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Memorandum M-458

STRESS INTENSITY FACTORS OF HOLE EDGE CRACKS.  
COMPARISON BETWEEN ONE CRACK AND TWO SYMMETRIC  
CRACKS.

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INTRODUCTION

Numerical results for the stress intensity factor of a crack at the edge of a hole in an infinite sheet were published by Bowie [1], Newman [2] and Tweed and Rooke [3]. Bowie solved this problem for both a single crack and two symmetric cracks, see inset of Figure 1. Newman treated the results for two symmetric cracks and Tweed and Rooke for a single crack. The results of Bowie will be compared to those of the other two sources. The prime reason for doing so was a comparison between K-factors for a single crack and two symmetric cracks. The ratio of the two K-factors:

$$F_{1/2} = \left( \frac{K_1 \text{ crack}}{K_2 \text{ cracks}} \right)_{\text{same } c/R} \quad (1)$$

is referred to as the Shah correction factor because Shah [4] proposed an approximate relation for this factor. The relation is:

$$F_{1/2} = \sqrt{\frac{2+c/R}{2+2 c/R}} \quad (2)$$

This relation is asymptotically correct if  $c/R$  becomes very large. In that case the "effective" crack length is:

$$\text{For 2 cracks: } a = R + c \rightarrow K_2 \text{ cracks} \sim S \sqrt{\pi(R+c)} \quad (3a)$$

$$\text{For 1 crack: } 2a = 2R + c \rightarrow K_1 \text{ crack} \sim S \sqrt{\pi(2R+c)/2} \quad (3b)$$

From these equations the ratio  $F_{1/2}$  in (2) is directly obtained. Shah proposed that this equation should be a reasonable approximation for small cracks as well, because the Bowie data do not deviate very much from the equation. This will be further analysed in the present note.

ANALYSIS OF DATA FROM THE LITERATURE

Data from Bowie, Newman and Tweed and Rooke are presented numerically in Table 1. The data are given as the geometry correction factor  $F$  defined by:

$$K = F S \sqrt{\pi C} \quad (4)$$

where  $c$  is the crack length measured from the edge of the hole. The Bowie data do not occur in his paper [1]. However, values derived from his publication were cited in the literature. A comparison between the results for two cracks shows that the Bowie data are somewhat higher than the Newman data in most cases. The differences do not show a regular pattern. For a single crack the Bowie data are larger than the Tweed/Rooke data for small cracks, and smaller for large cracks. Also in this case a regular pattern is not clear. A more interesting comparison is made by considering the ratio  $F_{1/2}$  derived from the Bowie data and the same ratio derived from the Newman data and the Tweed/Rooke data (see Table 1). The results have been plotted in Figure 1, which reveals an obvious difference between the two sets of data. The Bowie data show an irregular decreasing ratio  $F_{1/2}$  for increasing crack length. The picture emerging from the Newman - Tweed/Rooke data is a systematic and continuous decreasing function. The latter data are more appropriate for a comparison to the Shah correction factor (2). This factor has also been plotted in Figure 1. The difference between this factor and the Newman - Tweed/Rooke data has been plotted in the lower part of Figure 1. The maximum difference is about 3 percent.

The Newman - Tweed/Rooke data can be represented by the following equation obtained by data fitting:

$$F_{1/2} = \sqrt{\frac{2+c/R}{2+2c/R}} \left[ 1 + \frac{0.2c/R}{(1 + c/R)^3} \right] \quad (4)$$

The equation still satisfies the limit condition:

$$\lim_{c/R \rightarrow \infty} F_{1/2} = \frac{1}{\sqrt{2}}$$

The Newman data for two symmetric cracks are accurately represented by an equation also obtained by data fitting

$$F_2 = 1 + \frac{1}{2x^2 + 1.93x + 0.539} + \frac{1}{2(x+1)} \quad \text{with } x = c/R \quad (5)$$

The differences between this equation and the Newman data (① in table 1) are smaller than 0.2 %. By combining (4) and (5) the geometry correction factor for a single crack  $F_1$  is obtained as:

$$F_1 = F_{1/2} * F_2 \quad (6)$$

The differences between  $F_1$  values thus obtained and the Tweed/Rooke data (② in table 1) are also very small (maximum 0.4 %).

## DISCUSSION

It should be expected that the Newman data and the Tweed/Rooke data are much more accurate than the Bowie data. As a consequence a significantly more systematic behaviour is obtained as illustrated by Figure 1. The difference between the  $F_{1/2}$  ratio and the Shah correction factor is small but systematic. The difference is mainly found for small cracks ( $c/R < 1$ ). However, in this range the larger part of the crack growth life is spent. Anyhow, it is useful to know how large the difference is, and equation (4) can easily be applied, if it is undesirable to ignore the difference.

