Progress of Theoretical Physics, Vol. 103, No. 6, June 2000

String Theory and the Space-Time Uncertainty Principle

Tamiaki YONEYA*)

Institute of Physics, University of Tokyo, Komaba, Tokyo 153-8902, Japan

(Received April 14, 2000)

The notion of a space-time uncertainty principle in string theory is clarified and further developed. The motivation and the derivation of the principle are first reviewed in a reasonably self-contained way. It is then shown that the nonperturbative (Borel summed) high-energy and high-momentum transfer behavior of string scattering is consistent with the space-time uncertainty principle. It is also shown that, as a consequence of this principle, string theories in 10 dimensions generically exhibit a characteristic length scale which is equal to the well-known 11 dimensional Planck length $g_s^{1/3}\ell_s$ of M-theory as the scale at which stringy effects take over the effects of classical supergravity, even without involving D-branes directly. The implications of the space-time uncertainty relation in connection with D-branes and black holes are discussed and reinterpreted. Finally, we present a novel interpretation of the Schild-gauge action for strings from the viewpoint of noncommutative geometry. This conforms to the space-time uncertainty relation by manifestly exhibiting a noncommutativity of quantized string coordinates between, dominantly, space and time. We also discuss the consistency of the space-time uncertainty relation with S and T dualities.

§1. Introduction

Since the time that string theory [1)] was first recognized to be the prime candidate for the unified theory including gravity, [2), 3)] we have been discovering a multitude of facets of the theory which increasingly reveal its richness in unexpected ways. We are more or less convinced that the theory must have some hidden but firm foundation behind many surprising phenomena we observe on its surface. However, present string theory remains essentially as a collection of rules for building the S-matrix in perturbation theory. We do not know why such a perturbation theory can arise and what the basic principles leading to the symmetry of string perturbation theory are. Uncovering the underlying principles of string theory is an important necessary step toward a non-perturbative and completely well-defined formulation of the theory, based on which we should be able to pose various physically relevant questions that the ultimate unified theory has to answer, but hitherto have not been meaningfully dealt with.

There have been several attempts towards constructing nonperturbative formulations of string theory. The first and most traditional one is the field theory of strings, which has been pursued quite actively since more than fifteen years ago. [4)∼7)] A related approach involves various studies of the geometry of loop space and conformal field theories. A notable example is an approach based on an abstract complex geometry [8)] or, more physically, on the renormalization group in the theory space of two-dimensional field theories. [9)] All of these approaches are closely connected

*) E-mail: tam@hep1.c.u-tokyo.ac.jp

to each other in a variety of ways, depending on the manner in which we compare them. Curiously enough, however, it is generally not easy, despite their apparent similarities, to establish concrete connections among these attempts at different formulations into a unified scheme in which we can formally go back and forth among them. It seems that, from the viewpoint of representing string amplitudes, the difference between these approaches essentially lies in the manners that the moduli space of Riemann surfaces is decomposed. [10)∼12)] For example, the gauge invariance of string field theory expresses the requirement of smooth joining of the decomposed pieces of moduli space. The transition among different approaches therefore amounts to connecting theories with different schemes of the decomposition into one single theoretical framework. Such a 'transformation theory', if successfully established, could suggest some crucial structure behind string theory by extracting possible principles behind conformal symmetry.

A slightly different approach involves the 'old' matrix models [13)] as toy models for studying the theory in which the entire string perturbation series are summed over in lower space-time dimensions. Moreover, in recent years, after the discovery of D-branes [14)] and their effective description in terms of Yang-Mills theory, [15)] a new approach that we may call the 'new' matrix models, which can be formulated in higher space-time dimensions, has been advocated. Except for some special circumstances, such as the infinite-momentum limit and some sort of large N scaling limit analogous to the old matrix models, the new matrix models are regarded at best as effective low-energy descriptions of D-branes in terms of the lowest string excitation modes alone. The new matrix models have suggested some unexpected relations among local field theories embedded in string theory as low-energy approximations. The notable exceptions to this interpretation of the new matrix models might be the so-called Matrix theory [16)] and the IIB matrix model. [17)] They have been conjectured to be exact theories. However, at the present stage of development, it seems fair to say that we have not yet succeeded in showing convincingly their validity as the fundamental formulations of string theory.

Independently of these specific attempts, one thing is evident. Namely, the structure of string theory is governed by the conformal symmetry of world-sheet dynamics, exhibited by the present perturbative rules. Indeed, almost all merits of perturbative string dynamics, such as the emergence of the graviton, elimination of ultraviolet divergences, critical dimensions, complete bootstrap between states and interactions, and so on, are direct consequences of the world-sheet conformal invariance. This is true even when we take into account various brane excitations, since the interactions of the branes are mediated by the exchange and fluctuations of strings. The motion and interaction of D-branes are described by the ordinary elementary (or 'fundamental') strings and their vertex operators in terms of the world-sheet dynamics.

The exact conformal invariance of world-sheet dynamics of strings actually corresponds to the fact that the genuine observables of string theory are solely on-shell S-matrix elements. Given only an S-matrix, however, it is in general not easy to determine the real symmetries and degrees of freedom which are appropriate to express the content of the theory off shell. In particular, it is not obvious what is the appropriate generalization of the world-sheet conformal invariance to the off-shell formulation of string theory. This problem is a long-standing example among the several major obstacles we encounter in trying to formulate the nonperturbative dynamics of string theory. The nature of the problem is somewhat reminiscent of that which characterized the well-known history of physics evolving from early quantum theory to quantum mechanics. The quantized atomic spectra were derived by imposing the Bohr-Sommerfeld quantization conditions, characterized by the adiabatic invariance, which select particular orbits from the continuous family of allowed orbits in the phase space of classical mechanics. Then quantum mechanics replaced the Bohr-Sommerfeld condition by a deeper and universal structure, namely, Hilbert space and the algebra of observables defined on it, characterized by the superposition principle and the canonical commutation relations, respectively. We should perhaps expect something similar in string theory. Namely, the condition of conformal invariance, as the analog of the adiabatic invariance of the quantum condition, must be generalized to a more fundamental and universal structure which allows us to construct the concrete nonperturbative theory.

However, in view of the status of past approaches to nonperturbative definitions of string theory as briefly summarized above, it seems that the correct language and mathematical framework for formulating string theory remain yet to be discovered. Given this situation, it is worthwhile to attempt extracting the most characteristic qualitative properties of string theory originating from the world-sheet conformal invariance which are universally valid, irrespective of the particular formulations of string theory. In particular, in order to seek some clear basis behind conformal symmetry, it seems more advantageous to express directly the smooth nature of the moduli space of the Riemann surface without making decompositions of it, as is usually done in string field theories, for example.

Although such universal qualitative properties may, by definition, be too crude to make any quantitative predictions, as opposed to simply giving rough order estimates for some simple and typical phenomena, they might be of some help in pursuing the underlying principles of string theory *if* they characterize critically the departure of string theory from the physics governed by the traditional framework of local quantum field theories. The proposal of a space-time uncertainty principle [18), 19)] was motivated by this manner of thinking. The purpose of the present paper is to clarify and further develop the space-time uncertainty principle of string theory from a new perspective. We first review the original derivation with some clarifications in §2. We also make comparison of our space-time uncertainty relation with other proposals of similar nature, such as the notion of a modified uncertainty relation with stringy corrections. We explain why the latter cannot be regarded as a universal relation in string theory, in contrast to our space-time uncertainty relation, which we argue here to be valid nonperturbatively. Its implications are then discussed with regard to some aspects of string theory, mainly the high-energy limit of the string S-matrix in §3, and the characteristic physical scales in the physics of microscopic black holes and D-particle scatterings in §4. It is argued that the high-energy behavior with fixed scattering angle almost saturates the space-time uncertainty relation after summation over the genus by the Borel sum technique. This suggests strongly that the space-time uncertainty relation is valid nonperturbatively, being obeyed independently of the strength of the string coupling, at least for a certain finite range of the string coupling, including the weak coupling regime. It is shown that the M-theory scale is a natural consequence of the space-time uncertainty relation combined with the properties of microscopic black holes, even without invoking D-branes directly. The saturation of the space-time uncertainty relation is also shown to be one of the characteristic features of D-particle scatterings. It is argued that D-particle and anti-D-particle scattering, in general, do not saturate the space-time uncertainty relation. Section 4 also contains some remarks on the possible roles of the space-time uncertainty relation in connection with some developments of string theory, such as black-hole complementarity, holography and UV-IR correspondence. In particular, a simple interpretation of the Beckenstein-Hawking entropy for the macroscopic Schwarzschild black hole is given from the viewpoint of the space-time uncertainty relation. Then, in §5 we proceed to suggest the possibility of formulating the space-time uncertainty principle by quantizing string theory in a way which conforms to noncommutative geometry, exhibiting manifestly a noncommutativity between space and time. The argument is based on a novel interpretation of the string action in the Schild gauge, but its completion toward a concrete calculable scheme is left for future works. The final section is devoted to discussion of some issues which are not treated in the main text, including the interpretation of S and T dualities from the viewpoint of the space-time uncertainty principle, and of future prospects.

In addition to developing the ideas of the space-time uncertainty relation further, it is another purpose of the present paper to discuss most of the various relevant issues in a more coherent fashion from a definite standpoint, since previous discussions regarding the space-time uncertainty principle, mainly in works by the present author and including some works by other authors, are scattered in various different papers. We would like to lay a foundation for further investigation by discussing their meaning and usefulness in understanding the nature of string theory. The present author also hopes that this exposition will be useful to straighten up some confusion and awkward prejudices prevailing in the literature and to clarify the standpoint and, simultaneously, the limitations and remaining problems of the present qualitative approach. By so doing, we may hope to envisage some hints of a truly satisfactory formulations of the basic principles of string theory.

§2. Derivation of the space-time uncertainty relation

The first proposal [18)] of the space-time uncertainty relation in string theory came from an elementary space-time interpretation of the mechanism of eliminating ultraviolet divergencies in string theory. As is well known, the main reason that string amplitudes are free from ultraviolet divergences is that the string loop amplitudes satisfy the so-called modular invariance. The latter symmetry, which is a remnant of the conformal symmetry of Riemann surfaces after a gauge fixing, automatically generates the cutoff for the short-distance parts of the integrations over the proper times of the string propagation in loop diagrams. In traditional field theory approaches, the introduction of an ultraviolet cutoff suffers, almost invariably, from the violation of unitarity and/or locality. However, string perturbation theory is perfectly consistent with (perturbative) unitarity, preserving all the important axioms for a physically acceptable S-matrix, including the analyticity property of the S-matrix. It should be recalled that the analyticity of the S-matrix is customarily attributed to locality, in addition to unitarity, of quantum field theories. However, locality is usually not expected to be valid in theories with extended objects. From this point of view, it is not at all trivial to understand why string theory is free from the ultraviolet difficulty, and it is important to give correct interpretations to its mechanism.

2.1. *A reinterpretation of energy-time uncertainty relation in terms of strings*

The approach which was proposed in Ref. 18) is to reinterpret the ordinary energy-time uncertainty relation in terms of the space-time extension of strings:[*)]

$$\Delta E \Delta t \gtrsim 1. \tag{2·1}$$

The basic reason why we have ultraviolet divergencies in local quantum field theories is that in the short time region, $\Delta t \to 0$, the uncertainty with respect to energy increases indefinitely: $\Delta E \sim 1/\Delta t \to \infty$. This in turn induces a large uncertainty in momentum $\Delta p \sim \Delta E$. The large uncertainty in the momentum implies that the number of particles states allowed in the short distance region $\Delta x \sim 1/\Delta p$ grows indefinitely as $(\Delta E)^{D-1}$ in D-dimensional space-time. In ordinary local field theories, where there is no cutoff built-in, all these states are expected to contribute to amplitudes with equal strengths. This consequently leads to UV infinities.

