaction potential' to explain the evolution of a large zone of damage in the early stages of tensile failure. The large zone of anelastic damage predicted by extrapolation from the laboratory scale of Cox & Scholz (1988) and Rundle & Klein (1989) is borne out by the spatial extent of earthquake precursors.

The overall properties of such an ensemble of cracks can be determined by considering local stress intensities on each crack, and integrating the total effect over the range of crack lengths according to their size–frequency distribution. For example, this approach was adopted by Das & Scholz (1981), who assumed a constant applied stress and a random distribution of critical stress intensities (or fracture toughnesses) on different patches of a mechanically heterogeneous fault. They further assumed that the form of the laws of stress corrosion cracking derived from tensile failure laboratory experiments also held for shear failure, and hence derived Omori's aftershock law. Yamashita & Knopoff (1989) also applied a distribution of stress intensities and a constitutive law of the form (1) to model the occurrence of foreshock sequences. Main (1991), this issue), in a companion paper to the present work, derived the following expression for an effective energy release rate G', based on a Griffith energy balance criterion, modified for a weakly interacting ensemble of aligned tensile cracks of varying semilengths c, under conditions of constant applied stress

$$G' = \frac{\pi}{2}(\sigma^2/E)\partial\langle c^2\rangle/\partial\langle c\rangle, \tag{4a}$$

and compared its use to that of a mean strain energy release rate

$$\langle G\rangle = \pi(\sigma^2/E)\langle c\rangle. \tag{4b}$$

$\langle G\rangle$ is the expectation value of the strain energy release rate (sometimes called the crack extension force), integrated over the range of crack half-lengths ($c_{\min}, c_{\max}$), σ is the applied stress, and E is Young's modulus. The potential strain energy release rate G for a single crack is proportional to the square of the stress intensity, so averaging over the parameter G is effectively equivalent to integrating over K as in the models of Yamashita & Knopoff (1989). For a fractal distribution of crack lengths, G' and $\langle G\rangle$ both show a positive correlation with the total number of cracks contributing to the potential energy release rate, denoted N_T here, and a negative correlation with seismic b-values. The predicted negative correlation with seismic b-values is important because this is the basis of the semi-quantitative model of Meredith *et al.* (1990), and is consistent with observation of both tensile (Meredith & Atkinson 1983) and compressional (Meredith *et al.* 1990) failure of intact rock specimens. Although G' represents a minimum in free energy for the ensemble of cracks as a whole, and is therefore more appropriate on a physical basis, the gross features are in fact not sensitively dependent on the use of G' or $\langle G\rangle$ as a variable. Also G' can only be defined uniquely when either N_T or b are held constant. The use of the simpler and more generally applicable parameter $\langle G\rangle$ is therefore preferred here. An added advantage for comparative purposes is that the predicted range of effective stress corrosion indices n' when N_T is plotted against $\langle G\rangle$, turns out to be very similar to those observed in the laboratory when the fracture system is dominated by a single macrocrack and the event rate N is the appropriate variable (Main 1991).

The implication of the similarity of the predicted and observed values for n' is that observed event rates N during tensile fracture in the laboratory are proportional to the total number of potentially active cracks N_T for a theoretical distribution of aligned, elliptical cracks. Such proportionality is also likely to hold for a system with a time constant for crack healing similar to the time interval for counting the event rate. For example Jin & Aki (1989) compared temporal variations in seismic b-values, and coda-Q^{-1} from the southern California instrumental record. The two temporal anomalies were positively correlated with a statistical significance level of 97 per cent, but most significantly from the point of view of the present discussion is that there was no phase shift due to the b-value sampling the 'most recent' new fractures, whereas the scattering attenuation would be expected to result from the cumulative effect of fracturing, on a smaller scale, over a longer time period. The time interval for calculating b-values is of the order of a few months, indicating that the mechanisms responsible for producing temporal variations in coda-Q^{-1}, thought to be variations in microcrack density due to dilatancy, microcrack growth or crack healing, occur over similar time periods.