With equations (4), (5) and (6) very accurate K-values will be obtained. It should be recognized, that these values apply to through cracks at holes in an infinite sheet. Shah assumed that equation (2) can also be applied to part through cracks if the part through crack is replaced by a through crack (crack length  $c_{\text{eff}}$ ) with the same crack surface area. For a quarter elliptical crack (axes  $a$  and  $c$ ) it implies:

$$\frac{\pi}{4} a c = c_{\text{eff}} t \quad (7)$$

( $t$  = thickness). Substitution of  $c_{\text{eff}}$  in (2) gives:

$$F_{1/2} = \sqrt{\frac{2 + \frac{\pi}{4} \frac{a}{t} \frac{c}{R}}{2 + \frac{\pi}{2} \frac{a}{t} \frac{c}{R}}} \quad (8)$$

It was also assumed that this factor is independent of the specimen width. It appears that the two assumptions can only be checked by finite element calculations. In view of applications to realistic problems there is a real need for such calculations. The data recently generated by Raju and Newman [5] could then have a much wider application.

## CONCLUSION

The stress intensity factor calculated by Newman for a single crack at the edge of a hole and by Tweed and Rooke for two symmetric cracks are considered to be significantly more accurate than the Bowie data. Equations are presented which closely represent the former data.

## REFERENCES

- [1] O.L. Bowie, J. Math. Phys. 35(1956) 60-71
- [2] J.C. Newman, Jr., "An Improved Method of Collocation for the Stress Analysis of Cracked Plates with Various Shaped Boundaries", NASA Technical Note D-6376, (1971).
- [3] J. Tweed and D.P. Rooke, Int. J. Engng Sci. 11(1973) 1185-1195.
- [4] R.C. Shah in Mechanics of Crack Growth, ASTM Spec. Tech. Publ. 590 (1976) 425-459.
- [5] I.S. Raju and J.C. Newman, Jr. in Fracture Mechanics, ASTM Spec. Techn. Publ. 677 (1979) 411-430.

Table 1

Comparison between two edge cracks and one edge crack at a hole in infinite sheet.



c/R	Values of $F = \frac{K}{S \sqrt{\pi c}}$				Differences %			Ratios $\frac{F_1 \text{ crack}}{F_2 \text{ cracks}}$		
	Newman 2 cracks ①	Tweed/Rooke 1 crack ②	Bowie		2 cracks ① - ③ ③	1 crack ② - ④ ④	Newman Tweed/Rooke ② / ①	Bowie ④ / ③	Shah $\frac{2+c/R}{2+2c/R}$	Eq (4)
			2 cracks ③	1 crack ④						
0.01	3.272 (a)	3.291								
0.02	3.224	3.223					1.000		0.998	0.999
0.04	3.101	3.095					0.998		0.995	0.997
0.06	2.986	2.978					0.997		0.990	0.996
0.08	2.882	2.870					0.996		0.986	0.994
0.10	2.786	2.771	2.73	2.73	2.0	1.5	0.995	1.000	0.977	0.992
0.15	2.581	2.554 (b)					0.990		0.967	0.990
0.20	2.413	2.373	2.41	2.30	0.1	3.1	0.983	0.954	0.957	0.980
0.25	2.274	2.221					0.977		0.949	0.973
0.3	2.156	2.092	2.15	2.04	0.3	2.5	0.970	0.949	0.941	0.966
0.4	1.971	1.884	1.96	1.86	0.6	1.3	0.956	0.949	0.926	0.952
0.5	1.833	1.727	1.83	1.73	0.2	-0.2	0.942	0.945	0.913	0.940
0.6	1.726		1.71	1.64	0.9			0.959	0.901	0.928
0.8	1.574		1.58	1.47	-0.4			0.930	0.882	0.906
1	1.472	1.306	1.45	1.37	1.5	-4.9	0.887	0.945	0.866	0.888
1.2	1.400								0.853	.872
1.5	1.324	1.127	1.29	1.18	2.6	-4.7	0.851	0.915	0.837	0.853
2	1.244	1.030	1.21	1.06	2.7	-2.9	0.828	0.876	0.816	0.829
3	1.164	0.930	1.14	0.94	2.1	-1.1	0.799	0.825	0.791	0.798
5		0.845	1.07	0.81		-4.1		0.757	0.764	0.767

