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# Stress–strain model for concrete under cyclic loading

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**A hysteretic stress–strain model is developed for unconfined concrete with the intention of providing efficient modelling for the structural behaviour of concrete in seismic regions. The proposed model is based on the findings of previous experimental and analytical studies. The model for concrete subjected to monotonic and cyclic loading comprises four components in compression and tension – an envelope curve (for monotonic and cyclic loading), an unloading curve, a reloading curve and a transition curve. Formulations for partial unloading and partial reloading curves are also presented. The reliability of the proposed constitutive model is investigated for a reinforced concrete member using a non-linear finite-element analysis program. Comparisons with test results showed that the proposed model provides a good fit to a wide range of experimentally established hysteresis loops.**

## Notation

| | |
|---|---|
| $E_c$ | tangent modulus of elasticity of concrete |
| $E_{sec}$ | secant modulus of elasticity |
| $f'_c$ | specified concrete compressive strength |
| $n$ | material parameter that depends on the shape of the stress–strain curve |
| $n_1$ | modified material parameter at ascending branch |
| $n_2$ | modified material parameter at descending branch |
| $\varepsilon_c$ | axial concrete strain in general |
| $\varepsilon'_c$ | tensile strain corresponding to tensile strength |
| $\varepsilon_{pl}$ | plastic strain |
| $\varepsilon_{ro}$ | reloading concrete strain |
| $\varepsilon_{un}$ | unloading concrete strain |
| $\varepsilon'_{ct}$ | tensile concrete strain in general |
| $\sigma_c$ | concrete stress in general |
| $\sigma_f$ | crack closure stress |
| $\sigma_{new}$ | degraded concrete stress |
| $\sigma_{pr}$ | partial unloading stress |
| $\sigma_{ro}$ | initial concrete stress on reloading branch |
| $\sigma_{un}$ | reversal envelope stress |

## Introduction

Experimental programmes in laboratories produce real results for studying the non-linear behaviour of reinforced concrete (RC) structures but they are limited to knowledge of particular cases under restricted structural dimensions, sizes, shapes, loading and boundary conditions. The computational simulation approach, on the other hand, has no limit to its application (Maekawa *et al.*, 2003). Interest in materials modelling and analysis of concrete structures has increased because of the need to accurately predict the non-linear response of concrete structures under monotonic and cyclic loads. With rapid improvements in computer technology and numerical methods, considerable research has focused on the realistic simulation of concrete structures (Kwon, 2000).

Over recent decades, numerous simple and more sophisticated models for describing concrete behaviour under various stress states have been developed. However, many of these models have greater conceptual importance than practical significance since they only describe certain aspects of concrete behaviour and their implementation is limited to examples of small practical interest. Important features of a concrete constitutive model should include not only the essentially accurate representation of actual concrete behaviour but also clarity of formulation and efficient implementation in a robust and stable non-linear state-determination algorithm (Kwon, 2000).

Constitutive models for concrete are based on three approaches – the theory of elasticity, the theory of plasticity and the theory of fracture mechanics (CEB, 1996). Some combinational models based on plasticity and fracture mechanics theory have also been developed. Although plasticity and fracture mechanics models can correctly simulate the observed behaviour of concrete, their application in engineering practice is limited due to the great amount of parameters usually needed and the difficulty in obtaining them through conventional laboratory tests (Sima *et al.*, 2008).

In the context of this study, only the simplified models that are essentially mathematical formulations derived from the generalisation of test results for concrete under various loading histories are considered. Many of these models have been documented in the literature by, for example, Sinha *et al.* (1964), Karsan and Jirsa (1969), Buyukozturk and Tesng (1984), Yankelevsky and Reinhardt (1987a), Chang and Mander

(1994), Bahn and Hsu (1998), Elmorsi *et al.* (1998), Palermo and Vecchio (2003), Mansour and Hsu (2005) and Sima *et al.* (2008). Most refer only to the compressive cyclic behaviour of concrete and only a few consider the cyclic tension response. Other authors, such as Hordijk (1991), Okamura and Maekawa (1991) and Palermo and Vecchio (2003) have provided an accurate approximation of the complete unloading–reloading cycle in tension.

## Research significance

A constitutive model for the description of the response of concrete under general cyclic loading is presented. The model has several advantages over previous approaches.

(*a*) It allows consideration of all the hysteretic characteristics of the complex behaviour of concrete in a simple and practical way.
(*b*) It can be used to simulate the cyclic response of concrete subject to general load conditions, including partial unloading or reloading or mixed hysteretic loops involving the transition from compression to tension stresses or vice versa.
(*c*) All the required input data can be obtained through conventional laboratory monotonic compression and tension tests. This is an important feature issue that determines the applicability of the model in engineering practice.

The model is verified by comparison with available experimental results from other research.

## Existing constitutive models for cyclic loading

One of the first experimental investigations into the behaviour of plain concrete under cyclic loading was conducted by Sinha *et al.* (1964). The experiment comprised a series of 48 tests performed on concrete cylinders with compressive strengths of 20–28 MPa and subjected to cyclic axial compressive loading in order to determine the main factors governing the cyclic response of concrete. To investigate the effects of load history, load cycles were applied in two different ways, including complete and partial unloading. To represent the concrete response analytically, a polynomial relationship was adopted for the envelope curve (Palermo, 2002). The unloading and reloading paths were modelled using parabolic and linear equations respectively independent of the previous load history, although subsequent studies by Karsan and Jirsa (1969) and Bahn and Hsu (1998) showed the dependency of unloading and reloading response on the previous load history. The analytical cyclic response could show the test results qualitatively. Sinha *et al.* (1964) introduced some main characteristics of the cyclic behaviour of concrete and established a sound basis for the future studies in this area (Palermo, 2002).

Shah and Winter (1966a, 1966b) carried out a series of tests on prismatic specimens subjected to cyclic axial compressive loading. Their results indicated that the shakedown limit is approximately equivalent to the critical load at which the number of microcracks in mortar begins to increase sharply and a continuous pattern of microcracks begins to form. As a result, undamaged portions that carry the load are reduced and the stress–strain relationship becomes even more non-linear. The onset of major microcracking was reported at 70–90% of ultimate load (Dabbagh, 2006).

To gain further insight into the response of plain concrete under different cyclic compressive loading histories, Karsan and Jirsa (1969) performed an experimental study on 46 short rectangular concrete columns with cylinder compressive strengths ranging from 24 to 35 MPa. The specimens were tested under four different loading regimes: monotonic increasing loading to failure; cycles to envelope curve; cycles to envelope curve adding a specified strain increment during each cycle; and cycles between maximum and minimum stress levels. They found out that unloading and reloading curves are not unique but depend on the previous load history. Introducing the concept of non-recoverable strain (or plastic strains) as the strain corresponding to zero stress on the unloading or reloading curves, the shape of these curves was found to be significantly influenced by this factor (Dabbagh, 2006).

