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Stress Testing the Corporate Loans Portfolio of the Canadian Banking Sector

by

Miroslav Misina, David Tessier, and Shubhasis Dey

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Acknowledgements

Abstract

Stress testing, at its most general level, is an investigation of the performance of an entity under abnormal operating conditions. The authors focus on one set of entities—the Canadian banking sector—and investigate losses in the loans portfolio of this sector as a function of changing circumstances in the different industries in which these loans reside. These circumstances are characterized by means of one summary measure—sectoral probabilities of default—and this measure is modelled as a function of macroeconomic variables. Using this model, the authors assess the interrelationship between the macroeconomic environment and sectoral defaults, and perform a series of stress tests under different scenarios that are thought to be most pertinent to Canada. The tools underlying the authors' analysis are general and can be applied to other countries, as well as to other macroeconomic scenarios.

JEL classification: C15, G21, G33
Bank classification: Financial stability; Financial institutions

Résumé

La conduite de simulations de crise (*stress testing*) sert de façon générale à évaluer la performance d'une entité dans des conditions de fonctionnement anormales. Les auteurs centrent leur analyse sur le secteur bancaire canadien, en reliant les pertes sur le portefeuille des prêts bancaires aux modifications de l'environnement dans lequel les différents secteurs d'activité bénéficiaires de ces prêts évoluent. Les caractéristiques de l'environnement sont résumées au moyen d'un indicateur — la probabilité de défaut dans chaque secteur — qui est modélisé sous la forme d'une fonction de variables macroéconomiques. À l'aide de ce modèle, les auteurs évaluent les liens réciproques entre le contexte macroéconomique et les défaillances d'entreprises dans chacun des secteurs, et mènent une série de simulations selon divers scénarios de crise jugés pertinents dans le cas canadien. Les outils sur lesquels repose l'analyse des auteurs sont de nature générale et peuvent être appliqués à d'autres pays ainsi qu'à d'autres scénarios macroéconomiques.

Classification JEL : C15, G21, G33
Classification de la Banque : Stabilité financière; Institutions financières

1. Introduction

Stress testing, at its most general level, is an investigation of the performance of an entity under abnormal operating conditions. The objective of such an exercise is to identify potential vulnerabilities. Precise specification of the key components—entity, performance indicators, and operating conditions—depends on the nature of the exercise and one's concerns.

From a financial stability viewpoint, the entity of interest is the financial system, and the ultimate objective of stress tests is to assess the performance of the financial system under abnormal operating conditions. The financial system, however, is a complex entity consisting of a wide range of financial institutions, financial markets, and payments and settlement systems, and it is not easily amenable to aggregate analysis. In practice, the analysis of financial system stability focuses on individual components, most often the financial institutions, to arrive at an overall assessment of the financial system.

In this paper, the focus of analysis is the Canadian banking sector—a natural place to start, especially in Canada, given the size of this sector and its importance in the financial system. Whereas it might be debatable whether a particular financial institution is "systemically important," the systemic importance of the banking sector as a whole is not controversial.[1]

In investigating the performance of the banking sector, one can follow either a bottom-up or a top-down approach. The bottom-up approach consists of examining the performance of individual banks and aggregating the results, whereas the top-down approach involves looking at the banking sector as a whole. Both approaches have their strengths and weaknesses, and the decision regarding which to pursue depends to some extent on one's views about the nature and causes of financial instability.[2] In this work, we follow the top-down approach, on the premise that systemic vulnerabilities can result from common exposures, whether from exposures to similar classes of assets or, ultimately, similar risk factors.

It is important to emphasize that the two approaches are best viewed as complements rather than substitutes. The implications of the common risk factors for individual institutions can be assessed only on a case-by-case basis. Collecting these results and comparing them with the top-down approach would help us identify the externalities, if any, not taken into account at the individual level. The reverse is true as well: sources of vulnerabilities

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[1]See Gauthier and St-Amant (2005) for a discussion of systemic risk and financial instability.
[2]Borio (2005) contains an excellent description of two different views on financial stability and its causes.

identified by individual institutions might be quite informative, since they may turn out to be systemic.

Key to the identification of vulnerabilities is scenario selection. By scenario we mean an event (for example, an increase in interest rates)—and, possibly, its broader implications—that is believed to represent abnormal operating conditions. Scenarios can be chosen based on historical experience, or they can be hypothetical. In either case, the objective is to select as scenarios those rare but plausible events that have caused abnormal operating conditions in the past, or that could cause them in the future. The scenarios chosen in this paper reflect sources of vulnerabilities that are commonly considered 'typical' for Canada, rather than a reflection of the 'concerns of the moment.' Of course, at a given point in time, the views on the possible sources of risk may necessitate an analysis of different scenarios. These can be accommodated within the model developed in this paper in a straightforward manner.

The impact of different scenarios on the financial system is quantified by examining a set of performance indicators. The choice of indicators is closely related to the approach one takes to stress testing. In this paper, we opt for the portfolio approach and use two performance indicators—expected losses, and maximum losses at a given confidence level—based on the loss distribution for the loans portfolio of the banking sector.[3]

In this paper, we consider the aggregate loans portfolio of Canadian banks as a function of the macroeconomic environment and conduct stress tests at the aggregate level. The work most closely related to ours is that of Virolainen (2004), who performs a similar exercise for the Finnish banking sector. Our work differs from Virolainen's as follows: (i) we do not model macroeconomic variables in isolation, but rather use a vector-autoregression (VAR) model, which allows us to capture the impact of a change in one variable on the rest of the economy, (ii) we introduce lags in the default rate regression, allowing for the delayed impact of changes in macroeconomic variables on defaults, and (iii) we use industry-level simulation to assess the impact on portfolio losses, rather than generate defaults at the individual-company level. These modifications and their bearing on the results are explained in sections 2–4.

This paper is organized as follows. In section 2 we describe the main approaches to stress testing and place some of the existing literature in a broader context. In section 3 we describe the details of our model. Section 4 contains a detailed description of the data and a preliminary statistical analysis. In section 5 we describe the macroeconomic scenarios selected for the stress-testing exercise. In section 6 the main results are presented and discussed. In

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[3]Details of this approach are provided in section 2.

section 7 we offer a summary and outline possible extensions of this work.

2. An Overview of the Approaches to Stress Testing

As mentioned earlier, the choice of performance indicators is closely related to the approach used for stress testing. To make this fact transparent, the approaches to stress testing found in the literature are divided into two groups:

   (i) a balance-sheet approach, and

  (ii) a portfolio approach.

    The common trait of these approaches is that they seek to explain a set of indicators of financial position for an individual institution or a sector in terms of some underlying variables. These variables are commonly referred to as risk factors. The risk factors we are interested in are macroeconomic variables, and the following discussion will describe these approaches as relating different indicators to these variables.

    Balance-sheet approach

    The main idea behind this approach is to explain a chosen set of balance-sheet indicators in terms of a set of macroeconomic variables. The nature of indicators and explanatory variables varies from model to model and depends on one's interests and priors. Dey (2006) seeks to explain the returns on equity of the banking sector; Hoggarth et al. (2005) seek to explain the writeoffs-to-loans ratio; Kalirai and Scheicher (2002) use loan-loss provisions. Monnin (2005) and Misina and Tkacz (2006), rather than relying on a single indicator, use an index that is composed of balance-sheet and financial indicators, and try to explain it in terms of macro variables. The explanatory variables considered differ across studies, but commonly include domestic GDP, a measure of inflation, and short-term interest rates. To these are occasionally added exchange rates, a measure of monetary aggregates, and various asset prices, depending on the features of the economy, as well as on the indicator whose behaviour the author seeks to explain.

Portfolio approach

Under the portfolio approach, asset holdings of an institution or a sector are viewed as a portfolio whose risk characteristics are summarized in terms of a loss distribution. The idea behind this approach is to relate loss distributions of this portfolio to a set of macroeconomic variables, and then investigate the impact of changes in these variables on the loss distribution. Summary measures of the impact typically reported are the expected loss (mean of the distribution) and the maximum losses that can occur for a given probability, usually 95 or 99 per cent.

This approach is quite close to the value-at-risk approach used by individual institutions. There are two key differences, however: (i) the focus of our work is not on individual institutions but on sectors; (ii) we are interested in relating loss distributions to macroeconomic variables in an attempt to assess longer-term performance and potential risks, rather than focusing on potential short-term (daily or weekly) losses.

There is a wide range of commercial models in current use that could be of potential interest (both for market-risk and credit-risk assessment). The closest in spirit to our objectives is the Credit Portfolio View (CPV) model by Mackenzie and Associates.[4] The key difference between this model and other commercial models is that it makes default probabilities dependent on a set of future values of macroeconomic variables, and these are then used in derivations of loss distributions. Virolainen (2004) and Sorge and Virolainen (2006) use a version of this model and apply it to the analysis of the credit risks in the Finnish banking sector.

3. Model

The goal of the portfolio approach is to arrive at the loss distribution for a particular portfolio. At any given point in time, the expected loss in industry s, due to defaults of some of the individual companies in that industry, is

$$El_t^s = \pi_t^s \times ex_t^s \times l_t^s,$$

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[4]The model is described in detail in Wilson (1997a) and Wilson (1997b).

and the expected loss of the portfolio is

$$El_t = \sum_{s=1}^{S} \pi_t^s \times ex_t^s \times l_t^s,$$

where π_t^s denotes the default probability in industry s at time t, ex_t^s is the portfolio exposure to industry s at time t, and l_t^s is loss given default in industry s at time t.[5] The results obtained in this model will depend on the specification of these three components. We will discuss each component separately.

3.1 Probability of default

In modelling default probabilities as functions of macroeconomic variables, two issues need to be addressed:

(i) the nature of the statistical relationship between default probabilities and macroeconomic variables, and

(ii) the dynamics of default probabilities as a function of the dynamics of the underlying macroeconomic variables.