What is the difference, in string theory, regarding this general argument? Actually, the number of the allowed states with a large energy uncertainty ΔE behaves as $e^{k\ell_s \Delta E} \sim e^{k\ell_s/\Delta t}$ with some positive coefficient k, and $\ell_s \propto \sqrt{\alpha'}$ being the string length constant, where α' is the traditional slope parameter. This increase of the degeneracy is much faster than that in local field theories. The crucial difference with local field theories, however, is that the dominant string states among these exponentially degenerate states are not the states with large center-of-mass momenta, but can be the massive states with higher excitation modes along strings. The excitation of higher modes along strings contributes to the large spatial extension of string states. It seems reasonable to expect that this effect completely cancels the short distance effect with respect to the center-of-mass coordinates of strings, provided that these higher modes contribute appreciably to physical processes. Since the order of magnitude of the spatial extension corresponding to a large energy uncertainty ΔE is expected to behave as $\Delta X \sim \ell_s^2 \Delta E$, we are led to a remarkably simple relation for the order of magnitude ΔX for fluctuations along spatial directions of string states participating within the time interval $\Delta T = \Delta t$ of interactions:

$$\Delta X \Delta T \gtrsim \ell_s^2. \tag{2·2}$$

It is natural to call this relation the 'space-time uncertainty relation'. It should be emphasized that this relation is *not* a modification of the usual uncertainty relation,

*) Throughout the present paper, we use units in which $\hbar = 1, c = 1$.

but simply a *reinterpretation* in terms of strings. Note that as long as we remain in the framework of quantum mechanics, the usual Heisenberg relation $\delta x \delta p \gtrsim 1$ is also valid if it is correctly interpreted. For example, the latter is always valid for the center-of-mass momentum and the center-of-mass position of strings. The space-time uncertainty relation, on the other hand, gives a new restriction on the short-distance space-time structure, which comes into play because of the intrinsic extendedness of strings. In general, therefore, we have to combine the ordinary uncertainty relation and the space-time uncertainty relation appropriately in estimating the relevant scales in string theory, as is elucidated in discussion given below.

To avoid a possible misunderstanding, we remark that, as is evident in this simple derivation, the spatial direction is dominantly along the longitudinal direction of strings. Therefore, it should not be confused as the more familiar transverse spread of a string. That the longitudinal size indeed grows linearly with energy, at least in perturbation theory, is most straightforwardly seen as follows. For simplicity, let us consider the case of open strings. The interaction of open strings is represented by the vertex operator $\exp ipx(\tau, 0)$, inserted as the endpoint $\sigma = 0$ of one of the strings. If the string before the insertion is made is in the ground state with some moderate momentum, the effect of the vertex operator is to change the state after the insertion to a coherent state of the form $\exp(p\alpha_{-n}\ell_s/n)|0\rangle$ for each string mode n. This induces a contribution to the spatial extension of the string coordinate along the spatial components $\vec{p}$ of the momentum vector p_μ of order $\langle x_n^2 \rangle \sim |\vec{p}|^2 \ell_s^4/n^2$, which in the high-energy limit $|\vec{p}| \to \infty$ leads to $\Delta X \sim \sqrt{|\vec{p}|^2 \ell_s^4 \sum_{n<n_s}(1/n^2)} \sim E\ell_s^2$. Note that here we have adopted as the measure of string extension the quantity $\sqrt{\langle \int d\sigma\, (x(\sigma) - x_0)^2 \rangle}$. This apparently shows that there is a large extension with respect to the time direction too. However, the interaction time ΔT should be defined with respect to the center-of-mass coordinate of strings, and hence the apparent large extension along the time direction does not directly correspond to the time uncertainty in the energy-time uncertainty relation (2·1). Furthermore, we should expect the existence of some limit $n \lesssim n_s$ for the excitation of string modes, depending on the specific region that the string scattering is probing. In the Regge limit, for example, where the momentum transfer is small, we can show that $\ell_s/\Delta T \sim n_s \lesssim s\ell_s^2$ (see §3.3). In this case, in addition to the growth along the longitudinal direction, we have an intrinsic transverse extension of order $\ell_s \sqrt{\sum_{n \lesssim n_s}(1/n)} \sim \ell_s \sqrt{\log(E/\ell_s)}$ for all directions corresponding to the zero-point extension of the ground state wave function. The logarithmic transverse extension is negligible compared to the linear growth. The mechanism of suppressing the ultraviolet divergence, as exhibited by the modular invariance, cannot be attributed to the logarithmic growth of the extension of the ground state wave function. It is clearly the effect that is dominantly associated with the longitudinal extension of strings.

We will later see that in some cases, such as the case of high-energy-high-momentum transfer scattering of strings and D-particle scattering with small velocities, we can effectively neglect the effect of string higher modes. This is not directly in contradiction with the role of string higher modes which we emphasized above in connection with the enormous degeneracy of string states associated with the higher modes. The degeneracy is with regard to the standard string modes of free strings with standard boundary conditions. Situations in which the string higher modes are effectively negligible occur with different backgrounds or different boundary conditions for the string coordinates as fields on the string world sheet. In terms of the standard free strings, such cases are represented by a coherent state with excitation of higher string modes.

The main purpose of the present paper is to present several arguments which suggest that the space-time uncertainty relation (2·2) may be a universal principle which is valid nonperturbatively in string theory. It should be emphasized that the space-time uncertainty principle has yet only been qualitatively formulated. We cannot give a rigorous definition for the uncertainties ΔX and ΔT at the present stage of development. For example, one might ask how to define the time uncertainty if the string stretches linearly with energy. We always assume that the time is measured with respect to some preferred point, most naturally at the center-of-mass of a string. Also, there is no point-like probe with which we can measure the spatial uncertainty of a string: A string itself has an intrinsic extension depending on the scale of resolution if we are allowed to imagine a point-like probe.

The point we would like to stress is, however, that this simple looking relation has quite universal applicability both perturbatively and nonperturbatively, at least in some qualitative sense, if it is interpreted appropriately. Also, involving both time and space intrinsically, the relation is not just a kinematical constraint which decreases the number of degrees of freedom, but in principle may place a strong constraint on the dynamics of the system. Its precise role and the correct mathematical formulation can only be found after the proper framework of string theory is established. The prime motivation for this viewpoint was a general belief that any theory of quantum gravity must impose some crucial restrictions at short distance scales near the Planck length, beyond which the classical space-time geometry, on which general relativity is based on, is invalidated. It is then important to ascertain how such a restriction is realized in string theory, since it would suggest the precise manner in which string theory departs from the usual framework of quantum field theories. The present author is aware of many past attempts at the construction of a formal 'space-time quantization'. The standpoint in proposing the space-time uncertainty principle is not to propose yet another version of the formal theory of quantized space-time, but to possibly uncover some secrets that would help lead to the unification of quantum theory and general relativity from string theory which possesses several ideal properties as a candidate for the unified theory in a quite surprising and unexpected fashion.

2.2. *The nature of the space-time uncertainty relation*

Now an important characteristic of the relation (2·2) is that it demands a duality between the time-like and space-like distance scales. Whenever we attempt to probe the short distance region $\Delta T \to 0$ in a time-like direction, the uncertainty with respect to the space-like direction increases. In addition, we propose that the relation is also valid in the opposite limit. Namely, if we attempt to probe the short distance region $\Delta X \to 0$ in space-like directions, then the uncertainty ΔT in the time-like direction increases. In other words, the smallest distances probed in string theory cannot be made arbitrarily small with respect to both time-like and space-like directions *simultaneously.* It is proposed in Ref. 19) that the phenomena of minimal distance[20), 31)] in string perturbation theory can be interpreted in this way. However, it should be kept in mind that our space-time uncertainty relation does not forbid the possibility of probing shorter distance regions than the ordinary string scale in string theory, quite contrary to the implication following the usual notion of minimal possible distance in string theory.[*)] Rather, it only imposes a new condition that the short and large distances are dual to each other. We note that in some of the recent developments of nonperturbative string theory associated with D-branes, the regime of short open strings much below the string scale is a crucial ingredient.

What is the physical interpretation of the opposite limit, namely the short spatial distance which implies a large time uncertainty $\Delta T \to \infty$? Is it really possible to probe distance scales ΔX smaller than ℓ_s? Since any string state with a definite mass has an intrinsic spatial extension of order ℓ_s, it seems at first sight impossible to do this. It turns out that the D-particle, instead of the fundamental strings, plays precisely such a role as shown later. Moreover, the fact that the asymptotic string states can be represented by vertex operators coupled with local external fields may be interpreted as a consequence of the relation $\Delta X \sim \ell_s^2/\Delta T \to 0$. In this sense, the space-time uncertainty relation can also be viewed as a natural expression of the s-t duality, which has been the basic background for string theory. Roughly speaking, the resonance poles near on-shell in the s-channel correspond to $\Delta T \to \infty$, while the t-channel massless pole exchange to $\Delta X \to \infty$, with vanishingly small momentum transfer, if the exchange is interpreted in terms of pair creation and annihilation of open strings. In fact, the s-t duality was another motivation for proposing the space-time uncertainty relation in Ref. 19).

The fact that the string theory has a short distance cutoff built-in in this way might be somewhat counter intuitive, since strings have a much larger number of particle degrees of freedom than any local field theories or ordinary nonlocal field theories with multi-local fields and/or some nonlocal interactions. But precisely because of this counter-intuitive nature of string theory, we must study the short distance structure of string theory carefully. For example, the growth of the string size along the longitudinal direction with energy might seem to be quite contrary with the familiar idea of Lorentz contraction of a projectile. However, this is one of the properties that allows, at least in perturbation theory, string theory to contain gravity, as we will discuss in §3. Also, the large degeneracy of particle states should rather be interpreted as implying that string theory suggests an entirely new way for counting the physical degrees of freedom in the region of the smallest possible distance scales. We hope that the discussion given here will provide a basis for the concrete formulation of this general idea.

Before proceeding further, it is appropriate here to comment on the difference between our space-time uncertainty relation and the other proposal of a related un-

*) The possible relevance of shorter length scales has been later suggested in Ref. 21).

certainty relation with stringy corrections. In parallel to the original suggestion[*)] of the space-time uncertainty relation, the high-energy behavior of the string amplitudes has been studied. On the basis of such investigations, it was proposed independently of the proposal (2·2) that in the high-energy limit the space-time extensions of strings are proportional[22)] to energy and momentum as

$$x^{\mu} \propto \ell_s^2 \, p^{\mu}.$$

The reason behind this proposal is that the classical solution for the string world-sheet trajectory for given wave functions with momenta p_i^{μ} corresponding to external asymptotic states takes the following form

$$x^{\mu}(z, \overline{z}) = \ell_s^2 \sum_i p_i^{\mu} \log |z - z_i| \tag{2·3}$$

in the lowest tree approximation, where the z_i are the positions of the vertex operators on the Riemann surface corresponding to the asymptotic states with on-shell momenta p_i^{μ}. This seems also to be consistent with what we have discussed using the vertex operator in our derivation of the space-time uncertainty relation. Combined with the Heisenberg relation $|\delta x^{\mu}| \sim 1/|\delta p^{\mu}|$, the above proposal suggests a modified uncertainty relation[23)] for each space-time component (no summation over μ),

$$|\delta x^{\mu}||\delta p^{\mu}| \gtrsim 1 + \ell_s^2 \, |\delta p^{\mu}|^2, \tag{2·4}$$

which leads to $|\delta x^{\mu}| \gtrsim \ell_s$ for *each* component of the space-time coordinates separately. Our space-time uncertaity relation (2·2) is weaker than this relation, and it does not lead directly to the minimul distance, unless we assume some relation between time and space uncertainties : For example, if we set $\Delta T \sim \Delta X$, we immediately have the miminum distance relation $\Delta X \gtrsim \ell_s$. This is a crucial difference.

It appears that this particular form (2·4) cannot be regarded as being universally valid in string theory. We can provide at least three reasons for this. First, the uniform proportionality between energy-momenta and the extensions of the string coordinates is not valid in the region in which the high-energy behavior is dominated by the Riemann surfaces where the positions of the vertex operators approach the boundary of the moduli space. Second, even when the dominant contribution comes from a region which is not close to the boundary of the Riemann surface, it is known that the amplitudes after summing up the entire perturbation series using the Borel-sum technique behave differently from the tree approximation for high-energy fixed angle scattering. The known behavior is incompatible with the relations such as (2·4) demanding that the string extension grows indefinitely, while it turns out to be consistent with our relation (2·2). Third and most importantly, the relation (2·4) is

[*)] Unfortunately, since the proposal (2·2) was initially made in a paper[18)] written for the volume commemorating Prof. Nishijima's 60th birthday and has not been published in popular journals, it has long been ignored. The earliest discussion of the space-time uncertain relation in the popular journals was presented in Ref. 19). It, however, seems that even this reference has been largely ignored to date. The author hopes that the present exposition is useful in drawing attention to these papers.

not effective for explaining the short-distance behavior of D-brane interactions. In particular, a naive relation such as $|\delta x^\mu| \gtrsim \ell_s$, expressing the presence of a minimal distance, clearly contradicts the decisive role of the familiar characteristic spatial scale $g_s^{1/3}\ell_s$ in D-particle scattering, which is much smaller than the string length ℓ_s in the weak-coupling regime, and more generally in the conjecture of M-theory.[24] All of these points will be discussed fully in later sections.

