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Stress Increment Solution Charts for Soil Consolidation Analysis

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Abstract - Current practice of estimating average stress increment required for consolidation settlement computations employs mid-depth stress approach or multiple application of sublayer technique, which are tedious and difficult methods to implement for hand calculations. This paper presents simplified charts to estimate such a stress. The influence factor needed to estimate the average stress increment is calculated based on the integration of Boussinesq's equations for common foundations and various soil configurations. The results are presented in a series of normalized nondimensional charts, which are independent of structural loads and soil characteristics. The derived charts are useful especially when the compressible layer is not directly located underneath the loaded foundation and they avoid the necessity of dividing the soil into a series of sublayers to obtain a realistic value of average stress increment. They can be readily implemented into design allowing accurate prediction of consolidation settlement or can serve as a powerful tool for optimizing and proportioning the dimensions of footings under certain allowable settlement where otherwise an iterative tedious solution is required. Illustrative examples are presented to demonstrate the applicability and efficiency of the suggested charts for consolidation settlement computations.

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I. INTRODUCTION

Primary consolidation is the time-dependent settlement of soil resulting from squeezing out of water from the voids, due to the dissipation of the excess pore water pressure, following the application of the load increment. The resulting settlement can be particularly large when the drainage is not impeded, but its magnitude is of engineering significance only when reference is made to a tolerable settlement for a given type of structure (Balasubramaniam and Brenner 1981). The magnitude of consolidation settlement depends largely on load and soil characteristics. Thus, a reliable settlement analysis requires accurate determination of the induced stress in the soil layer in addition to reliable consolidation parameters.

The variation of the stress produced below the foundation is non-linear in nature as schematically shown in Figure 1. The intensity of the stress decreases from a maximum value just underneath the loaded area to about zero at a very large distance from the foundation. The calculation of the stress increment in a compressible layer is commonly dealt with by the mid-depth stress approach as suggested in the literature (Terzaghi 1943; Dunn et al. 1980; Holtz and Kovacs 1981; Cernica 1994; Bowles 1995; Budhu 2000; Craig 2004; Coduto et al. 2010; Das 2010). Usually, the average stress increment of the entire soil stratum is assumed to be the one calculated at the middle of layer ignoring the non-linear variation of the stress, which may produce a substantial error.

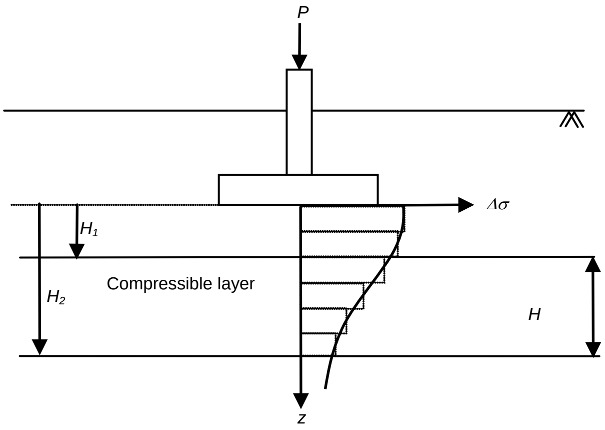

Fig. 1 : Stress increment distribution below the center of a uniformly loaded footing

Calculations of average stress increment in soil mass are improved by subdividing the soil stratum into a number of horizontal sublayers as illustrated in Figure 1. The technique involves replacing the smoothly varying stress distribution within a soil by a staircase-like distribution. The technique assumes a constant stress over each sublayer and the value at the mid-depth provides an approximation of the stress increment for every sublayer. The stress at the mid-depth of each sublayer is determined and the settlement within every sublayer is separately calculated, and then summed to obtain the total settlement. Although this multiple application of the sublayer technique is recommended in the literature, it is not widely used since it is impractical for manual computations, and the calculations are time-intensive and tedious.