There is also a possible causal explanation for the proportionality between N and N_T implied by the exponential increase in event rate during the early stages of compressional failure and the lack of time lag between b-values and coda-Q^{-1} in the field. In the early stages of failure cracks are thought to grow from smaller pre-existing flaws known as Griffith cracks. In this phase the number of new crack growth events capable of being recorded as acoustic emissions will depend on the applied stress, the minimum crack length for subcritical crack growth, the minimum threshold of complete detection of acoustic events, and most importantly on the total number of pre-existing cracks. Assuming the latter is the dominant effect, and neglecting any crack–crack interactions, i.e. in the early stages of failure, the simplest dependence is linear proportionality

$$N = dN_T/dt = \lambda N_T. \tag{5}$$

After integration it follows that

$$N_T = N_{T_0}\exp(\lambda t), \tag{6a}$$

or

$$N = N_0 \exp(\lambda t), \tag{6b}$$

where N_{T_0} is the total number of potentially active cracks at time $t = 0$. Such an exponential increase in event rate is exactly what is observed in the early stages of compressional failure of intact specimens of rock (e.g. Meredith *et al.* 1990). This observation is therefore strong indirect evidence of the proportionality between event rate and the total number of potentially active cracks in the early phase of rock failure in the laboratory.

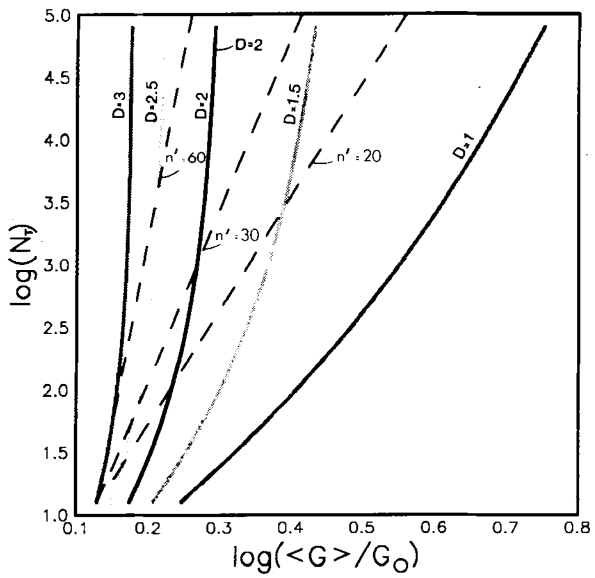

Figure 3. Dependence of the total number of potentially active cracks N_T on $\langle G \rangle$ for constant D, using equations (8) and (9). The slope of the straight lines on this diagram represent half of the effective stress corrosion index n', assuming the event rate is proportional to the total number of potentially active cracks. The straight lines correspond to applying the form of Charles' law for subcritical crack propagation velocities (equation 1) to seismic event rates (equation 2).

To illustrate this further Fig. 3 shows a graph of the predicted total number of cracks $\log(N_T)$ against $\log(\langle G \rangle / G_0)$, assuming a power-law length distribution of flat, elliptical tensile cracks of varying size and constant aspect ratio. Such a power-law distribution is the only form which is consistent with a scale-invariant or fractal crack distribution (Turcotte 1989). The theory on which this is based is presented separately in the Appendix. The seismic b-value (the slope of the log-linear frequency magnitude relation for earthquakes or acoustic emissions) is proportional to the negative exponent D of the crack length distribution via

$$D = 3b/C, \quad (7)$$

(Aki 1981) and hence D can be inferred from seismicity or AE frequency–magnitude statistics. C is a scaling parameter between magnitude and the common logarithm of the seismic moment, and is equal to 3/2 for intermediate-sized earthquakes (Kanamori & Anderson 1975). For most background seismicity, or when seismicity is averaged over a long time period, $b \approx 1$, or $D \approx 2$ in earthquake zones, but reaching a critical value in the short term, $b \approx 1/2$, or $D \approx 1$, both for foreshock sequences (von Seggern 1980) and for the final nucleation stage of dynamic failure in intact laboratory samples under both tensile and compressional loading (Meredith *et al.* 1990). If D can be regarded as a fractal dimension (Aki 1981; Mandelbrot 1982; Turcotte 1989), it must be restricted to the range $1 \leq D \leq 3$. This prediction is borne out both by the direct observation and by indirectly determing D from the seismic b-value (Main *et al.* 1990), and is the justification for the range of D modelled in Fig. 3.