One might naively think that when the spatial extension becomes large the interaction time would also increase, as expressed in (2·4), since the spatial region for interaction grows. This intuition might be correct if we were dealing with ordinary extended objects, such as polymers, which may interact with each other in the bulk of the spatial extension. However, the nature of the interaction of elementary strings is strongly constrained by conformal invariance. Elementary strings have no bulk-type forces among their parts. Thus, the ordinary intuition for the extended object is not necessarily applicable to string theory. For this reason, whether the interaction time should also increase as the spatial extension or not must depend on specific situations and cannot be stated as a general property.

As the final topic of this subsection, let us ask whether and how the space-time uncertainty relation (2·2) can be compatible with kinematical Lorentz invariance. The answer is that the relation as an *inequality* can be consistent with Lorentz invariance. Suppose that the relation is satisfied in some preferred Lorentz frame which we call the proper frame, where the uncertainties are $\Delta T = \Delta T_0$ and $\Delta X = \Delta X_0$, and, in particular, the spatial uncertainty can be estimated as being at rest. In most physical applications discussed later in this paper, we always assume such a preferred frame in deriving the relation. Let us make a Lorentz boost of the frame of reference with velocity v and measure the same lengths in the boosted frame. Then the uncertainty in time is $\Delta T = \Delta T_0/\sqrt{1-v^2}$, while the spatial interval is contracted as $\Delta X = \sqrt{1-v^2}\Delta X_0$ or is not affected $\Delta X = \Delta X_0$ depending on the directions of the characteristic spatial scale. This shows that the inequality (2·2) is preserved in any Lorentz frame provided that it is satisfied in some proper Lorentz frame, after averaging over the spatial directions.

We can arrive at the same conclusion from more formal considerations too. Let us temporarily suppose the existence of an algebraic realization of the space-time uncertainty relation, by introducing the space-time (hermitian) operators X^μ, transforming as Lorentz vectors, as some effective agents measuring the observable distance scales in each Minkowski direction μ. Then, as has been discussed in a previous paper,[25] the space-time uncertainty relation may correspond to an operator constraint which is manifestly Lorentz invariant, as given by

$$\frac{1}{2}[X^\mu, X^\nu]^2 \sim \ell_s^4, \tag{2·5}$$

where the contracted indices are summed over as usual.[*] By decomposing into time

[*] Similar constraints are studied from a different viewpoint in Ref. 26).

and space components, we have

$$\sqrt{\langle -[X^0, X^i]^2\rangle} = \sqrt{\frac{1}{2}\langle -[X^i, X^j]^2\rangle + \ell_s^4} \gtrsim \ell_s^2. \qquad (2\cdot6)$$

This shows that the inequality (2·2) of the space-time uncertainty relation can in principle be consistent with Lorentz invariance, conforming to the first argument. This also suggests a possible definition of the proper frame such that the noncommutativity of space-like operators is minimized. To avoid a possible misconception, however, it should be noted that the present formal argument is *not* meant to imply that the author is proposing that the operator constraint (2·5) is the right way for realizing the space-time uncertainty principle. In particular, it is not at all obvious whether the uncertainties can be defined using Lorentz vectors, since they are not local fields. Here it is only used for an illustrative purpose to show schematically the compatibility of the space-time uncertainty relation with Lorentz invariance. There might be better way of formulating the principle in a manifestly Lorentz invariant manner. We will later present a related discussion (§5) from the viewpoint of noncommutative geometric quantization of strings based on the Schild action.

2.3. *Conformal symmetry and the space-time uncertainty relation*

For the sake of completeness, we now explain an independent derivation of the space-time uncertainty relation on the basis of conformal invariance of the world-sheet string dynamics, following an old work.[19] This derivation seems to support our proposal that the space-time uncertainty relation should be valid universally in both short-time and long-time limits.

All the string amplitudes are formulated in terms of path integrals as weighted mappings from the set of all possible Riemann surfaces to a target space-time. Therefore, any characteristic property of the string amplitudes can be understood from the property of this path integral. The absence of the ultraviolet divergences in string theory from this point of view is a consequence of the modular invariance. We will see that the space-time uncertainty relation (2·2) can be regarded as a natural generalization of the modular invariance for arbitrary string amplitudes in terms of the direct space-time language.

Let us start by briefly recalling how to define the distance on a Riemann surface in a conformally invariant manner. For a given Riemannian metric $ds = \rho(z,\overline{z})|dz|$, an arc γ on the Riemann surface has length $L(\gamma,\rho) = \int_\gamma \rho|dz|$. This length is, however, dependent on the choice of the metric function ρ. If we consider some finite region Ω and a set of arcs defined on Ω, the following definition, called the 'extremal length' in mathematical literature,[27] is known to give a conformally invariant definition for the length of the set Γ of arcs:

$$\lambda_\Omega(\Gamma) = \sup_\rho \frac{L(\Gamma,\rho)^2}{A(\Omega,\rho)} \qquad (2\cdot7)$$

with

$$L(\Gamma,\rho) = \inf_{\gamma\in\Gamma} L(\gamma,\rho), \quad A(\Omega,\rho) = \int_\Omega \rho^2 dz d\overline{z}.$$

Since any Riemann surface corresponding to a string amplitude can be decomposed into a set of quadrilaterals pasted along the boundaries (with some twisting operations, in general), it is sufficient to consider the extremal length for an arbitrary quadrilateral segment Ω. Let the two pairs of opposite sides of Ω be α, α' and β, β'. Take Γ be the set of all connected set of arcs joining α and α'. We also define the conjugate set of arcs Γ^* be the set of arcs joining β and β'. We then have two extremal distances, $\lambda_\Omega(\Gamma)$ and $\lambda_\Omega(\Gamma^*)$. The important property of the extremal length for us is the reciprocity

$$\lambda_\Omega(\Gamma)\lambda_\Omega(\Gamma^*) = 1. \tag{2·8}$$

Note that this implies that one of the two mutually conjugate extremal lengths is larger than 1.

The extremal lengths satisfy the composition law, which partially justifies the naming "extremal length": Suppose that Ω_1 and Ω_2 are disjoint but adjacent open regions on an arbitrary Riemann surface. Let Γ_1 and Γ_2 consist of arcs in Ω_1 and Ω_2, respectively. Let Ω be the union $\Omega_1 + \Omega_2$, and let Γ be a set of arcs on Ω.

1. If every $\gamma \in \Gamma$ contains a $\gamma_1 \in \Gamma_1$ and $\gamma_2 \in \Gamma_2$, then
$$\lambda_\Omega(\Gamma) \geq \lambda_{\Omega_1}(\Gamma_1) + \lambda_{\Omega_2}(\Gamma_2).$$
2. If every $\gamma_1 \in \Gamma_1$ and $\gamma_2 \in \Gamma_2$ contains a $\gamma \in \Gamma$, then
$$1/\lambda_\Omega(\Gamma) \geq 1/\lambda_{\Omega_1}(\Gamma_1) + 1/\lambda_{\Omega_2}(\Gamma_2).$$

These two cases correspond to two different types of compositions of open regions, depending on whether the side where Ω_1 and Ω_2 are joined does not divide the sides which $\gamma \in \Gamma$ connects, or do divide, respectively. One consequence of the composition law is that the extremal length from a point to any finite region is infinite and the corresponding conjugate length is zero. This corresponds to the fact that the vertex operators describe the on-shell asymptotic states whose coefficients are represented by local external fields in space-time. We also recall that the moduli parameters of world-sheet Riemann surfaces are nothing but a set of extremal lengths with some associated angle variables, associated with twisting operations, which are necessary in order to specify the joining of the boundaries of quadrilaterals.

Conformal invariance allows us to conformally map any quadrilateral to a rectangle on the Gauss plane. Let the Euclidean lengths of the sides (α, α') and (β, β') be a and b, respectively. Then, the extremal lengths are given by the ratios

$$\lambda(\Gamma) = a/b, \quad \lambda(\Gamma^*) = b/a. \tag{2·9}$$

For a proof, see Ref. 27).

Let us now consider how the extremal length is reflected by the space-time structure probed by general string amplitudes. The euclidean path-integral in the conformal gauge is essentially governed by the action $\frac{1}{\ell_s^2}\int_\Omega dz d\bar{z}\, \partial_z x^\mu \partial_{\bar{z}} x^\mu$. Take a rectangular region as above and the boundary conditions ($z = \xi_1 + i\xi_2$) as

$$x^\mu(0,\xi_2) = x^\mu(a,\xi_2) = \delta^{\mu 2} B\xi_2/b,$$
$$x^\mu(\xi_1,0) = x^\mu(\xi_1,b) = \delta^{\mu 1} A\xi_1/a.$$

The boundary conditions are chosen such that the kinematical momentum constraint $\partial_1 x^\mu \cdot \partial_2 x^\mu = 0$ in the conformal gauge is satisfied for the classical solution.*) The path integral then contains the factor

$$\exp\Big[-\frac{1}{\ell_s^2}\Big(\frac{A^2}{\lambda(\Gamma)}+\frac{B^2}{\lambda(\Gamma^*)}\Big)\Big].$$

This indicates that the square root of extremal length can be used as the measure of the length probed by strings in space-time. The appearance of the square root is natural, as suggested from the definition (2·7):

$$\Delta A = \sqrt{\langle A^2\rangle} \sim \sqrt{\lambda(\Gamma)}\ell_s, \quad \Delta B = \sqrt{\langle B^2\rangle} \sim \sqrt{\lambda(\Gamma)}\ell_s.$$

In particular, this implies that probing short distances along both directions simultaneously is always restricted by the reciprocity property (2·8) of the extremal length, $\Delta A \Delta B \sim \ell_s^2$. In Minkowski metric, one of the directions is time-like and the other is space-like, as required by the momentum constraint. This conforms to the space-time uncertainty relation, as derived in the previous subsection from a very naive argument. Also note that the present discussion clearly shows that the space-time uncertainty relation is a natural generalization of modular invariance, or of open-closed string duality, exhibited by the string loop amplitudes.

Since our derivation relies on conformal symmetry and is applicable to arbitrary quadrilaterals on arbitrary Riemann surfaces, which in turn can always be constructed by pasting quadrilaterals appropriately, we expect that the space-time uncertainty relation is robust with respect to possible corrections to the simple setup of the present argument. In particular, the relation, being independent of the string coupling, is expected to be universally valid to all orders of string perturbation theory. Since the string coupling cannot be regarded as the fundamental parameter of nonperturbative string theory, it is natural to expect that any universal principle should be formulated independently of the string coupling.

We have assumed a smooth boundary condition at the boundary of the rectangle. This leads to a saturation of the inequality of the uncertainty relation. If we allow more complicated 'zigzag' shapes for boundaries, it is not possible to establish such a simple relation as that above between the extremal distance and the space-time uncertainties. However, we can expect that the mean values of the space-time distances measured along the boundaries of complicated shapes in general increase, due to the entropy effect, in comparison with the case of smooth boundaries (namely the zero mode) obtained as the average of given zigzag curves. Although there is no general proof, any reasonable definition of the expectation value of the space-time distances conforms to this expectation, since the fluctuations contribute positively to the expectation value. Thus the inequality (2·2) should be the general expression of the reciprocity relation (2·8) in a direct space-time picture. Since the relation is symmetric under the interchange $\Gamma \leftrightarrow \Gamma^*$, it is reasonable to suppose that the space-time uncertainty relation is meaningful in both limits $\Delta T \to 0$ and $\Delta T \to \infty$, as we proposed in the previous subsection.