The error resulting between the application of mid-depth stress approach for a soil stratum and multiple application of sublayer technique, which might be misleading and unacceptable, depends largely on the size and shape of foundation, thickness of the compressible layer and its location relative to the applied load, and the number of sublayers as

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demonstrated by McPhail et al. (2000) and Ismeik (2012). Obviously, as the thickness of sublayer decreases, the precision of the computed settlement becomes greater. However, using a large number of sublayers is not propitious to hand calculations. Thus, practically the soil stratum is usually divided into few sublayers with the intention of providing a reasonable answer with a moderate amount of effort.

II. STUDY MOTIVE

Current practice of estimating average stress increment required for consolidation settlement computations usually employs conventional methods such as mid-depth stress approach or multiple application of sublayer technique, which are both tedious and difficult to implement for hand calculations. In addition, they do not consider the case where the compressible soil layer is not directly located below the loaded foundation.

This paper enables the average stress increment beneath the center of a uniformly loaded foundation to be obtained as opposed to the stress increment at a specific depth. A series of normalized non-dimensional charts are developed to estimate the influence factor of a finite soil layer based on size and shape of foundation, thickness of compressible layer, and its location relative to the applied load. Numerical examples are included to illustrate the effectiveness and applicability of the derived charts for settlement computations. A comparison is made between the results obtained by these charts and conventional methods.

III. DERIVATION

As proposed by Terzaghi (1943), the magnitude of consolidation settlement of a compressible layer is determined as:

$$dS = m_v(z)\Delta\sigma(z)dz \tag{1}$$

in which dS is the differential settlement due to compression of soil thickness dz, $m_v(z)$ is the soil coefficient of volume compressibility, and $\Delta\sigma(z)$ is the vertical stress increment produced below the loaded foundation at a particular depth z.

If the coefficient of volume compressibility $m_v(z)$ is taken as a constant, at least at certain depths, the total consolidation settlement S, over the entire thickness of soil stratum H, is the integration of Equation (1) as:

$$S = m_v \int_0^H \Delta\sigma(z)dz \tag{2}$$

Based on the theory of elasticity, Boussinesq (1885) provided the equations needed to calculate the stress increment $\Delta\sigma(z)$ in a soil mass. The equations consider a point load on the surface of a semi-infinite, homogeneous, isotropic, weightless and elastic half-space. The integration of vertical stress at a depth below a uniformly loaded area was originally described by Newmark (1935). Then the solutions were later improved by Steinbrenner (1936) and graphically represented and summarized by Fadum (1948), and Poulus and Davis (1974). Despite of all the unrealistic assumptions used to develop such solutions, they are still traditionally being used in the literature to obtain the stress increment $\Delta\sigma(z)$ under foundation loads.

Using Boussinesq (1885) solutions, the calculation of the stress increment beneath the center of a uniformly loaded foundation is computed as:

$$\Delta\sigma(z) = qI(z) \tag{3}$$

where q is the surface contact stress at the foundation level. I is a non-dimensional influence factor defined as:

$$I = \frac{2}{\pi}\left[\frac{mn}{\sqrt{1+m^2+n^2}}\,\frac{1+m^2+2n^2}{(1+n^2)(m^2+n^2)} + \sin^{-1}\frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right] \tag{4}$$

where m and n are dimensionless shape and depth factors, respectively, defined as a function of the rectangular foundation dimensions width B, and length L, as:

$$m = \frac{L}{B} \text{ and } n = \frac{z}{0.5B} \tag{5}$$

If the loaded foundation is circular, of diameter D, the influence factor I is defined as:

$$I = \left[1 - \frac{1}{\left[1 + \left[\frac{D}{2z}\right]^2\right]^{3/2}}\right] \tag{6}$$

Substituting the value of stress increment $\Delta\sigma(z)$, as defined by Equation (3), into Equation (2), the settlement of the compressible soil stratum is thus computed as:

$$S = m_v q I_{ave} H \tag{7}$$

where I_{ave} is the average influence factor of soil stratum defined as:

$$I_{ave} = \frac{1}{H}\int_0^H I(z)dz \tag{8}$$

The integration of Equation (8) is commonly dealt with numerically since the influence factor I has a complex non-linear variation, which is a function of shape and size of the foundation, and depth of soil layer as given by Equations (4) and (6). Accuracy is improved when the integration is calculated over an infinite number of sublayers each of an infinitesimal uniform thickness *dz*.