G_0 is the minimum energy release rate for which quasi-static crack growth is possible. However, in most cases where the number of cracks are counted seismically G_0 is effectively higher than this physical minimum due to the incomplete reporting of small events against a background of seismic noise. Assuming the proportionality between N_T and N justified above, the straight lines drawn on Fig. 3 have slopes equal to $n'/2$, because G is proportional to the square of the stress intensity K (Lawn & Wilshaw 1975). For n' in the range 20–60, these theoretical lines imply that D (and by implication b) decreases as $\langle G \rangle$ increases, a result also consistent with Meredith & Atkinson's (1983) empirical observations.

Analogy with stress corrosion

Comparison of Fig. 2 with Fig. 3 implies that the behaviour of a population of fractally distributed cracks can be regarded as analogous to that of stress corrosion in a system dominated by a single macrocrack. That is the constitutive laws have a similar form, except that the stress intensity K is replaced by a mean energy release rate $\langle G \rangle$ as the effective variable. Alternatively stress corrosion can be seen as just one example, albeit a very important one, of a universal class of quasi-static, subcritical processes which produce a scale-invariant fracture geometry. (In this sense 'subcritical' implies prior to the dynamic failure of the whole sample, even if the smaller cracks are produced by dynamic rupture on a much smaller scale.) This universality may explain why the predictions of seismic b-value anomalies from empirical observation of tensile subcritical crack growth were so successful in predicting anomalies during the compressional failure of intact laboratory rock samples in Meredith *et al.* (1990).

Theory and experiment can be compared directly (e.g. Figs 2 and 3) as long as (a) the seismic event rate is proportional to the total number of active cracks in the population, and (b) the seismic b-value is proportional to the power-law exponent of the length distribution. The analogy between stress corrosion concentrated on a single dominant macrocrack and the more general behaviour of a fractal population of fractures can also be shown in an analytic way. From the Appendix, the mean energy release rate defined in equation (4b) is given by

$$\langle G \rangle = (\pi\sigma^2 c_{min}/E)[D/(D-1)][1 - x^{1-D}]/[1 - x^D], \quad (8)$$

where x is c_{max}/c_{min} and $x \geq 1$. When $D = 1$

$$\langle G \rangle = (\pi\sigma^2 c_{min}/E) \ln(x)/(1 - x^{-1}). \quad (9)$$

Assuming there is one crack of the maximum size c_{max} it follows that for large N_T (see Appendix)

$$N_T = \exp[\langle G \rangle / (\pi\sigma^2 c_{min}/E)]. \quad (10)$$

By associating a minimum energy release rate G_0 with the smallest crack size c_{min} equation (10) can be simplified to

$$N_T = \exp(\langle G \rangle / G_0). \quad (11)$$

Apart from the model ignoring any temperature dependence, this accords very well with the exponential relationship (equation 3) derived by Wiederhorn & Boltz (1970) from reaction rate theory. If the fractal range is a few orders of magnitude in length, then N_T or N is very sensitive to small changes in $\langle G \rangle$, as observed in laboratory experiments where G or K of a dominant macrocrack is the measured variable. This makes the form of (11) indistinguishable from a power law in practice (Meredith & Atkinson 1983).

QUANTITATIVE PREDICTIONS OF SEISMIC QUIESCENCE

The above discussion has shown that seismic event rates are very sensitively dependent either on the stress intensity K for stress corrosion cracking dominated by a single macrocrack, or on the mean energy release rate $\langle G \rangle$ for a fractal pupulation of cracks produced by any mechanism including stress corrosion. The degree of sensitivity is very similar, as evidenced by the similar magnitudes of the non-linear exponent n' for both cases. In this section we use this sensitivity to predict the magnitude of seismic quiescence which can be achieved by reductions in K or $\langle G \rangle$, using the form of equation (2) for K, or

$$N/N_0 = (\langle G \rangle / \langle G \rangle_0)^{n'/2}, \qquad (12)$$

for $\langle G \rangle$. If we take $n' \cong 30$ from the experimental results of Fig. 2, then a 45 per cent reduction in N can occur with only a 2 per cent reduction in K, and a 90 per cent reduction occurs for a 7 per cent reduction in K. Even when there is no dominant macrocrack, a similar reduction in N follows theoretically from equation (12) by considering equally small reductions in the mean energy release rate $\langle G \rangle$ for a fractal distribution of cracks, because of the similar values for n' calculated for the theoretical model.