*) The Hamilton constraint $(\partial_1 x)^2 = (\partial_2 x)^2$ is satisfied after integrating over the moduli parameter, which in the present case of a rectangle is the extremal length itself.

§3. High-energy scattering of strings and the space-time uncertainties

We now proceed to study how the space-time uncertainty relation derived in the previous section is reflected in the high-energy behavior of string scattering. To the author's knowledge, a careful comparative investigation of the space-time uncertainty relation with the high-energy (and/or high-momentum transfer) behavior of string scattering has not yet been made. We hope that the present section fills this gap.

3.1. *How do we detect the interaction region from S-matix?*

In general, there are various difficulties in extracting precise space-time structure from on-shell S-matrix. This is so even in ordinary particle theories, since it is not possible, quantum mechanically, to define the trajectories of interacting particles unambiguously only from the S-matrix element. In string theory, the difficulties are compounded, since strings themselves have intrinsic extendedness. Therefore it is not completely clear how to extract the space-time uncertainties from scattering amplitudes. The only conceivable way at present is to just treat a string state as a particle state and approximately trace its trajectory by forming a wave packet in space-time with respect to the center-of-mass coordinate. The effect of extendedness would then be approximately reflected upon the uncertainties of the interaction region with respect to the center-of-mass coordinates of strings without referring to their internal structure. In our case, we have to separate the distance scales into temporal and spatial directions. We will see that the high-energy behavior of the scattering matrix alone does not allow us to carry out this completely. But we will be able to check whether the space-time uncertainty relation is consistent with the high-energy behavior.

Let us consider the elastic scattering of two massless particles $1+2 \to 3+4$. The wave packet of each particle can be written as

$$\psi_i(x_i,p_i) = \int d^9\vec{k}_i\, f_i(\vec{k}_i - \vec{p}_i)\, e^{i(\vec{k}_i\cdot\vec{x}_i - |\vec{k}_i|t_i)}, \tag{3·1}$$

where $f_i(\vec{k})$ is any function with a peak at $\vec{k}=0$. Here and in what follows, we assume a 10 dimensional flat space-time, unless otherwise specified, neglecting the issue of compactification, in particular. The inverse of the width at the peak gives the spatial extension of the wave packet. The scattering amplitude is then given as

$$\langle 3,4|S|1,2\rangle = \Big(\prod_{i=1}^{4}\int d^9\vec{k}_i\Big) f_3^*(3)f_4^*(4)f_1(1)f_2(2)\,\delta^{(10)}(k_1+k_2-k_3-k_4)A(s,t), \tag{3·2}$$

where $s=-(k_1+k_2)^2, t=-(k_2+k_3)^2$, and, for brevity, the momentum variables in the wave functions $f_i(\vec{k}_i-\vec{p}_i)$ are suppressed. The uncertainty of the interaction region is measured by examining the response of the S-matrix under appropriate shifts of the particle trajectories in space-time. The wave packet, after given shifts Δt and $\Delta\vec{x}$ of time and space coordinates, respectively, is

$$\psi_i(x_i,p_i;\Delta t_i,\Delta\vec{x}_i) = \int d^9\vec{k}_i\, f_i(\vec{k}_i-\vec{p}_i)\, e^{i(\vec{k}_i\cdot\vec{x}_i-|\vec{k}_i|t_i)}e^{i(\vec{k}\cdot\Delta\vec{x}_i-|\vec{k}_i|\Delta t_i)}. \tag{3·3}$$

To measure the uncertainty of the interaction region with respect to time, it is sufficient to choose $\Delta t_1 = \Delta t_2 = -\Delta t_3 = -\Delta t_4 = \Delta t/2$ and $\Delta x_i = 0$ for all i. Thus, the uncertainty ΔT can be estimated by examining the decay of the matrix element (3·2) under the insertion of the additional phase factor $\exp(-i(|\vec{k}_1| + |\vec{k}_2|)\Delta t)$ in the integrand comparing it with that without the insertion. On the other hand, to measure the uncertainty of the interaction region with respect to spatial extension, it is sufficient to choose $\Delta\vec{x}_1 = -\Delta\vec{x}_2 = \Delta\vec{x}^I/2$ and $\Delta t_i = 0$ and $\Delta\vec{x}_3 = -\Delta\vec{x}_4 = \Delta\vec{x}^F/2$, corresponding to the relative spatial positions of the trajectories of initial and final states, respectively. In this case, the additional phase factor is $\exp[i(\vec{k}_1 - \vec{k}_2)\cdot\Delta\vec{x}^I/2 - i(\vec{k}_3 - \vec{k}_4)\cdot\Delta\vec{x}^F/2)]$.

Let us choose the center-of-mass system for the momenta k_i. Assuming that the scattering takes place in the 1-2 plane and that the particles are all massless, we set

$$\begin{aligned} k_1 &= (-E\sin\phi/2, E\cos\phi/2, 0, \ldots, 0, E), \\ k_2 &= (E\sin\phi/2, -E\cos\phi/2, 0, \ldots, 0, E), \\ k_3 &= (E\sin\phi/2, E\cos\phi/2, 0, \ldots, 0, E), \\ k_4 &= (-E\sin\phi/2, -E\cos\phi/2, 0, \ldots, 0, E). \end{aligned} \tag{3·4}$$

Thus,

$$k_1 + k_2 = (0, 0, 0, \ldots, 0, 2E), \tag{3·5}$$
$$k_1 - k_2 = (-2E\sin\phi/2, 2E\cos\phi/2, 0, \ldots, 0, 0), \tag{3·6}$$
$$k_3 - k_4 = (2E\sin\phi/2, 2E\cos\phi/2, 0, \ldots, 0, 0), \tag{3·7}$$
$$k_1 - k_3 = (-2E\sin\phi/2, 0, 0, \ldots, 0, 0), \tag{3·8}$$
$$k_1 - k_4 = (0, 2E\cos\phi/2, 0, \ldots, 0, 0). \tag{3·9}$$

The order of magnitude of the decay width with respect to $|\Delta t|$ is estimated by taking a small variation with respect to the center-of-mass energy E, fixing the scattering angle ϕ, since the variation of the additional phase is just $E\Delta t$ and is independen of the angle ϕ. As Δt increases, the decay of amplitude becomes appreciable when the absolute value of the variation of the logarithm of the amplitude is exceeded by the variation of the additional phase $E\Delta t$. Therefore, we can roughly set

$$\Delta T \sim \langle|\Delta t|\rangle \sim \frac{1}{2}\left|\frac{\partial}{\partial E}\log A(s,t)\right|. \tag{3·10}$$

This way of measuring the uncertainty should perhaps be regarded as giving a lower bound, since it does not take into account the extendedness of the initial and final string states. We must evaluate this quantity at the peak values of the momenta. Note that this expression is similar to the well-known Wigner formula for time delay, for which we usually take only the phase of the amplitude. For the spreading of the interaction region, the variation of the modulus of the amplitude plays an equally important role as its phase.*)

Similarly, the decay width with respect to $|\Delta x|$ can be estimated by taking variations with respect to both the energy and scattering angle, since the additional phase now behaves as $(k_1-k_2)\cdot\Delta\vec{x}^I/2-(k_3-k_4)\cdot\Delta\vec{x}^F/2 = E(-(\Delta\vec{x}^I+\Delta\vec{x}^F)_1\sin\frac{\phi}{2}+(\Delta\vec{x}^I-\Delta\vec{x}^F)_2\cos\frac{\phi}{2})$, where the lower indices refer to the components in the 1-2 plane. The order of magnitude of the allowed spatial uncertainty is constrained by the conditions obtained by identifying the first variations of the modulus of the logarithm of the amplitude and of the additional phase

$$\left|\delta\Big(E\Big(-\Delta x_1^{(+)}\sin\frac{\phi}{2}+\Delta x_2^{(-)}\cos\frac{\phi}{2}\Big)\Big)\right| \sim |\delta\log A| \qquad (3\cdot11)$$

for $\Delta\vec{x}^{(\pm)}\equiv\Delta\vec{x}^I\pm\Delta\vec{x}^F$. This gives two equations for determining the components of the vector $(\Delta x_1^{(+)},\Delta x_2^{(-)})\equiv\Delta\vec{\tilde{x}}$ from the coefficients with respect to the variations δE and $\delta\phi$. This relation shows that there are limitations in estimating the spatial uncertainties. First, since the energy variation essentially gives the same contribution to $|\Delta x^{(\pm)}|$ as Δt, the high-energy scattering of massless particles can only probe the region in which $\Delta X \gtrsim \Delta T$. This limitation is inevitable, since, for particles moving with the light velocity, a time uncertainty necessarily contributes to a spatial uncertainty of the same order. Second, and more importantly, we can only probe the *vector* sum or difference of the spatial uncertainties for initial and final states. However, the spatial uncertainty for the space-time uncertainty relation should be defined to be the average of the uncertainties of the initial and final states as $\Delta X \sim (|\Delta x^I|+|\Delta x^F|)/2$. Due to the triangle inequality, we at least have a lower bound for the spatial uncertainty:

$$\Delta X > \Delta\tilde{x}. \qquad (3\cdot12)$$

Note that we cannot in general expect this equality to be saturated, except for the very peculiar case where either the initial or final spatial uncertainty vector vanishes. Complete information on the space-time structure could only be obtained if one could completely convert the scattering matrix into the coordinate representation. Of course, for each term of the perturbation series, we already have such a picture in the form of the world-sheet path-integral representation. But nonperturbatively, in general, we cannot expect to have such a picture.

We remark here that the power-law behavior for high-energy fixed-angle scattering necessarily leads to the decrease by a factor of $1/E$ for both spatial and temporal uncertainties in the above sense. This is, of course, typical behavior for the high-energy limit of local field theories. Our task is to examine how string scattering deviates from such typical behavior of particle scattering in local field theories.

3.2. *High-energy and high-momentum transfer behavior of string scattering*

Fortunately, the behavior of string scattering in the high energy limit with fixed scattering angle is studied in detail in Refs. 22) and 28). We study how far we

*) When the first variation with respect to the integration variables is small, we must be careful in checking whether the higher variations are negligible. For measuring the transverse size $|\delta x_t|$ corresponding to the shift of the form $\exp i(k_1-k_3)\delta x_t$, the second variation is indeed important. We also note that the present method is reliable only when $\log A(s,t)$ is a smooth function.

can probe the short distance space-time structure using the results of these works. Throughout the present section, we use the string unit $\ell_s = 1$.

At the tree level, the leading behavior is

$$A_{\rm tree}(s,\phi) \sim ig^2 2^9 s^{-1}(\sin\phi)^{-6}\exp\Big(\frac{-s}{4}f(\phi)\Big), \tag{3·13}$$

where

$$f(\phi) = -\sin^2(\frac{\phi}{2})\Big|\log\,\sin^2(\frac{\phi}{2})\Big| - \cos^2(\frac{\phi}{2})\Big|\log\,\cos^2(\frac{\phi}{2})\Big|. \tag{3·14}$$

Although this particular form is for bosonic closed strings, the main feature that the amplitude falls off exponentially is due only to the Riemannian nature of the world sheet, and hence the exponential behavior including the function (3·14) is completely universal for any perturbative string theory.

The exponential fall off of (3·13) has been regarded as one of the features of string theory which is clearly distinctive from local field theories. This has been the main motivation for the suggestion of the modification of Heisenberg uncertainty relation as (2·4). Indeed, if we apply the above method for estimating the width to the tree behavior directly with finite angle ϕ, we would get $\Delta T \sim \Delta X \sim E$, corresponding to the dominant classical world sheet configuration (2·3). However, the exponential fall off of tree amplitudes only means that the tree approximation is quite poor for representing the high-energy behavior of string scattering. In fact, for $N-1$ loop amplitudes, the exponential factor is replaced by $\exp(-sf(\phi)/N)$. Thus, for any small but finite string coupling, the high-energy limit is dominated by large N contributions. The nonperturbative high-energy behavior is derived in Ref. 28) by performing the Borel-sum over N. The final result there is summarized as

$$|A_{\rm resum}(s,\phi)| \sim \exp\Big(-\sqrt{4sf(\phi)\log(1/g^2)}\Big) \tag{3·15}$$

for $1 \ll \log(1/g^2) \ll s \ll 1/g^{4/3}$, and

$$|A_{\rm resum}(s,\phi)| \sim \exp\Big(-\sqrt{6\pi^2 sf(\phi)/\log s}\Big) \tag{3·16}$$

for $s \gg 1/g^{4/3}$. The tree behavior (3·13) with a much faster decreasing exponential is valid only for $1 \ll s \ll \log(1/g^2)$.

