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STRESSES IN ADHESIVELY BONDED JOINTS:
A CLOSED-FORM SOLUTION

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STRESSES IN ADHESIVELY BONDED JOINTS: A CLOSED-FORM SOLUTION(*)

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Abstract

In this paper the general plane strain problem of adhesively bonded structures which consist of two different orthotropic adherends is considered. Assuming that the thicknesses of the adherends are constant and are small in relation to the lateral dimensions of the bonded region, the adherends are treated as plates. Also, assuming that the thickness of the adhesive is small compared to that of the adherends, the thickness variation of the stresses in the adhesive layer is neglected. However, the transverse shear effects in the adherends and the in-plane normal strain in the adhesive are taken into account. The problem is reduced to a system of differential equations for the adhesive stresses which is solved in closed form. A single lap joint and a stiffened plate under various loading conditions are considered as examples. To verify the basic trend of the solutions obtained from the plate theory and to give some idea about the validity of the plate assumption itself, a sample problem is solved by using the finite element method and by treating the adherends and the adhesive as elastic continua. It is found that the plate theory used in the analysis not only predicts the correct trend for the adhesive stresses but also gives rather surprisingly accurate results. The solution is obtained by assuming linear stress-strain relations for the adhesive. In the Appendix the problem is formulated by using a nonlinear material for the adhesive and by following two different approaches.

1. Introduction

Generally an adhesively bonded structure consists of three components of different mechanical properties, namely the two adherends and the adhesive layer. Even though under most operating loads and environmental conditions the adherends may behave in a linearly elastic manner,

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under relatively severe loading and temperature the adhesive may exhibit viscoelastic and/or nonlinear behavior. However, because of the material nonhomogeneity and the geometric complexity of the medium, the exact analytical treatment of the related structural problem is hopelessly complicated. The existing analytical solutions have, therefore, been carried out under certain simplifying assumptions in formulating the problem. The primary factors influencing the choice of a particular idealized model for the adhesive and the adherends appear to be the adhesive-to-adherend and adherend-to-adherend thickness ratios and the ratio of the adherend thickness to the lateral joint dimensions. Thus, in [1,2] the adhesive is neglected and the adherends are treated as membranes, in [3,4] it is assumed that the adherends are membranes and the adhesive is a shear spring, in [5-8] the adherends are assumed to be plates and the adhesive a tension-shear spring, and in [9-11] one or both adherends are treated as an elastic continuum. One should, of course, add that by using the finite element method, it is possible to treat all three components of the adhesively bonded structure as elastic continua.

In this paper it is assumed that the thickness of the adhesive is small compared to the thicknesses of the adherends which, in turn, are small compared to the length of the joint. Thus, the problem is formulated under the following simplifying assumptions: (i) the adherends are orthotropic plates for which a transverse shear theory is used, (ii) the adhesive is an elastic layer in which the thickness variation of stresses is neglected, and (iii) the bonded structure is in a state of plane strain, i.e., $\varepsilon_z = 0$ for the entire structure (Figure 1). The main purpose of the paper is to show that under the stated assumptions (a) the adhesively bonded joint problems can be solved in closed-form and, (b) by comparing the solution with that of the finite element method, the analytical results thus found are quite realistic. In formulating the adhesive a slight improvement is made over the standard tension-shear spring model used in [5-8] by taking into account the effect of the average in-plane strain ε_x (Figure 1). It should be pointed out that taking into account any material nonlinearity for the adhesive layer in solving the problem appears to be quite difficult. The Appendix shows the formulation of the problem by using two different approaches to account for the material nonlinearity. In both cases the problem is reduced to a system of nonlinear differential equations. The questions to be resolved in this regard, however, are (a) whether the lengthy and tedious effort necessary to solve the complicated nonlinear problem is justified for the type of structural problems under consideration, and (b) whether a linear (or linearized) rheological behavior of the adhesive would not be a more important factor than the nonlinear elastic behavior affecting the stress distribution and failure in bonded structures [12].

2. Formulation of the Problem

Referring to Figure 1 for notation, the equilibrium equations for the adherends which are assumed to be plates under cylindrical bending and normal membrane loading may be expressed as

$$\frac{dN_{1x}}{dx} = \tau , \qquad \frac{dN_{2x}}{dx} = - \tau$$

$$\frac{dQ_{1x}}{dx} = \sigma , \qquad \frac{dQ_{2x}}{dx} = - \sigma$$

$$\frac{dM_{1x}}{dx} = Q_{1x} - \frac{h_1+h_0}{2}\tau , \qquad \frac{dM_{2x}}{dx} = Q_{2x} - \frac{h_2+h_0}{2}\tau , \qquad \text{(1a-f)}$$

where N_{ix}, Q_{ix}, M_{ix}, $(i=1,2)$ are the stress and moment resultants and $\tau = \tau_{xy}$ and $\sigma = \sigma_y$ are the shear and the normal stress in the adhesive. Note that in the problem under consideration $N_{xz} = 0 = M_{xz}$ and, aside from the shear resultants Q_{ix} acting on the boundaries, no other external transverse loads are applied to the adherends. The stress and moment resultants are related to the x,y-components of the displacements u_i, v_i and to the rotation β_{ix}, $(i=1,2)$ by

$$\frac{du_1}{dx} = C_1 N_{1x} , \qquad \frac{du_2}{dx} = C_2 N_{2x} ,$$

$$\frac{dv_1}{dx} + \beta_{1x} = Q_{1x}/B_1 , \qquad \frac{dv_2}{dx} + \beta_{2x} = Q_{2x}/B_2 ,$$

$$\frac{d\beta_{1x}}{dx} = D_1 M_{1x} , \qquad \frac{d\beta_{2x}}{dx} = D_2 M_{2x} , \tag{2a-f}$$

where

$$C_i = \frac{1-\nu_{ix}\nu_{iz}}{h_i E_{ix}} , \; B_i = \frac{5}{6} h_i G_{ixy} , \; D_i = \frac{12(1-\nu_{ix}\nu_{iz})}{h_i^3 E_{ix}} . \tag{3a-c}$$

Assuming that y-dependence of the strains ε_x, ε_y, and γ_{xy} in the adhesive is negligible; from simple kinematical considerations we obtain

$$\varepsilon_y = (v_1 - v_2)/h_o ,$$

$$\gamma_{xy} = (u_1 - \frac{h_1}{2}\beta_{1x} - u_2 - \frac{h_2}{2}\beta_{2x})/h_o ,$$

$$\varepsilon_x = (\frac{du_1}{dx} - \frac{h_1}{2}\frac{d\beta_{1x}}{dx} + \frac{du_2}{dx} + \frac{h_2}{2}\frac{d\beta_{2x}}{dx})/2 , \tag{4a-c}$$

where h_o, h_1, and h_2 are the thicknesses of the adhesive and the adherends 1 and 2, respectively. Since only the plane strain problem is considered, the remaining strains in the structure are zero. If E, ν, and G denote the elastic constants of the adhesive, its stress strain relations may be expressed as

$$\varepsilon_y = \frac{1-\nu-2\nu^2}{E(1-\nu)}\sigma - \frac{\nu}{1-\nu}\varepsilon_x , \quad \gamma_{xy} = \frac{\tau}{G} . \tag{5a,b}$$

Through simple eliminations equations (1), (2), (4) and (5) may be reduced to the following system of differential equations for the adhesive stresses $\sigma_y = \sigma(x)$ and $\tau_{xy} = \tau(x)$:

$$\frac{d^3\tau}{dx^3} + \alpha_1 \frac{d\tau}{dx} + \alpha_2\sigma = 0,$$

$$\frac{d^4\sigma}{dx^4} + \beta_1 \frac{d^2\sigma}{dx^2} + \beta_2\sigma + \beta_3 \frac{d^3\tau}{dx^3} + \beta_4 \frac{d\tau}{dx} = 0 \ , \qquad (6a,b)$$

where

$$\alpha_1 = - \frac{G}{h_o} (C_1 + C_2 + \frac{h_1(h_o+h_1)}{4} D_1 + \frac{h_2(h_o+h_2)}{4} D_2) \ ,$$

$$\alpha_2 = \frac{G}{h_o} (\frac{h_1D_1}{2} - \frac{h_2D_2}{2}) \ ,$$

$$\beta_1 = \frac{E(1-\nu)}{(1-\nu-2\nu^2)} [\frac{\nu}{2(1-\nu)} (\frac{h_1D_1}{2} + \frac{h_2D_2}{2}) - \frac{1}{h_o} (\frac{1}{B_1} + \frac{1}{B_2})],$$

$$\beta_2 = \frac{E(1-\nu)(D_1+D_2)}{h_o(1-\nu-2\nu^2)} \ ,$$

$$\beta_3 = - \frac{\nu E}{2(1-\nu-2\nu^2)} [C_1 - C_2 + \frac{h_1(h_o+h_1)}{4} D_1 - \frac{h_2(h_o+h_2)}{4} D_2] \ ,$$

$$\beta_4 = - \frac{(1-\nu)E}{2h_o(1-\nu-2\nu^2)} [(h_1+h_o)D_1 - (h_2+h_o)D_2] \ . \qquad (7)$$

Eliminating σ, (6) may further be reduced to

$$\frac{d^7\tau}{dx^7} + a_5 \frac{d^5\tau}{dx^5} + a_3 \frac{d^3\tau}{dx^3} + a_1 \frac{d\tau}{dx} = 0 \ , \qquad (8)$$

where

$$a_5 = \alpha_1 + \beta_1, \ a_3 = \alpha_1\beta_1 + \beta_2 - \alpha_2\beta_3, \ a_1 = \alpha_1\beta_2 - \alpha_2\beta_4. \qquad (9)$$

At the boundaries of the adherends $x = \mp \ell$, the following conditions may be prescribed

$$N_{1x}(\ell) = N_1(\ell) , \qquad N_{2x}(\ell) = N_2(\ell) , \qquad N_{2x}(-\ell) = N_2(-\ell) ,$$

$$M_{1x}(\ell) = M_1(\ell) , \qquad M_{2x}(\ell) = M_2(\ell) , \qquad M_{2x}(-\ell) = M_2(-\ell),$$

$$Q_{1x}(\ell) = Q_1(\ell) , \qquad Q_{2x}(\ell) = Q_2(\ell) , \qquad Q_{2x}(-\ell) = Q_2(-\ell) , \tag{10}$$

where $N_i(\mp\ell)$, $Q_i(\mp\ell)$, and $M_i(\mp\ell)$, $(i=1,2)$, are known constants. Note that at $x=-\ell$ the boundary conditions for the adherend 1 need not be prescribed as they must be such that the gross static equilibrium conditions of the composite plate are satisfied.