To address the first issue, we start by assuming that the default probability in each industry is a function of a set of macroeconomic variables:

$$\pi^s = f(\mathbf{x}),$$

where $\mathbf{x}_t$ is a vector of macroeconomic variables. The problem is to find a suitable function, f, which relates macroeconomic variables to default probabilities. It is well known that when the dependent variable is a probability, postulating a linear relationship between the explanatory and the dependent variables is, in general, inappropriate. A common approach is to specify the log of the odds ratio, $\frac{\pi_t^s}{1-\pi_t^s}$, as a linear function of the explanatory variables,

$$\ln\left(\frac{\boldsymbol{\pi}_t^s}{1-\boldsymbol{\pi}_t^s}\right) = \mathbf{X}_{t-l}\boldsymbol{\beta}^s + \mathbf{e}_t^s, \;\; s = 1, ..., S, \tag{1}$$

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[5]π_t^s should not be viewed as the probability of default of industry s, since an entity called industry cannot default. It should be understood either as the average default probability of companies in industry s, or as the number of companies in industry s that are expected to default at time t. In this paper, we use the second interpretation, due to the nature of the data used as a proxy for π_t^s.

where $\mathbf{X}_{t-l} \equiv \left[1, x_t^1, ..x_{t-L}^1..., x_t^M, .., x_{t-L}^M\right]$ is a $1 \times (ML+1)$ vector of macroeconomic variables and their lags, and $\boldsymbol{\beta}^s \equiv \left[\beta_0^s, ..., \beta_{ML}^s\right]$ is an $(ML+1) \times 1$ coefficient matrix. It can be shown that this is equivalent to specifying the relationship between the original probabilities and the macroeconomic variables as a logistic function[6]:

$$\boldsymbol{\pi}_t^s = \left(1 + e^{-\mathbf{X}_{t-l}\mathbf{b}^s}\right)^{-1}. \tag{2}$$

The marginal effects of each macroeconomic variable on the default probability, given by individual elements of $\mathbf{b}^s \equiv \left[b_0^s, ..., b_{ML}^s\right]$ in (2), can be obtained from (1) and the relationship

$$\boldsymbol{\beta}_m^s = b_m^s \pi_t^s \left(1 - \pi_t^s\right).$$

This formulation allows for inclusion of industry-specific variables as explanatory variables. Indeed, inclusion of such variables in (1) will, in general, improve the fit of these regressions. There is, however, a trade-off between the inclusion of industry-specific variables and the goal of explaining default probabilities using a set of common risk factors.

The key difference between (1) and the default probability model in Wilson (1997a) and Sorge and Virolainen (2006) is that the latter postulate a contemporaneous relationship between default probabilities and macroeconomic variables. Whether this is warranted depends on the nature of the data used to infer default probabilities. If default probabilities are inferred from ratings, the assumption implies that the ratings reflect macroeconomic developments promptly. If, on the other hand, default probabilities are proxied by the observed number of bankruptcies, the assumption of contemporaneous relationship may not be warranted. As explained in section 4.1, our data are of the latter type. Consequently, we opt for the specification with lags.

To deal with the second issue—the dynamics of default rates as a function of the dynamics of macroeconomic variables—one needs to model the evolution of the macroeconomic variables. From a statistical viewpoint, one of the following two basic approaches can be chosen:

(i) model the dynamics of each variable separately and then do the estimation using the seemingly unrelated regressions (SUR) method, or

(ii) model the behaviour of macro variables jointly using a VAR model.

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[6] See Davidson and MacKinnon (1993), chapter 15, page 515.

The CPV and Sorge and Virolainen (2006) use the first approach, and assume that each macro variable follows an autoregressive process,[7]

$$x_{m.t} = \alpha_0 + \sum_i \alpha_i x_{m,t-i} + v_{m,t}, \quad m = 1, ..., M, \tag{3}$$

where $v_{m,t} \sim n.i.d.(\mathbf{0}, \boldsymbol{\Sigma}_v)$. Interdependencies among macro variables are admitted by not restricting $\boldsymbol{\Sigma}_v$ to a diagonal matrix.

Under the VAR approach, the interdependencies among the macroeconomic variables are explicitly taken into account by specifying the relationship of the form

$$\mathbf{X}_t = \boldsymbol{\phi}_1 \mathbf{X}_{t-1} + ... + \boldsymbol{\phi}_p \mathbf{X}_{t-p} + \mathbf{u}_t, \tag{4}$$

with $\mathbf{u} \sim n.i.d.(\mathbf{0}, \boldsymbol{\Sigma_u})$.[8] The advantage of the VAR approach is that it offers two channels of impact of a macroeconomic shock on default probabilities: the direct impact of a change in X_t^m on default probabilities, and an indirect impact via the impact on other macroeconomic variables. Owing to these advantages, in our empirical work we opt for VAR.

Our default probability model is thus given by

$$\ln\left(\frac{\boldsymbol{\pi}_t^s}{1 - \boldsymbol{\pi}_t^s}\right) = \mathbf{X}_{t-l}\boldsymbol{\beta}^s + \mathbf{e}_t^s, \ s = 1, ..., S, \tag{5}$$

and

$$\mathbf{X}_t = \boldsymbol{\phi}_1 \mathbf{X}_{t-1} + ... + \boldsymbol{\phi}_p \mathbf{X}_{t-p} + \mathbf{u}_t, \tag{6}$$

with

$$\mathbf{e}_t \equiv \begin{bmatrix} e_t^1 & ... & e_t^S \end{bmatrix} \sim N(\mathbf{0}, \boldsymbol{\Sigma_e}).$$

This model is estimated to obtain $\hat{\boldsymbol{\beta}}, (\hat{\boldsymbol{\phi}}_1, ..., \hat{\boldsymbol{\phi}}_p)$, and $\hat{\boldsymbol{\Sigma}}_\mathbf{e}$. The estimated parameters are then used to simulate future realizations of default rates by dynamic simulation:

(i) starting from a set of initial values of the variables $\mathbf{X}_{t-1}$, use (6) to obtain the values of these variables in the next period. K iterations of this procedure will result in a

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[7]The CPV and Sorge and Virolainen use AR(2) processes.

[8]The constant term is implicitly present, and for each autoregression, m, is given by $c_m \equiv \sum_{j=1}^p \phi_{m,j}^0$.

K-period-ahead path for the macroeconomic variables:

$$\mathbf{X}_{t+1} = \hat{\phi}_1 \mathbf{X}_t,$$
$$\mathbf{X}_{t+2} = \hat{\phi}_1 \mathbf{X}_{t+1} + \hat{\phi}_2 \mathbf{X}_t,$$
$$...$$
$$\mathbf{X}_{t+K} = \hat{\phi}_1 \mathbf{X}_{t+K-1} + ... + \hat{\phi}_K \mathbf{X}_t,$$

(ii) substitute the results of the previous step into (5) to obtain a vector of realizations of default probabilities for all industries, $\tilde{\boldsymbol{\pi}}_t$.

3.2 Exposures

The second component of the model is the exposure of the banking sector to different industries. This information is needed in order to construct the credit portfolio that is representative of the banks' lending.

If the only credit instruments in the portfolio under consideration are plain loans, the definition of exposure is straightforward, and can be taken as the book value of loans to individual institutions, or industries. The situation is much more complicated if the credit instruments include off-balance-sheet over-the-counter contracts. In these cases, it is often not even clear where the credit-risk exposure really lies.[9] In the macro stress-testing literature, the focus is typically on plain loans, and in the empirical part of our work we follow that tradition.

In constructing the representative bank's credit portfolio, Virolainen (2004) focuses on lending to the corporate sector, and builds the portfolio by using financial information from individual companies. Firms with loans from financial institutions are divided into industries, and the value of loans to each industry is obtained by adding the appropriate values of individual loans.[10]

In constructing the credit portfolio for our exercise, we focus on sectoral non-mortgage loans to individuals for business purposes. The data are collected by the Office of the Superintendent of Financial Institutions. A detailed description of the data is provided in section 4.2.

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[9] See, for example, Saunders and Allen (2002), chapter 14 for details.

[10] For details, see Virolainen (2004), page 20, or Sorge and Virolainen (2006), section 3.2.3.

3.3 Loss given default

The last component of the model is loss given default, specifying the amount of money that is likely to be lost in the event of the default of an obligor. For an individual obligor, loss given default at time t is defined as

$$l_t = 1 - rr_t,$$

where rr_t is the recovery rate—the amount of money that can be recovered on defaulted loans. It is typically expressed as a percentage of the par amount of the loan. For a given industry, the recovery rate is the average recovery rate on loans in that industry. The recovery rate for a credit portfolio is defined in a similar manner.

Whereas the notion of loss given default and recovery rates is conceptually clear, there are a number of practical challenges in estimating recovery rates and losses in the event of a default.[11] First, there is no clear agreement on what constitutes default. There are several possibilities[12]:

(i) the market definition relates defaults to financial instruments and corresponds to principal or interest past due,

(ii) Basel II defines default as past due 90 days on financial instruments or provisioning, but default can also be based on a judgmental assessment of a firm by the bank,

(iii) the legal definition links defaults to the bankruptcies of a firm.

Second, recovery rates depend on a number of other factors, such as the debt renegotiation process, quality of collateral attached to loans, level of seniority of bank loans, and industry of the issuer.

Third, there is no standard measure of the level of recovery. Two measures are used:

(i) ultimate recovery—the amount that the debt holder will recover eventually after the debt is settled, and

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[11] For an in-depth discussion of the issue, see Servigny and Renault (2004), chapter 4.

[12] See Servigny and Renault (2004), page 119, or Needham and Verde (2006) for an example of a definition used by practitioners.

(ii) the price of debt immediately after default, keeping in mind that this price will be available only for the fraction of debt that is publicly traded.

Ultimate recovery may be difficult to measure in cases where claims are not settled in cash, but it is the only way to measure recovery rates to illiquid bank loans.[13]

In practice, recovery rates are either assumed to be constant or are assumed to be stochastic and drawn from a particular distribution. In both cases, recovery rates are assumed to be independent of default rates. The evidence, however, seems to suggest that the recovery rates are not constant, and, more importantly, that there is a link between default rates and recovery rates. There seems to have been little work done on this issue to date. Sorge and Virolainen (2006) assume that the recovery rates are constant, and, in light of the issues discussed above, this may not be a bad place to start.[14]

3.4 Generating loss distributions

The basic features of the model we will use for stress testing are:

- default probability is a function of macroeconomic variables

- exposures are given for each industry

- losses given default are assumed to be equal across industries and constant

To generate a loss distribution, we proceed as follows:

(i) Generate a path for the macroeconomic variables in the manner described in section 3.1.

(ii) Generate a vector of s random variables with the variance-covariance matrix given by $\hat{\boldsymbol{\Sigma}}_{\mathbf{e}}$. This is done by first generating a vector $\mathbf{Z} \sim N\left(\mathbf{0}, \mathbf{I}\right)$, and then calculating $\tilde{\mathbf{e}}_t = A' Z_t$, where $\hat{\boldsymbol{\Sigma}} = AA'$.