Now let us estimate the space-time uncertainties exhibited in the nonperturbative high-energy behavior (3·16) for fixed string coupling. For our purpose of estimating the order of magnitude for the decay width of the amplitudes with respect to the shift of the particle trajectories, we can neglect the imaginary part of the logarithm $\log A(s,\phi)$, since it only contributes to the present qualitative estimation at most to the same order as the real part, and hence it only affects the numerical multiplicative factor for the width.

Using (3·10), the uncertainty of the interaction region with respect to time is

$$\Delta T \sim \sqrt{f(\phi)}. \tag{3·17}$$

We neglect the logarithms as well as the numerical factor with respect to the energy E, since our method (or any other conceivable method) is not sufficiently precise to include them. Note that in the limit of small scattering angle, we have $\Delta T \sim \phi\sqrt{\log\phi} \sim (\sqrt{t}/E)\sqrt{\log(E/\sqrt{t})} \to 0$. The dependence on the momentum transfer is strange, compared with $\Delta T \sim 1/E$ for the standard Regge behavior for fixed t. In reality, the approximation used in the derivation of the high-energy limit will break down in the small angle limit, since in that case the saddle point approaches a singular boundary of the moduli space. Therefore we can trust our result only for moderate scattering angles.

In order to obtain the uncertainty of the spatial interaction region, we use (3·11), which leads to

$$\epsilon_1\sqrt{f(\phi)} = -\Delta\tilde{x}_1 \sin\frac{\phi}{2} + \Delta\tilde{x}_2\cos\frac{\phi}{2}, \tag{3·18}$$

$$\epsilon_2\frac{|f'(\phi)|}{\sqrt{f(\phi)}} = \Delta\tilde{x}_1 \cos\frac{\phi}{2} + \Delta\tilde{x}_2\sin\frac{\phi}{2}, \tag{3·19}$$

where ϵ_1 and ϵ_2 are arbitrary sign factors, arising in making the comparison (3·11). At $\phi = \pi/2$, the first variation with respect to the scattering angle vanishes. It is, however, easy to check that taking account of the second variation does not change the final conclusion in the limit $E \to 0$. We then obtain

$$4(\Delta\tilde{x})^2 \sim f(\phi) + \frac{f'(\phi)^2}{f(\phi)}. \tag{3·20}$$

Because of the inequality (3·12) and the relation (3·17), this gives a lower bound for the space-time uncertainty relation as

$$\begin{aligned}\Delta T\Delta X > \Delta T\Delta\tilde{x} &\sim \frac{1}{2}\sqrt{f(\phi)^2 + f'(\phi)^2} \\ &= \sqrt{\sin^2\frac{\phi}{2}\Big(\log\sin\frac{\phi}{2}\Big)^2 + \cos^2\frac{\phi}{2}\Big(\log\cos\frac{\phi}{2}\Big)^2}.\end{aligned} \tag{3·21}$$

For moderate values of the scattering angle which are not close to 0 or π, the right-hand side is of order 1, independent of energy. This is consistent with our space-time uncertainty relation. In particular, this shows that we cannot probe arbitrarily short distances, even if both energy and momentum transfer increase without limit. The fact that the right hand side vanishes in either limit $\phi \to 0$ or $\phi \to \pi$ implies only that this inequality (3·12) is far from being saturated, namely, $\Delta X \gg \Delta\tilde{x}$. For example, if we use (3·18) and (3·19) in the limit $\phi \to 0$ of forward scattering, we find $\Delta\tilde{x}_2 \to \sin\frac{\phi}{2} \to 0$ and $\Delta\tilde{x}_1 \sim O(1)$ which indicate that the components of the spatial uncertainty match for the initial and final states, i.e. $\Delta x_2^I \sim \Delta x_2^F$, along the longitudinal direction while along the transverse direction there is a spread of order one. In any case, however, we cannot trust our formulas for such small scattering angle, as emphasized above. For a generic scattering angle, it seems reasonable to regard the inequality as almost saturated, namely, $\Delta X \sim \Delta\tilde{x}$, since there is no preferred direction for the spatial uncertainty.

3.3. *The Regge limit and space-time uncertainties*

We have studied the high-energy limit for fixed scattering angle, which means high-momentum transfer $s \sim t \to \infty$. Let us briefly consider the case of fixed momentum transfer $t = -(k_1 - k_3)^2 = -4E^2 \sin^2 \frac{\phi}{2}$.*) Since this corresponds to the limit of small scattering angle, the above discussion suggests that we cannot expect information other than some matching conditions between initial and final spatial uncertainties. The high energy behavior is dominated by the exchange of Regge poles. In string theory, the leading Regge trajectory is that of the graviton. Hence, the tree (invariant) amplitude is given by

$$A_{\rm tree}(s,t) \sim g^2 \frac{1}{t}(-is/8)^{2+t/4}. \tag{3·22}$$

As is well known, however, this behavior is actually incompatible [30] with unitarity for sufficiently high energies. To recover unitarity, it is again necessary to resum the whole perturbation series. This problem is investigated in Ref. 31) using the method of Reggeon calculus. It was shown that the series can be summed into an (operatorial) eikonal form in the region of large impact parameter, or equivalently, in the region of small momentum transfer in the present momentum representation. In particular, the tree form (3·22) is justified only when the eikonal is very small, i.e. when $1/b \sim \sqrt{t} < (g^2 s)^{-1/(D-4)} \ll 1$ is satisfied. This is essentially the classical region. By reapplying the method used above to the tree amplitude (3·22) in this region, we obtain the uncertainty in time,

$$\Delta T \sim \frac{\partial}{\partial E} \log s \sim 1/E \ll 1. \tag{3·23}$$

For the uncertainty in the spatial direction, we can only obtain the following constraints

$$|\Delta \tilde{x}_1| \sim \frac{2}{E}\Big(\Big(1+\frac{t}{2}\log E\Big)\sin\frac{\phi}{2} + \frac{\epsilon}{\sin\frac{\phi}{2}}\Big(1-\frac{t}{2}\log E\Big)\Big) \sim \frac{1}{\sqrt{t}}\Big(1-\frac{t}{2}\log E\Big), \tag{3·24}$$

$$|\Delta \tilde{x}_2| \sim \frac{2}{E}\Big(\Big(1+\frac{t}{2}\log E\Big)\sin\frac{\phi}{2} - \frac{\epsilon}{\sin\frac{\phi}{2}}\Big(1-\frac{t}{2}\log E\Big)\Big) \sim \frac{4}{E} \text{ or } \frac{-2t}{E}\log E, \tag{3·25}$$

where ϵ is the choice of relative sign between the energy and angle variations in extracting the orders of magintude for the uncertainties using (3·11). In conformity with the tendency found in the fixed-angle case, the spatial uncertainties along the longitudinal direction 2 match for initial and final states. This is expected, since the space-time uncertainty relation requires that the longitudinal spatial uncertainty increase with energy (or decrease of interaction time). The growth of the longitudinal size of a string with decreasing time uncertainty would be impossible unless the uncertainties match for the initial and final states along that direction. On the other hand, along the transverse directions, (3·24) indicates that the average uncertainty spreads without limit as the momentum transfer vanishes, corresponding to the exchange of a massless graviton. The singular behavior of (3·24) in t is produced by the pole at $t = 0$ of the Regge amplitude. This is also consistent with the growth of the longitudinal length of strings. From the s-channel viewpoint, it is very difficult

*) For a previous analysis of high-energy string scattering with fixed momentum transfer, see Ref. 29).

to imagine the generation of long-range interactions without the rapid growth of the string extension.

That the high-energy Regge behavior of string amplitudes, at least with only its simplest 2-2 elastic scattering, can only be utilized for a consistency check of the space-time uncertainty relation might seem somewhat disappointing. However, this is inevitable in view of the number of the variables available in the scattering amplitude and its kinematical structure.

We note that our method of estimating the interaction region directly from the high-energy behavior is not sensitive enough to fix the Regge intercept: For the property $\Delta T \sim 1/E$, only the power behavior with respect to energy with fixed momentum transfer is relevant, and the value of intercept, including its sign, cannot be detected. Actually, in the Regge limit, this information of the Regge intercept, namely, the existence of massless states such as the graviton and photon in string theory, may be regarded as a consequence of the space-time uncertainty relation. It has long been known [32] that in light-cone string theory there is a simple geometrical explanation for the intercept of the Regge trajectory of string theory. We can adapt this geometrical interpretation to the space-time uncertainty relation as follows.

Consider the elastic scattering of two strings, $p_1 + p_2 \to p_3 + p_4$ in the extreme forward region, where the longitudinal momenta p_1^+ and p_3^+ are very large compared with others. Namely, we choose a sort of a laboratory frame instead of the center-of-mass frame. If we treat the high-momentum state as the target string and the low-momentum state as the projectile string, it is natural to represent the interaction by the insertion of the vertex operators corresponding to the absorption and emission of the projectile string onto the target string state. In this case, the projectile string can effectively be treated as a probe with small longitudinal extension, since the momentum associated with the vertex operator is small. On the other hand, by reversing the roles of projectile and target strings we see that the intermediate state induced by the interaction has a large longitudinal extension. Also, by repeating the above analysis of the Regge limit in the present frame, we can see that the interaction time is small and the longitudinal extension associated with initial and final states must match each other in the Regge limit. Note that the main difference between this situation and that in the center-of-mass frame is only that $s \sim p_1^+ p_2^-$ instead of $s \sim E^2$. This means that the probability for the interaction with forward scattering is proportional to its longitudinal length, which can be regarded as being proportional to the longitudinal momenta, since the interactions of strings are regarded as occurring independently at each segment of the target string.[*] With the identification of the longitudinal length and the longitudinal momentum in accordance with the space-time uncertainty relation, this means that the probability amplitude in the tree approximation linearly grows with large longitudinal momentum $p_1^+ \sim p_3^+$. For the invariant amplitude, this amounts to the Regge intercept $\alpha(0) = 2$. If we

[*] Here it is important that the string is a continuous object. If, for example, we consider some extended object with only a discrete and finite number of degrees of freedom, we cannot expect to generate a graviton or any massless particles naturally. It seems very difficult to construct a reasonable theoretical framework other than string theory that contains gravity and satisfies the space-time uncertainty relation.

only consider the open string interactions, neglecting the closed string, the same argument leads to $\alpha(0) = 1$, since the interaction only occurs at the string endpoint, and hence the probability amplitude is constant in the high-energy limit.

3.4. *A remark: minimum nonlocality ?*

Finally we remark that there is no guarantee that the Borel summation of the leading behavior of the perturbation series gives a unique nonperturbative result. Therefore, the formula which we have relied upon for studying the fixed angle scattering may not be completely correct, due to some nonperturbative effects that have not been taken into account in the Borel summation.

However, at least for a certain finite range of the string coupling including the weak coupling regime, it seems reasonable to regard the properties found here as providing evidence for the following viewpoint: The space-time uncertainty relation is a natural principle which characterizes string theory nonperturbatively as being *minimally but critically* departed from the usual framework of local field theory for resolving ultraviolet difficulties. This view may be supported by recalling that the high-energy behavior (3·16) with fixed scattering angle almost realizes the fastest allowed decrease of the form,[33] $\exp(-f(\phi)\sqrt{s}\log s)$. The proof of this theorem uses, apart from the usual analyticity and unitarity, assumptions of polynomial boundedness in the energy for fixed t and also of the existence of a mass gap. Obviously, the latter is not satisfied in the presence of graviton. However, that this lower bound just represents the behavior, up to logarithms, corresponding to the saturation of the bound expressed in the space-time uncertainty relation, as is exhibited by (3·21), is very suggestive. We may say that 'locality' is almost satisfied in some effective sense in string theory from the viewpoint of the analyticity property of scattering amplitudes.[*] The space-time uncertainty relation may be interpreted as the basic principle for introducing nonlocality in a way that does not contradict the analyticity property of the scattering amplitude, whose validity is usually assumed for local field theories.