A constitutive model for concrete consistent with a compression field approach was proposed by Palermo and Vecchio (2003). This concrete cyclic model considers concrete in both compression and tension. The unloading and reloading curves were linked to the envelope curves, which were represented by monotonic response curves. Reloading was modelled as a linear curve with degrading reloading stiffness. This model also considers the case of partial unloading–reloading and a linear crack-closing function. All the model parameters were statistically derived from tests developed by other authors.

Sima *et al.* (2008) developed a constitutive model for normal-strength concrete subjected to cyclic loading in both compression and tension. Particular emphasis was paid to the description of the strength and stiffness degradation produced by the load cycling in tension and compression, the shape of unloading and reloading curves and the transition between opening and closing of cracks. Aslani (2010), Aslani and Bastami (2011) and Aslani and Nejadi (2012) introduced two independent damage parameters in compression and in tension to model concrete degradation due to increasing loads.

## Proposed compressive stress–strain model for concrete under cyclic loading

### Envelope curve

It is commonly accepted by most researchers (e.g. Bahn and Hsu, 1998; Karsan and Jirsa, 1969; Yankelevsky and Reinhardt, 1987a) that the envelope curve for concrete subjected to axial cyclic compression can be approximated by the monotonic stress–strain curve (Sima *et al.*, 2008). The proposed compressive envelope curve is based on the model of Carreira and Chu (1985), as given by Equations 1–9 (the equations in this study are in metric units)

1. $$\frac{\sigma_c}{f'_c} = \frac{n(\varepsilon_c/\varepsilon'_c)}{n - 1 + (\varepsilon_c/\varepsilon'_c)^n}$$

2. $$n = n_1 = [1{\cdot}02 - 1{\cdot}17(E_{sec}/E_c)]^{-0{\cdot}74} \text{ if } \varepsilon_c \leqslant \varepsilon'_c$$

3. $$n = n_2 = n_1 + (a + 28b) \text{ if } \varepsilon_c \geqslant \varepsilon'_c$$

where

4. $$a = 3{\cdot}5(12{\cdot}4 - 1{\cdot}66 \times 10^{-2} f'_c)^{-0{\cdot}46}$$

5. $$b = 0{\cdot}83 \exp(-911/f'_c)$$

6. $$E_{sec} = f'_c/\varepsilon'_c$$

7. $$E_c = 3320\,(f'_c)^{0{\cdot}5} + 6900$$

8. $$\varepsilon'_c = \left(\frac{f'_c}{E_c}\right)\left(\frac{r}{r-1}\right)$$

9. $$r = \frac{f'_c}{17} + 0{\cdot}8$$

### Unloading and reloading curves

As observed by many researchers (Bahn and Hsu, 1998; Karsan and Jirsa, 1969; Sinha *et al.*, 1964), when a concrete specimen is monotonically loaded up to a certain strain level and then unloaded to zero stress level in a typical cyclic test, the unloading curve is concave from the unloading point and is characterised by high stiffness at the start. A power-type equation is proposed here for the unloading curve of concrete and a linear type equation is used for the reloading curve (Aslani, 2010).

Unloading curve

10. $$\sigma_c = \left(\frac{1 - [(\varepsilon_c - \varepsilon_{un})/(\varepsilon_{pl} - \varepsilon_{un})]}{1 + 1{\cdot}2[(\varepsilon_c - \varepsilon_{un})/(\varepsilon_{pl} - \varepsilon_{un})]}\right)^{1{\cdot}2} \sigma_{un}$$

11. $$\varepsilon_{pl} = \varepsilon_{un} - \frac{\sigma_{un}}{E_r}$$

12. $$E_r = E_c\left(\frac{(\sigma_{un}/E_c\varepsilon'_c) + 0{\cdot}57}{(\varepsilon_{un}/\varepsilon'_c) + 0{\cdot}57}\right)$$

Reloading curve

13. $$\sigma_c = \sigma_{ro} + E_{c1}(\varepsilon_c - \varepsilon_{ro})$$

14. $$E_{c1} = \frac{\sigma_{ro} - \sigma_{new}}{\varepsilon_{ro} - \varepsilon_{un}}$$

15. $$\sigma_{new} = \sigma_{un}[1 - 0{\cdot}09\,(\varepsilon_{un}/\varepsilon_{ro})^{0{\cdot}5}]$$

The equation proposed for the unloading branch includes the mean features of the unloading curves obtained experimentally, such as the curvature of the unloading curve, the initial unloading stiffness, the final unloading stiffness and the unloading strain–plastic strain ratio.

### Partial unloading and reloading curves

There is a lack of information considering the case where partial unloading is followed by partial reloading to strains less than the previous maximum unloading strain. This more general case was modelled using the experimental results of Bahn and Hsu (1998). For the case of partial unloading followed by reloading to a strain in excess of the previous maximum unloading strain, the reloading path is defined by the expressions governing full reloading (Aslani, 2010), as given by Equation 16

16. $$\sigma_c = \sigma_{pr} + \left(\frac{1 - [(\varepsilon_c - \varepsilon_{un})/(\varepsilon_{pl} - \varepsilon_{un})]}{1 + 1{\cdot}2[(\varepsilon_c - \varepsilon_{un})/(\varepsilon_{pl} - \varepsilon_{un})]}\right)^{1{\cdot}2} \times (\sigma_{un} - \sigma_{pr})$$

The partial reloading is based on the partial reloading proposed by Palermo and Vecchio (2003).