Thus the intermediate-term seismic quiescences observed before earthquakes can easily be explained by a small strain softening effect where significant nucleation of the dominant macrocrack has not yet taken place. In this early phase of strain softening the mean energy release rate $\langle G \rangle$ is dominated by the array of cracks around this ultimate dynamic nucleation point. In this phase $\langle G \rangle$ is reducing because the stress σ is decreasing whilst the mean crack semilength $\langle c \rangle$ is either decreasing, or increasing at a rate slower than σ^2 is falling. If the theory presented above is correct, the reason such intermediate-term quiescence was not seen in the phase of strain softening in the compressional laboratory experiments of Meredith *et al.* (1990) is that the decrease in stress was accompanied by an increase in accelerated nucleation due to strain energy stored in a rather soft loading configuration. That is the reduction in stress intensity (or mean energy release rate) caused by decreasing stress was offset by an increase, in the macrocrack length (or mean crack length), which was forced artificially by the strain energy contained in the loading ram. In the Earth we would expect a much stiffer loading, and more recent laboratory experiments (P. R. Sammonds 1991, personal communication), which do indicate a small quiescence in the strain softening phase, were carried out under much stiffer loading conditions determined by servo-control.

QUIESCENCE ACCOMPANIED BY A CHANGE IN *b*-VALUE

So far the discussion has concentrated primarily on changes in event rate N rather than the complete 2-D phase space $\langle G \rangle = f(N, b)$. This phase space can be determined from the theory presented above if the event rate N is proportional to the total number of active cracks N_T, and the seismic b-value is proportional to the power-law exponent D. We have concentrated on event rates so far because it is often difficult to show a statistically significant change in b-value with the small number of seismic events detected above the background noise level by conventional seismic networks. However we have seen from applying equation (12) to Fig. 3 that a changing b-value is predicted as $\langle G \rangle$ increases. Moreover both Smith (1981, 1986) and Wyss *et al.* (1990) report that intermediate-term quiescence is often associated with an increasing b-value, whereas Main & Meredith (1989) report a short-term quiescence preceding the Western Nagano earthquake of September, 1984, which was accompanied by a short-term drop in b-value to its critical value of 0.5 ($D = 1$). Smith (1981, 1986) reports several examples of such short-term drops in b-value in the final stages of earthquake nucleation. The interpretation given by Main *et al.* (1989) for this behaviour is that the former is due to stress drop predominantly by smaller earthquakes in the fracture system around the dominant fault, and the latter by accelerating quasi-static stress drop on a growing dominant crack in the final stages of the earthquake cycle. Thus quiescence can occur either by long-range interactions in the early phase of deformation (intermediate-term quiescence), or by short-range stress concentrations nearer the dynamic failure time (short-term quiescence). Although the model of Main (1991) is not strictly applicable in the case of strong short-range interactions, we show below that its gross features are consistent with both types of quiescence described above.

When the theory described in the previous section is extended fully to two dimensions, $\langle G \rangle = f(N, b)$, it can be shown (Main 1991) that N, is positively correlated, and b is negatively correlated, to $\langle G \rangle$. This combined result had already been obtained empirically for one dominant macrocrack using K directly as the appropriate variable (Meredith & Atkinson 1983). Thus intermediate-term quiescence accompanied by an increase in b-value is consistent with a *reduction* in K or $\langle G \rangle$, whereas short-term quiescence accompanied by decreasing b is consistent with an *increasing* K or $\langle G \rangle$ in the final stages of dynamic rupture nucleation. The reduction in K or $\langle G \rangle$ has a stabilizing effect because the system is moved further from the failure criterion $K = K_c$ or $\langle G \rangle = \langle G \rangle_c$. In contrast short-term quiescence occurs when K or $\langle G \rangle$ are increasing towards this failure criterion, and represents an unstable state. These predictions are consistent with the relatively long duration of intermediate-term quiescence (months to years) and the relatively short duration of short-term quiescence (hours to days). This is illustrated schematically in Fig. 4, which shows quiescence associated with increasing, constant and decreasing b-value respectively. The first two cases produce reductions in K or $\langle G \rangle$, and represent stable, intermediate-term quiescence, whereas unstable short-term quiescence occurs only when b is decreasing. An interesting feature of Fig. 4 is that a similar order of magnitude change in $\langle G \rangle$, or K, has a much greater effect on the combined seismicity statistics for the case of concomitantly decreasing N and b. This highlights the inherent instability and unpredictability of the final nucleation phase of