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[13]Servigny and Renault (2004), page 123.

[14]Another interesting possibility is that recovery rates are procyclical: high in expansions and low in recessions. In that case, the impact on defaults on portfolio losses would be magnified and the assumption of constant recovery rates would lead to underestimation of losses. While this is an interesting possibility, strong empirical evidence to support it is lacking.

(iii) Substitute the results of the previous two steps into

$$\ln\left(\frac{\pi_t^s}{1-\pi_t^s}\right) = \mathbf{X}_{t-l}\boldsymbol{\beta}^s + \tilde{\mathbf{e}}_t^s, \forall s,$$

to obtain the values of default probabilities for each industry at a given point in time.

(iv) For each value of simulated default probability for industry s, compute the expected loss in that industry according to

$$El_t^s = \pi_t^s \times ex_t^s \times l_t^s. \tag{7}$$

The expected loss at time t on the portfolio is computed by summing the expected losses in all industries:

$$El_t = \sum_{s=1}^{S} \pi_t^s \times ex_t^s \times l_t^s.$$

To obtain a loss distribution, a large number of realizations of $\boldsymbol{\pi}_t$ are generated and, for each realization, steps (i)–(iv) are repeated.

4. Data

The key issue is which industries to include in the aggregate credit portfolio. Data on the number of defaults and exposures are available for the following industries: (i) accommodation and food services, (ii) agriculture, (iii) construction, (iv) manufacturing, (v) mining, quarrying, and oil wells, (vi) retail, (vii) transportation, (viii) wholesale trade, (ix) utility, and (x) others (private non-profit institutions, religious, health and education institutions, etc.).

Unfortunately, not all the data are usable, either because of incomplete exposure coverage or a large number of periods with no bankruptcies in particular industries.[15] In making a decision on which industries to include, we are guided primarily by the occurrence of bankruptcies and include the industries in which bankruptcies have been most frequent over the sample period: (i) manufacturing (**MAN**), (ii) construction (**CON**), (iii) retail (**RET**), and (iv) accommodation and food services (**ACC**). These are the industries with, on average, the highest credit exposures as well. The data used are quarterly, spanning the period from 1987Q1 to 2005Q4.[16]

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[15]The latter problem results in difficulties in estimating default probability regressions.
[16]Details regarding data definitions, sources, and sample sizes are provided in Appendix A.

4.1 Defaults by industry

We construct this series by relying on two different series: the total number of bankruptcies by industry, and the total number of establishments in each industry.[17,18] The frequencies of these two series do not coincide: the number of bankruptcies is available monthly, whereas establishment counts are available semi-annually. The quarterly bankruptcy rate series is obtained as follows:

- determine the total number of bankruptcies occurring in each quarter by summing the monthly numbers for each industry

- convert the semi-annual establishment counts into quarterly by taking the average of two adjacent semi-annual values to fill the missing quarterly value

- at each point in time, divide the total number of bankruptcies in each industry by the establishment count for that industry

The question is whether the bankruptcy rates constructed in this way correspond to bankruptcy rates applicable to individual banks.[19] It is very likely that the denominator of our series overestimates the number of establishments in the loans portfolio of banks, since it implies that all establishments in an industry are in the loans portfolio. This would lead to underestimation of bankruptcy rates. On the other hand, the number of companies that go bankrupt may be overestimating the number of bankruptcies observed by individual banks. This would result in an upward bias in the bankruptcy rates. The net effect is difficult to predict.[20]

A plot of bankruptcy rates for the selected industries is shown in Figure 1.

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[17]Statistics Canada defines an establishment in the following way. "A statistical establishment is the production entity or the smallest grouping of production entities which:
(a)   Produces a homogeneous set of goods or services;
(b)   Does not cross provincial boundaries; and
(c)   Provides data on the value of output together with the cost of principal intermediate inputs used along with the cost and quantity of labour resources used to produce the output.
For example, a plant in the manufacturing industry which provides accounting information regarding the value of shipments (sales), direct costs and labour costs is considered a single establishment. However, two stores in the retail industry may be considered one establishment if the accounting information, described in item (c) above, is not available separately, but is combined at a higher level."

[18]The number of establishments in each industry is obtained from Statistics Canada's Business Register database.

[19]We thank Meyer Aaron and Jim Armstrong for drawing our attention to this issue.

[20]Note, however, that Fitch, a rating agency, uses a similar method to compute default rates; see Needham and Verde (2006) for details.
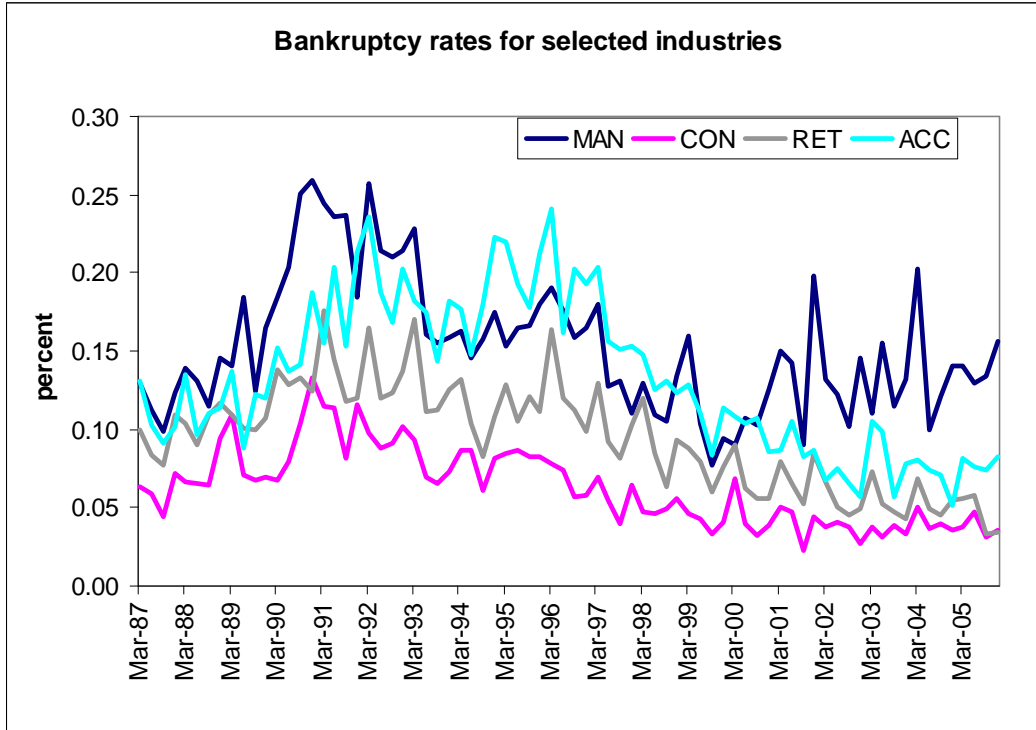
Figure 1: Bankruptcy rates

For each industry, the bankruptcy rate at a point in time represents the fraction of firms that went bankrupt out of the total number of firms in that industry. For example, in the manufacturing industry, the value at 2005Q4 is 0.157, meaning that 0.157 per cent of the firms in that industry went bankrupt in the last quarter of 2005.[21]

There are several notable features to this data:

- there was an overall increase in bankruptcy rates in all industries during the recession in the early 1990s

- bankruptcy rates have, in general, been declining over the past 10 years, with the exception of accommodation—which experienced a second peak in the mid-1990s—and manufacturing, which has been on an upward trend over the past five years

- bankruptcy rates in the construction industry were low and stable over the sample period

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[21] For a recent analysis of bankruptcies in Canada, see Lecavalier (2006).

Summary statistics are provided in Table 1.

	MAN	CON	RET	ACC
Mean (%)	0.15	0.06	0.09	0.13
Std. dev.	0.04	0.03	0.03	0.05
Skewness	0.71	0.59	0.22	0.33
Kurtosis	-0.08	-0.32	-0.51	-0.92

Table 1: Descriptive statistics for bankruptcy rates by industry

Manufacturing and accommodation bankruptcy rates display the highest volatility, with standard deviations of 0.04 and 0.05, respectively. Preliminary statistical analysis of the data indicates that there is some evidence of asymmetry, summarized by the positive skewness coefficient. Interestingly, all of the variables are platykurtic; i.e., they have flatter peaks and less probability mass in tails than the normal density. The latter property is not surprising, given that the data are bounded to a $[0, 1]$ interval.

The assumption of normality cannot be rejected for the retail industry, but it is rejected for the other three industries.[22]

The correlation matrix for the industries is

$$
\begin{bmatrix}
1 & & & \\
0.73 & 1 & & \\
0.71 & 0.84 & 1 & \\
0.59 & 0.72 & 0.81 & 1
\end{bmatrix}.
$$

All values are statistically significant at the 5 per cent level. Default rates are clearly highly correlated across industries. This may have important systemic consequences, especially if the portfolio is heavily exposed to industries with highly correlated defaults.

In using these data to compute losses in the loans portfolio of the banking industry, we assume that each firm has an equal impact of each bankruptcy on losses. This implies that the exposures are equally divided among the firms so that, for given recovery rates, there is no difference in impact of any individual firm's bankruptcy on the losses. This assumption is clearly false: Virolainen (2004) finds that, of over 58,000 companies in Finland, the largest 3,000 account for roughly 94 per cent of the outstanding loans—but the only solution is to

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[22]The results are based on the Shapiro-Wilk's W test, 95 per cent confidence interval. Normality is weakly rejected for the construction industry—the associated p-value is 0.04.