## Proposed tensile stress–strain model for concrete under cyclic loading

### Envelope curve

Much less attention has been directed towards the modelling of concrete under cyclic tensile loading. Some researchers consider little or no excursions into the tension stress regime and those who have proposed models assume, for the most part, linear unloading/reloading responses with no plastic strains. Several expressions have been documented in the literature to represent the softening branch, including straight lines (Bažant and Oh, 1983), polylinear curves (Gustafsson, 1985; Gylltoft, 1983; Hillerborg *et al.*, 1976; Petersson, 1981; Rots *et al.*, 1985), exponential curves (Gopalaratnam and Shah, 1985; Sima *et al.*, 2008), polynomial curves (Lin and Scordelis, 1975; Yankelevsky and Reinhardt, 1987b, 1989), combinations of them (Cornelissen *et al.*, 1985), a continuous damage-based formulation to represent post-peak stress–strain curves of concrete (Mazars, 1981) and tension softening in terms of prescribed drops (Scanlon, 1971). The proposed tensile envelope curve given by Equation 17 is a very simple model (Aslani, 2010)

17. $$\sigma_{\mathrm{c}} = \begin{Bmatrix} f'_{\mathrm{c}} E_{\mathrm{c}} & \varepsilon_{\mathrm{c}} \leqslant \varepsilon'_{\mathrm{ct}} \\ f'_{\mathrm{c}} (\varepsilon'_{\mathrm{ct}}/\varepsilon_{\mathrm{c}})^{0\cdot 85} & \varepsilon_{\mathrm{c}} > \varepsilon'_{\mathrm{ct}} \end{Bmatrix}$$

### Unloading and reloading curves

The response of concrete under cyclic tension has been studied in detail by Reinhardt (1984) and Reinhardt *et al.* (1986). A straight line is used for the unloading branch in tension. The same curve is considered for the reloading branch when there is no incursion in compression during a cycle. The plastic strain in the proposed tension model is used to define the shape of the unloading curve, the slope and damage of the reloading path and the point at which cracked surfaces come into contact. Similar to concrete in compression, plastic strains in tension seem to be dependent on the unloading strain from the backbone curve. The proposed plastic strain model is expressed as

18. $$\varepsilon_{\mathrm{pl}} = 0\cdot 725\, \varepsilon_{\mathrm{un}}$$

## Crack-closing model

A series of tests attempting to characterise the effect of damage in tension when the specimen is loaded in compression were developed by Ramtani *et al.* (1992). These test results have shown that completely closing the cracks requires a certain amount of compression. Once the crack is closed, the stiffness of the concrete is not affected by accumulated damage in tension. The crack closure mechanism is governed by the 'crack closure stress' $\sigma_{\mathrm{f}}$, which is the stress at which the crack is supposed to be completely closed. It has been observed that the crack closure stress is strongly affected by the concrete strength and placement methods (crack roughness). For monolithic structures with no previous damage in compression, $\sigma_{\mathrm{f}}$ is in the range of the tensile strength (Légeron *et al.*, 2006) and can be taken as

19. $$\sigma_{\mathrm{f}} = f'_{\mathrm{c}}/10$$

## Model verification: comparison with test results

### Repeated compressive loading

Several uniaxial cyclic test results have been compared with predictions obtained by means of the model presented. These tests cover several concrete strengths and a variety of cyclic histories, including both cyclic compression and cyclic tension. In the case of cyclic compression, results from works performed by Sinha *et al.* (1964), Okamoto *et al.* (1976), Tanigawa and Uchida (1979) and Bahn and Hsu (1998) were considered; Figures 1–4 show these experimental tests for cyclic compressive loading compared with the proposed model. In Figure 5, an experimental test for partial cyclic compressive loading carried out by Bahn and Hsu (1998) is compared with the proposed model. Based on the comparisons shown in Figures 1–5, the following conclusions can be made.

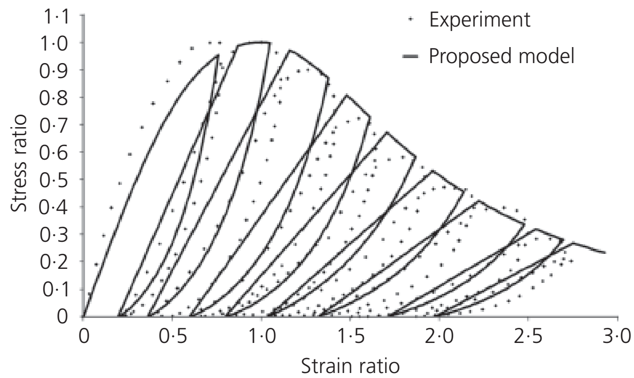



Figure 1. Comparison of experimental data of Sinha *et al.* (1964) with proposed model

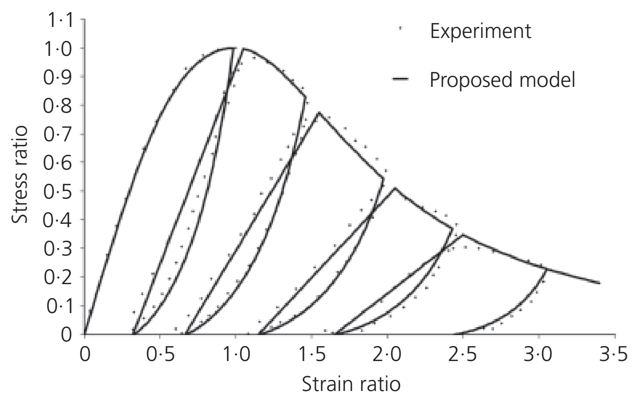



Figure 2. Comparison of experimental data of Okamoto *et al.* (1976) with proposed model