3. General Solution

The differential equation (8) is valid for any composite plate in $-\ell < x < \ell$ which consists of two layers and is subjected to cylindrical bending,and membrane and transverse shear loading given by (10). Hence, it may be used to solve any bonded joint problem with two adherends. Looking for a solution of the form $\tau(x) = e^{mx}$, the characteristic equation of the problem is found to be

$$m^7 + a_5 m^5 + a_3 m^3 + a_1 m = 0 \quad . \tag{11}$$

From (11) it is seen that

$$m_1 = 0 , \tag{12}$$

$$\theta^3 + a_5\theta^2 + a_3\theta + a_1 = 0 \tag{13}$$

where $m^2 = \theta$. Let θ_1, θ_2, θ_3 be the roots of (13) and let

$$\phi_j = \theta_j^{1/2} , \; (j=1,2,3). \tag{14}$$

Then, the roots of (11) may be written as

$$m_1 = 0, \; m_2 = \phi_1, \; m_4 = \phi_2, \; m_6 = \phi_3 , \tag{15}$$
$$m_3 = -\phi_1, \; m_5 = -\phi_2, \; m_7 = -\phi_3 \; ,$$

where ϕ_1^2 is real and ϕ_2^2 and ϕ_3^2 are, in general, complex conjugates. The general solution of (8) may now be expressed as follows:

$$\tau(x) = K_o + K_1 \sinh \phi_1 x + K_2 \cosh \phi_1 x + K_3 \sinh \phi_2 x$$
$$+ K_4 \cosh \phi_2 x + K_5 \sinh \phi_3 x + K_6 \cosh \phi_3 x. \qquad (16)$$

where $K_o, \ldots, K_6$ are the integration constants. From (6) and (16) it then follows that

$$\sigma(x) = -\frac{1}{\alpha_2}[(\alpha_1\phi_1 + \phi_1^3)(K_1 \cosh \phi_1 x + K_2 \sinh \phi_1 x)$$
$$+ (\alpha_1\phi_2 + \phi_2^3)(K_3 \cosh \phi_2 x + K_4 \sinh \phi_2 x)$$
$$+ (\alpha_1\phi_3 + \phi_3^3)(K_5 \cosh \phi_3 x + K_6 \sinh \phi_3 x)]. \qquad (17)$$

It can be shown that expressed in terms of σ and τ alone the boundary conditions (10) are equivalent to the following relations:

$$\int_{-\ell}^{\ell} \tau \, dx = N_2(-\ell) - N_2(\ell), \qquad (18)$$

$$\int_{-\ell}^{\ell} \sigma \, dx = Q_2(-\ell) - Q_2(\ell), \qquad (19)$$

$$\int_{-\ell}^{\ell} \sigma \, x \, dx = M_2(\ell) - M_2(-\ell) - \ell Q_2(\ell) - \ell Q_2(-\ell)$$
$$- \frac{h_o+h_2}{2}[N_2(\ell) - N_2(-\ell)], \qquad (20)$$

$$\frac{d}{dx}\tau(\ell) = \frac{G}{h_o}[C_1N_1(\ell) - C_2N_2(\ell) - \frac{h_1D_1}{2}M_1(\ell) - \frac{h_2D_2}{2}M_2(\ell)], \qquad (21)$$

$$\frac{d^2}{dx^2}\,\tau(\ell) = \frac{G}{h_o}\,\{[C_1+C_2 + \frac{h_1(h_o+h_1)}{4}\,D_1 + \frac{h_2(h_o+h_2)}{4}\,D_2]\tau(\ell)$$

$$- \frac{h_1D_1}{2}\,Q_1(\ell) - \frac{h_2D_2}{2}\,Q_2\,(\ell)\}\ , \qquad (22)$$

$$[\frac{1}{h_o}\,(\frac{1}{B_1} + \frac{1}{B_2}) - \frac{\nu}{4(1-\nu)}\,(h_1D_1 + h_2D_2)]\sigma(\ell)$$

$$- \frac{1-\nu-2\nu^2}{E(1-\nu)}\,\frac{d^2}{dx^2}\,\sigma(\ell) + \frac{\nu}{2(1-\nu)}\,[C_1 - C_2 + \frac{h_1(h_o+h_1)}{4}\,D_1$$

$$- \frac{h_2(h_o+h_2)}{4}\,D_2]\,\frac{d}{dx}\,\tau(\ell) = \frac{D_1}{h_o}\,M_1(\ell) - \frac{D_2}{h_o}\,M_2(\ell)\ , \qquad (23)$$

$$[\frac{1}{h_o}\,(\frac{1}{B_1} + \frac{1}{B_2}) - \frac{\nu}{4(1-\nu)}\,(h_1D_1 + h_2D_2)]\,\frac{d}{dx}\,\sigma(\ell)$$

$$- \frac{1-\nu-2\nu^2}{E(1-\nu)}\,\frac{d^3}{dx^3}\,\sigma(\ell) + \frac{1}{2h_o}\,[(h_o+h_1)D_1 - (h_o+h_2)D_2]\tau(\ell)$$

$$+ \frac{\nu}{2(1-\nu)}\,[C_1-C_2 + \frac{h_1(h_o+h_1)}{4}\,D_1 - \frac{h_2(h_o+h_2)}{4}\,D_2]\,\frac{d^2}{dx^2}\,\tau(\ell)$$

$$= \frac{D_1}{h_o}\,Q_1(\ell) - \frac{D_2}{h_o}\,Q_2(\ell)\ . \qquad (24)$$

Technically, the integration constants $K_o,\ldots,K_6$ may be determined by substituting from (16) and (17) into (18-24), which become

$$2\ell K_o + \frac{2K_2}{\phi_1}\,\sinh\,\phi_1\ell + \frac{2K_4}{\phi_2}\,\sinh\,\phi_2\ell + \frac{2K_6}{\phi_3}\,\sinh\,\phi_3\ell$$

$$= N_2(-\ell) - N_2(\ell)\ , \qquad (25)$$

$$- \frac{2}{\alpha_2}\,[K_1(\alpha_1+\phi_1{}^2)\,\sinh\,\phi_1\ell + K_3(\alpha_1+\phi_2{}^2)\,\sinh\,\phi_2\ell$$

$$+ K_5(\alpha_1+\phi_3{}^2)\,\sinh\,\phi_3\ell] = Q_2(-\ell) - Q_2(\ell)\ , \qquad (26)$$

$$- \frac{2}{\alpha_2} [K_2(\alpha_1+\phi_1{}^2)(\ell \cosh \phi_1\ell - \frac{1}{\phi_1} \sinh \phi_1\ell)$$

$$+ K_4 (\alpha_1+\phi_2{}^2)(\ell \cosh \phi_2\ell - \frac{1}{\phi_2} \sinh \phi_2\ell)$$

$$+ K_6(\alpha_1+\phi_3{}^2)(\ell \cosh \phi_3\ell - \frac{1}{\phi_3} \sinh \phi_3\ell)] = M_2(\ell)$$

$$- M_2(-\ell) - \ell Q_2(\ell) - \ell Q_2(-\ell) - \frac{h_o+h_2}{2} [N_2(\ell) - N_2(-\ell)], \quad (27)$$

$$K_1\phi_1 \cosh \phi_1\ell + K_2\phi_1 \sinh \phi_1\ell + K_3\phi_2 \cosh \phi_2\ell + K_4\phi_2 \sinh \phi_2\ell$$

$$+ K_5\phi_3 \cosh \phi_3\ell + K_6\phi_3 \sinh \phi_3\ell = \frac{G}{h_o} [C_1N_1(\ell) - C_2N_2(\ell)$$

$$- \frac{h_1D_1}{2} M_1(\ell) - \frac{h_2D_2}{2} M_2(\ell)] , \quad (28)$$

$$K_o\alpha_1 + (\alpha_1+\phi_1{}^2)(K_1\sinh\phi_1\ell + K_2\cosh\phi_1\ell) + (\alpha_1+\phi_2{}^2)(K_3\sinh\phi_2\ell$$

$$+ K_4\cosh\phi_2\ell) + (\alpha_1+\phi_3{}^2)(K_5\sinh\phi_3\ell + K_6\cosh\phi_3\ell)$$

$$= - \frac{G}{2h_o} [h_1D_1Q_1(\ell) + h_2D_2Q_2(\ell)], \quad (29)$$

$$[(\alpha_1\phi_1+\phi_1{}^3)(\lambda_1+\lambda_2\phi_1{}^2) - \alpha_2\lambda_3\phi_1](K_1 \cosh \phi_1\ell + K_2 \sinh \phi_1\ell)$$

$$+ [(\alpha_1\phi_2+\phi_2{}^3)(\lambda_1+\lambda_2\phi_2{}^2) - \alpha_2\lambda_3\phi_2](K_3\cosh \phi_2\ell + K_4 \sinh \phi_2\ell)$$