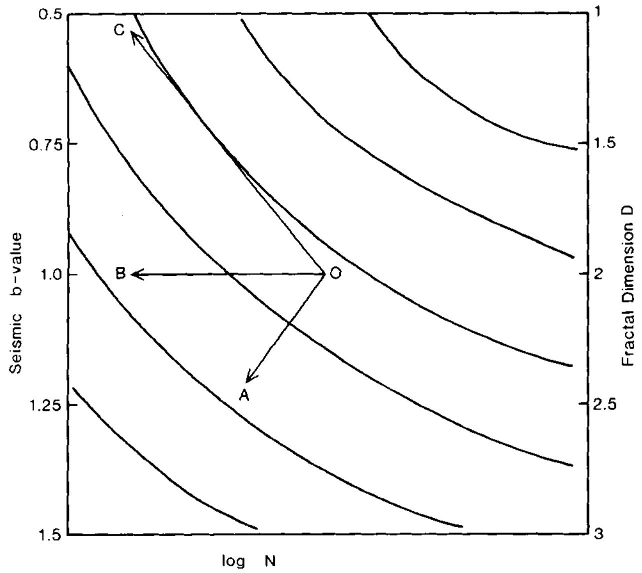

Figure 4. Schematic illustration of quiescence in a 2-D phase space (N, b) where $\langle G \rangle$ is positively correlated to N and negatively correlated to b. $\langle G \rangle$ increases monotonically towards the top right of the phase space, and quiescence can occur when the b-value is increasing (OA), remaining constant (OB), or decreasing (OC). The diagram assumes the event rate N is positively correlated to the total number of potentially active cracks N_T.

earthquakes. The predicted effects of the three different paths OA, OB and OC on the observed seismicity statistics is shown schematically in Fig. 5. In the cases where b is varying, it is important that the effect of the arbitrary range of complete reporting represented by the lower cut-off magnitude m_c and the dashed lines of Fig. 5 are considered in any interpretation of the results; i.e. if b varies the two lines are not parallel and will intersect somewhere, meaning that the measured value of $N(m_c)$ will depend on where the arbitrary cut-off m_c occurs in relation to this apparent fulcrum.

Figure 5 implies that intermediate-term quiescence of the type OA is due to the stress relaxation being dominated by many small microseismic events below the threshold of complete reporting, possibly including brittle microcracking. Type OB is consistent with a general reduction in activity at all length scales, and a reduction in the size the largest potentially active fracture. However short-term quiescence (OC) is consistent with stress release predominantly on a few larger fractures late in the seismic cycle, thereby inhibiting stress relaxation on a smaller scale. The latter argument is similar to that used in a previous qualitative model based on time-dependent stress corrosion introduced by Ohnaka (1985). However the model presented in the present paper is quantitative, more generally applicable, and is also supported by a large body of laboratory and field data. Moreover the general theory presented above predicts the occurrence of short-term as well as intermediate-term quiescence.

CONCLUSIONS

Intermediate-term seismic quiescences of a magnitude similar to those observed before major earthquakes can be predicted from both laboratory results and a theoretical model based on tensile failure. In particular the theoretical model predicts intermediate-term quiescence when a small strain softening effect occurs but *before* significant nucleation of a dominant macrocrack has taken place. For systems dominated by a single macrocrack only a few per cent reduction (2–7 per cent) in stress intensity (concomitant with a reduction in stress) is required to produce a large reduction (45–90 per cent respectively) in the event rate for an effective stress corrosion index n' of 30 or so. Such quiescence is also quantitatively consistent with a reduction in the mean strain energy release rate $\langle G \rangle$ for a fractal crack population not dominated by a single macrocrack. This general result applies to *any* underlying mechanism which produces a fractal array of subsidiary fractures, though stress corrosion is known to be an important mechanism in the Earth, and provides a useful analogy for testing the predictions of the general theory.