Share of foreign-currency loans in loans to the business sector
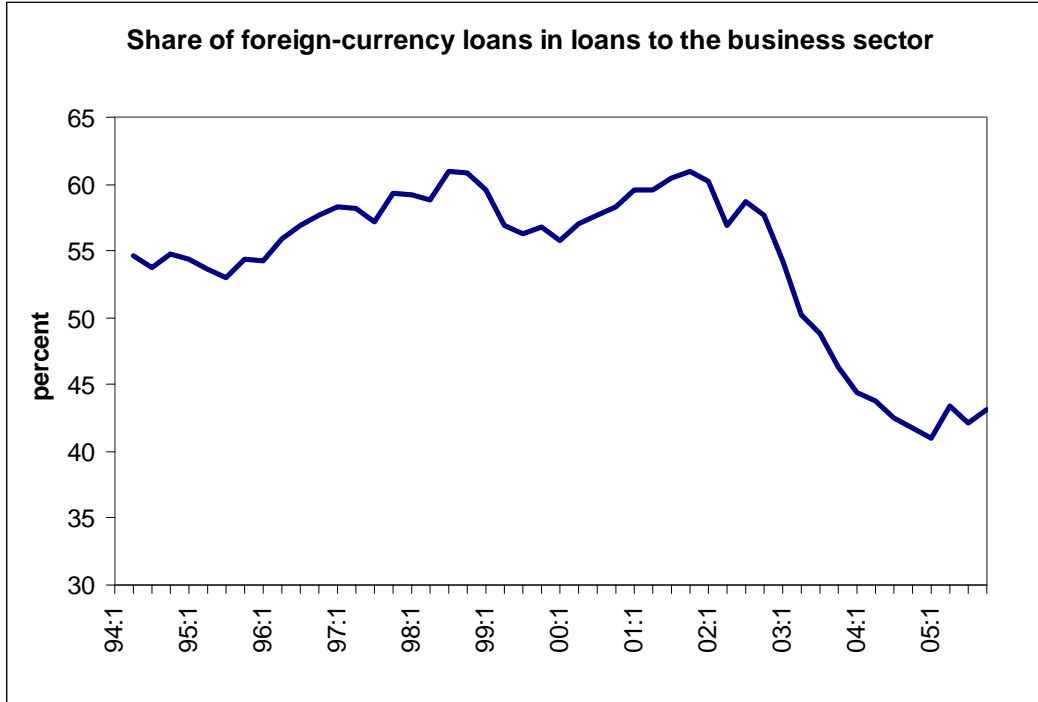
Figure 2: Share of foreign currency loans in total loans to non-financial business sector

construct the data using the bottom-up approach and verify the extent to which the violation of this assumption impacts the results. This issue is discussed further in section 7.

4.2 Sectoral exposures

Sectoral exposure data used in the empirical exercise consist of non-mortgage loans issued to individuals and others for business purposes. The available data span the period 1994Q1 to 2005Q4. The loans reported refer to loans in both domestic and foreign currencies.

Figure 2 shows the relative share of foreign currency loans in the credit portfolio of the Big Six Canadian banks over the sample period. Whereas the share clearly fluctuates, with a downward trend over the last three to four years, it is still relatively high, in the range of 40–45 per cent.[23]

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[23]The reason for this decrease is most likely in the appreciation of the Canadian dollar relative to the U.S. dollar over this period. This hypothesis is based on two considerations:
(i) the majority of loans are either to the United States or are denominated in U.S. dollars, and
(ii) whereas the book value of these loans is fixed, their value in domestic currency each period is calculated using the market exchange rate for that period.

It is clear that a comprehensive analysis of credit risks in the aggregate loans portfolio of Canadian banks should be based on total exposures, but this is far from straightforward. At the most basic level, one would have to trace these loans to each country and each industry of that country, and then model the sectoral default rates in foreign industries in a way that follows the procedure for the domestic sector. Depending on the geographical allocation of the loans, this would necessitate a multi-country model of macroeconomic variables. Simpler but potentially less satisfactory solutions, such as focusing on only the major geographical exposure, might be feasible.

In addition, the value of foreign exposures will depend on fluctuations in the foreign exchange rate. Whether this is of concern depends on the extent to which the banks hedge the foreign exchange (FX) risks. Ideally, one would like to have a measure of "FX-hedged" exposures. These data are, unfortunately, not available, and the extent to which the banks engage in hedging this risk, or how successful they are in hedging it, is not clear.

In light of these considerations, we focus on the domestic currency portion of the loans portfolio.[24,25] Figure 3 shows the evolution of the shares of domestic currency loans exposures by industry.

Of the four industries analyzed, the exposure to the manufacturing industry is the largest, with values in the range of 18–26 per cent and a slight upward trend over the sample period. Exposures to retail and construction have ranged between 10 and 15 per cent; the exposure to the accommodation industry has been stable, at around 5 per cent.

In the stress-testing exercise, one has to decide whether to use actual exposures at a particular point in time, or historical averages. We opt to use the actual exposures at the end of 2004Q3. However, given the nature of the data in our sample, the difference between using averages and actual exposures is, with the possible exception of manufacturing, not significant.

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[24]Virolainen (2004) does not discuss this issue, but, given the procedure used to construct the portfolio, it appears that the focus is on the domestic currency loans only. No information is provided to assess the extent to which the Finnish sector is engaged in foreign currency lending activities.

[25]If we want to investigate the impact of changes in exchange rates on credit risks, there are two channels:
(i) the impact of FX on default probabilities of the domestic sectors, and
(ii) the impact of FX on the exposures of foreign currency loans.
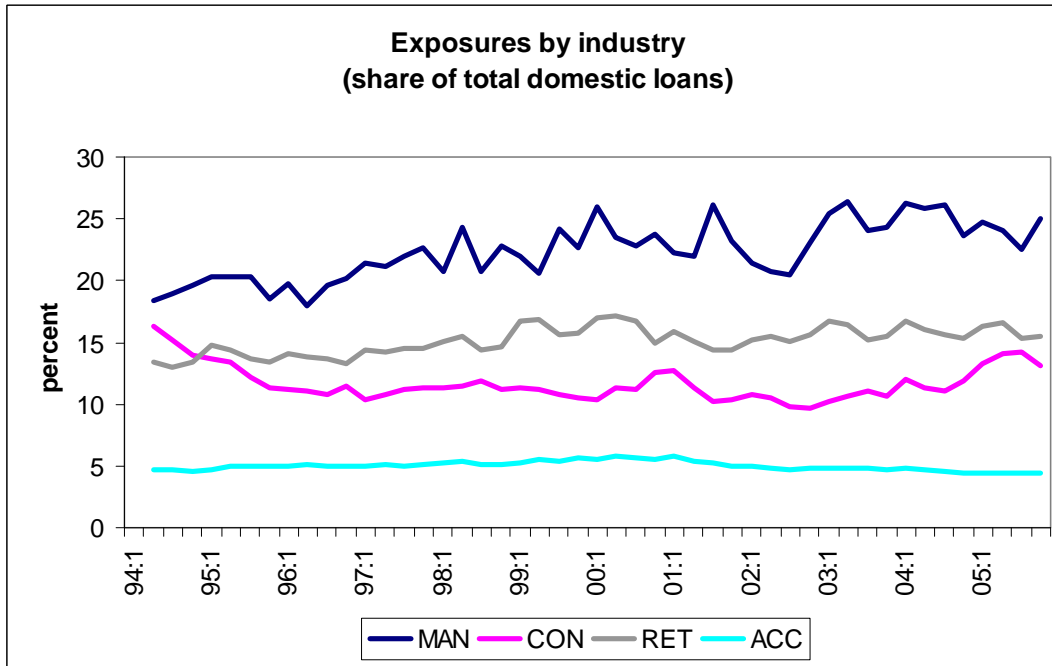By focusing on domestic currency loans, we ignore the second channel.

Figure 3: Share of loans to selected industries in total domestic currency loans

4.3 Macroeconomic variables

The macroeconomic variables selected for the exercise are divided into endogenous and exogenous, according to their role in the VAR model. The endogenous variables selected are Canada's GDP, which is converted into the GDP growth rate,[26] and Canada's real interest rate. These variables are typically present in macro stress-testing exercises regardless of the approach used.

In addition to these, we selected a set of exogenous variables that might constitute important sources of stress for the Canadian financial industry:

- the Bank of Canada commodity price index, converted into the growth rate

- the U.S. real GDP growth rate

- the U.S. real interest rate

The selection of exogenous variables reflects the close ties of the Canadian economy with

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[26]The growth rate for variable x is computed as $g_x = \log(x_t) - \log(x_{t-1})$.

the U.S. economy, and the importance of commodity prices in Canada. All series are quarterly, with the sample spanning the period 1987Q1 to 2005Q4. Various statistical tests suggest that the VAR(5) model is the best specification.

5. Macroeconomic Scenarios

The key component of stress tests is scenario analysis. The type of scenario selected depends on our priors regarding the possible causes of deviations from the normal operating conditions. Although there is a wide range of possibilities, we select the ones that are most pertinent to Canada. The scenarios considered are:

(i) a 20 per cent increase in the commodity price index,

(ii) a 4 per cent decrease in the growth rate of U.S. real GDP,

(iii) a 200 basis points increase in the U.S. real interest rate,

(iv) a combination scenario, which examines the effects of the joint occurrence of the first two scenarios.

The magnitudes of change in the scenario variable are set based on historical precedents, with the focus on infrequent but historically plausible events that would be deemed as deviations from normal operating conditions.

As described in section 3, the key assumptions used in our exercise are: (i) the recovery rate is constant and the value is set at 0.5, and (ii) the exposures are set at the value that prevailed in 2004Q3. Under these conditions, the properties of the loss distribution will depend on the stochastic properties of the default rates, given by equation (5). In each scenario, there are, in principle, two channels of impact on default probabilities:

(i) the direct impact of a change in the scenario variable on the default probability, and

(ii) an indirect impact, via the impact of the change in scenario variable on other macroeconomic variables.[27]

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[27]The AR-based approach described in section 3.2 does not allow for the presence of the second channel.

In our experiments, we assume that both channels operate under the scenarios involving domestic macroeconomic variables. The impact of foreign macroeconomic variables on default probabilities is assumed to occur via the impact of these variables on domestic macroeconomic variables.

In terms of implementation of scenarios, the question of the initial values for the macroeconomic variables arises. One could use either the values prevailing at a given point in time, or historical averages over past values. For the purposes of monitoring going forward, the former approach is preferable, since it assesses the impact of a scenario taking the most recent levels as starting points. The latter approach is useful when one is trying to assess potential losses under normal operating conditions. In implementing the scenarios, we opt for a combination of the approaches, and set the initial values for macroeconomic variables at their average value over 12 months preceding the stress event. Starting from these values, each scenario is implemented by setting the scenario variable to its postulated value, tracing its impact on other variables through VAR, and taking the realized values of the variables k periods ahead. The impact of different scenarios on the loss distribution reported in the next section is at the one-year horizon.