IV. Results and Application

Hand calculations of the average influence factor I_{ave} of Equation (8), over a series of several sublayers, is impractical and tedious even for a single soil layer. Alternatively, a computer code is developed to evaluate the integral numerically and the results are presented graphically. The solution charts, which are independent of structural loads and soil properties, consider a relative configuration of the compressible layer H_2/H_1 ranging from 1 to 10, and common foundation types such as square ($L/B = 1$), rectangular ($L/B = 2$ and 3), strip ($L/B > 10$), and circular ones as presented in Figures 2, 3, 4, 5, and 6, respectively.

The presented charts are the exact solutions of average influence factor and they can be used confidently in geotechnical design. They enable the average stress increment, beneath the center of a uniformly loaded foundation, to be obtained directly as opposed to the stress value at a specific depth, as provided by Boussinesq's (1885) solutions. The charts, which agree well with the results of Ismeik (2012), have two powerful and practical advantages for preliminary foundation design when hand calculations are carried out, and especially if the compressible layer is not directly located below the loaded foundation. Firstly, the estimation of the average influence factor is far easier when obtained from the charts and thus avoids the use of mid-depth stress approach, which may produce a large error. Secondly, the charts can be used efficiently to optimize the required dimensions of a footing constrained by a tolerable settlement, as an alternative to classical mid-depth stress approach where an iterative method is required to find minimum dimensions.

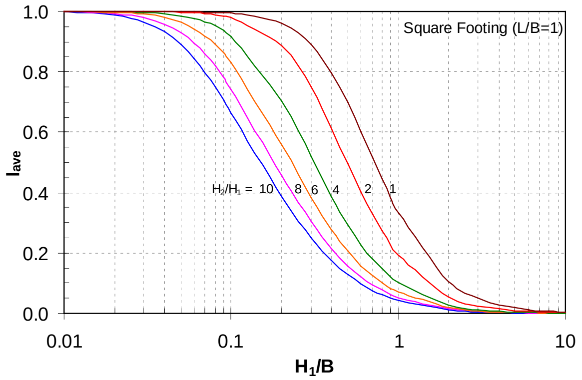

Fig. 2 : Influence factor under the center of a uniformly loaded square footing (L/B = 1)

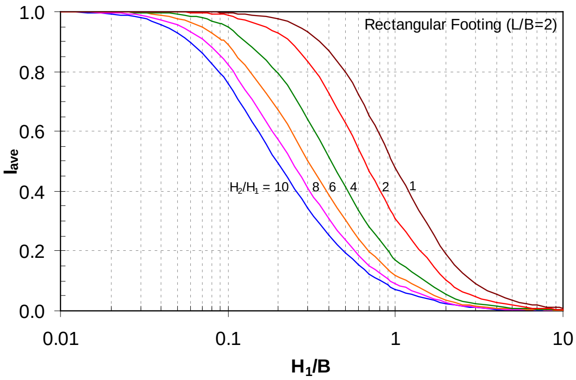

Fig. 3 : Influence factor under the center of a uniformly loaded rectangular footing (L/B = 2)