Short-term quiescences can occur when an increasing $\langle G \rangle$ combines with a decreasing b-value, as stress relaxation on a few large fractures around a nucleating macrocrack dominate the strain softening. In contrast intermediate-term quiescences are often accompanied by increasing b-values in field results, consistent with a drop in both stress and $\langle G \rangle$, due to stress relaxation on many small faults and cracks below the lower magnitude cut-off. Such intermediate-term quiescence has not yet been seen unequivocally in laboratory experiments, possibly because of the difficulty in obtaining low enough strain rates.

Although the Kaiser effect has been suggested as a

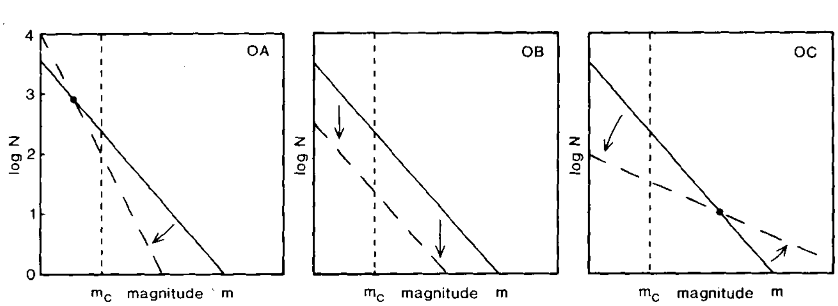

Figure 5. Changes in frequency–magnitude statistics predicted by Fig. 4. OA—quiescence accompanied by an increasing b-value (e.g. Smith 1986). OB—quiescence with constant b-value (e.g. Wyss & Habermann 1988). OC—quiescence accompanied by a decreasing b-value, applicable to short-term quiescence (e.g. Scholz 1988). OA and OB are consistent with stable, intermediate-term quiescence, and OC with unstable, short-term quiescence.

possible mechanism of intermediate-term and short-term quiescences, it cannot by definition be applied to the case of strain softening, except on a local scale where both stress and strain relaxation can be demonstrated. The discussion in this paper suggests that constitutive laws based on experimental observation and the theoretical modelling of seismicity accompanying tensile subcritical crack growth provides a useful, quantitative alternative to the interpretation of compressional laboratory and field data, as long as an appropriate normalization is applied for variations in fracture toughness. However this should be tempered with an acknowledgment of the difficulty in determing statistically significant variations in frequency–magnitude statistics in the field.

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REFERENCES

Aki, K., 1981. A probabilistic synthesis of precursory phenomena, in *Earthquake Prediction—an International Review*, Maurice Ewing Ser. 4, pp. 566–574, eds Simpson, D. W. & Richards, P. G., American Geophysical Union, Washington, DC.

Atkinson, B. K. & Meredith, P. G., 1987. The theory of subcritical crack growth with application to minerals and rocks, in *Fracture Mechanics of Rock*, pp. 111–166, ed. Atkinson, B. K., Academic Press, London.

Charles, R. J., 1958. Static fatigue of glass, *J. appl. Phys.*, **29,** 1549–1560.

Cox, S. J. D. & Scholz, C. H., 1988. Rupture initiation in shear fracture of rocks: an experimental study, *J. geophys. Res.*, **93,** 3307–3320.

Das, S. & Scholz, C. H., 1981. Theory of time-dependent rupture in the Earth, *J. geophys. Res.*, **86,** 6039–6051.

Habermann, R. E., 1988. Precursory seismic quiescence: past, present and future, *Pageoph.*, **126,** 279–318.

Jin, A. & Aki, K., 1986. Temporal change in coda Q before the Tangshan earthquake of 1976 and the Haicheng earthquake of 1975, *J. geophys. Res.*, **91,** 665–673.

Jin, A. & Aki, K., 1989. Spatial and temporal correlation between coda-Q^{-1} and seismicity and its physical mechanism, *J. geophys. Res.*, **94,** 14 041–14 059.

Kaiser, J., 1953. Erkenntnise und folgerungen aus der Messung von gerauschen bei Zugbeanspruchung von Mettalischen Werkstoffen, *Archiv. fur das Eisenhuttenwessen*, **24,** 43–45.