$$+ [(\alpha_1\phi_3+\phi_3{}^3)(\lambda_1+\lambda_2\phi_3{}^2) - \alpha_2\lambda_3\phi_3](K_5 \cosh \phi_3\ell + K_6 \sinh \phi_3\ell)$$

$$= \frac{\alpha_2}{h_o} [D_2M_2(\ell) - D_1M_1(\ell)] , \quad (30)$$

$$\lambda_4 K_o - [\frac{\alpha_1\phi_1+\phi_1^3}{\alpha_2}(\lambda_1\phi_1+\lambda_2\phi_1^3) - \lambda_4-\lambda_3\phi_1^2](K_1\sinh\phi_1\ell+K_2\cosh\phi_1\ell)$$

$$- [\frac{\alpha_1\phi_2+\phi_2^3}{\alpha_2}(\lambda_1\phi_2+\lambda_2\phi_2^3)-\lambda_4-\lambda_3\phi_2^2](K_3\sinh\phi_2\ell+K_4\cosh\phi_2\ell)$$

$$- [\frac{\alpha_1\phi_3+\phi_3^3}{\alpha_2}(\lambda_1\phi_3+\lambda_2\phi_3^3)-\lambda_4-\lambda_3\phi_3^2](K_5\sinh\phi_3\ell+K_6\cosh\phi_3\ell)$$

$$= \frac{1}{h_o}[D_1Q_1(\ell) - D_2Q_2(\ell)] , \tag{31}$$

where

$$\lambda_1 = \frac{1}{h_o}(\frac{1}{B_1}+\frac{1}{B_2}) - \frac{\nu}{4(1-\nu)}(h_1D_1 + h_2D_2) ,$$

$$\lambda_2 = -\frac{1-\nu-2\nu^2}{E(1-\nu)} ,$$

$$\lambda_3 = \frac{\nu}{2(1-\nu)}[C_1 - C_2 + \frac{h_1(h_o+h_1)}{4}D_1 - \frac{h_2(h_o+h_2)}{4}D_2],$$

$$\lambda_4 = \frac{1}{2h_o}[(h_o + h_1)D_1 - (h_o + h_2)D_2] . \tag{32}$$

After determining $K_o,\ldots,K_6$ from (25-31) the adhesive stresses τ and σ may be obtained from (16) and (17), and the stress and moment resultants from (1).

From equations (7) and (6) it is seen that if the adherends have the same thickness and the same material constants, then $\alpha_2 = \beta_3 = \beta_4 = 0$ and the differential equations uncouple. Hence, the problem becomes considerably simpler. The elastic results for this case have been given in [12] and are recovered numerically from the present formulation by selecting the same material constants for the adherends and a very small constant for $(h_1-h_2)/h_1$.

4. Examples

A single lap joint and a stiffened plate under various loading conditions shown in Figure 1 are considered as examples. For the five problems shown in Figure 1 the constants which appear in the boundary conditions (10) may be expressed as follows:

(a) Lap joint under tension

$$N_1(\ell) = M_1(\ell) = Q_1(\ell) = N_2(-\ell) = M_2(-\ell) = Q_2(-\ell) = Q_2(\ell) = 0$$

$$N_2(\ell) = N_0, \; M_2(\ell) = N_0 \left(\frac{h_0}{2} + \frac{h_1+h_2}{4}\right) . \tag{33}$$

(b) Lap joint under bending

$$M_2(\ell) = M_0 , \tag{34}$$

and all remaining eight constants are zero.

(c) Lap joint under transverse shear

$$Q_2(\ell) = Q_0 \; , \; M_2(\ell) = \ell Q_0 \; , \tag{35}$$

and all remaining seven constants are zero.

(d) Stiffened plate under tension

$$N_2(\ell) = N_0, \; N_2(-\ell) = N_0 \; , \tag{36}$$

and all remaining seven constants are zero.

(e) Stiffened plate under bending

$$M_2(\ell) = M_0, \; M_2(-\ell) = M_0 \; , \tag{37}$$

and all remaining seven constants are zero.

The following material constants and dimensions are used in the examples:

Adherend 1 (a boron-epoxy laminate)

$E_{1x} = 3.24 \times 10^7$ psi $(2.234 \times 10^{11} N/m^2)$,

$E_{1z} = 3.50 \times 10^6$ psi $(2.413 \times 10^{10}\ N/m^2)$,

$G_{1xy} = 1.23 \times 10^6$ psi $(8.481 \times 10^9 \ N/m^2)$,

$\nu_{1x} = 0.23$, $h_1 = 0.03$ in $(7.62 \times 10^{-4}$ m) .

Adherend 2 (aluminum plate)

$E_2 = 10^7$ psi $(6.895 \times 10^{10} \ N/m^2)$,

$\nu_2 = 0.3$, $h_2 = 0.09$ in $(0.229 \times 10^{-2}$ m) .

Adhesive (epoxy)

$E = 4.45 \times 10^5$ psi $(3.068 \times 10^9 \ N/m^2)$,

$G = 1.65 \times 10^5$ psi $(1.138 \times 10^9 \ N/m^2)$,

$h_o = 0.004$ in $(1.016 \times 10^{-4}$ m).

The length of the bonded region 2ℓ is 1 in. for the lap joint and 2 in. for the stiffened plate.

The calculated results are shown in Tables 1-4. For three basic loading conditions, Table 1 gives the normalized adhesive stresses τ and σ in a single lap joint. The variation of these stresses with ℓ is shown in Tables 2 and 3. Table 4 gives the adhesive stresses in the stiffened plate. A limited amount of information is also displayed in Figure 2 in order to show the trends in the distribution of $\tau(x)$ and $\sigma(x)$. In the membrane loading of the lap joint, if $N_{1x}(-\ell) = N_{2x}(\ell) = N_o$, then to satisfy the static equilibrium conditions additional bending moments and/or transverse shear loads equivalent to a couple $N_o(h_o+h_1/2+h_2/2)$ must be applied to the structure. For example, if the structure is loaded through pin connections, then Q_{1x} and Q_{2x} at the pins would be nonzero and M_{1x} and M_{2x} would be zero. In the example considered, the equilibrium is satisfied by applying two equal moments at the ends of the structure. Needless to say, the results will be dependent on the secondary loads applied to the structure to maintain its static equilibrium.

5. Discussion and the Finite Element Solution

From Tables 1-3 and Figure 2 it may be observed that in a lap joint the adhesive stresses will be concentrated in the end regions of the joint provided the length of the joint 2ℓ is large compared to the adherend thicknesses h_1 and h_2. However, as shown by Tables 2 and 3 for relatively smaller values of ℓ the stresses in the mid-region of the joint may no longer be negligible. By and large, the results found in this paper are in agreement with those reported in [6]. The differences are mainly due to the approximation resulting from the method used for the numerical integration of the differential equations in [6] and partly due to the differences in the adhesive models used in [6] and in this paper(*). Note that because of the smaller bending stiffness of the adherend 1, the peak values of σ and τ at $x= -\ell$ are greater than those at $x=\ell$ (see Figures 1,2 and Tables 1-3). This is the physically expected result.

In the solution given in this paper, the peak values of shear as well as that of the normal stress in the adhesive are found to be at the end points $x = \mp\ell$. This is inherent in the type of the adhesive model used in this type of studies. First, referring to the results given in [13] with regard to the stress singularities in bonded wedges of two dissimilar elastic materials one may observe that if the adherends and the adhesive are treated as elastic continua then generally the interface stresses σ and τ would have a power singularity at the end points $\mp\ell$. On the other hand, if the adherends are treated as elastic continua and the adhesive is assumed to be an uncoupled tension-shear spring, then the kernels of the related integral equations would have only logarithmic singularities and consequently the stresses σ and τ would be bounded everywhere, including the ends. Furthermore, the examples given in [10] and [11] show that in this case the maximum stresses are at the end points. In this sense, the condition that the

(*)For some joint geometries, significant differences were observed between the two sets of calculated results. Within the stated assumptions, the solution given in this paper is exact. Hence, these differences must be due to the numerical integration.

shear stress be zero at the end points of the joint as imposed in some investigations is not consistent with the tension-shear spring adhesive model (see, for example, [7]). Partly to investigate the overall trend of the solution and partly to give some idea about the suitability of the plate model for the adherends, the finite element method is used to solve a sample problem in which the adherends and the adhesive are assumed to be elastic continua.

The problem considered for the finite element solution is that of the stiffened plate shown in Figure 3. For simplicity it is assumed that the adherends 1 and 2 are of the same material. Material constants and dimensions are shown in the figure. The external loads are assumed to be either tension or bending applied at the ends of the plate (i.e., at $x = \mp 1$ in.). Because of symmetry only one half of the structure is considered. Due to the Saint Venant's principle, since the length of the extended portion (0.5 in) of the plate is considerably greater than its thickness (0.06 in), the details of the distribution of the applied loads at the ends have no effect on the stresses in the stiffener and in the plate away from the end regions. Rectangular four-node isoparametric finite elements with incompatible modes[14,15] are used in the plane strain solution. The final results given in this paper are obtained from the mesh assembled with 544 elements interconnected at 620 nodal points. Because of its extremely small thickness, only one layer of finite elements is used for the adhesive. The calculated stresses are those at the midpoint of the elements.

The results calculated by using the finite element method along with those obtained from the plate solution given in this paper are shown in Figures 4-7. Figures 4 and 5 show the shear and normal stresses in the adhesive for a stiffened plate under tension. The bending results are shown in Figures 6 and 7. The finite element results obtained in this paper are believed to be reasonably accurate. From the figures it is seen that the agreement between the two sets of results not only with regard to their trends but also from a quantitative viewpoint is surprisingly very good. One may therefore conclude that in analyzing the adhesively bonded structures with relatively small adherend thicknesses a plate theory properly accounting for the transverse shear effects may be used with a certain degree of confidence. For the case of two orthotropic adherends under plane strain conditions, this paper gives the general closed form solution.