6. Results

The results reported in this section are based on the models given by (5) and (6), which are reproduced here for convenience:

$$\ln\left(\frac{\boldsymbol{\pi}_t^s}{1 - \boldsymbol{\pi}_t^s}\right) = \mathbf{X}_{t-l}\boldsymbol{\beta}^s + \mathbf{e}_t^s, \ s = 1, ..., S$$

and

$$\mathbf{X}_t = \boldsymbol{\phi}_1\mathbf{X}_{t-1} + ... + \boldsymbol{\phi}_p\mathbf{X}_{t-p} + \mathbf{u}_t.$$

In implementing each scenario, the paths for default probabilities under stress are arrived at in the manner described in section 3.1. Loss distributions are generated according to the procedure outlined in section 3.4. For each scenario, two sets of results are reported. The first set describes the impact of a given shock on the macro variables implied by the estimated VAR model; the second set shows the impact on loss distribution, expected and unexpected losses.

	GDP_{CAN}	Int. rate_{CAN}
MAN	-0.106	0.040
CON	-0.080	0.138
RET	-0.052	0.143
ACC	-0.048	0.116

Table 2: The impact of explanatory variables in logistic regressions

The coefficient values and diagnostics associated with the VAR model (6) are provided in Appendix B. The impact of macroeconomic variables on default probabilities in each industry is summarized in Table 2. The values in the table are computed by summing the value of coefficients associated with each macro variable at all estimated lags. Tests for the number of lags suggest the specification with four lags for each macro variable. The explanatory power of the resulting industry-level logistic regression is high, with R^2 0.48 in accommodation to 0.72 in the construction industry. The impact of macroeconomic variables on default probabilities is consistent across regressions and intuitively plausible. A decrease in GDP is associated with increases in default rates in all industries, with the greatest impact felt by manufacturing. Increases in domestic interest rates are accompanied by increases in default rates, with the greatest impact felt by the retail and construction industries.

6.1 Increase in commodity prices

An increase in commodity prices is a scenario that has been deemed quite relevant in light of recent developments in the world commodity markets. In our model, commodity prices are measured using the commodity price index. This index consists of energy and non-energy subcomponents. The weight of the former in the index is roughly one-third. More importantly, most of the volatility in the index is accounted for by the volatility of its energy subcomponent. In light of this, one may interpret the commodity-price increase in this scenario as being due to an increase in energy prices.

Commodity prices are assumed to be determined in the world markets, and, from the viewpoint of the Canadian economy, are taken as exogenous. This is reflected in the status of commodity prices as an exogenous variable in the VAR model. A change in commodity prices impacts the default probabilities in the domestic industries via its impact on domestic macroeconomic variables. The impact of a commodity-price shock on Canada's GDP and interest rate is summarized in Figures 5 and 6. For each variable, the figures show two paths—under no stress and under stress—over two years following the initial shock. The paths are generated using the VAR model (6). An increase in commodity prices results in a
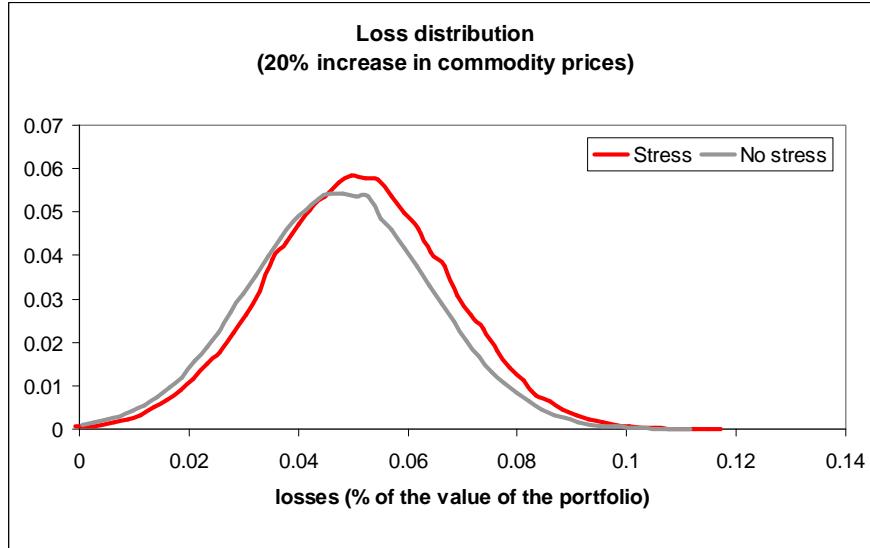
Figure 4: Loss distribution under scenario 1

moderate but somewhat persistent decrease in output, and an increase in the interest rate. The loss distribution under this scenario is shown in Figure 4. Statistics given in Table 3 suggest that the impact of this scenario on expected and unexpected losses is minor: expected losses increase by roughly 6 per cent, and the 99 per cent value-at-risk (VaR) increases by 3 per cent. The relatively low impact of this scenario on losses is not surprising, in light of the fact that the impact of a change in commodity prices on the macroeconomic variables that drive industry-level default rates is small.

Commodity-price shock	No stress	Stress
Expected loss	0.046	0.050
Unexpected loss (99% VaR)	0.085	0.088
Unexpected loss (99.9% VaR)	0.097	0.100

Table 3: Expected and unexpected losses under scenario 2

6.2 Decrease in the U.S. real GDP growth rate

A 4 per cent decrease in the U.S. GDP growth rate may seem an extreme scenario, but in light of the growth rates of the U.S. economy over the past few years it would amount to no more than a mild recession. Given that the initial value for the U.S. GDP growth rate (the average over 12 months preceding the stress scenario) was 3.47 per cent, the decrease considered here would bring that rate to -0.53 per cent. Smaller rates of decrease would represent merely a slowdown. The performance of the U.S. economy is taken as exogenously given, and its impact
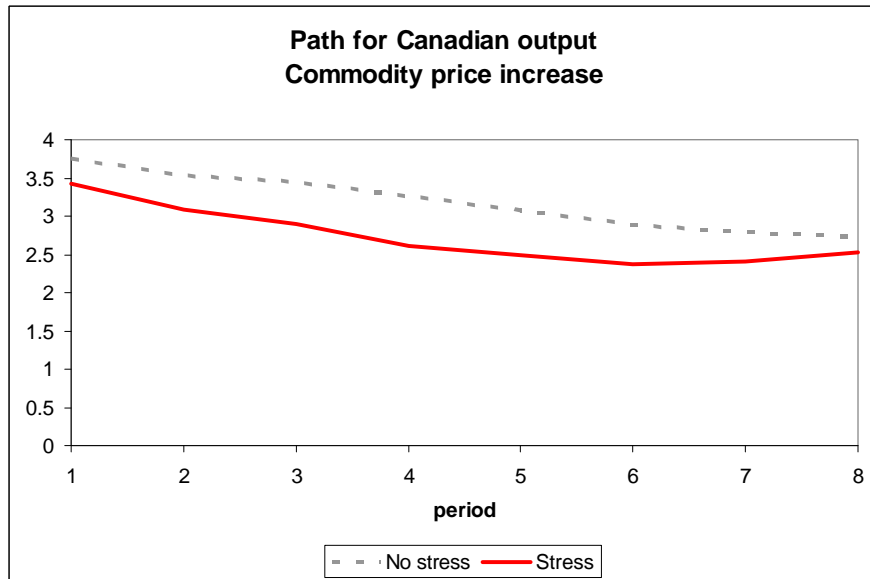
Figure 5: Impact of commodity prices on Canada's output
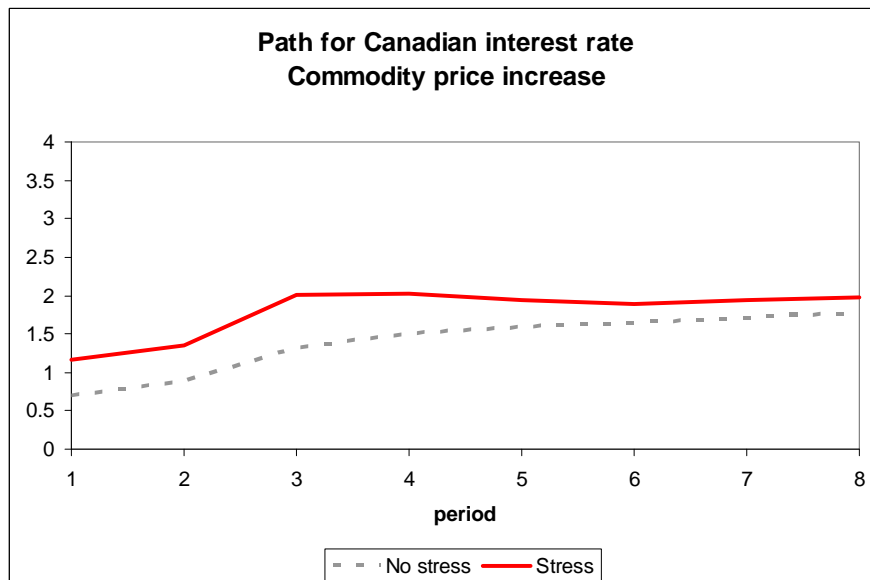
Figure 6: Impact of commodity prices on Canada's interest rate
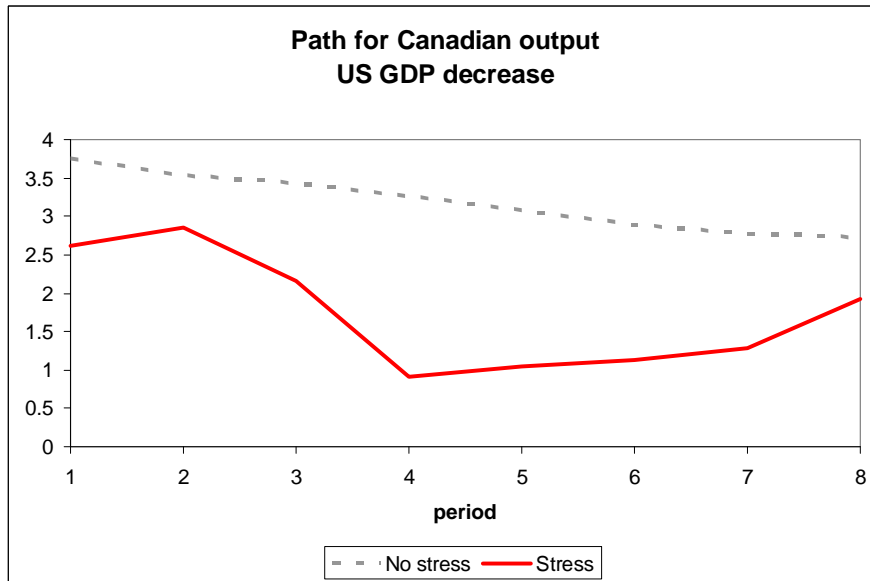
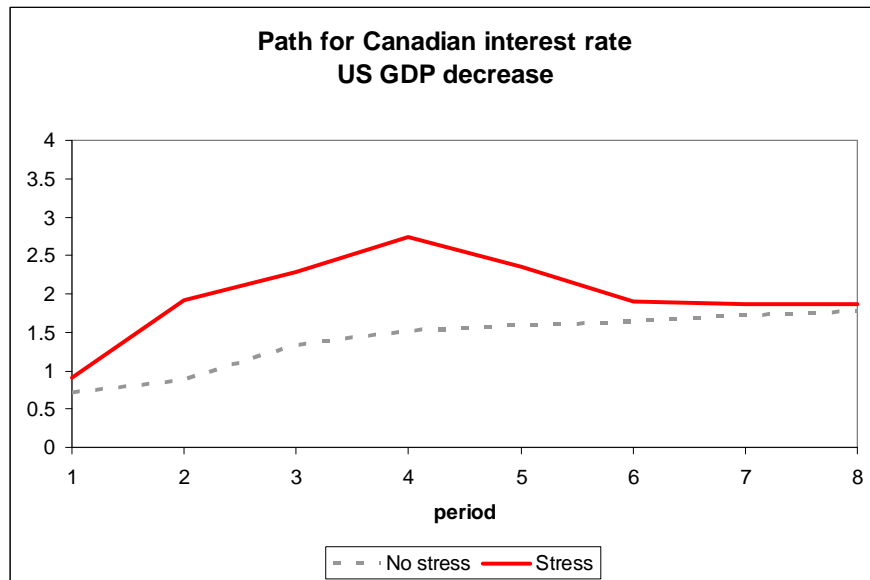Figure 7: Impact of a U.S. GDP shock on Canada's output