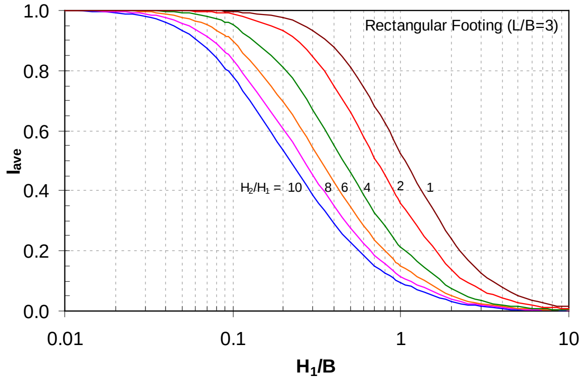

Fig. 4 : Influence factor under the center of a uniformly loaded rectangular footing (L/B = 3)

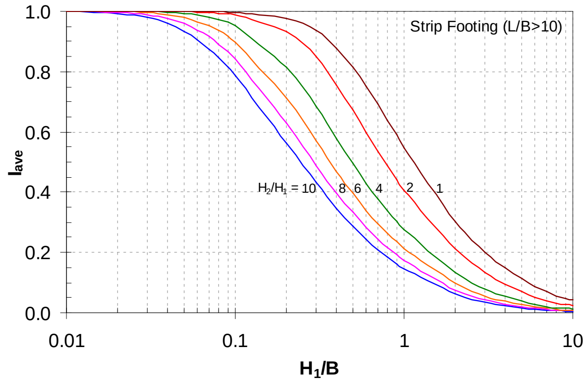

Fig. 5 : Influence factor under the center of a uniformly loaded strip footing (L/B > 10)

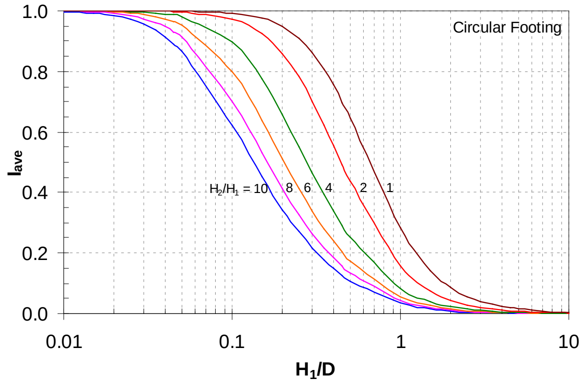

Fig. 6 : Influence factor under the center of a uniformly loaded circular footing

V. EXAMPLES

The use of the charts in settlement computations is illustrated by considering the 2 m width square footing as shown in Figure 7. The soil profile is composed of a 5 m clay layer overtopped by a 3 m sand layer and underlain by an impermeable hard base. Soil characteristics are $\gamma_s = 17$ kN/m^3, $\gamma_c = 19$ kN/m^3, $m_v = 0.0002$ m^2/kN, and $H_1 = 1$ m, $H_2 = 6$ m, and $H = 5$ m. The structural loading values are $P = 600$ kN and $q = 150$ kN/m^2. Exact values of average influence factor and settlement are 0.2059 and 30.885 mm, respectively.

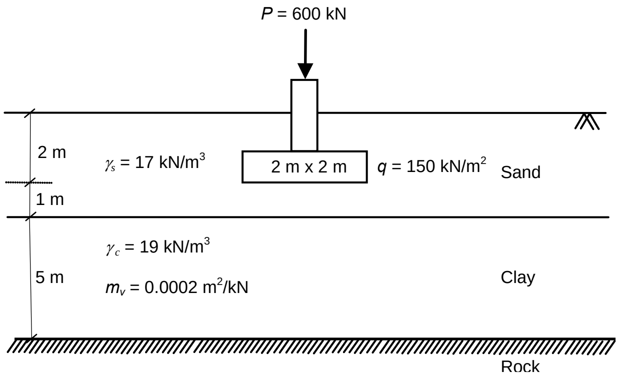

Fig. 7 : Loading and soil profile data of the examples

With $H_2/H_1 = 6$ and $H_1/B = 0.5$, the average influence factor I_{ave} is picked up from Figure 2 as about 0.21. Thus, the consolidation settlement is computed by Equation (7) as:

$$S = 0.0002 \times 150 \times 0.21 \times 5000 = 31.5mm \tag{9}$$

The error produced by the use of the proposed charts is about 1.99% of actual settlement, which is quite acceptable.