6. References

1. J.N. Goodier and C.S. Hsu, "Transmission of tension from a bar to a plate", J. Applied Mechanics, Vol. 21, Trans. ASME, pp. 147-150, 1954.

2. R. Muki and E. Sternberg, "On the stress analysis of overlapping bonded elastic sheets", Int. J. Solids Structures, Vol. 4, pp. 75-94, 1968.

3. F. Erdogan and M. Ratwani, "Stress distribution in bonded joints", J. of Composite Materials, Vol. 5, pp. 378-393, 1971.

4. F. Erdogan and K. Arin, "A sandwich plate with a part-through and debonding crack", J. Engineering Fracture Mech., Vol. 4, pp. 449-458, 1972.

5. M. Goland and E. Reissner, "The stresses in cemented joints", Journal of Appl. Mechanics, Trans. ASME, Vol. 1, No. 1, pp. (A.17)-(A.27), 1944.

6. U. Yuceoglu and D.P. Updike, "Stress analysis of bonded plates and joints", Journal of the Engineering Mechanics Division, ASCE, Vol. 106, pp. 37-56, 1980.

7. W.J. Renton and J.R. Vinson, "The efficient design of adhesive bonded joints", J. Adhesion, Vol. 7, pp. 175-193, 1975.

8. J.H. Williams, Jr., "Stresses in adhesive between dissimilar adherends", J. Adhesion, Vol. 7, pp. 97-107, 1975.

9. F. Erdogan, "Analysis of elastic cover plates", Developments in Mechanics, Vol. 6, pp. 817-829, 1971.

10. F. Erdogan and M.B. Civelek, "Contact problem for an elastic reinforcement bonded to an elastic plate", Journal of Applied Mechanics, Vol. 41, pp. 1014-1018, 1974.

11. F. Erdogan, "Fracture problems in composite materials", J. Engineering Fracture Mechanics, Vol. 4, pp. 811-840, 1972.

12. F. Delale and F. Erdogan, "Viscoelastic analysis of adhesively bonded joints", NASA Langley Research Center Contractor Report 159294, March 1980.

13. V.L. Hein and F. Erdogan, "Stress singularities in a two-material wedge", Int. J. of Fracture Mechanics, Vol. 7, pp. 317-330, 1971.

14. K.J. Bathe et al., "SAP IV, a structural analysis program for static and dynamic response of linear systems", Report No. EERC 73-11, University of California, Berkeley, 1974.

15. K.J. Bathe and E.L. Wilson, "Numerical Methods in Finite Element Analysis", Prentice-Hall, Inc., 1976.

16. V.V. Novozhilov, Foundations of the Nonlinear Theory of Elasticity, Graylock Press, Rochester, NY, 1953.

17. W. Ramberg and W.R. Osgood, NACA TN. 902, 1943.

18. J.W. Hutchinson, "Singular behavior at the end of a tensile crack in a hardening material", J. Mech. Phys. Solids, Vol. 16, pp. 13-31, 1968.

APPENDIX

Formulation of a Bonded Joint with Nonlinear Adhesive

Assuming that the adherends are linearly elastic and may be modeled as "plates", equations (1-3) are always valid. Again, if we assume that the thickness variation of the strains in the adhesive layer is negligible, the kinematical relations (4) also remain valid even if the adhesive is not a linearly elastic material. However, in this case, the stress-strain relations (5) must be replaced by a system of nonlinear constitutive equations. This may be done in several ways. One approach would be to define a strain energy function in terms of the strain invariants which would, in turn, give the stress-strain relations necessary for the formulation of the problem [16]. Thus, if a_2, a_1, a_0 are the strain invariants, defining a strain energy function $\phi(a_2, a_1, a_0)$ the constitutive relations for the adhesive may be expressed as

$$\sigma_{ij} = \frac{\partial \phi}{\partial \varepsilon_{ij}} , \; (i,j = x,y,z) \; . \qquad (A.1)$$

If the material has the same behavior in tension and in compression, in its simplest form ϕ must have at least six constants. Let us, therefore, assume that

$$\phi = A_1 a_2^2 + A_2 a_1 + c_1 a_2^4 + c_2 a_2^2 a_1 + c_3 a_2 a_0 + c_4 a_1^2 , \qquad (A.2)$$

where

$$a_2 = \varepsilon_x + \varepsilon_y + \varepsilon_z , \qquad (A.3)$$

$$a_1 = \varepsilon_x \varepsilon_y + \varepsilon_x \varepsilon_z + \varepsilon_y \varepsilon_z - \frac{1}{4} (\varepsilon_{xy}^2 + \varepsilon_{xz}^2 + \varepsilon_{yz}^2) , \qquad (A.4)$$

$$a_0 = \varepsilon_x \varepsilon_y \varepsilon_z - \frac{1}{4}(\varepsilon_x \varepsilon_{yz}^2 + \varepsilon_y \varepsilon_{xz}^2 + \varepsilon_z \varepsilon_{xy}^2 - \varepsilon_{xy} \varepsilon_{xz} \varepsilon_{yz}) , \qquad (A.5)$$

and A_1, A_2, c_1, c_2, c_3, and c_4 are the material constants. For the plane strain problem under consideration, we have

$$\varepsilon_z = 0,\ \varepsilon_{xz} = 0,\ \varepsilon_{yz} = 0\ , \tag{A.6}$$

from which it follows that

$$a_2 = \varepsilon_x+\varepsilon_y\ ,\ a_1 = \varepsilon_x\varepsilon_y - \varepsilon_{xy}^2/4\ ,\ a_o = 0. \tag{A.7}$$

From (A.1), (A.2) and (A.7) the stress-strain relations relevant to the formulation of the adhesive joint problem are found to be

$$\tau_{xy} = \tau = -\frac{1}{2}A_2\varepsilon_{xy} + \frac{c_4}{4}\varepsilon_{xy}^3 - \varepsilon_{xy}\left[\frac{c_2}{2}(\varepsilon_x^2+\varepsilon_y^2+2\varepsilon_x\varepsilon_y) + c_4\varepsilon_x\varepsilon_y\right] \tag{A.8}$$

$$\begin{aligned}\sigma_{yy} = \sigma = {} & 2A_1(\varepsilon_x+\varepsilon_y) + A_2\varepsilon_x + 4c_1(\varepsilon_x^3+\varepsilon_y^3+3\varepsilon_x^2\varepsilon_y+3\varepsilon_x\varepsilon_y^2)\\ & + 2c_2(\varepsilon_x\varepsilon_y - \frac{1}{4}\varepsilon_{xy}^2)(\varepsilon_x+\varepsilon_y) + c_2\varepsilon_x(\varepsilon_x^2+\varepsilon_y^2+2\varepsilon_x\varepsilon_y)\\ & + 2c_4\varepsilon_x(\varepsilon_x\varepsilon_y - \frac{1}{4}\varepsilon_{xy}^2).\end{aligned} \tag{A.9}$$

Noting that $\varepsilon_{xy} = \gamma_{xy}/2$ and using the kinematical relations (4), (A.8) and (A.9) may be expressed as

$$\tau = f_1(u_i,\ v_i,\ \beta_{ix},\ \frac{du_i}{dx},\ \frac{d\beta_{ix}}{dx})\ , \tag{A.10}$$

$$\sigma = f_2(u_i,\ v_i,\ \beta_{ix},\ \frac{du_i}{dx},\ \frac{d\beta_{ix}}{dx}),\ (i = 1,2)\ , \tag{A.11}$$

where f_1 and f_2 are known nonlinear functions. Thus, equations (A.10) and (A.11) along with the equilibrium equations (1 a-f) and the stress resultant-displacement relations (2 a-f) give 14 equations to determine the 14 unknown functions τ, σ, u_i, v_i, β_{ix}, N_{ix}, Q_{ix}, and M_{ix}, $(i=1,2)$. Since $a_o = 0$ the constant c_3 does not appear in the formulation of the problem. However, even if it were possible to determine the five remaining material constants with sufficient accuracy, because of the highly nonlinear nature of the functions f_1 and f_2, even the numerical solution of the problem becomes extremely complicated.