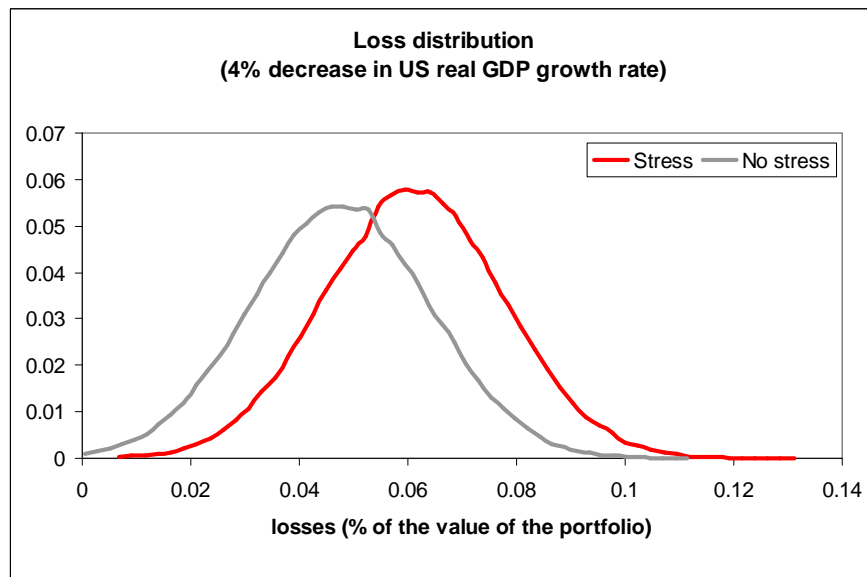Figure 8: Impact of a U.S. GDP shock on Canada's interest rate

Figure 9: Loss distribution under scenario 2

on Canadian industries is assumed to occur via domestic macroeconomic variables. The plausibility of this assumption depends on the sectors considered. For industries that export heavily to the United States, the direct impact of U.S. economic conditions is probably non-trivial. To the extent that this is the case, our assumption would lead to an underestimation of the overall impact on those industries.

Paths for Canada's output and interest rate following a decrease in the U.S. GDP growth rate are shown in Figures 7 and 8. A decrease in Canada's GDP growth rate is much larger than under the first scenario, although not sufficient to push the Canadian economy into a recession. The impact on Canada's interest rate is more pronounced than under the first scenario, but the impact is negligible after six quarters.

Given the relatively strong impact of the U.S. GDP shock on both Canadian macroeconomic variables, one would expect a more pronounced impact on losses. This is confirmed by the results shown in Figure 9 and Table 4. Loss distribution under stress is shifted to the right, with expected losses increasing by some 30 per cent relative to the no-stress case, and 99 per cent VaR increasing by 17 per cent from 0.085 to 0.099.
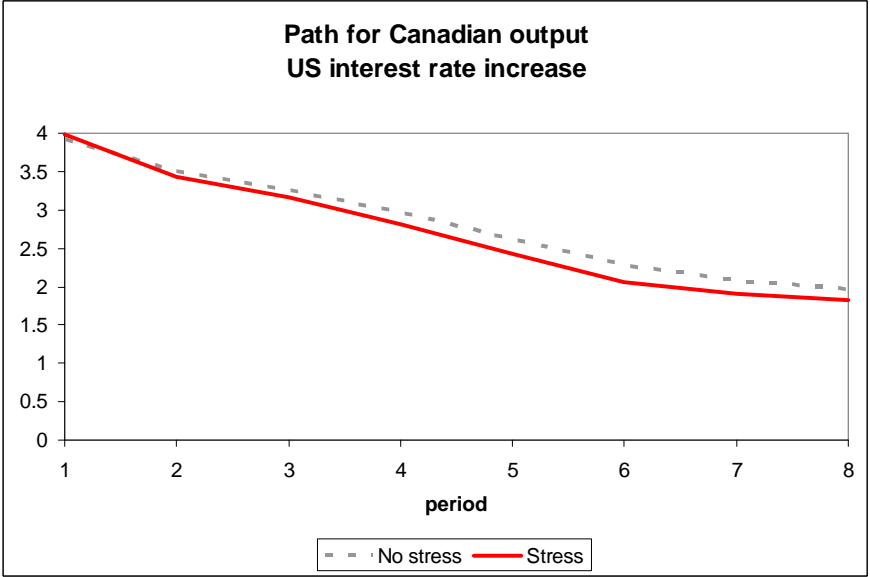
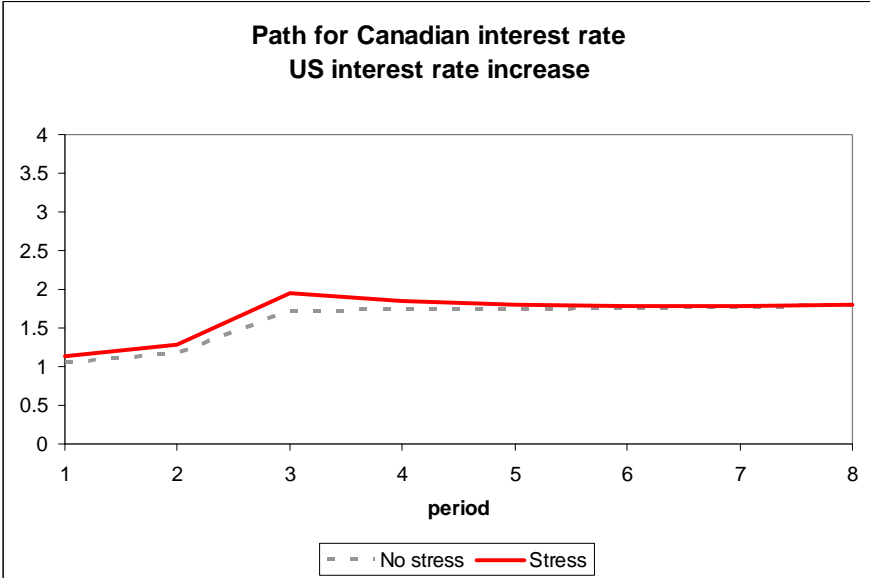Figure 10: Impact of a change in the U.S. interest rate on Canada's output

Figure 11: Impact of a change in the U.S. interest rate on Canada's interest rate

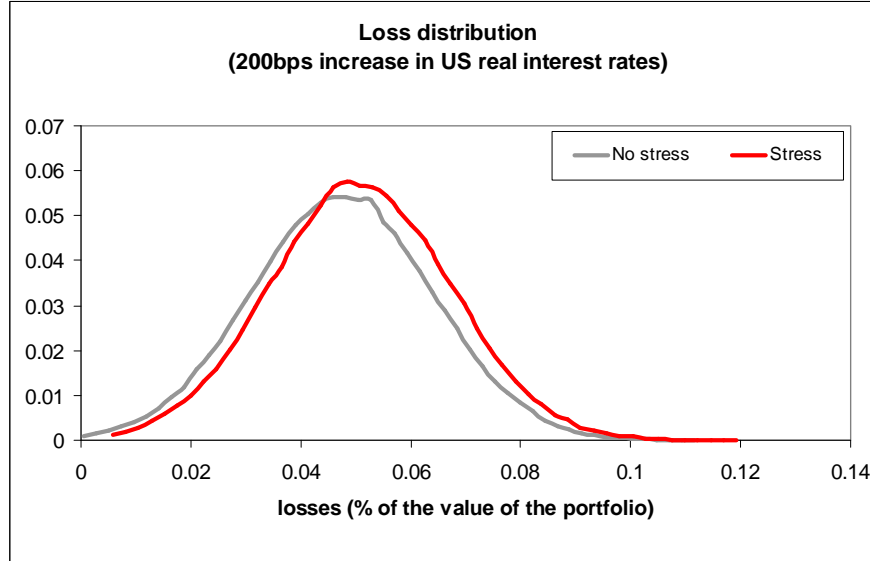| U.S. GDP shock	No stress	Stress
Expected loss	0.046	0.060
Unexpected loss (99% VaR)	0.085	0.099
Unexpected loss (99.9% VaR)	0.097	0.111

Table 4: Expected and unexpected losses under scenario 3

Figure 12: Loss distribution under scenario 3

6.3 Increases in U.S. real interest rates

In this scenario, we consider a tightening of 200 basis points in the U.S. real interest rate. The initial value of the real interest rate is 0.18 per cent (the average over the 12 months preceding the stress scenario), so that the tightening of the magnitude considered here would put the real interest rate at 2.18 per cent. This is a situation not unlike the one experienced over the past couple of years in the United States. As is clear from Figures 10 and 11, the impact on Canadian variables is minimal, and similar to the one under the first scenario. Figure 12 contains loss distributions for both scenarios; summary statistics are given in Table 5. As anticipated, the impact on expected and unexpected losses is small and similar to the one under the first scenario.