Had the mid-depth stress approach been used to calculate the influence factor I, for $m = 1$ and $n = 3.5$, the computations for settlement predication using Equations (4) and (7) would be:

$$I = \frac{2}{\pi}\left[\frac{1\times 3.5}{\sqrt{1+1^2+3.5^2}}\frac{1+1^2+2\times 3.5^2}{(1+3.5^2)(1^2+3.5^2)} + \sin^{-1}\frac{1}{\sqrt{1^2+3.5^2}\sqrt{1+3.5^2}}\right] = 0.1371 \tag{10}$$

$$S = 0.0002 \times 150 \times 0.1371 \times 5000 = 20.565mm \tag{11}$$

Such a settlement value yields a significant error of about 33.41%, which is definitely unsatisfactory in geotechnical design. Thus, the direct use of mid-depth stress approach may provide inaccurate results and can be misleading when compared with actual settlement values. As seen, the provided charts simplified the computations and can be used confidently to predict the average stress increment with acceptable accuracy.

Another powerful application of the proposed charts would be to determine the minimum dimensions of a footing required to satisfy an allowable settlement. If design code permits a tolerable settlement of 25.4 mm (1 inch) for the above footing, the average influence factor I_{ave} can be obtained directly from Figure 2 for several trials of width B. Then the corresponding settlement is calculated from Equation (7) as shown below.

B (m)	H_1/B	q (kN/m^2)	I_{ave}	S (mm)
2.4	0.41	104.16	0.26	27.08
2.6	0.38	88.75	0.29	25.73
2.8	0.35	76.53	0.32	24.48

As seen, a foundation width B of about 2.6 m would satisfy the settlement requirement. Exact value of width is 2.642 m obtained by an iterative tedious solution of Equations (7) and (8) simultaneously. The resulting error is 1.59%, which is reasonably acceptable. Such an application of the charts saves effort and time when reliable fast values of dimensions are required by manual hand calculations.

VI. SUMMARY AND CONCLUSIONS

Solution charts to predict the average vertical stress increment needed for consolidation settlement analysis are presented based on the numerical integration of Boussinesq's solutions. A software code is developed to provide relationships between the influence factor and shape and size of foundation, thickness of compressible layer, and its depth relative to the location of applied load.

The suggested charts provide a refined estimate of the stress increment, which could only be obtained with a large number of sublayers in the routinely used multiple application of the sublayer technique. In addition, if the soil is considered as one layer system, the mid-depth stress approach may provide inaccurate results.

The presented charts can be used as an alternative to current conventional methods. They represent an efficient and powerful solution to calculate the average stress increment especially when the compressible layer is not directly located below the loaded foundation, or can serve as a useful tool for optimizing and proportioning the dimensions of footings under an allowable settlement.

The most important advantages of these charts, when compared to conventional solutions, are their speed, ease of implementation, and versatility for routine hand settlement calculations required for geotechnical design of shallow foundations.

Notation

Symbol	Unit	Definition
B	m	Width of foundation
D	m	Diameter of foundation
H	m	Thickness of compressible stratum
H_1	m	Depth to upper boundary of compressible layer
H_2	m	Depth to lower boundary of compressible layer
I	-	Influence factor
I_{ave}	-	Average influence factor
L	m	Length of foundation
m	-	Shape factor
m_v	m^2/kN	Coefficient of volume compressibility
n	-	Depth factor
P	kN	Vertical load at the center of foundation
q	kN/m^2	Surface contact stress
S	m	Consolidation settlement
z	m	Depth below the loaded area
$\Delta\sigma$	kN/ m^2	Vertical stress increment
γ	kN/m^3	Unit weight of soil
π	-	PI constant 3.141592654

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