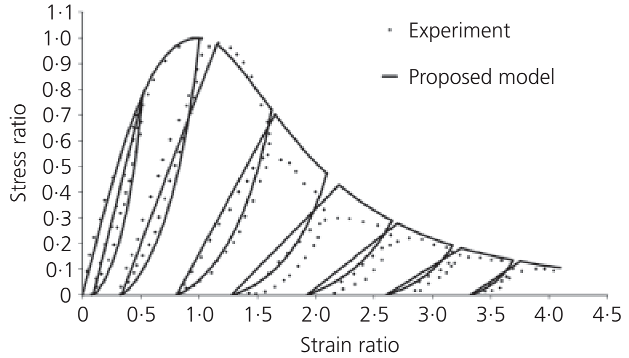



**Figure 3.** Comparison of experimental data of Tanigawa and Uchida (1979) with proposed model

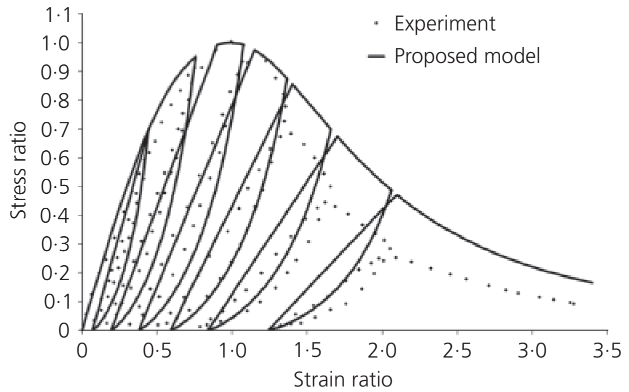



**Figure 4.** Comparison of experimental data of Bahn and Hsu (1998) with proposed model

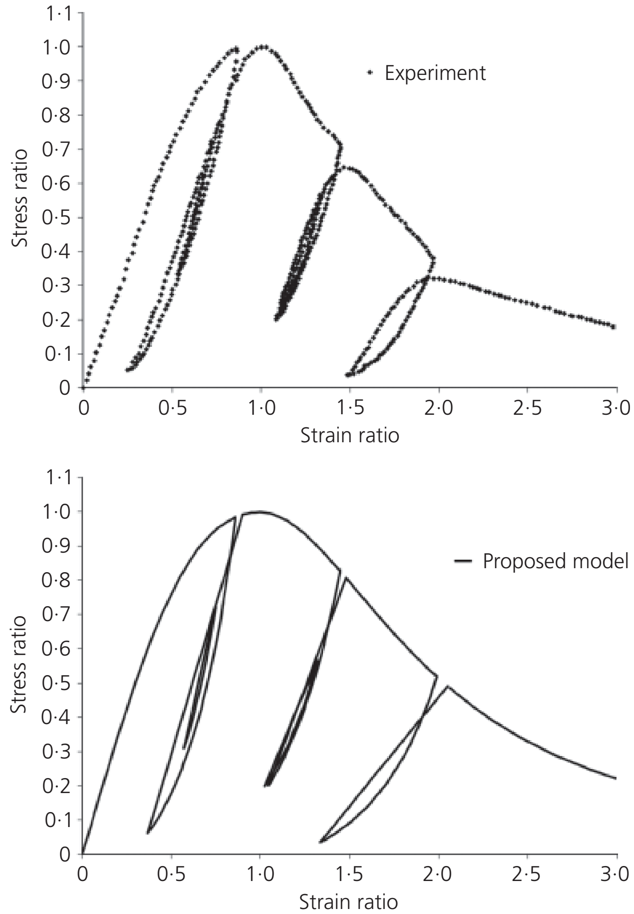



**Figure 5.** Comparison of partial experimental data of Bahn and Hsu (1998) with proposed model

(*a*) The envelope curve for cyclic loading could be represented by the response of concrete to monotonic loading.
(*b*) The residual strains are a function of the strain at unloading; an increase in unloading strain causes approximately the same increase in the accumulated residual strain.
(*c*) The unloading and reloading curves do not coincide and are not parallel to the initial loading curve. The average slope of the unloading and reloading curves is inversely proportional to the plastic strain. This result is based on the overall observations of several experimental results compared with available models. This suggests that there is stiffness degradation for the entire stress–strain beyond elastic.
(*d*) Continuous degradation of the concrete is reflected in the decrease of the slopes of the reloading curves.
(*e*) Reloading curves are nearly linear up to the intersection with the unloading curve, after which there is a softening in the response.
(*f*) The shape of the unloading curve is strongly dependent on the location of unloading plastic strain rather than the envelope unloading strain.
(*g*) There is no additional strain accumulation in the partial reloading curve until the stress level exceeds a certain limit (stability limit).
(*h*) Concrete exhibits typical hysteretic behaviour where the area within the hysteresis loops, representing the energy dissipated during a cycle, becomes larger as the unloading strain increases.
(*i*) Based on previous test results for full unloading and full reloading, and random cyclic loading, the envelope reloading strain is always greater than the envelope unloading strain regardless of partial or full unloading.

### Repeated tensile loading

In the case of cyclic tension and cyclic tension with small incursions in compression, the model is compared with the test results reported by Reinhardt (1984) (Figure 6) and Yankelevsky and Reinhardt (1987b) (Figure 7). In both cases, the present model shows satisfactory agreement with the experimental results.

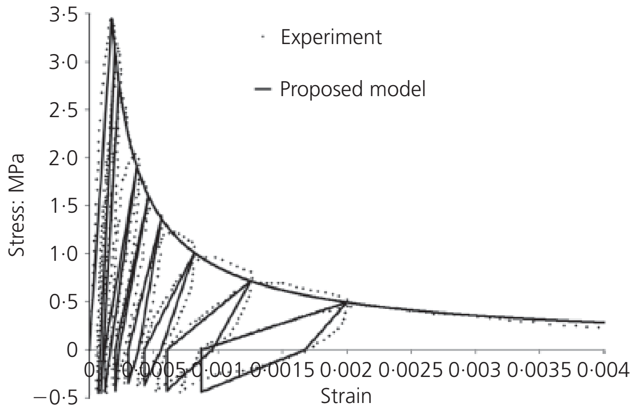



**Figure 6.** Comparison of experimental data of Reinhardt (1984) with proposed model

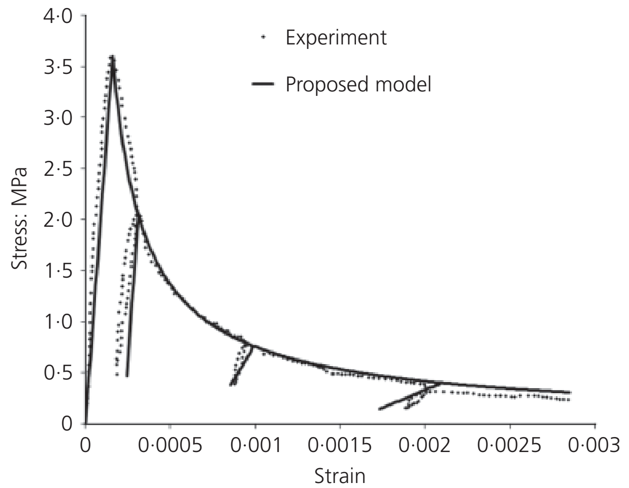



**Figure 7.** Comparison of experimental data of Yankelevsky and Reinhardt (1987b) with proposed model

In Figure 6, the unloading and reloading curves in the model coincide and there is no energy dissipation during a cycle. However, in the experimental results it can be observed that the amount of energy dissipated in a cycle is very small. In Figure 7, the unloading and the reloading path of one cycle are significantly different, exhibiting a large hysteresis loop. This feature can be accurately simulated with the model by considering an adequate crack closure stress. The following conclusions can be drawn from Figures 6 and 7.

(*a*) In cyclic loading tests, one may define the envelope curves as the line on which both the starting points of unloading and the end points of reloading lie.
(*b*) Comparison of the monotonic loading curve in uniaxial compression with that in tension shows that the descending branch in tension immediately beyond the peak is considerably steeper than in compression and the ratio between the ultimate strain corresponding to the peak stress in tension is considerably larger.
(*c*) The unloading curve softens gradually while stress is decreasing and the stiffness of the unloading curve at a given stress level is smaller for larger strains.
(*d*) The unloading curve in tension becomes a loading curve in compression, which becomes stiffer with increasing compressive stresses.