6.4 Combination scenario

The last scenario considered is a combination scenario, consisting of a simultaneous increase in commodity prices and a decline in the U.S. GDP growth rate. This is clearly the worst-
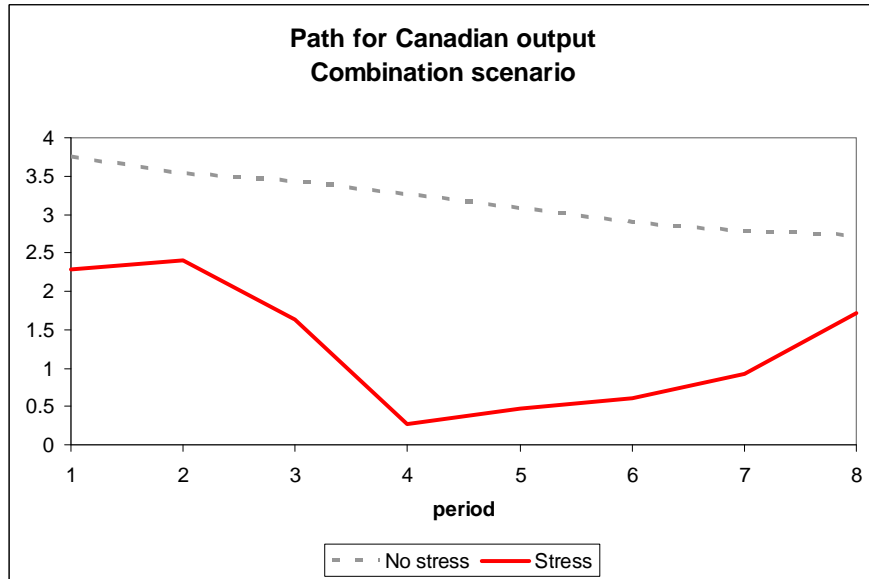
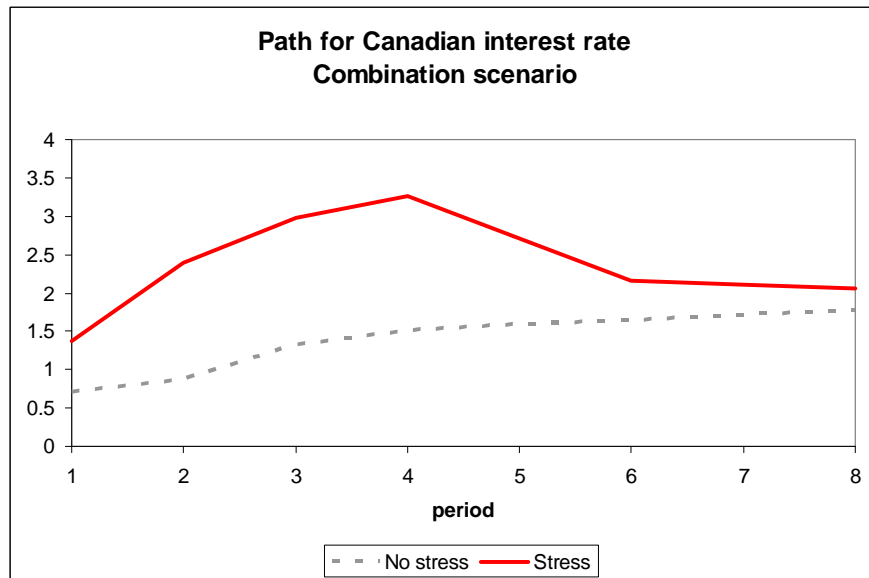Figure 13: Impact on output under combination scenario

Figure 14: Impact on interest rate under combination scenario

Interest rate shocks	No stress	Stress
Expected loss	0.046	0.049
Unexpected loss (99% VaR)	0.085	0.088
Unexpected loss (99.9% VaR)	0.097	0.102

Table 5: Expected and unexpected losses under scenario 3

case scenario of all considered in this paper, since it combines the negative impact of the two exogenous variables considered in scenarios 1 and 2. The magnitudes of the shocks are the same as in those scenarios.

The combination scenario may seem counterintuitive: everything else being the same, one would expect that a decline in the U.S. GDP growth rate would be accompanied by a decrease in world commodity prices, due to a lower demand. This need not occur if the growth in other parts of the world continues to be robust, not only sustaining the demand but even putting some upward pressure on world commodity prices.

The impact of this scenario on Canada's output and interest rates is summarized in Figures 13 and 14. Canada's output declines over two years following a decline in U.S. output. The slowdown is particularly sharp in the first year. The trend reverses in the second year, but the output is still well below the no-stress case at the end of the two-year period. The interest rate initially increases above the no-stress level, but the impact is minor after six quarters.

The loss distribution under this scenario is shown in Figure 15, and the summary statistics are reported in Table 6. Perhaps not surprisingly, the impact of this scenario on the loss distribution is significant. The expected losses increase by 41 per cent, and the unexpected losses by 22 per cent (99 per cent VaR). In terms of the overall impact, this is clearly the 'worst-case scenario.' Note that the overall impact in this scenario is more than the sum of the impacts of scenarios 1 and 2.

Multiple shocks	No stress	Stress
Expected loss	0.046	0.065
Unexpected loss (99% VaR)	0.085	0.104
Unexpected loss (99.9% VaR)	0.097	0.117

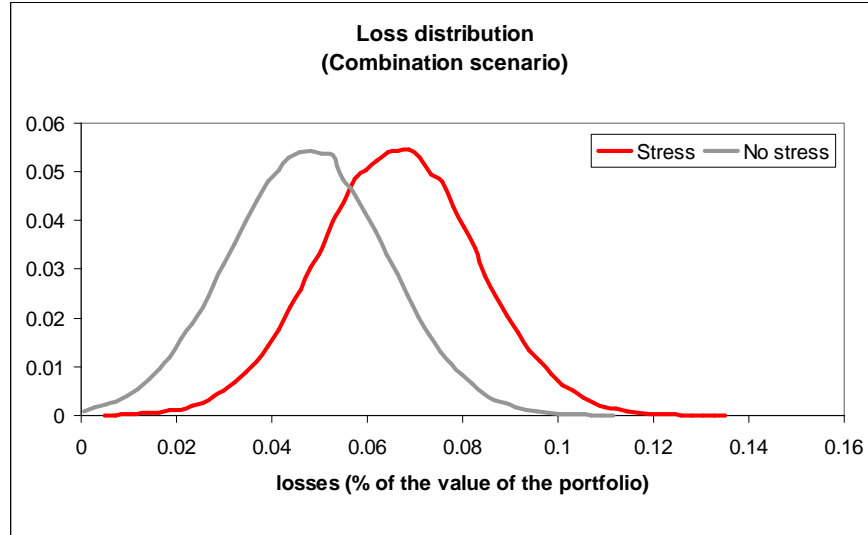Table 6: Expected and unexpected losses under scenario 4

Figure 15: Loss distribution under scenario 4

6.5 Discussion

An assessment of the results of the foregoing exercises depends on one's criteria. Have we uncovered sources of risk that could pose serious threats to the banking sector? That depends on the ability of the banking sector to deal with losses of the magnitudes that occur in these scenarios. One way to gauge the banking sector's ability to withstand the shocks is to look at the loan-loss provisions—the amounts set aside by the banks to cover losses. Loan-loss provisions are forward looking and reflect banks' views on the magnitudes of risk, if not its sources. Figure 16 depicts the data on average historical loan-loss provisions, together with the range of losses generated in our experiments. Both measures are expressed as a ratio to total loans (exposures). The rectangular area on the graph represents the range of losses that we are able to generate under no stress (lower bound) and under the worst-case stress scenario (upper bound). The horizontal line at 0.09 represents historical average loan-loss provisions over the sample period. As the graph shows, the results generated under various scenarios are close to the average loan-loss provisions. Furthermore, the relationship between historical loan-loss provisions and stress-induced losses is characterized by three distinct periods: (i) from 1996Q1 until 2000Q3, the actual loan-loss provisions are in the range of the stress-induced losses, (ii) from 2000Q3 to 2003Q1, the provisions exceed even the worst-case scenario losses, and (iii) during the most recent period, starting in 2003Q3, the provisions are below the expected losses.

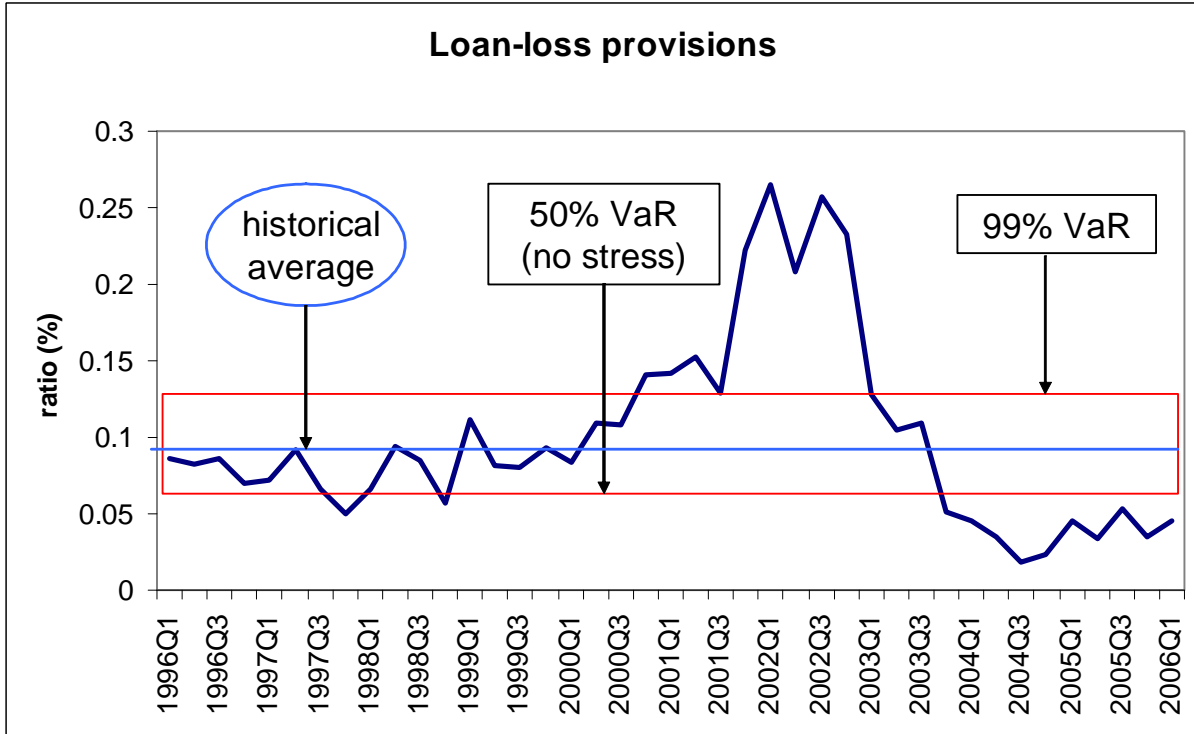One might be tempted to interpret these results as implying that the banks have been