§4. The meaning of space-time uncertainty relation

Now that we have checked the consistency of the space-time uncertainty relation with high-energy string scattering, let us study its implications from a more general standpoint. Since the relation expresses a particular way by which string theory deals with the short distance structure of space-time, we expect that it should predict (or explain) some characteristic features of string theory, when combined with other physical characteristics of the theory.

4.1. *The characteristic scale for microscopic black holes in string theory*

We first consider an implication for microscopic gravitational phenomenon. Usually, the characteristic scale of quantum gravity is assumed to be the Planck scale,

[*] In the literature, we can find another approach to locality based on the commutation relation of string fields.[34),35)] It would be an interesting problem to connect the latter approach to our space-time uncertainty relation.

which in ten-dimensional string theory is equal to $\ell_P \sim g_s^{1/4}\ell_s$, corresponding to the ten-dimensional Newton constant $G_{10} \sim g_s^2\ell_s^8$. Indeed, if we neglect the effect of higher massive modes of string theory, this would be the only relevant scale. Let us consider the limitation of the notion of classical space-time from this viewpoint in light of the possible formation of black holes in the short distance regime. Suppose that we probe the space-time structure at a small resolution of order δT along the time direction. This necessarily induces an uncertainty $\delta E \sim 1/\delta T$ of energy. Let us require ordinary flat space-time structure to be qualitatively preserved at the microscopic level by demanding that no virtual horizon is encountered, associated with this uncertainty of energy. Then we have to impose the condition that the minimum resolution along spatial directions must be larger than the Schwarzschild radius corresponding to this energy:

$$\delta X \gtrsim (G_{10}/\delta T)^{1/7}.$$

This leads to the 'black-hole uncertainty relation' *)

$$\delta T(\delta X)^7 \gtrsim G_{10}. \tag{4·1}$$

This expresses a limitation, for observers at asymptotic infinity, with respect to spatial and temporal resolutions, below which the naive classical space-time picture without the formation of microscopic black holes can no longer be applied. If we assume that the spatial and temporal scales are of the same order, this would lead to the familiar looking relation $\delta T \sim \delta X \gtrsim \ell_P$. However, in the presence of some stable very massive particle state in probing the short distance scales, such as a D-particle, this assumption may not necessarily be valid, and we should in general treat the two scales independently.

Furthermore, it is important to remember that the relation (4·1) does *not* forbid *smaller* spatial scales than δX *in principle*. Suppose we use as a probe a sufficiently heavy particle, such as a D-particle in the weak string-coupling regime. We can then neglect the extendedness of the wave function and localize the particle in an arbitrarily small region. In this limit, classical general relativity can be a good approximation. But general relativity only requires the existence of local time, and hence we cannot forbid the formation of black holes. This only stipulates that we cannot read the clock on the particle inside the black hole from an asymptotic region at infinity. If we suppose a local observer (namely just another particle) sitting somewhere apart in a local frame which falls into the black hole, it is still meaningful to consider the local space-time structure at scales which exceed the condition (4·1), since the extendedness of the wave packet of a sufficiently heavy particle can, in principle, be less than the limitation set by (4·1). In connection with this, it should

*) Similar relations have been considered by other authors, independently of string theory. However our interpretation is somewhat different from those of other works. (See for a recent example Ref. 36).) We also note that the power $1/7\,(=1/(D-3))$ on the right hand side depends on the space-time dimensions. In particular, for $D=4$ the left-hand side of the black hole uncertainty relation takes the same form as the stringy one (2·2). In connection with this, see an interesting paper.[37] The author would like to thank M. Li for bringing this last reference to his attention.

be kept in mind that the above condition only corresponds to the restriction on the formation of microscopic black holes. For example, for a light probe, instead of a very heavy one, we have to take into account the usual quantum mechanical spread of the wave function, as we will do below in deriving the characteristic scale of D-particle scattering.

Despite a similarity in its appearance to (4·1), the space-time uncertainty relation of full string theory places a limitation in principle on the scale beyond which we can never probe the space-time structure by any experiment allowed in string theory,

$$\Delta T \Delta X \gtrsim \ell_s^2. \tag{4·2}$$

Note that such a strong statement is acceptable in string theory, because it is a well-defined theory resolving the ultraviolet problems. The nature of the condition (4·1) is therefore quite different from (4·2). In this situation, the most important scale corresponding to truly stringy phenomena is where these two limitations of different kinds meet. Namely, beyond this crossover point, it becomes completely meaningless to talk about the classical geometry of a black hole, and hence it is where the true limitation on the validity of classical general relativity must be set. The critical scales ΔT_c and ΔX_c corresponding to the crossover are obtained by substituting the relation $\Delta T_c \sim \ell_s^2/\Delta X_c$ into (4·1):

$$(\Delta X_c)^6 \sim \frac{G_{10}}{\ell_s^2} = g_s^2 \ell_s^6. \tag{4·3}$$

This gives

$$\Delta X_c \sim g_s^{1/3} \ell_s, \quad \Delta T_c \sim g_s^{-1/3} \ell_s. \tag{4·4}$$

Interestingly enough, we have derived the well-known eleven dimensional M-theory scale

$$\ell_M = g_s^{1/3} \ell_s = \Delta X_c \tag{4·5}$$

as the critical spatial scale, without invoking D-branes and string dualities directly. Note that this critical scale crucially depends on 10 dimensional space-time. For example, in 4 space-time dimensions, there is no such critical scale for arbitrary values of string coupling: Namely, there is only a 'critical coupling' $g_s \sim 1$ at which the Planck scale and string scale coincide.

To appreciate the meaning of the critical scales, it is useful to look at the diagram in Fig. 1. We see clearly that for $\Delta t < \Delta T_c$ there is no region where the concept of the microscopic black hole associated with quantum fluctuations is meaningful. On the other hand, in the region $\Delta t > \Delta T_c$, there is a small region where $(\Delta t)^{-1}\ell_s^2 < \Delta X < \Delta X_c$ is satisfied, and hence black hole formation at the microscopic level may be meaningful in string theory. The importance of this region increases as the string coupling grows larger. In the limit of weak string coupling, where $\Delta T_c \to \infty$ and $\Delta X_c \to 0$, there is essentially no fluctuation of the space-time metric corresponding to the formation of microscopic black holes. Unfortunately, the space-time uncertainty relation alone cannot predict more detailed properties of stringy black holes at microscopic scales. It is an important problem to explore

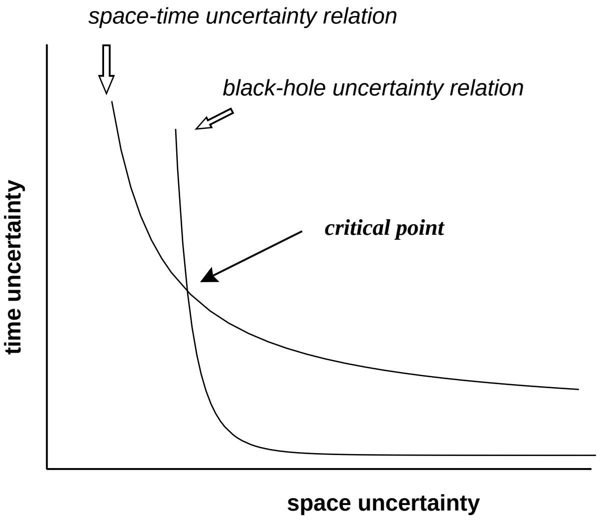

Fig. 1. This diagram schematically shows the structure of the space-time uncertainty relation and the black hole uncertainty relation. The critical point is where the two relations meet.

the physics in this region in string theory. The above relation between the space-time uncertainty relation and the black hole uncertainty relation suggests that in the strong string-coupling regime and in the region $\Delta T > \Delta T_c (\ll \ell_s)$, the black-hole uncertainty relation essentially governs the physics at long distatances $\Delta X > R_S$, with R_S being the Schwarzschild radius, since $R_S > \ell_s^2/\Delta T$ with $\Delta T \sim 1/E$ there.

4.2. *The characteristic scale of D-particle dynamics*

In the case of high-energy string scattering, we could not probe the region $\Delta X < \Delta T$. To overcome this barrier, we need massive stable particles. The point-like D-brane, i.e. a D-particle, of the type IIA superstring theory is an ideal agent in this context, at least for a sufficiently weak string coupling, since its mass is proportional to $1/g_s$ and its stability is guaranteed by the BPS property. The derivation of the characteristic scale of D-particle interactions has been given in two previous works.[38),39)] However, for the purpose of selfcontainedness and for comparison with the result of the previous subsection, we repeat the argument here with some clarifications.

Suppose that the region we are trying to probe by the scattering of two D-particles is of order ΔX. Since the characteristic spatial extension of open strings mediating the D-particles is then of order ΔX, we can use the space-time uncertainty relation. The space-time uncertainty relation demands that the characteristic velocity v of D-particles is constrained by

$$\Delta T \Delta X \sim \frac{(\Delta X)^2}{v} \gtrsim \ell_s^2,$$

since the period of time required for the experiment is of order $\Delta T \sim \Delta X/v$. Note that the last relation is due to the fact that ΔT is the time interval during which the length of the open string is of order $\varDelta X$. This gives the order of the magnitude for the minimum possible distance probed by D-particle scattering with velocity v:

$$\varDelta X \gtrsim \sqrt{v}\ell_s. \tag{4·6}$$

Thus to probe spatial distances shorter than the time scale, i.e. $\varDelta X \ll \varDelta T$, we have to use D-particles with small velocity $v \ll 1$. However, the spreading of the wave packet increases with decreasing velocity as

$$\varDelta X_w \sim \varDelta T \varDelta_w v \sim \frac{g_s}{v}\ell_s, \tag{4·7}$$

since the ordinary time-energy uncertainty relation asserts that the uncertainly of the velocity is of order $\varDelta_w v \sim g_s v^{-1/2}$ for time interval of order $\varDelta T \sim v^{-1/2}\ell_s$. To probe a range of spatial distance $\varDelta X$, we must have $\varDelta X \gtrsim \varDelta X_w$. Combining these two conditions, we see that the shortest spatial length is given by

$$\varDelta X \sim g_s^{1/3}\ell_s, \tag{4·8}$$

and the associated time scale is

$$\varDelta T \sim g_s^{-1/3}\ell_s. \tag{4·9}$$

Thus the minimal scales of D-particle-D-particle scattering coincide with the critical scales for microscopic black holes derived above. In other words, the minimal scales of D-particle scattering is just characterized by the condition that the fluctuation of the metric induced by the D-particle scattering is automatically restricted so that no microscopic black holes are formed during a scattering process. Indeed, we can derive the same scales from the black-hole uncertainty relation (4·1) by using the restriction $\delta T/m\delta X \sim \delta X$ for the spreading of the wave packet of a free particle with mass $m \sim 1/g_s\ell_s$ which is localized within a spatial uncertainty of order δX, conforming to the above agreement.

In view of this interpretation of the scale of D-particle dynamics, the agreement between D-particle scales and those for microscopic black hole formation is consistent with the seemingly surprising fact that the effective supersymmetric Yang-Mills quantum mechanics, which is formulated on a flat Minkowski background and does not, at least manifestly, have any degrees of freedom of the gravitational field, can reproduce[40)~42)] the gravitational interaction of type IIA supergravity, or equivalently, of the 11 dimensional supergravity with vanishingly small compactification radius $R_{11} = g_s\ell_s$, in the weak string-coupling (perturbative) regime. Naively, we expect that the supergravity approximation to string theory is only valid at scales which are larger than the string scale ℓ_s. On the other hand, the Yang-Mills approximation, keeping only the lowest string modes, is in general regarded as being reliable in the regime where the lengths of open strings connecting D-particles are small compared with the string scale. However, the consideration given in the previous subsection indicates that truly stringy gravitational phenomena are characterized by the critical scales $\varDelta T_s \sim g_s^{-1/3}\ell_s \gg \ell_s, \varDelta X_c \sim g_s^{1/3}\ell_s \ll \ell_s$. Given the fact that the Yang-Mills approximation to string theory is characterized precisely by the same scales, the compatibility of Yang-Mills approximation with supergravity can naturally be accepted as a consistency check of our chain of ideas at a 'phenomenological' level.