Another approach to the formulation of the problem would be to assume that the adhesive has a nonlinear elastic behavior of the type suggested by Ramberg and Osgood [17] for the deformation theory of plasticity. In such materials, the stress-strain relation for normal loading is

$$\bar{\varepsilon} = \bar{\sigma} + \alpha\, \bar{\sigma}^n , \tag{A.12}$$

where $\bar{\varepsilon}$ is the strain normalized with respect to σ_Y/E, $\bar{\sigma}$ is the stress normalized with respect to σ_Y, α is a constant (generally 0.02), n is the strain hardening coefficient, E is the initial slope of the stress-strain curve, and σ_Y is a constant (the conventional yield strength in plasticity). In the three-dimensional case, one may write (see, for example, [18])

$$\bar{\varepsilon}_{ij} = (1+\nu)\bar{s}_{ij} + \frac{1-2\nu}{3}\bar{\sigma}_{kk}\,\delta_{ij} + \frac{3}{2}\alpha\,\bar{\sigma}_e^{\,n-1}\,\bar{s}_{ij},\ (i,j=x,y,z), \tag{A.13}$$

where ν is a known material constant and

$$\bar{s}_{ij} = \bar{\sigma}_{ij} - \frac{1}{3}\bar{\sigma}_{kk}\,\delta_{ij},\quad \bar{\sigma}_e^2 = \frac{3}{2}\bar{s}_{ij}\,\bar{s}_{ij},\ (i,j=x,y,z). \tag{A.14}$$

Noting again that for the problem under consideration

$\varepsilon_z = 0$, $\varepsilon_{xz} = 0$, $\varepsilon_{yz} = 0$, $\sigma_{xz} = 0$, $\sigma_{yz} = 0$, from (A.14) we obtain

$$E\varepsilon_x = \sigma_x - \nu\sigma_y - \nu\sigma_z + \frac{1}{2}\alpha\,\sigma_e^{\,n-1}(2\sigma_x - \sigma_y - \sigma_z), \tag{A.15}$$

$$E\varepsilon_y = \sigma_y - \nu\sigma_x - \nu\sigma_z + \frac{1}{2}\alpha\,\sigma_e^{\,n-1}(2\sigma_y - \sigma_x - \sigma_z), \tag{A.16}$$

$$0 = \sigma_z - \nu\sigma_x - \nu\sigma_y + \frac{1}{2}\alpha\,\sigma_e^{\,n-1}(2\sigma_z - \sigma_x - \sigma_y), \tag{A.17}$$

$$\frac{E}{2}\gamma_{xy} = (1+\nu)\sigma_{xy} + \frac{3}{2}\alpha\,\sigma_e^{\,n-1}\sigma_{xy} , \tag{A.18}$$

where

$$\sigma_e^2 = 3\left(\frac{\sigma_{xy}}{\sigma_Y}\right)^2 + \left(\frac{\sigma_x}{\sigma_Y}\right)^2 + \left(\frac{\sigma_y}{\sigma_Y}\right)^2 - \frac{1}{\sigma_Y^2}(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)\ . \tag{A.19}$$

Using now the kinematical relations (4), from (A.15)-(A.18) it follows that

$$\frac{E}{2}\left(C_1 N_{1x} - \frac{h_1 D_1}{2} M_{1x} + C_2 N_{2x} + \frac{h_2 D_2}{2} M_{2x}\right)$$

$$= \sigma_x - \nu\sigma_y - \nu\sigma_z + \frac{\alpha}{2}(2\sigma_x - \sigma_y - \sigma_z)\,\sigma_e^{n-1}\,, \qquad (A.20)$$

$$\frac{E}{h_o}(v_1 - v_2) = \sigma_y - \nu\sigma_x - \nu\sigma_z + \frac{\alpha}{2}(2\sigma_y - \sigma_x - \sigma_z)\sigma_e^{n-1}\,, \qquad (A.21)$$

$$0 = \sigma_z - \nu\sigma_x - \nu\sigma_y + \frac{\alpha}{2}(2\sigma_z - \sigma_x - \sigma_y)\sigma_e^{n-1}\,, \qquad (A.22)$$

$$\frac{E}{2h_o}\left(u_1 - \frac{h_1}{2}\beta_{1x} - u_2 - \frac{h_2}{2}\beta_{2x}\right) = (1+\nu)\sigma_{xy} + \frac{3\alpha}{2}\sigma_e^{n-1}\sigma_{xy}, \qquad (A.23)$$

where σ_x, $\sigma_y = \sigma$, σ_z and $\sigma_{xy} = \tau$ are the stresses in the adhesive layer. Thus, (A.20)-(A.23) along with (1 a-f) and (2 a-f) give a system of 16 equations to determine σ_x, σ_y, σ_z, σ_{xy}, u_i, v_i, β_{ix}, N_{ix}, Q_{ix}, and M_{ix}, $(i = 1,2)$.

Table 1. Adhesive stresses for a single lap joint, ℓ = 0.5 in.

	$N_0 \neq 0$, $M_0 = 0 = Q_0$		$M_0 \neq 0$, $N_0 = 0 = Q_0$		$Q_0 \neq 0$, $N_0 = 0 = M_0$	
x/ℓ	$\tau/(N_0/2\ell)$	$\sigma/(N_0 2\ell)$	$\tau/(M_0/4\ell^2)$	$\sigma/(M_0/4\ell^2)$	$\tau/(Q_0/2\ell)$	$\sigma/(Q_0/2\ell)$
-1.0	-13.436	41.191	345.70	-1328.5	-160.45	588.10
-0.9	- 3.690	- 4.295	83.789	137.25	- 30.571	-68.272
-0.8	- 0.858	- 1.713	12.929	56.239	4.535	-27.432
-0.7	- 0.141	- 0.555	- 1.820	18.880	11.944	- 9.184
-0.6	0.011	- 0.183	- 3.190	6.516	12.703	- 3.163
-0.5	0.028	- 0.063	- 2.188	2.357	12.253	- 1.142
-0.4	0.020	- 0.023	- 1.233	0.897	11.803	- 0.434
-0.3	0.011	- 0.009	- 0.645	0.356	11.520	- 0.175
-0.2	0.005	- 0.004	- 0.335	0.143	11.362	- 0.075
-0.1	0.001	- 0.002	- 0.194	0.051	11.274	- 0.038
0.	- 0.002	- 0.002	- 0.161	- 0.001	11.213	- 0.029
0.1	- 0.007	- 0.003	- 0.223	- 0.051	11.145	- 0.038
0.2	- 0.017	- 0.006	- 0.418	- 0.132	11.030	- 0.071
0.3	- 0.036	- 0.012	- 0.865	- 0.292	10.800	- 0.147
0.4	- 0.078	- 0.026	- 1.835	- 0.632	10.316	- 0.313
0.5	- 0.166	- 0.056	- 3.908	- 1.362	9.285	- 0.671
0.6	- 0.356	- 0.119	- 8.327	- 2.939	7.090	- 1.443
0.7	- 0.764	- 0.252	-17.713	- 6.369	2.417	- 3.113
0.8	- 1.641	- 0.526	-37.572	-13.767	- 7.502	- 6.691
0.9	- 3.538	- 0.851	-79.408	-23.586	-28.512	-11.695
1.0	- 7.806	9.693	-170.60	262.06	-74.409	109.72

Table 2. Variation of τ/N_0 with ℓ for a single lap joint under tension ($N_0 \neq 0$, $M_0 = 0 = Q_0$)

x/ℓ	ℓ (in) 0.5	0.4	0.3	0.2	0.1
-1.0	-13.436	-13.436	-13.438	-13.469	-14.111
-0.9	- 3.690	- 4.846	- 6.331	- 8.247	-11.253
-0.8	- 0.858	- 1.572	- 2.796	- 4.886	- 8.928
-0.7	- 0.141	- 0.446	- 1.170	- 2.843	- 7.092
-0.6	0.011	- 0.088	- 0.451	- 1.635	- 5.674
-0.5	0.028	0.010	- 0.149	- 0.939	- 4.601
-0.4	0.020	0.027	- 0.034	- 0.554	- 3.809
-0.3	0.011	0.021	- 0.000	- 0.359	- 3.245
-0.2	0.005	0.011	- 0.003	- 0.283	- 2.869
-0.1	0.001	0.001	- 0.024	- 0.285	- 2.647
0.	- 0.002	- 0.011	- 0.058	- 0.345	- 2.558
0.1	- 0.007	- 0.028	- 0.108	- 0.459	- 2.584
0.2	- 0.017	- 0.055	- 0.184	- 0.627	- 2.714
0.3	- 0.036	- 0.104	- 0.299	- 0.862	- 2.944
0.4	- 0.078	- 0.193	- 0.479	- 1.183	- 3.272
0.5	- 0.166	- 0.356	- 0.761	- 1.620	- 3.701
0.6	- 0.356	- 0.655	- 1.206	- 2.212	- 4.239
0.7	- 0.764	- 1.208	- 1.911	- 3.017	- 4.898
0.8	- 1.641	- 2.230	- 3.031	- 4.117	- 5.698
0.9	- 3.538	- 4.130	- 4.825	- 5.635	- 6.666
1.0	- 7.806	- 7.806	- 7.805	- 7.794	- 7.846

Table 3. Variation of σ/N_0 with ℓ for a single lap joint under tension ($N_0 \neq 0$, $M_0 = 0 = Q_0$)

x/ℓ	ℓ (in) 0.5	0.4	0.3	0.2	0.1
-1.0	41.191	41.191	41.187	41.117	39.711
-0.9	- 4.295	- 4.292	- 2.979	1.487	12.572
-0.8	- 1.713	- 2.634	- 3.818	- 4.301	1.169
-0.7	- 0.555	- 1.093	- 2.135	- 3.838	- 3.180
-0.6	- 0.183	- 0.444	- 1.095	- 2.656	- 4.458
-0.5	- 0.063	- 0.183	- 0.558	- 1.738	- 4.473
-0.4	- 0.023	- 0.078	- 0.289	- 1.127	- 4.030
-0.3	- 0.009	- 0.035	- 0.154	- 0.743	- 3.477
-0.2	- 0.004	- 0.017	- 0.088	- 0.508	- 2.956
-0.1	- 0.002	- 0.010	- 0.058	- 0.372	- 2.515
0.	- 0.002	- 0.009	- 0.049	- 0.302	- 2.165
0.1	- 0.003	- 0.012	- 0.054	- 0.279	- 1.898
0.2	- 0.006	- 0.020	- 0.072	- 0.294	- 1.702
0.3	- 0.012	- 0.036	- 0.106	- 0.343	- 1.560
0.4	- 0.026	- 0.065	- 0.163	- 0.427	- 1.447
0.5	- 0.056	- 0.119	- 0.254	- 0.548	- 1.322
0.6	- 0.119	- 0.217	- 0.394	- 0.702	- 1.106
0.7	- 0.252	- 0.394	- 0.605	- 0.847	- 0.639
0.8	- 0.526	- 0.689	- 0.839	- 0.732	0.420
0.9	- 0.851	- 0.726	- 0.283	0.862	2.803
1.0	9.693	9.693	9.691	9.654	8.075