Kanamori, H. & Anderson, D. L., 1975. Theoretical bases of some empirical relations in seismology, *Bull. seism. Soc. Am.*, **65,** 1073–1095.

Kisslinger, C., 1988. An experiment in earthquake prediction and the 7 May, 1986 Andreanof islands earthquake, *Bull. seism. Soc. Am.*, **78,** 218–229.

Langbein, J., 1991. Earthquake explanations, *Nature*, **349,** 287.

Lawn, B. R. & Wilshaw, T. R., 1975. *Fracture of Brittle Solids*, Cambridge University Press, Cambridge, UK.

Main, I. G., 1988. Prediction of failure times in the Earth for a time-varying stress, *Geophys. J.*, **92,** 455–464.

Main, I. G., 1991. A modified Griffith criterion for the evolution of damage with a fractal distribution of crack lengths: application to seismic event rates and *b*-values, *Geophys. J. Int.*, **107,** 353–362.

Main, I. G. & Meredith, P. G., 1989. Classification of earthquake precursors from a fracture mechanics model, *Tectonophysics*, **167,** 273–283.

Main, I. G., Meredith, P. G. & Jones, C., 1989. A reinterpretation of the seismic *b*-value anomaly from fracture mechanics, *Geophys. J.*, **96,** 131–138.

Main, I. G., Meredith, P. G., Sammonds, P. R. & Jones, C., 1990. Influence of fractal flaw distributions on rock deformation in the brittle field, in *Deformation Mechanisms, Rheology and Tectonics*, Geol. Soc. Lond. Special Publication 54, pp. 81–96, eds Knipe, R. J. & Rutter, E. H.

Mandelbrot, B. B., 1982. *The Fractal Geometry of Nature*, Freeman, San Francisco.

Matthews, M. V. & Reasenberg, P., 1987. Comment on Habermann's method for detecting seismicity rate changes, *J. geophys. Res.*, **92,** 9443–9445.

Meredith, P. G., 1989. Comparative fracture toughness testing of rocks, in *Fracture Toughness and Fracture Energy: Test Methods for Concrete and Rock*, pp. 265–277, eds Mihashi, H., Takahashi, H. & Wittman, F. H., A. A. Balkema, Rotterdam.

Meredith, P. G. & Atkinson, B. K., 1983. Stress corrosion and acoustic emission during tensile crack propagation in Whin Sill dolerite and other basic rocks, *Geophys. J. R. astr. Soc.*, **75,** 1–21.

Meredith, P. G., Main, I. G. & Jones, C., 1990. Temporal variations in seismicity during quasi-static and dynamic rock failure, *Tectonophysics*, **175,** 249–268.

Mogi, K., 1987. The relation between the duration of long-term precursors and the magnitude of the subsequent earthquake, *Tectonophysics*, **142,** 163–172.

Ohnaka, M., 1985. A sequence of seismic activity in the Kanto area precursory to the 1923 Kanto earthquake, *Pageoph.*, **122,** 848–862.

Ohtake, M., Matumoto, T. & Latham, G. V., 1977. Seismicity gap near Oaxaca, southern Mexico as a probable precursor to a large earthquake, *Pageoph.*, **115,** 375–385.

Raleigh, C. B., Bennet, G., Craig, H., Hanks, T., Molnar, P., Nur, A., Scholz, C. H., Turner, R. & Wu, F., 1977. Prediction of the Haicheng earthquake, *EOS, Trans. Am. geophys. Un.*, **58,** 236–272.

Rikitake, T., 1987. Earthquake precursors in Japan: precursor time and detectability, *Tectonophysics*, **136,** 265–282.

Rundle, J. B. & Klein, W., 1989. Nonclassical nucleation and growth of cohesive tensile cracks, *Phys. Rev. Lett.*, **63,** 171–174.

Scholz, C. H., 1988. Mechanisms of seismic quiescences, *Pageoph.*, **126,** 701–718.

Smith, W. D., 1981. The *b*-value as an earthquake precursor, *Nature*, **289,** 136–139.

Smith, W. D., 1986. Evidence for precursory changes in the frequency–magnitude *b*-value, *Geophys. J. R. astr. Soc.*, **86,** 815–838.