## Reversed cyclic loading

Two specimens reported by Dabbagh (2006) were selected to validate the model under reversed cyclic loading. These particular specimens were also selected in order to examine the analytical predictions of the proposed model for high-strength concrete for different axial loading. The properties of the two specimens (SW3 and SW4) studied experimentally by Dabbagh (2006) are shown in Figures 8 and 9 and Tables 1–4. These specimens are nominally identical in geometry to specimens tested by Gupta and Rangan (1996), so that a comparison between cyclic testing and monotonic testing can be investigated. The specimens were designed to fail in shear.

The scale of the test specimens used by Dabbagh (2006) was approximately one-third of shear walls used in a multi-storey building. The specimens were made up of web wall, edge elements and stiff top and bottom slabs. The height-to-width ratio of the walls was equal to 1, as shown in Figure 8(a). The geometrical dimensions of all specimens were identical. The top slab (1300 × 575 × 200 mm) was designed to be sufficiently stiff to distribute the lateral and axial loads on the test walls. The bottom slab (1800 × 575 × 400 mm) was also stiff and clamped to the test set-up representing a rigid foundation. To simulate columns or cross-walls that may exist at the ends of a wall in a multi-storey building, the shear wall specimens were constructed with edge elements (1000 × 375 × 100 mm). The clear dimensions of the web wall were 1000 mm height, 800 mm width and 75 mm thickness. The reinforcement arrangements for the specimens are shown in Figure 8(b). For all specimens, the reinforcement details for the top slab, bottom slab and edge elements were the same.

The wall specimens were subjected to a combination of constant axial load and cyclic lateral loading as shown in Figure 9(a), except for specimen SW4, in which the axial load was zero. To study the pre-cracking as well as post-cracking behaviour of specimens, lateral loading was applied using a displacement control. The specimens were tested under reversed cyclic conditions displacing them laterally, along the axis of the web wall, in 4 mm increments in the negative (downward) and positive (upward) directions (Figure 9(a)). Since the wall specimens had high strength and stiffness and their behaviours were approximately brittle and linear until