Table 4. Results for a stiffened plate (ℓ = 1 in)

x/ℓ	$N_0 \neq 0,\ M_0 = 0 = Q_0$		$M_0 \neq 0,\ N_0 = 0 = Q_0$	
	$\tau/(N_0/\ell)$	$\sigma/(N_0/\ell)$	$\tau/(M_0/\ell^2)$	$\sigma/(M_0/\ell^2)$
0.	0	-3.06×10^{-7}	0	-3.01×10^{-5}
0.05	-7.49×10^{-7}	-3.99×10^{-7}	-7.36×10^{-5}	-3.92×10^{-5}
0.1	-1.95×10^{-6}	-7.33×10^{-7}	-1.92×10^{-4}	-7.20×10^{-5}
0.15	-4.34×10^{-6}	-1.51×10^{-6}	-4.26×10^{-4}	-1.48×10^{-4}
0.20	-9.35×10^{-6}	-3.20×10^{-6}	-9.18×10^{-4}	-3.15×10^{-4}
0.25	-2.00×10^{-5}	-6.84×10^{-6}	-1.97×10^{-3}	-6.72×10^{-4}
0.30	-4.29×10^{-5}	-1.46×10^{-5}	-4.21×10^{-3}	-1.44×10^{-3}
0.35	-9.17×10^{-5}	-3.12×10^{-5}	-8.99×10^{-3}	-3.07×10^{-3}
0.40	-1.96×10^{-4}	-6.66×10^{-5}	-0.019	-6.57×10^{-3}
0.45	-4.20×10^{-4}	-1.42×10^{-4}	-0.041	-0.014
0.50	-8.98×10^{-4}	-3.03×10^{-4}	-0.088	-0.030
0.55	-1.92×10^{-3}	-6.44×10^{-4}	-0.188	-0.064
0.60	-4.13×10^{-3}	-1.36×10^{-3}	-0.401	-0.138
0.65	-8.86×10^{-3}	-2.88×10^{-3}	-0.857	-0.295
0.70	-0.019	-6.02×10^{-3}	-1.831	-0.633
0.75	-0.041	-0.012	-3.907	-1.362
0.80	-0.090	-0.025	-8.326	-2.939
0.85	-0.197	-0.048	-17.713	-6.369
0.90	-0.439	-0.085	-37.571	-13.767
0.95	-0.997	-0.096	-79.408	-23.586
1.0	-2.346	1.307	-170.60	262.06

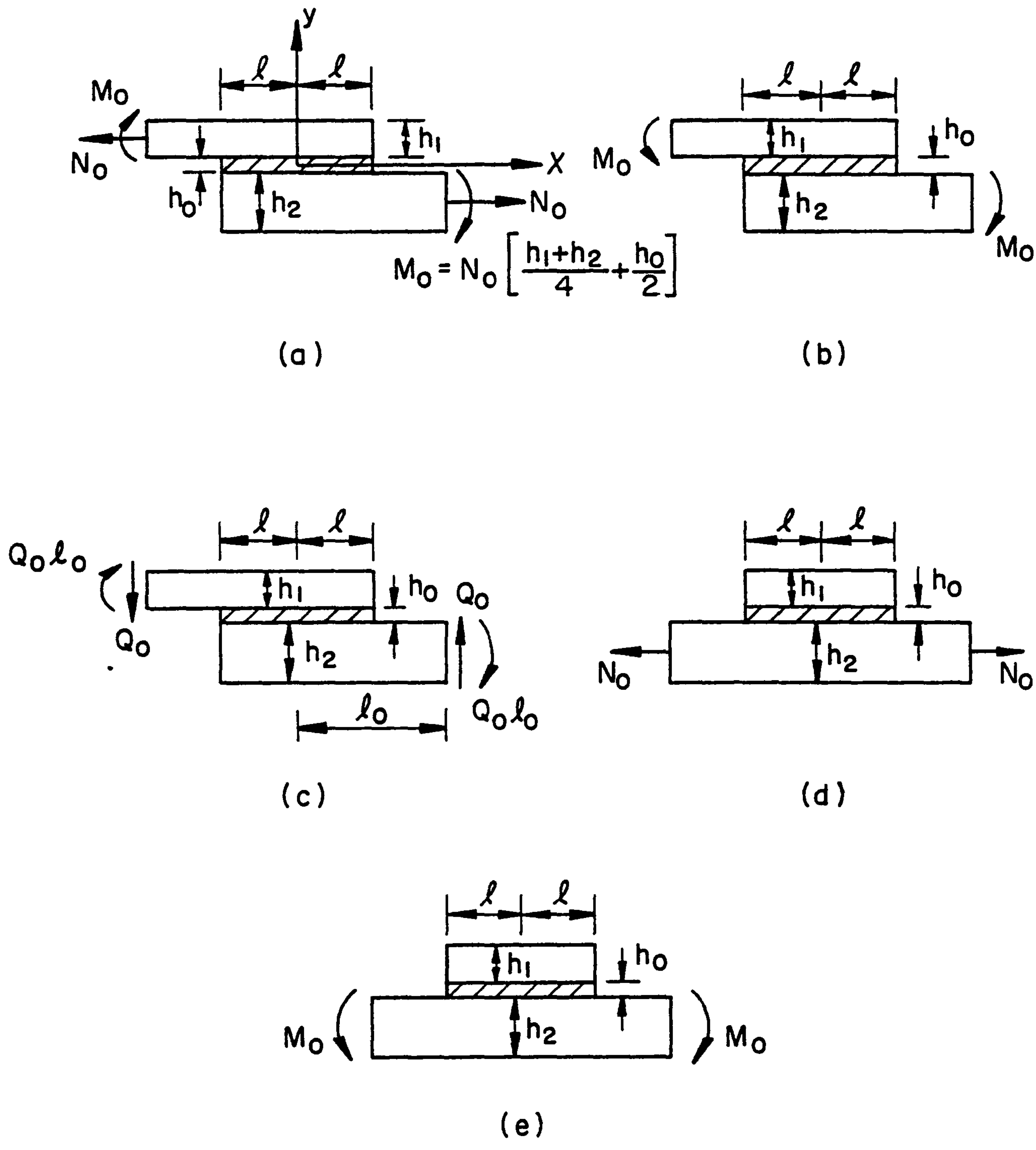

Figure 1. The notation and the geometry for adhesively bonded structures considered as examples.

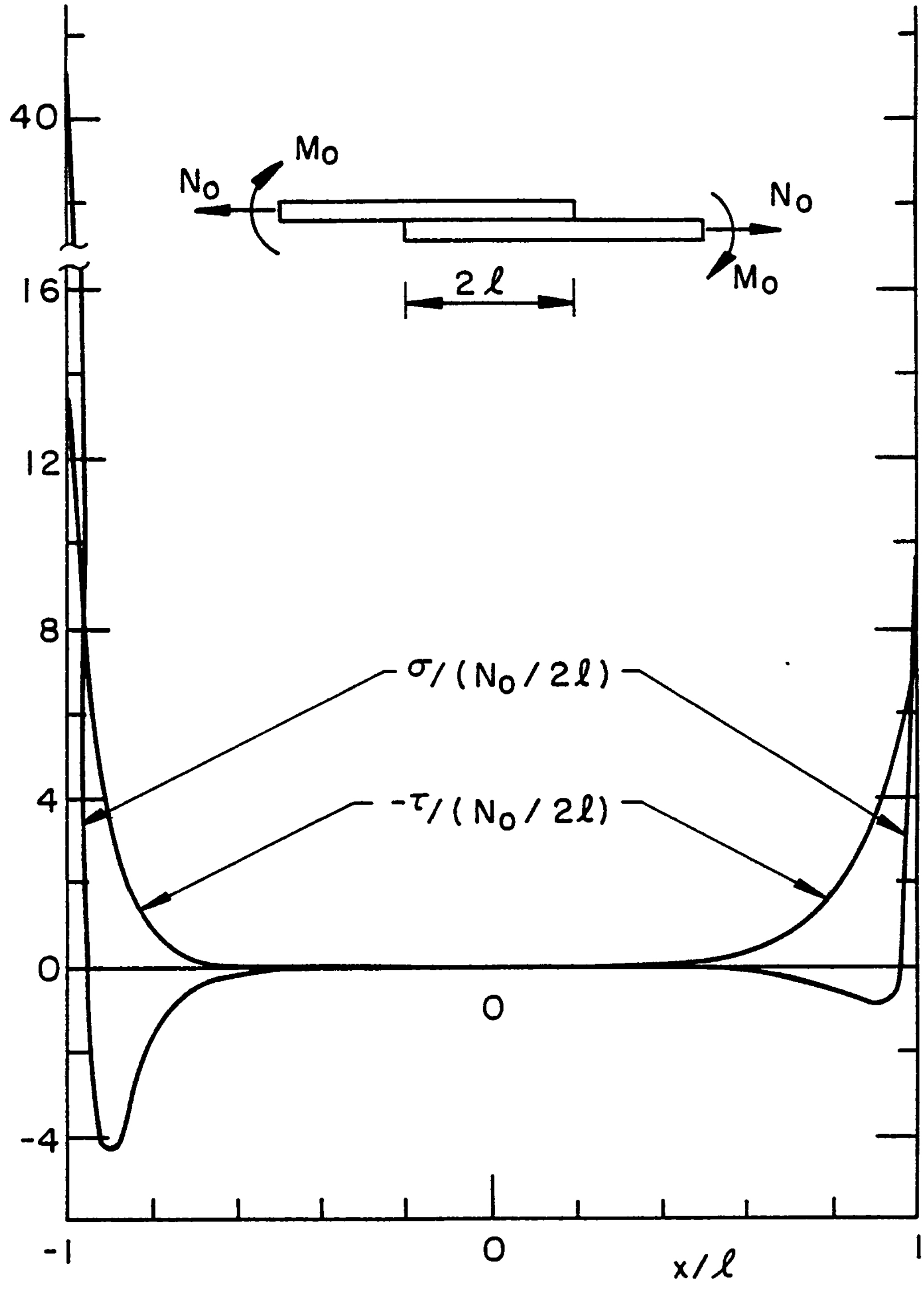

Figure 2. The normal (σ_y=σ) and shear (τ_{xy}=τ) stresses in the adhesive for a single lap joint under tension, ℓ=0.5 in.