Stuart, W. D., 1979. Strain softening prior to two-dimensional strike slip earthquakes, *J. geophys. Res.*, **84,** 1063–1070.

Thatcher, W., 1990. Order and diversity in the modes of circum-Pacific earthquake recurrence, *J. geophys. Res.*, **95,** 2609–2623.

Turcotte, D. L., 1989. Fractals in geology & geophysics, *Pageoph.*, **131,** 171–196.

von Seggern, D., 1980. A random stress model for seismicity statistics and earthquake prediction, *Geophys. Res. Lett.*, **7,** 637–640.

Wiederhorn, S. M. & Bolz, L. H., 1970. Stress corrosion and static fatigue of glass, *J. Am. Ceram. Soc.*, **53,** 543–548.

Wyss, M. & Burford, R. O., 1987. A predicted earthquake on the San Andreas fault, California, *Nature*, **329,** 323–325.

Wyss, M. & Habermann, R. E., 1988. Precursory seismic quiescence, *Pageoph.*, **126,** 319–332.

Wyss, M., Bodin, P. & Habermann, R. E., 1990a. Seismic quiescence at Parkfield: an impending indication of an imminent earthquake, *Nature*, **345**, 426–428.

Wyss, M., Slater, L. & Burford, R. O., 1990b. Decrease in deformation rate as a possible precursor to the next Parkfield earthquake, *Nature*, **345**, 428–431.

Yamashita, T. & Knopoff, L., 1989. A model of foreshock occurrence, *Geophys. J. Int.*, **96**, 389–399.

APPENDIX

If the distribution of crack semilengths c is a power law, consistent with a scale-invariant or fractal distribution between the limits $(c_{\min}, c_{\max})$, then the probability density distribution p has the form

$$p(c) = c^{-v} \Big/ \int_{c_{\min}}^{c_{\max}} c^{-v}\, dc, \tag{A1}$$

where the denominator normalises the cumulative probability in the range $(c_{\min}, c_{\max})$ to unity. The exponent v of the probability density distribution is related to the exponent D of the discrete frequency distribution by

$$v = D + 1. \tag{A2}$$

The expectation value $\langle c \rangle$ is defined by

$$\langle c \rangle = \int_{c_{\min}}^{c_{\max}} c p(c)\, dc. \tag{A3}$$

For $D > 1$, using (A1) and (A2)

$$\langle c \rangle = c_{\min}[D/(D-1)] \times [1 - (c_{\max}/c_{\min})^{1-D}]/[1 - (c_{\max}/c_{\min})^{-D}]. \tag{A4}$$

If we consider the cumulative frequency histogram of the length distribution as being characterized by the total number of cracks N_T greater or equal to $c_{\min}$, with only one crack in the range $c_{\max} - \delta c/2 \le c < c_{\max} + \delta c/2$, then we can show from (A1) and (A2) for large N_T that

$$c_{\max}/c_{\min} = (DN_T\delta c/c_{\min})^{[1/(D+1)]}. \tag{A5}$$

Since δc is arbitrary we define $c_{\max}$ for $\delta c = c_{\min}$. Thus $c_{\max}$ is defined statistically using all of the cracks in the distribution rather than a solitary largest event which may be subject to large statistical fluctuations in magnitude when its event rate is recorded seismically. Using this definition we can show from equation (4b) in the main text and equation (A5) that

$$\langle G \rangle = (\pi\sigma^2 c_{\min}/E)[D/(D-1)](1 - x^{1-D})/(1 - x^{-D}), \tag{A6}$$

where x is $c_{\max}/c_{\min}$ and $x \ge 1$. When $D = 1$ the solution of (A3) implies

$$\langle G \rangle = (\pi\sigma^2 c_{\min}/E)\ln(x)/(1 - x^{-1}). \tag{A7}$$

From (A5) $x = (DN_T)^{[1/(D+1)]}$, so that for large N_T it follows in this case that

$$N_T = \exp[\langle G \rangle/(\pi\sigma^2 c_{\min}/E)]. \tag{A8}$$

By associating a minimum energy release rate G_0 with the smallest crack (A8) can be simplified to

$$N_T = \exp(\langle G \rangle/G_0). \tag{A9}$$