It should be kept in mind that the present discussion is certainly not sufficient for explaining the agreement of the Yang-Mills description with supergravity in the long distance regime. Why such Yang-Mills models can simulate gravity is still largely in the realm of mystery, since Yang-Mills theory has no symmetry corresponding to general coordinate transformations, and also since it has no manifest Lorentz invariance, either as an effective 10-D theory or as an infinite-momentum frame description of 11-D theory following the Matrix-theory conjecture. At least in the lowest order one-loop approximation,[40] the agreement is explained by the constraint coming from supersymmetry. It seems hard to believe, however, that global supersymmetry *alone* can explain the quantitative agreement of 3-body interactions found in Ref. 42) which is a genuinely nonlinear effect of supergravity. But this might turn out to be an incorrect prejudice. For a recent detailed discussion on the role of supersymmetry in general Yang-Mills matrix models, see Ref. 43) and references cited therein.

As the next topic of this subsection, we consider the D-particle scales from a slightly different viewpoint of degrees-of-freedom counting. Although the space-time uncertainty relation is first derived by a reinterpretation of the ordinary quantum mechanical uncertainty relation between energy and time, the fact that it puts a limitation on the observable length scales suggests that it should also imply a drastic modification on the counting of physical degrees of freedom. Let us consider how it affects the quantum state itself in the case of D-particles. The discussion of the previous subsection on D-particle scales emphasized the possible scale probed by the dynamical process of scattering. It is not obvious whether the same scale is relevant for restricting the general quantum state. The following derivation of the scale suggests that the same scale indeed is important from this viewpoint too.

Consider the state of a D-particle which is localized within a spatial uncertainty ΔX. The ordinary Heisenberg relation $\Delta p \Delta X \gtrsim 1$, which is the usual restriction on the degrees of freedom in quantum theory, then gives the relation for the velocity

$$v \gtrsim \frac{g_s \ell_s}{\Delta X}.$$

On the other hand, the space-time uncertain relation, reflecting the interaction of D-particles through open strings, implies the condition (4·6) for the minimum meaningful distances among D-particles with given velocities of order v as

$$\Delta X \gtrsim \sqrt{v}\ell_s.$$

By eliminating the velocity, we obtain the same condition on the scale of localization $\Delta X \gtrsim g_s^{1/3}\ell_s$ of a D-particle state at a given instant of time. In the M-theory interpretation of the D-particle, this is consistent with the holographic behavior that the minimum bit of quantum information stored in a D-particle state is identified with the unit cell whose volume in the transverse dimensions is of the order of the 11-dimensional Planck volume $\ell_{11}^9 \sim g_s^3 \ell_s^9 \sim (\Delta X)^9$.

Finally, we explain how these characteristic scales of D-particle dynamics are embodied in the effective Yang-Mills quantum mechanics: It can best be formulated by a symmetry property, called 'generalized conformal symmetry', which is proposed in the Ref. 44) and further developed in Ref. 45)−47). Briefly, the effective action, suppressing the fermionic part,

$$S = \int dt \, \mathrm{Tr}\Big(\frac{1}{2g_s\ell_s} D_t X_i D_t X_i + i\theta D_t \theta + \frac{1}{4g_s\ell_s^5}[X_i, X_j]^2 - \cdots\Big) \qquad (4{\cdot}10)$$

of the supersymmetric Yang-Mills matrix quantum mechanics is invariant under the transformations

$$X_i \to \lambda X_i, \qquad t \to \lambda^{-1} t, \qquad g_s \to \lambda^3 g_s, \qquad (4{\cdot}11)$$

$$\delta_K X_i = 2\epsilon t X_i, \qquad \delta_K t = -\epsilon t^2, \qquad \delta_K g_s = 6\epsilon t g_s, \qquad (4{\cdot}12)$$

which together with the trivial time translation symmetry form an $SO(2,1)$ algebra. This shows that the characteristic scales of the theory are indeed (4·8) and (4·9). Combining this with the fact that the same symmetry is satisfied in the classical metric of the D0 solution of type-IIA supergravity and with the help of some constraints due to supersymmetry, it is demonstrated in Ref. 44) that the generalized conformal symmetry can determine the effective D0 action as a probe to all orders in the velocity expansion, within the eikonal approximation, neglecting time derivatives of the velocity.

An important point here is that the supersymmetry of the model plays a crucial role for ensuring that the D-particles can be free when the distances among them are sufficiently large. Without the supersymmetry we would have nonvanishing zero-point energies. The zero-point energies contribute to the effective static potential, which grows linearly with distances. This would render scattering experiment impossible. If we assume the scaling symmetry (4·11), the effective action for two-body scattering in general takes the form

$$S_{\mathrm{eff}} = \int dt \Big(\frac{1}{2g_s\ell_s} v^2 - \sum_{p=0}^{\infty} c_p \frac{v^{2p}\ell_s^{4p-2}}{r^{4p-1}} + O(g_s)\Big) \qquad (4{\cdot}13)$$

in the limit of weak coupling with c_p representing numerical constants. The zero-point oscillation corresponds to the first term, $p = 0$. It is well known that supersymmetry eliminates the next term, $p = 1$, too, and the effective interaction starts from the $p = 2$ term, v^4/r^7.

As a further remark, we note that the product $\delta X \delta t$ of small variations is invariant under the above transformations, suggesting that the generalized conformal symmetry may be a part of a more general set of transformations which characterize the algebraic structure associated with the space-time uncertainty relations. Just as the canonical structure of classical phase space is transformed into Hilbert space of physical states in quantum theory, which is characterized by the 'unitary structure', such a characterization might lead to some appropriate mathematical structure underlying the space-time uncertainty relation. Exploration of such ideas might be an important future direction. However, this issue will not be addressed in the present paper. To carry out such a study meaningfully, we need more data.

For example, the Yang-Mills models of above type cannot describe the system with both D-branes and anti D-branes. Once the D-branes and anti D-branes are both included, [48] we have no justification for the approximation retaining only the lowest string modes, as the following argument shows. In the simplest approximation in which we retain only the usual gravitational interaction, the effective action is

$$\int dt\Big(\frac{1}{2g_s\ell_s}v^2+\frac{\ell_s^6}{r^7}\Big).$$

If we assume that the space-time uncertainty relation is saturated at its lower bound, the relation $r^2 \sim v\ell_s^2$ leads to an estimate of the characteristic length scale as $r_c \sim g_s^{1/11}\ell_s$ which is smaller than the string scale ℓ_s in the weak coupling region, while it is somewhat larger than the critical spatial scale of D-particle-D-particle scattering. Since, however, the string scale ℓ_s is just the characteristic scale corresponding to the instability, we have to take into account tachyons, and all higher modes too which are characterized by the same string scale, in terms of open strings. In the case of pure D-particle systems, the validity of retaining only the lowest open string modes and consistency with supergravity at least in the lowest order approximation in the weak string coupling is ensured by the supersymmetry: It leads to the fact that both short-distance and long-distance forces are described by the lowest Yang-Mills modes alone in the approximation of one-graviton exchange. Without manifest supersymmetry, however, there is no such mechanism which may ensure the validity of a field theory approximation.

A conclusion of this simple argument is that the D-particle and anti-D-particle system cannot be assumed to saturate the lower bound of the space-time uncertainty relation. In fact, if we just apply the ordinary Heisenberg uncertainty relation for the Hamiltonian $H=\frac{g_s\ell_s p^2}{2}-\frac{\ell_s^6}{r^7}$, we get a much larger spatial scale of order $\Delta X \sim g_s^{-1/5}\ell_s \gg \ell_s$. If we further assume that the scattering occurs through a metastable resonant state, the characteristic time scale is $\Delta T \sim g_s^{-7/5}\ell_s$, which leads to $\Delta X\Delta T \sim g_s^{-8/5}\ell_s^2 \gg \ell_s^2$. It seems that the lower bound of the space-time uncertainty relation is expected only for some particularly symmetric systems, such as systems satisfying the BPS condition and the generalized conformal symmetry. This expectation is also in accord with the results of high-energy fixed angle (or high-momentum transfer) scattering of strings obtained in the previous seciton, if one supposes that through the high-energy fixed angle scattering we are probing a regime where the symmetry is much enhanced.

At this juncture, it is perhaps worth remarking also that the generalized conformal symmetry is regarded as the underlying symmetry for the so-called DLCQ interpretation of the Yang-Mills matrix model. We can freely change the engineering scales in analyzing the system. Thus, if we wish to keep the numerical value of the transverse dimensions, we perform the rescaling $t \to \lambda^{-1}t,\ X_i \to \lambda^{-1}X_i,\ \ell_s \to \lambda^{-1}\ell_s$ simultaneously with the generalized scaling transformation leading to the scaling $t \to \lambda^{-2}t$, $X_i \to X_i$, $R \to \lambda^2 R$ and $\ell_{11} \to \ell_{11}$, which can be interpreted as a kinematical boost transformation along the 11th direction compactified with radius $R_{11} = g_s\ell_s$. Alternatively, we can keep the numerical value of time or energy by making the rescaling $t \to \lambda t,\ X_i \to \lambda X_i,\ \ell_s \to \lambda \ell_s$, leading to the scaling $t \to t,\ R \to \lambda^4 R,\ X_i \to \lambda^2 X_i,\ \ell_{11} \to \lambda^2 \ell_{11}$ and $\ell_s \to \lambda \ell_s$, which is in fact equivalent to the 'tilde' transformation utilized in Ref. 49) in an attempt to justify the Matrix theory for finite N. Note that although the second case makes the string length ℓ_s small by assuming small λ, the length scale for transverse directions smaller than the string scale is always even smaller ($< \lambda^2 \ell_s$). For further discussions and applications of the generalized conformal symmetry in D-brane dynamics, we refer the reader to our previous papers cited above. Here we only mention that the generalized conformal symmetry provides a basis for the extension of the AdS/CFT correspondence for the Yang-Mills matrix model. The concrete computation of the correlators led to somewhat unexpected but suggestive results with regard to the question of the compatibility of Lorentz invariance and holography in Matrix-theory conjecture, as fully discussed in Refs. 46) and 47).

4.3. *Interpretation of black-hole complementarity and UV-IR correspondence*

In the first part of the present section, we have emphasized the relevance of the space-time uncertainty relation to the question of formation of microscopic black holes through the fluctuation of space-time geometry. Is this relation also relevant for macroscopic black holes? Qualitatively at least, one thing is clear: For an external observer sitting outside black holes, strings are seen to more and more spread as they approach the horizon, because of the infinite time delay near the horizon. Namely, for the observer far from the horizon, the uncertainty of time becomes small, $\Delta T \to 0$, without limit as strings approach the horizon. The space-time uncertainty relation then demands that the spatial uncertainty increases as $\Delta X \sim \ell_s^2/\Delta T \to \infty$ without limit. This phenomenon is the basis for the proposal of implementing the principle of 'black-hole complementarity' [50] in terms of string theory by Susskind [51] in 1993. The general space-time uncertainty relation (2·2) proposed earlier just conforms to this principle of black-hole physics. In fact, a version of the space-time uncertainty relation is rederived in light-cone string theory in Ref. 51) from the viewpoint of black-hole complementarity.

However, starting from microscopic string theory, it is in general an extremely difficult dynamical problem to deal with macroscopic black holes involving string interactions in essential ways and resulting in macroscopic scales quite different from the fundamental string scale. Thus we cannot be completely sure in identifying the concrete physical consequences of the above general property of strings near the horizon. In the present subsection, we give a reinterpretation of the Beckenstein-Hawking entropy of macroscopic black holes from the viewpoint of the space-time uncertainty relation, following the general idea of black-hope complementarity. Although most of what we discuss here may simply represent different ways of expressing points which have been discussed previously, we hope that our presentation at least has the merit of looking at important things from a new angle.

As already alluded to in our derivation of the space-time uncertainty relation, one of the crucial properties of a free string, which is responsible for the space-time uncertainty principle, is its large degeneracy $[d(E) \sim \exp k\ell_s E]$ as energy increases. It is reasonable to suppose that this property is not qualitatively spoiled by the interaction of strings, which must definitely be taken into account for the treatment of macroscopic phenomenon.