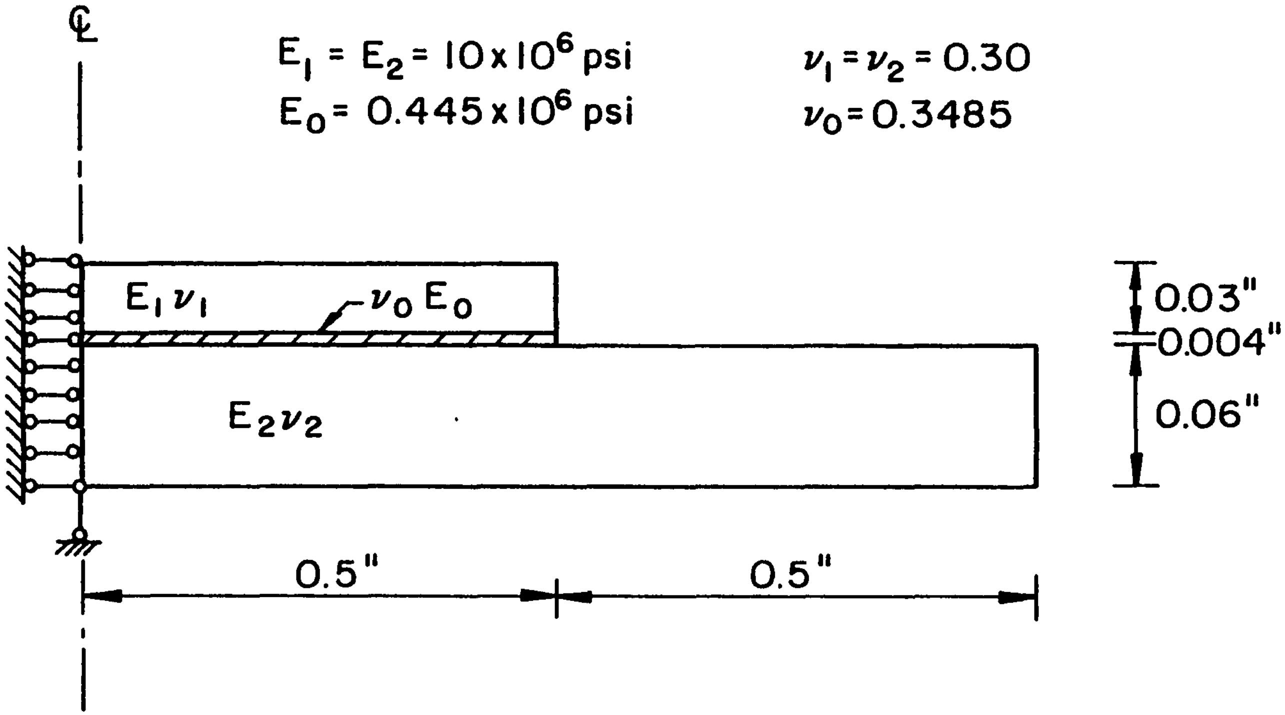

Figure 3. The dimensions and material constants for the stiffened plate which is solved by using the finite element method.

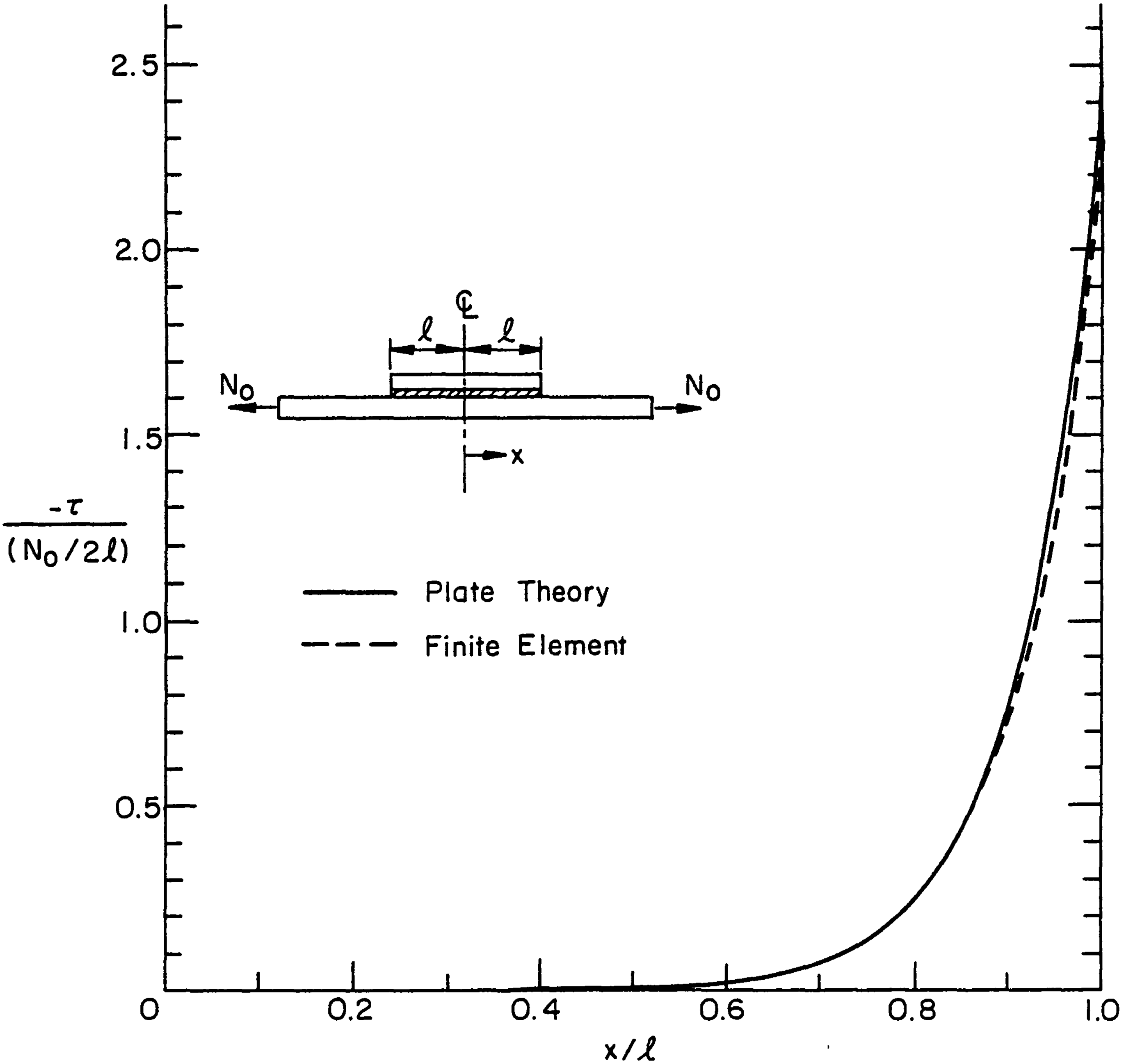

Figure 4. Comparison of the shear stresses in the adhesive for a stiffened plate under tension obtained from the plate theory and from the finite element method.