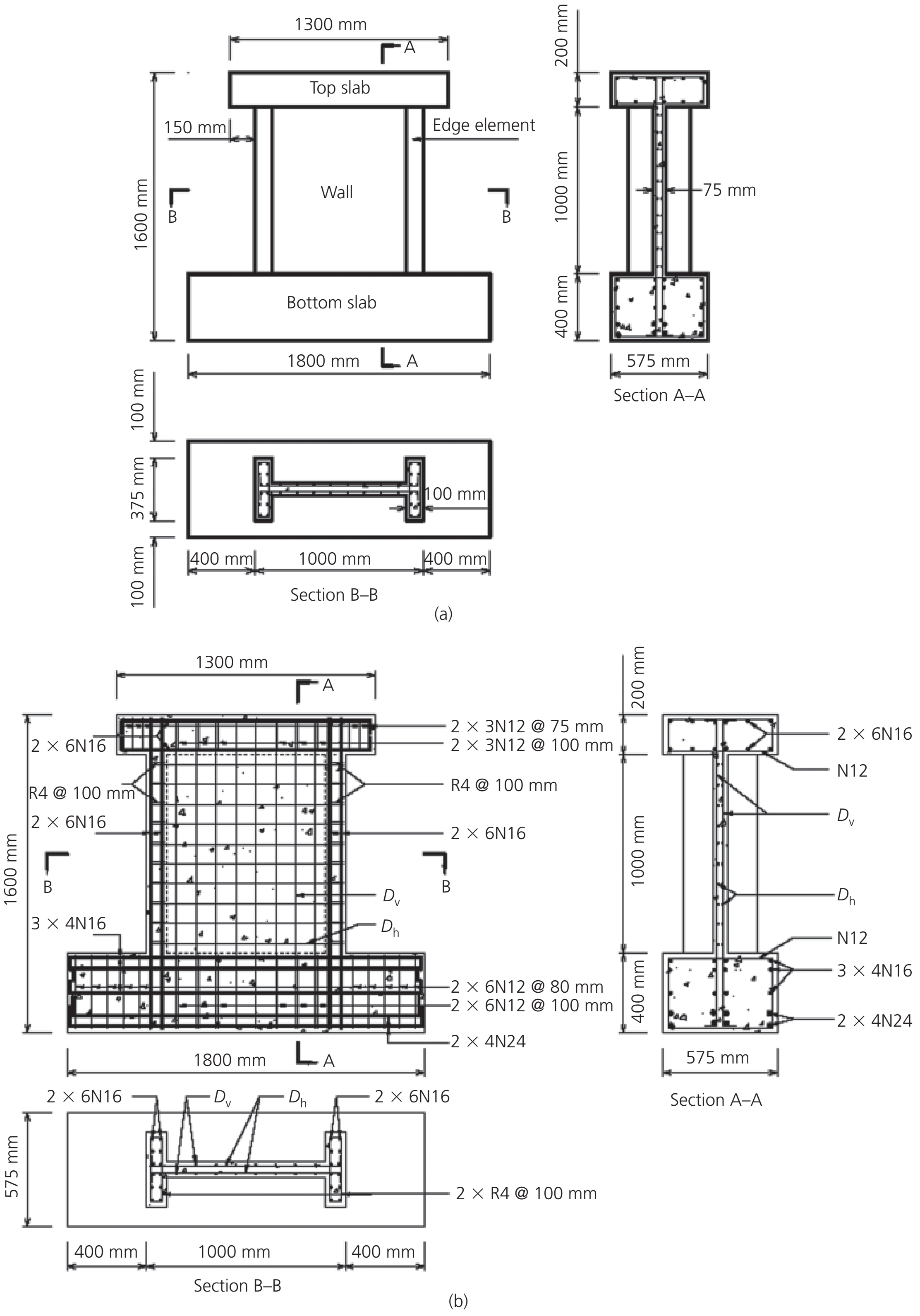



**Figure 8.** (a) Dimensions of wall specimens; (b) reinforcement details of shear wall specimens (Dabbagh, 2006)

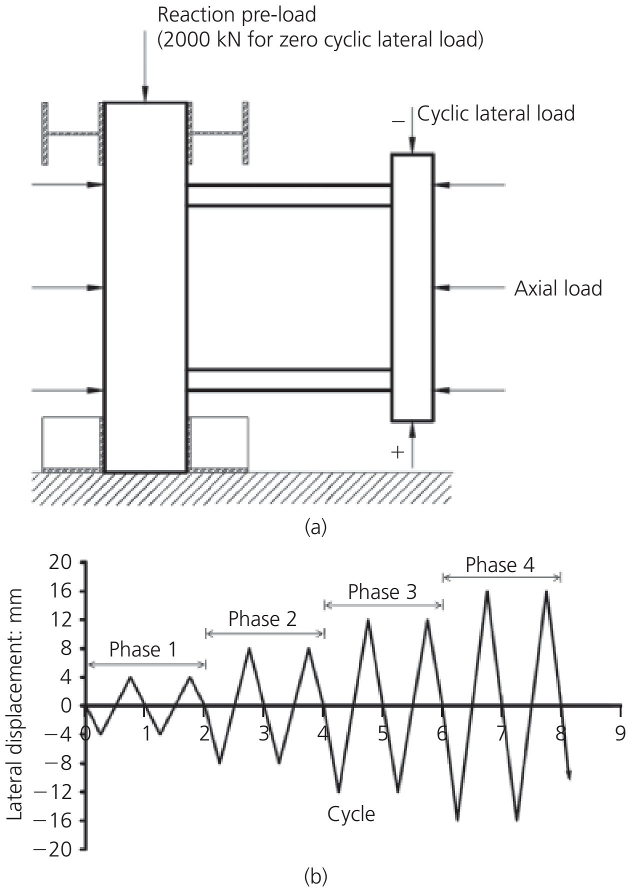



Figure 9. Reinforcement details of shear wall specimens (Dabbagh, 2006)

| Age: days | Compressive strength: MPa | Spitting tensile strength: MPa | Elastic modulus: MPa |
|---|---|---|---|
| 28 | 81 | 4·6 | 42 760 |
| 355 | 96 | 7·2 | 43 670 |

Table 2. Properties of concrete used for specimens SW3 and SW4 (Dabbagh, 2006)

failure, using displacement increments of 4 mm seems logical. A loading rate of 30 min per cycle was maintained until the specimens experienced significant loss of capacity. For the cyclic tests, two repetitions at each displacement level were imposed for each phase. The loading history applied to the specimens is shown in Figure 9(b).

Figures 10–13 show the load–displacement response of specimens SW3 and SW4; a summary of the results is given in Table 4. Specimens SW3 and SW4 had longitudinal and transverse reinforcement ratios of 0·8% with the specimens subjected to a combination of axial load and lateral reversed cyclic loading. The ages of SW3 and SW4 at the time of testing were 355 and 358 days respectively. The compressive strength of concrete on the day of test was 96 MPa.

Specimen SW3 was tested under displacement cycles accompanied by an axial load of 1200 kN. The lateral loading was applied to the specimen through complete phases of 4, 8 and 12 mm and

| Specimen | Transverse reinforcement | | Longitudinal reinforcement | |
|---|---|---|---|---|
| | $D_h$ | Equivalent reinforcement ratio: % | $D_v$ | Equivalent reinforcement ratio: % |
| SW3 | 2 × 10W6 @ 100 mm | 0·8 | 2 × 5W8 @ 160 mm | 0·8 |
| SW4 | 2 × 10W6 @ 100 mm | 0·8 | 2 × 5W8 @ 160 mm | 0·8 |

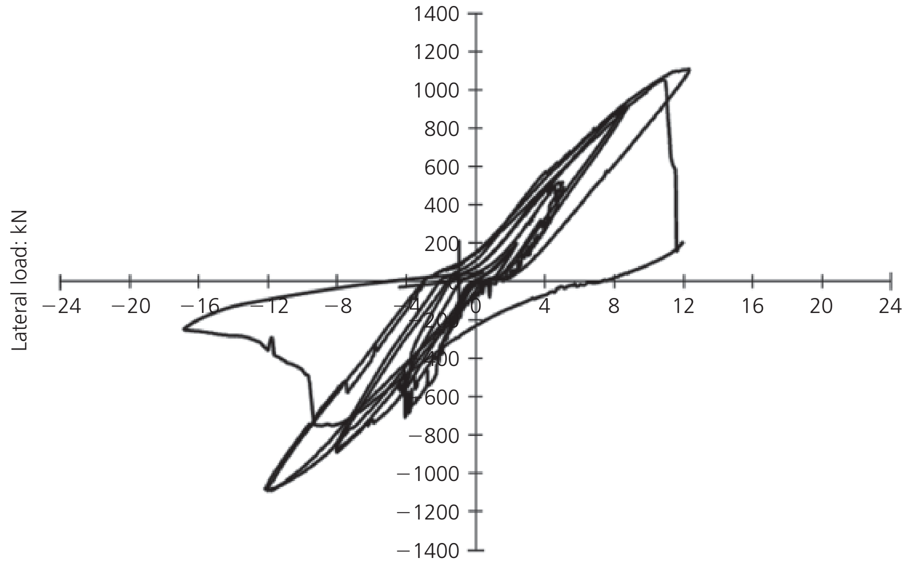
Table 1. Reinforcement details of web walls (Dabbagh, 2006)

| Reinforcement | Yield strain | Yield strength: MPa | Ultimate strength: MPa | Elastic modulus: GPa |
|---|---|---|---|---|
| W6 | 0·00320 | 536 | 597 | 198 |
| W8 | 0·00330 | 498 | 535 | 179 |
| N12 | 0·00330 | 571 | 649 | 199 |
| N16 | 0·00300 | 535 | 638 | 204 |
| N24 | 0·00340 | 524 | 623 | 195 |

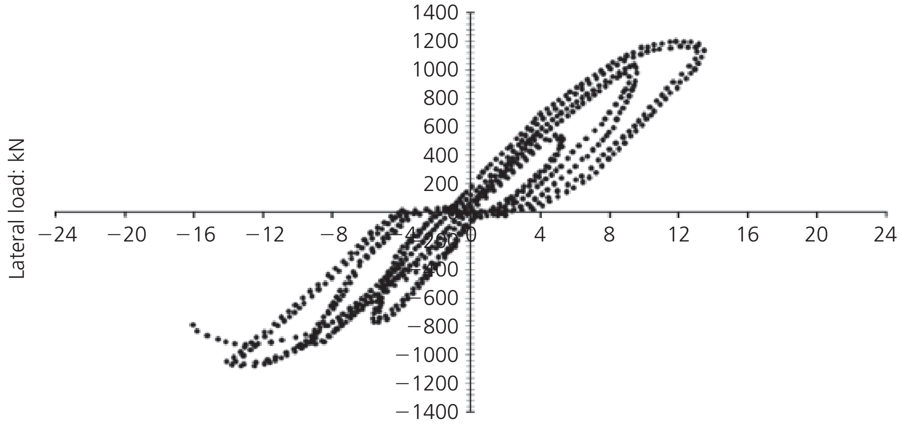
Table 3. Properties of reinforcing steel (Dabbagh, 2006)

| Specimen | Concrete strength: MPa | Reinforcement ratio: % | | Axial load: kN | Peak lateral load: kN | | Corresponding displacement: mm | |
|---|---|---|---|---|---|---|---|---|
| | | Transverse | Longitudinal | | Downward | Upward | Downward | Upward |
| SW3 | 96 | 0·8 | 0·8 | 1200 | −1090 | 1107 | −12·12 | 12·31 |
| SW4 | 96 | 0·8 | 0·8 | 0 | −683 | 753 | −7·94 | 12·32 |