Figure 16: Historical loan-loss provisions

underprovisioning over the past three years and that they overprovisioned in the period 2000Q3–2003Q1, but we urge extreme caution in this respect. The discrepancy between actual provisions and stress-induced losses may be a result of model deficiencies as much as of the problems with actual provisions. In fact, one could easily rationalize the apparent overprovisions by arguing that the result is due to the fact that, during the period in question, the banks were exposed to the high-tech sector, the collapse of which in early 2000 led to increases in provisions. Since our portfolio does not contain the high-tech sector, the model would not be able to capture this aspect of the banks' provisioning behaviour.

A simple solution would be to extend portfolio coverage, ultimately including all industries, all credit instruments, and all sources of risk, real or perceived, over the sample period. This is hardly feasible. The focus on a subset of the credit portfolio may be the only feasible way of performing these tests. It is, however, important to remember the implications of this strategy. First, the focus is a reflection of the view regarding the most likely source of potential vulnerabilities. The focus on the corporate loans part of the portfolio reflects the view that the problems are likely to originate there. Second, since we focus on one subset of the portfolio, a comparison with the historical loan-loss provisions may not be very informative,

and it is in general misleading to talk about over- or underprovisioning in this context.

Nonetheless, these types of exercises have a role to play in an overall assessment of the soundness of the banking sector, and, more broadly, the financial system. By focusing on different parts of the credit portfolio and performing these exercises, one may get a better sense of the relative soundness of different parts of the portfolio and possible sources of risk. In addition, stress tests under different scenarios may give us a better sense of the relative importance of different sources of risk, thus leading to a ranking of risks in terms of their impact on the losses. For example, our exercise implies that decreases in U.S. GDP pose the largest risks in terms of the losses on the loans portfolio. At the opposite extreme, this part of the banking sector portfolio displays very little sensitivity to interest rate movements, which indicates that interest rate developments may be relatively less important in terms of potential sources of risk.

7. Summary and Further Work

An assessment of financial system stability and potential vulnerabilities is a complex exercise, because of the nature of the entities comprising the financial system and the wide range of possible scenarios that may result in abnormal operating conditions. These complexities have been dealt with by focusing on a set of entities that are deemed to be "systemically important" for a given financial system, and by selecting as scenarios those rare but plausible events that have caused abnormal operating conditions in the past, or that could cause them in the future.

In this work, we have focused on one set of entities—the Canadian banking sector—and investigated losses in the domestic loans portfolio of this sector as a function of the changing circumstances of different industries in which these loans reside. These circumstances can be characterized using a variety of balance-sheet indicators, but we have opted for a different approach, by focusing on one summary measure—sectoral probabilities of default—and modelling this measure as a function of macroeconomic variables. This model has enabled us to assess the interrelationship between the macroeconomic environment and sectoral defaults in the past, as well as perform a series of tests under different possible scenarios. The scenarios selected reflect the sources of vulnerabilities that are commonly considered as 'typical' for Canada, rather than a reflection of the 'concerns of the moment.' The tools underlying the analysis in this paper are general and can be easily applied to other scenarios.

There are a number of possibilities for further work within the existing framework:

- Structural macroeconomic models. The interrelationships among the macro variables are summarized using a reduced-form statistical model. One would, ideally, like to have a structural model that would be flexible enough to incorporate all variables of interest. One possibility is to integrate this type of work with central banks' macroeconomic models.

- Sectoral-level simulations with a better measure of the average default probability in different sectors. We use bankruptcy rates as proxies for default probabilities. A different proxy could be obtained by inferring default probabilities from credit ratings.

- Firm-level simulations. Loss distributions in this paper are based on industry-level simulations of default probabilities. An alternative, used in Virolainen (2004), is to work with the microdata, and generate default events at the firm level. Results based on firm-level simulations of default probabilities are possibly closer to the experiences of individual banks. In addition, firm-level simulations would enable us to generate default events of a small number of companies with large loans, thus leading to the possibility of small-probability extreme losses.

- Enlarge the subset of the credit portfolio by including additional industries on the corporate loans side.

Other possibilities include enlarging the subset of the credit portfolio by including additional industries on the corporate loans side, including the personal sector loans, and analyzing different types of risk (such as market risk). We reiterate that the analysis presented herein is not a substitute for institution-level stress tests. The two approaches to stress testing—top-down and bottom-up—are best viewed as complements. The top-down approach can be used to identify macroeconomic scenarios that can lead to vulnerabilities. The impact on individual institutions must be assessed on a case-by-case basis.

A well-understood limitation of VaR-based stress-testing models is that they do not incorporate feedback effects. They simply provide implications of the 'first-round impact' of a scenario. The implicit assumptions are twofold:

(i) losses are assessed under the assumption that financial institutions do not do anything to alleviate the impact of shocks, and

(ii) losses in the financial sector have no impact on the macroeconomic variables.

These assumptions may not be problematic if one is interested in the very short run (e.g., one-day VaR), but they do matter from a macro-financial stability viewpoint. The first assumption implies that one cannot assess losses arising from situations in which exposures to common risk factors lead financial institutions to simultaneously adjust their portfolio positions, leading to an 'evaporation of liquidity' or 'liquidity black holes.'[28]   The second assumption precludes an assessment of the impact of phenomena such as stress-induced tightening of credit conditions on the financial position of businesses and households, and their ability to meet their financial obligations in the future.

A common feature of these examples is that a shock that initially has a local impact may result in the second-round systemic impact. Whether this type of assessment is within the domain of stress testing is debatable. It is clear that work along these lines is needed to deepen our understanding of systemic risk.

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[28]Persaud (2003) provides a good discussion of the issues.

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Appendix A: Data Definitions and Sources

- Sectoral Loan Exposures (1994Q2–2005Q4). Source: Office of the Superintendent of Financial Institutions (OSFI)

    - Manufacturing: Total domestic currency loans of the Big Six Canadian banks to the manufacturing sector.

    - Construction: Total domestic currency loans of the Big Six Canadian banks to the construction sector.

    - Retail: Total domestic currency loans of the Big Six Canadian banks to the retail sector.

    - Accommodation: Total domestic currency loans of the Big Six Canadian banks to the accommodation sector.

- Sectoral Bankruptcy Rates (1987Q1–2005Q4). Source: Office of the Superintendent of Bankruptcies (OSB) and Statistics Canada.

    - Manufacturing: Total number of bankruptcies in the manufacturing sector divided by the total number of establishments in that sector.

    - Construction: Total number of bankruptcies in the construction sector divided by the total number of establishments in that sector.

    - Retail: Total number of bankruptcies in the retail sector divided by the total number of establishments in that sector.

    - Accommodation: Total number of bankruptcies in the accomodation sector divided by the total number of establishments in that sector.

- Loan-Loss Provisions (1996Q1–2006Q1): Percentage of total loans set aside for provisioning by the Big Six Canadian banks. Source: OSFI.

- Macroeconomic Variables (1987Q1–2005Q4). Source: CANSIM.

    - Real GDP Growth Rate: Annualized quarterly growth rate of Canadian (U.S.) real GDP.

    - Real Interest Rate: 3-month Canadian (U.S.) real interest rate.

    - Commodity Price Index: Commodity Price Index (U.S. dollars); 1982–90 = 100.

Appendix B: VAR Estimation Results

VAR Lag Order Selection Criteria
Endogenous variables: RR GDPGR
Exogenous variables: C
Sample: 1987Q1 2005Q4
Included observations: 69

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-302.4456	NA	23.30714	8.824510	8.889266	8.850201
1	-167.1491	258.8280	0.518493	5.018815	5.213085	5.095889
2	-152.0279	28.05091	0.375768	4.696462	5.020245*	4.824918*
3	-150.7361	2.321578	0.406816	4.774959	5.228256	4.954797
4	-150.2089	0.916823	0.450614	4.875621	5.458431	5.106841
5	-138.9303	18.96123*	0.365817	4.664645	5.376969	4.947248
6	-133.7348	8.433150	0.354642*	4.629995*	5.471832	4.963980
7	-133.4297	0.477547	0.396706	4.737094	5.708444	5.122461

* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level) FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
H0: residuals are multivariate normal
Sample: 1987Q1 2005Q4
Included observations: 71

Component	Skewness	Chi-sq	df	Prob.
1	0.134286	0.213387	1	0.6441
2	-0.258878	0.793044	1	0.3732
Joint		1.006432	2	0.6046

Component	Kurtosis	Chi-sq	df	Prob.
1	2.478377	0.804936	1	0.3696
2	2.135908	2.208856	1	0.1372
Joint		3.013792	2	0.2216

Component		Jarque-Bera	df	Prob.
1		1.018323	2	0.6010
2		3.001901	2	0.2229
Joint		4.020224	4	0.4033

VAR Residual Serial Correlation LM Tests
H0: no serial correlation at lag order h
Sample: 1987Q1 2005Q4
Included observations: 71

Lags	LM-Stat	Prob
1	9.096398	0.0587
2	0.589712	0.9642
3	10.06609	0.0393
4	7.607204	0.1071
5	3.341550	0.5024
6	4.540604	0.3378
7	13.37960	0.0096
8	2.797702	0.5922
9	3.620223	0.4598
10	9.090933	0.0589
11	8.999599	0.0611
12	1.144758	0.8871
13	8.380324	0.0786
14	2.770866	0.5969
15	3.759201	0.4396
16	1.543449	0.8189

Probs from chi-square with 4 df.

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