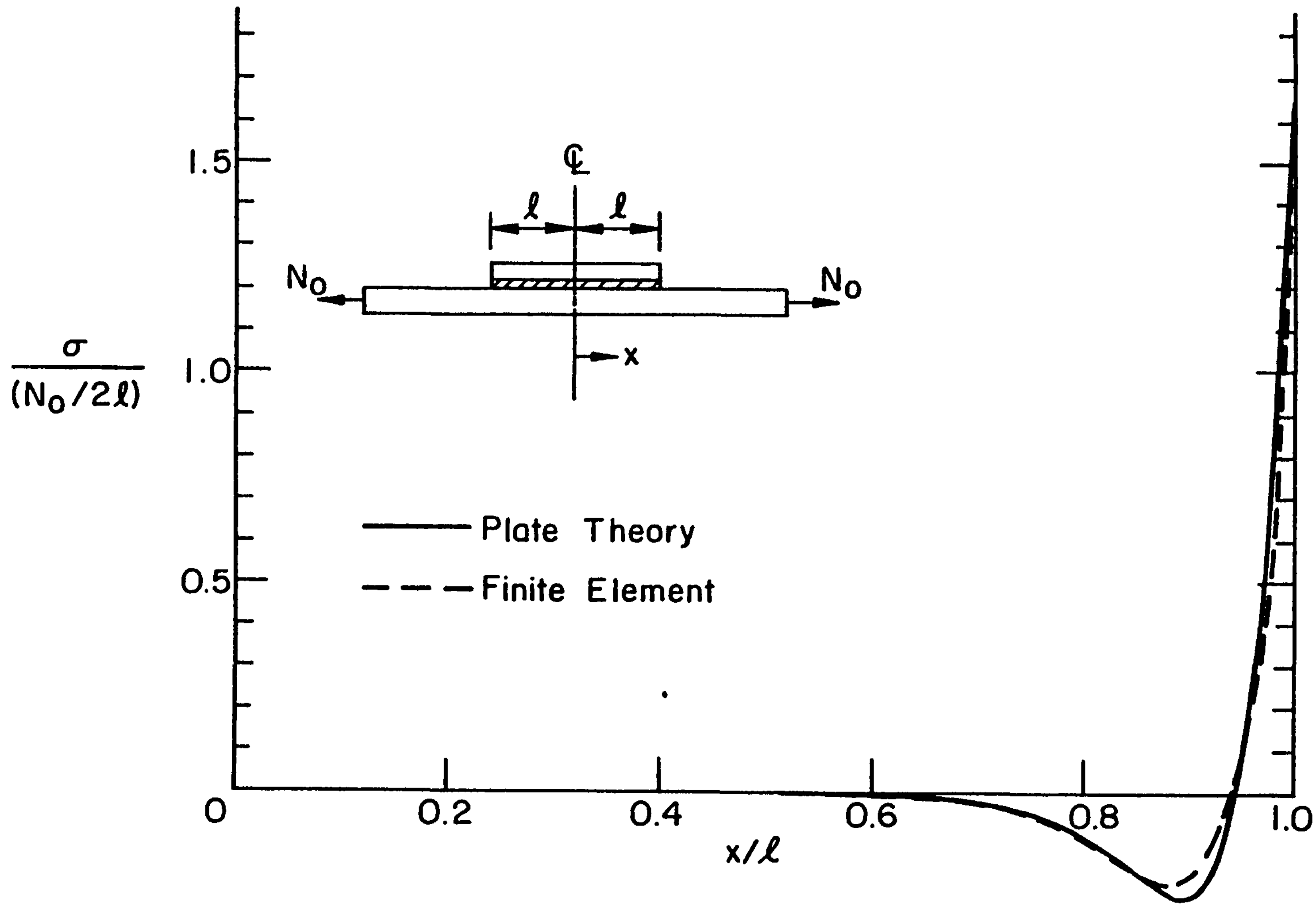

Figure 5. Comparison of the normal stresses in the adhesive for a stiffened plate under tension obtained from the plate theory and from the finite element method.

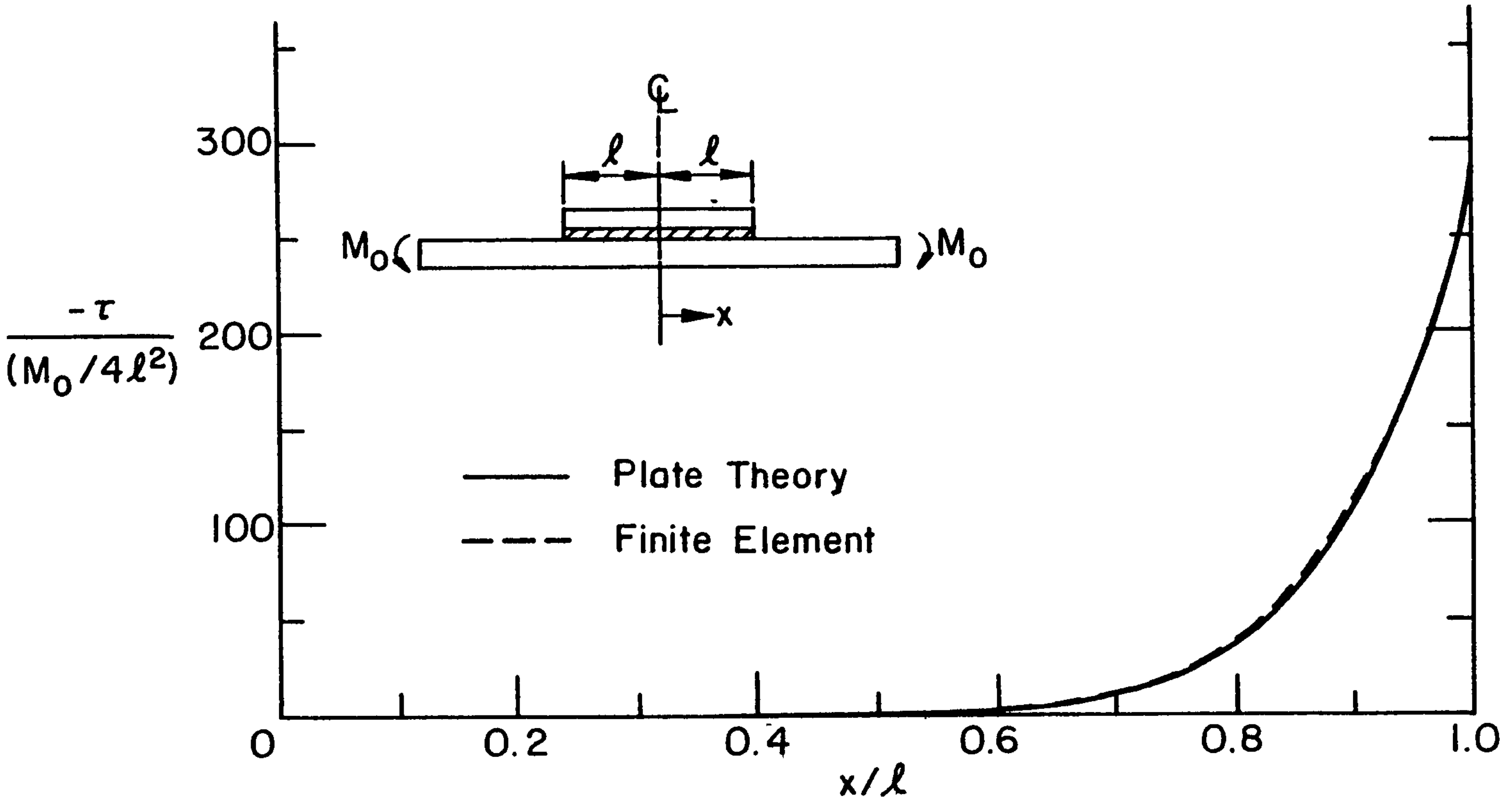

Figure 6. Same as Figure 4, for a plate under bending.

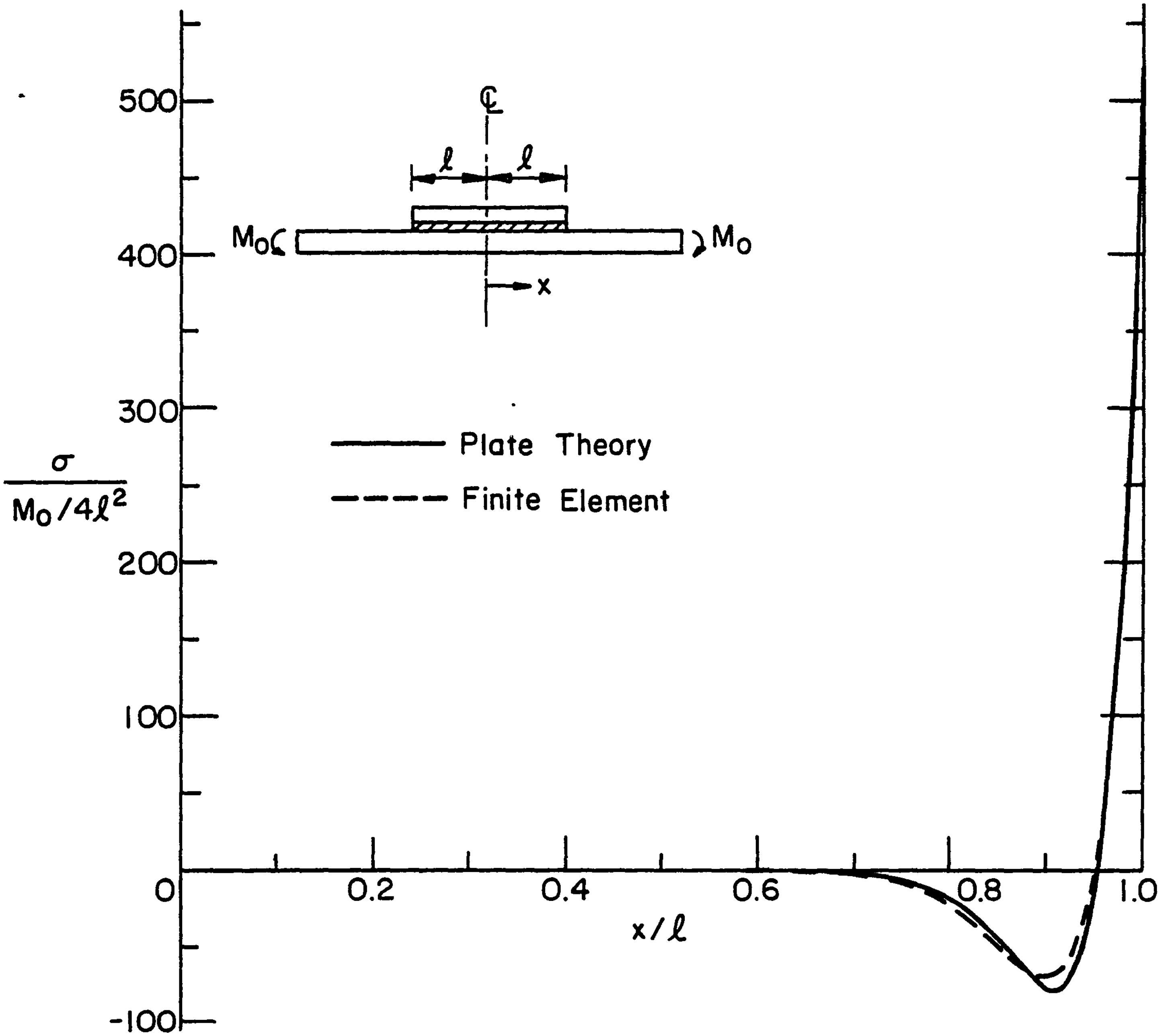

Figure 7. Same as Figure 5, for a plate under bending.

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