**Table 4.** Summary of experimental results (Dabbagh, 2006)

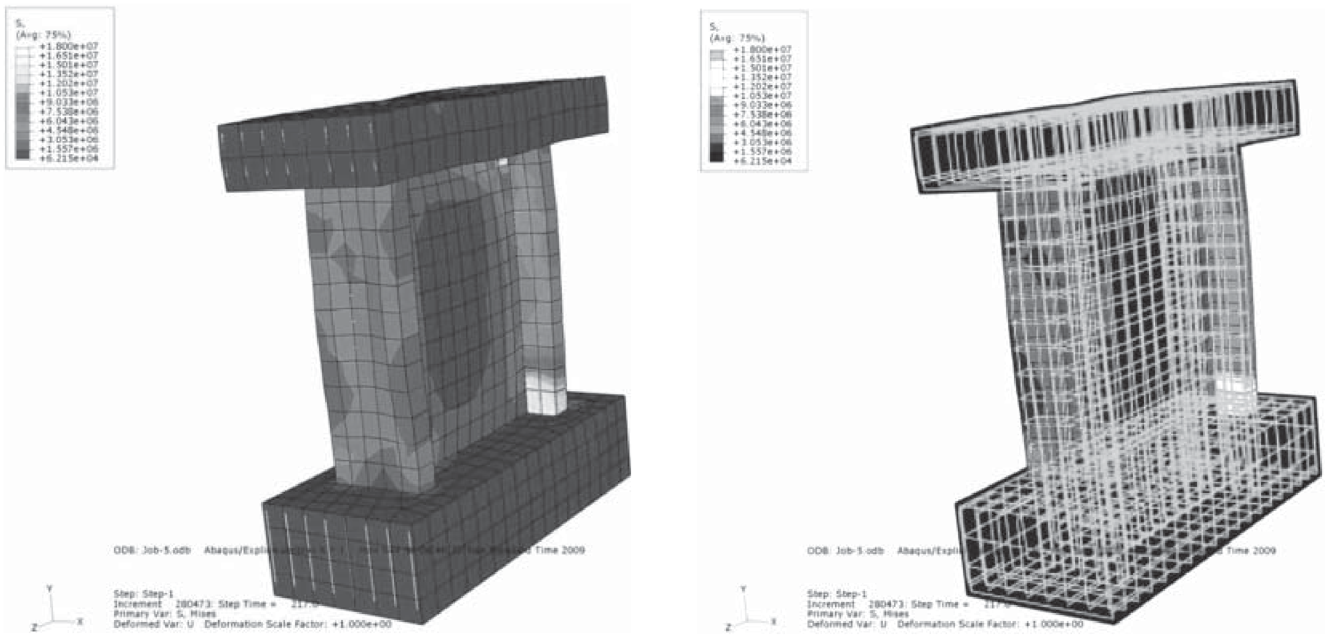



(a)




(b)

**Figure 10.** (a) Experimental load–displacement response of specimen SW3 (Dabbagh, 2006). (b) Analytical load–displacement response of SW3 using the proposed model



(a) (b)

**Figure 11.** (a) Stress of SW3 under revised cyclic loading. (b) Stress of SW3 with steel reinforcing detail under revised cyclic loading

a half-cycle in the negative direction at the 16 mm phase. At this stage the test was terminated as the wall had failed in both directions and its stiffness was significantly decreased. The behaviour of SW3 was dominated by shear action with little ductility. The failure was accompanied with a major crack at $P = 1052$ kN in the positive loading direction and corresponded to a displacement of 10·86 mm.

Specimen SW4 was subjected to only lateral loading with four phases of lateral loading completed with failure occurring during the first excursion into phase three. The maximum loads recorded in the negative and positive directions were $-684$ and 752 kN respectively, corresponding to displacements of $-7{\cdot}9$ and 12·1 mm respectively. The specimen failed during cycle five at a load of $-683$ kN, corresponding to a displacement of $-8{\cdot}0$ mm.

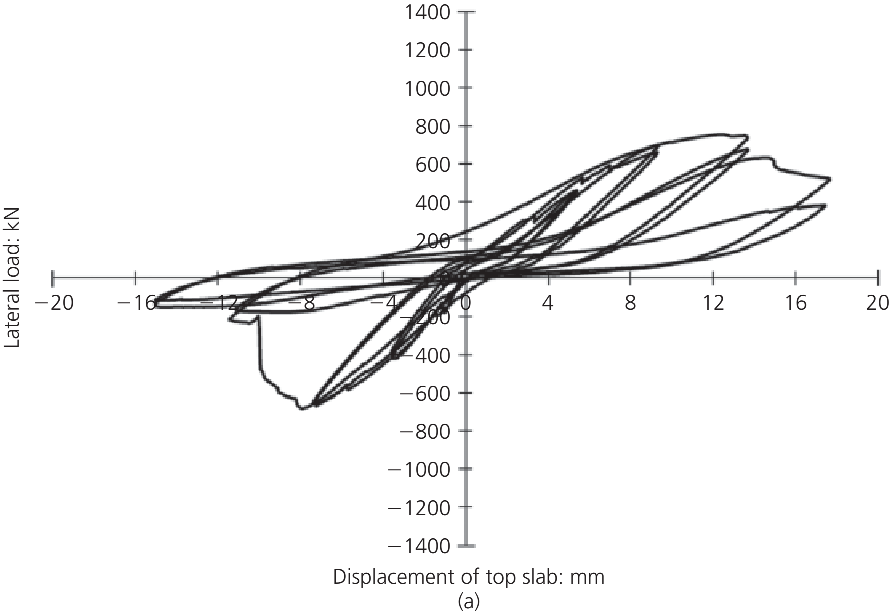
The finite-element analysis program Abaqus (DSSC, 2010) was employed for analytical modelling of SW3 and SW4 to determine that the proposed model can be used to introduce concrete properties under reversed cyclic loading. Abaqus/CAE was employed as a finite-element method solver. Interfaces between the Abaqus user's subroutine Umat and the Abaqus main code were developed to allow further extension of the current method. Umat allows the user to define the mechanical behaviour of a material and to interface with any externally defined programs. The stress–strain relations computed from these proposed models were used in Umat to define a cyclic constitutive material model. Based on these proposed mechanical properties of SW3 and SW4 in the modelling phase, the main final results of lateral load against displacement of the top slab were achieved by accurate lateral load and displacement history analysis.