(a) this value is supposed to be too low

(b) interpolated

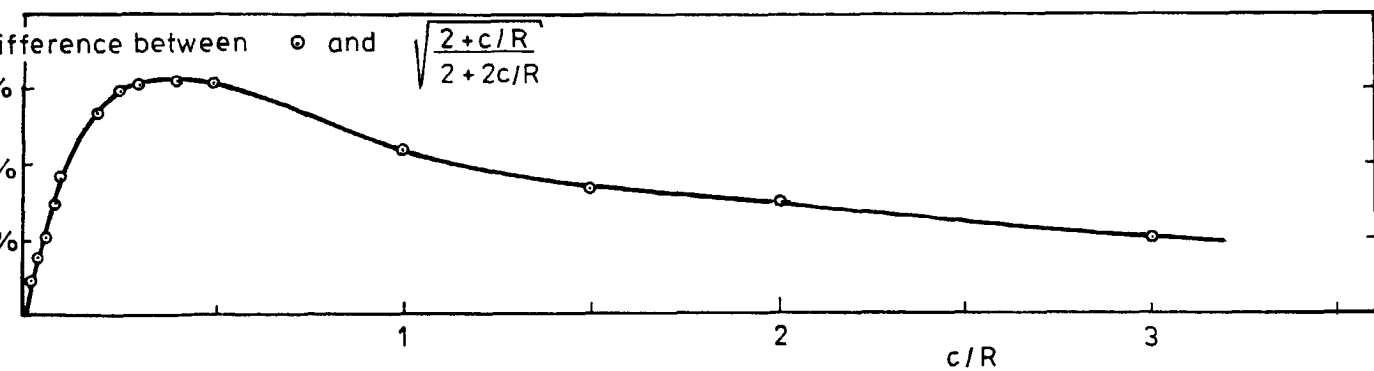
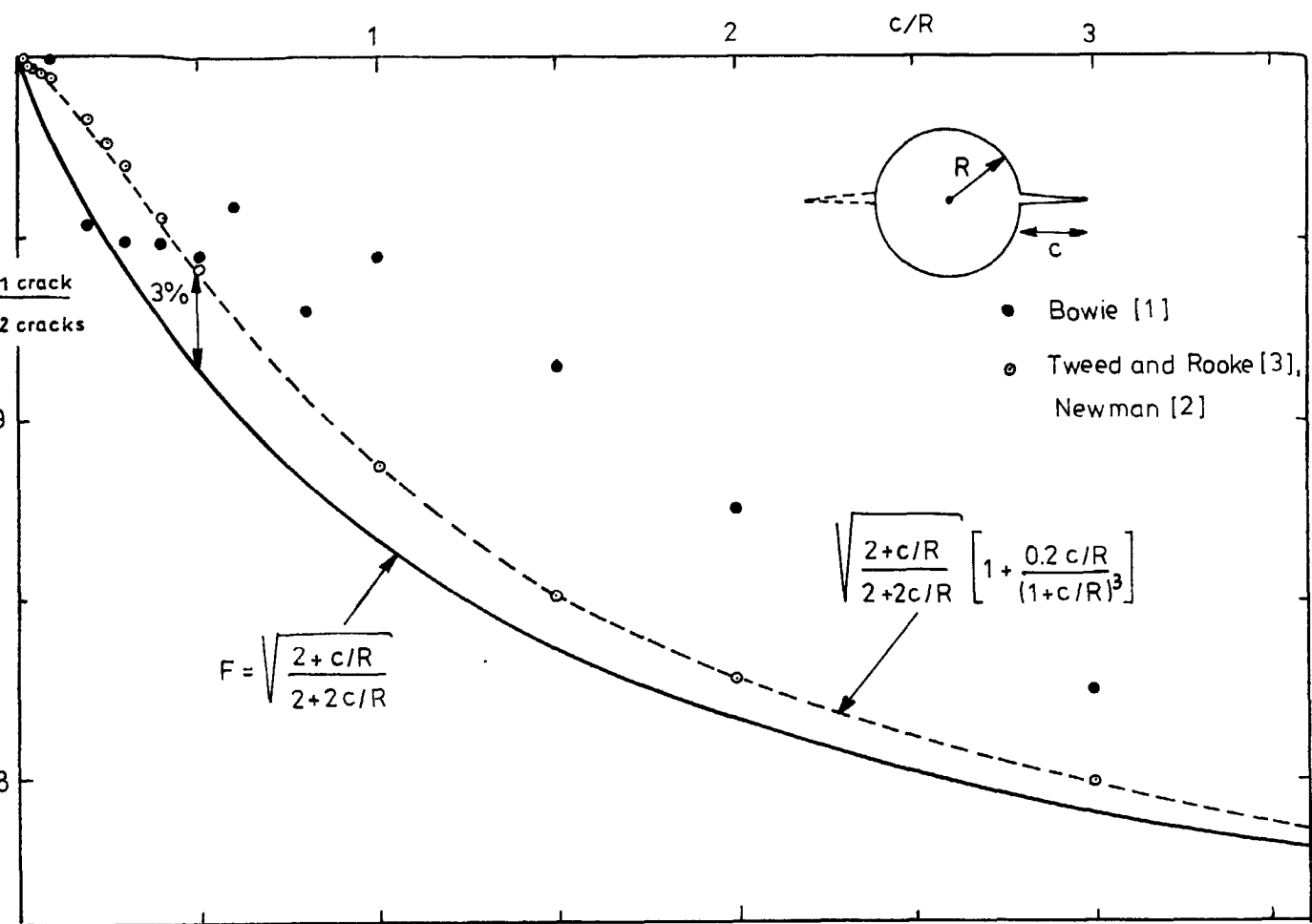


Figure 1: The ratio between K-factors for one and for two cracks  
Comparison between data of different sources

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