Figures 10(b) and 12(b) show analytical results of SW3 and SW4 compared with the experimental results (Figures 10(a) and 12(a)). Figures 11 and 13 show the Abaqus simulation using the proposed stress–strain relationship for concrete for specimens SW3 and SW4.

## Conclusions

A cyclic constitutive model has been developed for unconfined concrete. The following conclusions are drawn from the current study.

(*a*) The proposed constitutive model was developed for the simulation of the response of concrete subjected to cyclic loadings in both compression and tension.
(*b*) The model can reproduce the complex behaviour of concrete under any history of uniaxial cyclic loading (i.e. full loading and partial loading).
(*c*) Unloading is assumed to be non-linear and is modelled



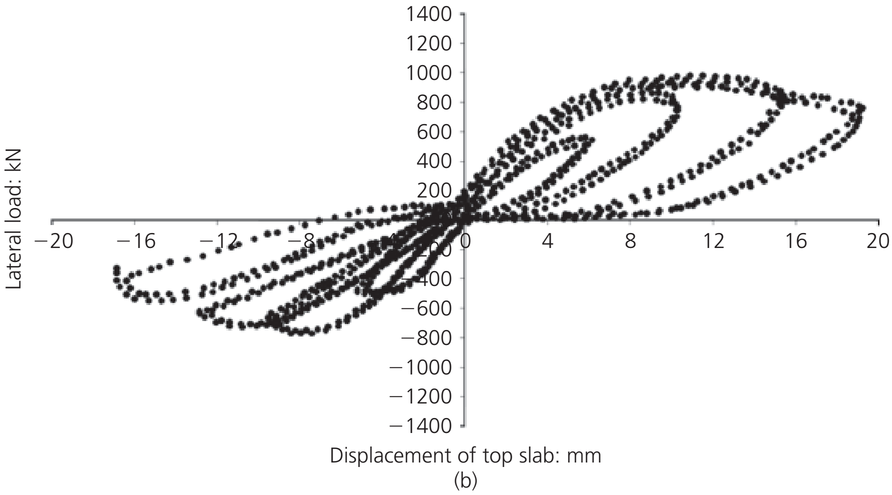




**Figure 12.** (a) Experimental load–displacement response of specimen SW4 (Dabbagh, 2006). (b) Analytical load–displacement response of SW4 using the proposed model

using a power-type equation that considers boundary conditions at the onset of unloading and at zero stress. Unloading, in the case of full loading, terminates at the plastic strain.

(*d*) The model was verified by comparing the results with a series of tests developed by other authors. In all cases, the proposed model shows satisfactory agreement with the experimental results.

(*e*) Reloading is modelled as linear with a degrading reloading stiffness. The reloading response does not return to the backbone curve at the previous unloading strain and further straining is required to intersect the backbone curve.

(*f*) The model also considers the general case of partial unloading and partial reloading in the region below the previous maximum unloading strain.

(*g*) The proposed model is capable of predicting reversed cyclic behaviour of high-strength concrete members.

(*h*) The proposed model is user friendly and is suitable for introduction into a finite-element program.

(*i*) Results from analytical studies comparing the proposed model with experimental results proved the capability of the model for analysis of high- and normal-strength concrete members.

(*j*) A remarkable feature of the model lies in the fact that all the input data required can be obtained through conventional monotonic compression and tension tests.

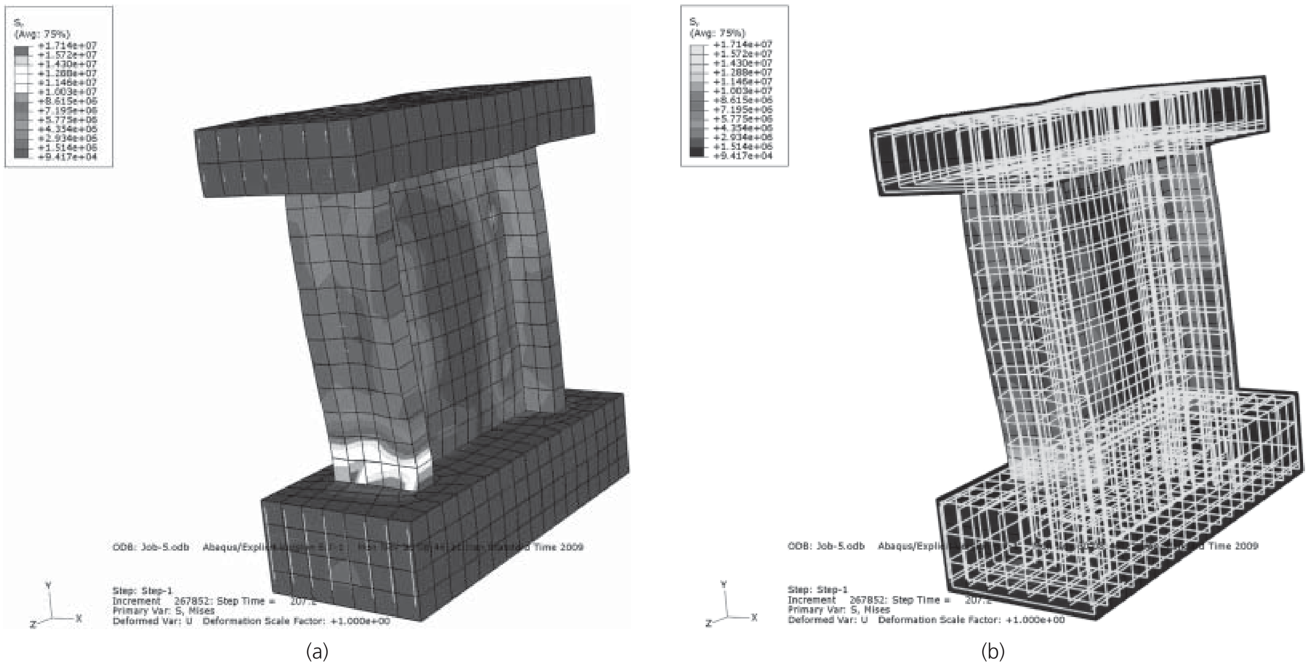



**Figure 13.** (a) Stress of SW4 under revised cyclic loading. (b) Stress of SW4 with steel reinforcing detail under revised cyclic loading

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