Based on this expectation, our fundamental assumption is that the entropy of macroscopic Schwarzschild black hole is given by

$$S = \log W \sim \Delta X_{\text{eff}}/\ell_s, \tag{4·14}$$

where ΔX_{eff} is the *effective* spatial uncertainty of the state. The space-time uncertainty relation then leads to a lower bound in terms of the effective uncertainty ΔT_{eff} along the time direction as

$$S \gtrsim \ell_s/\Delta T_{\text{eff}}. \tag{4·15}$$

Intuitively, the motivation for this proposal should be clear: We have replaced the energy by the uncertainties in the formula of the degeneracy $[W \sim d(E)]$ of a free string state. In particular, the form $\Delta X_{\text{eff}}/\ell_s$ is natural if we assume that the macroscopic state is effectively described as a single string state with effective longitudinal length ΔX_{eff} corresponding to the effective spatial uncertainty. The assumption that near a black hole horizon the state should be treated as a single string state seems natural in view of the exponentially large degeneracy, as previously argued, e.g. in Ref. 51).

The effective uncertainties in general should depend on how precisely the states are specified. That a state is macroscopic means that it is specified solely by the macroscopic variables of state, such as the mass, temperature, total angular momentum, and so on. In the case of a Schwarzschild black hole, such macroscopic parameters are only its mass M and its Schwarzschild radius R_S. We treat these two parameters as being independent, since the gravitational constant is regarded as an independent dynamical parameter corresponding to the vacuum expectation value of the dilaton. Now, on dimensional grounds, the effective spatial uncertainty must take the form

$$\Delta X_{\text{eff}} = \ell_s f\left(\frac{R_S}{\ell_s}, M\ell_s\right). \tag{4·16}$$

However, the entropy of a macroscopic state should be expressible only in terms of macroscopic parameters, the function $f(\frac{R_S}{\ell_s}, M\ell_s)$ actually depends only on the product of the variables without explicit dependence of the string length ℓ_s:

$$f\left(\frac{R_S}{\ell_s}, M\ell_s\right) = f(R_s M).$$

To fix the form of the function $f(x)$ of a single variable, we here invoke the 'correspondence principle'[51), 52)] that the black hole entropy must be reduced to $\log d(M) \sim \ell_s M$ at the point where the Schwarzschild radius becomes equal to the string scale R_S, namely in the limit $R_S \to \ell_s$. This immediately leads to $f(x) \sim x$. Thus we obtain the entropy of the Beckenstein form in D dimensional space-time,

$$S \sim R_S M \sim (G_D M)^{1/(D-3)} M \sim G_D^{-1} (G_D M)^{(D-2)/(D-3)}, \tag{4·17}$$

where G_D is the Newton constant in D-dimensions, $G_D \sim g_s^2 \ell_s^{D-2}$.

The characteristic effective time uncertainty $\Delta T \sim \ell_s/(R_S M)$ associated with this reinterpretation of the black hole entropy can be understood from the viewpoint of 'stretched horizon' which is assumed to be located at a distance of order ℓ_s. As is well known, the near horizon geometry of a large Schwarzschild black hole is approximated by the Rindler metric $ds^2 = -\rho^2 d\tau_R^2 + d\rho^2 + ds^2_{\text{transverse}}$ whose time τ_R is related to the Schwarzschild time (namely the time which is synchronized with a clock at infinity) by $\tau_R \sim t/R_S$. The time scale at the stretched horizon $\rho \sim \ell_s$ must be scaled by ℓ_s. Then, a Schwarzschild time scale of order $1/M$ is converted to a proper time scale $\ell_s/R_S M$ at the stretched horizon. Thus, the *effective* uncertainties are essentially the uncertainties at the stretched horizon measured in the Rindler frame[51] describing the near-horizon geometry of a macroscopic black hole.

Our arguments, though admittedly mostly the consequences of simple dimension counting and hence yet too crude, seem to show the basic conformity of the space-time uncertainty principle with black hole entropy, and perhaps with the property of holography,[53),54)] which is expected to be satisfied in any well-defined quantum theory of gravity. The information of a macroscopic black hole is encoded within the spatial uncertainty of order $\Delta X_{\text{eff}} \sim R_S M \ell_s$. Or in terms of time, this corresponds to the effective time resolution of order $\Delta T_{\text{eff}} \sim \ell_s/(R_S M)$ at the lower bound for the entropy. At first sight, the last relation may seem quite counter intuitive, since it suggests a time scale much smaller than the string scale for understanding a macroscopic object. But it is not so surprising if we recall that this is precisely where the black-hole horizon plays the role as the agent for producing an infinite delay with respect to time duration. Although the horizon is not singular at all in terms of classical *local* geometry, it plays a quite singular role in terms of quantum theory, which *cannot* be formulated in terms of local geometry alone because of the superposition principle. This is one of the fundamental conflicts between general relativity and quantum theory, from a conceptual viewpoint. The space-time uncertainty relation demands that this conflict should be resolved by abandoning the simultaneous locality with respect to both time and space. In the previous section, we have seen that such a weakening of locality does not directly contradict the analyticity of the S-matrix. Also, the argument in the first subsection of the present section shows that there is in principle a regime where the time scale associated with the black hole can be much smaller than the string scale ℓ_s in the srong string-coupling regime.

The proposed general form (4·14), and in particular its lower bound (4·15), suggests that to decode the information, it is in general necessary to make the time resolution large by appropriately averaging over the time scale, in accordance with a viewpoint expressed in Ref. 55) in the context of Matrix theory. The time averaging in turn liberates the information stored in the spatial uncertainties and hence reduces the value of the entropy. For an observer outside a black-hole horizon, decoding all the information stored inside requires an observation of infinitely long time.

In connection with holography, we finally remark on the connection of the space-time uncertainty relation with the so-called UV-IR correspondence,[57)] which is familiar in the recent literature of AdS/CFT correspondence.[58)] In brief, the UV-IR correspondence asserts that the UV behavior of the Yang-Mills theory (CFT) on the boundary corresponds to the IR behavior of supergravity in the bulk, and vice versa.

On the other hand, the space-time uncertainty relation for open strings mediating D-branes leads to a similar relation that a small spatial uncertainty ΔX in the bulk corresponds to large uncertainties ΔT along the time-like direction on the brane at the boundary. Thus, the space-time uncertainty relation and the UV-IR correspondence seem to be equivalent in the sense that UV and IR are correlated in the bulk and boundary. However, with a little scrutiny, we see that there is a small discrepancy in that the UV-IR relation is a statement involving classical supergravity and consequently that it only requires a macroscopic scale characterized by the curvature near the horizon, which is given as $R_{\rm ads} \sim (g_s N)^{1/4}\ell_s$. In contrast with this, the space-time uncertainty relation only involves the string scale ℓ_s. This puzzle can be resolved as suggested essentially in Ref. 56) if we convert the uncertainty along the time-like direction into a spatial uncertainty on the brane at the boundary. Since, for the brane, open strings behave as electric sources, the uncertainty ΔT with respect to time is translated typically into a self energy associated with the spatial uncertainty $\Delta X_{\rm brane}$ within the brane as

$$\Delta T \sim \Delta X_{\rm brane}/\sqrt{g_s N}, \qquad (4\cdot18)$$

by using the well known fact that the effective Coulomb coupling for the superconformal Yang-Mills theory is $(g_s N)^{1/4} \sim (g_{YM}^2 N)^{1/4}$. This leads to $\Delta X_{\rm bulk}\, \Delta X_{\rm brane} \sim R_{\rm ads}^2$ which is the relation, involving only the supergravity scale $R_{\rm ads}$ actually used in Refs. 57) and 56), for a derivation of the holographic bound for the entropy of D3-branes. Note that here we are using the standard AdS coordinate used in Ref. 58) instead of that of Ref. 57). The infrared cutoff of order $\Delta X_{\rm bulk} \sim R_{\rm ads}$ amounts to an ultraviolet cutoff of order $\Delta X_{\rm brane} \sim R_{\rm ads}$ for D-branes at the boundary. For D3 branes wrapping around a 3-torus of volume L^3, the degrees of freedom are then $N_{\rm dof} \sim N^2 L^3/R_c^3 = L^3 R_c^5/G_{10}$.

We emphasize that the holography and UV-IR correspondence are of macroscopic nature, involving only macroscopic parameters in their general expressions. In fact, the black-hole entropy bound and the more general holographic bound have been argued (see Ref. 59) and references therein) to follow from the second law of thermodynamics, generalized to gravitating systems. In contrast to this, the space-time uncertainty relation is a general principle of a microscopic nature, characterized directly by the string scale without any macroscopic variables. Hence, in applying the space-time uncertainty relation to macroscopic physics, it is in general necessary to make appropriate conversions of the scales in various ways, depending on different physical situations, as exemplified typically by (4·14), (4·16) and (4·18). The qualitative conformity of the microscopic space-time uncertainty relation with holography suggests that the former can be a consistent microscopic principle which underlies the required macroscopic properties. As emphasized above, the departure of string theory from the framework of local field theory seems to be minimal in its nature. But the nonlocality of string theory, as being represented by the space-time space-time uncertainty principle, appears to be sufficient for coping with black-hole complementarity and holography.

Finally, in connection with the problem of macroscopic black holes, there remains one big problem. That is the problem of space-time singularities. Customarily, we expect that classical geometry breaks down around the length scale near the string scale ℓ_s. From the point of view of the space-time uncertainty relation, however, we have to discriminate the scales with respect to time and space. If we take the typical example of a Schwarzschild black hole, the singularity is a space-like region. Any object after falling inside the horizon encounters the singularity within a finite proper time. If one asks precisely at what time it encounters the singularity, the time resolution of the clock on the object must be sufficiently small. But then the space-time uncertainty relation again tells us that the locality with respect to the spatial direction is completely lost. Thus the classical local-geometric formulation which the existence of singularity relies upon loses its validity. Similarly, if the singularity is time like, the locality along the time direction is completely lost. It seems thus certain that in string theory space-time singularities are resolved. However, it is unclear whether this way of resolving the problem of space-time singularities has any observable significance, characterizing string theory.[*)]

§5. Toward a noncommutative geometric formulation

We have emphasized the role of world-sheet conformal symmetry as the origin of the space-time uncertainty relation. As has already been alluded to at the end of §2.2, such a dual relation between time and space obviously suggests some mathematical formalism which exhibits noncommutativity between operators associated to space and time. However, the usual world-sheet quantum mechanics of strings does not, at least manifestly, exhibit such noncommutativity. In a sense, in the ordinary world-sheet formulation, use is made of a representation in which the time (center-of-mass time of a string) is diagonalized, and the spatial extension ΔX is measured by the Hamiltonian, as is evident in our first intuitive derivation of the space-time uncertainty relation. Thus the noncommutativity of space and time is indeed there in a hidden form. Are there any alternative formulations of string quantum mechanics which explicitly exhibit noncommutativity? Note that we are not asking a further extension of string theory with an additional requirement of space-time noncommutativity. What is in mind here is a different representation of string theory with manifest noncommutativity that is, however, equivalent, at the level of the on-shell S-matrix, to the usual formulation, at least perturbatively. A different representation may well be more suited for an off-shell non-perturbative formulation, hopefully.

The purpose of this section is to suggest a particular possibility in this direction. From the above consideration, we should expect the existence of a world-sheet picture which is quite different from the ordinary one with respect to the choice of gauge.

[*)] An interesting point is that, in both the cases of the black-hole horizon and the space-time singularity, the increase of spatial extendedness of strings in the short time limit is coincident with those of the spatial distances between the geodesic trajectories exhibited in the classical Schwarzschild metric.

Let us consider the so-called Schild action*) of the form ($\lambda = 4\pi\alpha', \xi = (\tau, \sigma)$)

$$S_{\text{schild}} = -\frac{1}{2}\int d^2\xi\, e\Big\{\frac{1}{e^2}\Big[-\frac{1}{2\lambda^2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2\Big] + 1\Big\} + \cdots, \tag{5·1}$$

where e is an auxiliary field necessary to maintain the reparametrization invariance. We only consider the bosonic part for simplicity. The relevance of this action to the space-time uncertainty relation has been discussed in a previous work [25] from a slightly different context. There, it is shown how to transform the action into the more familiar Polyakov formulation. Also the study of this action motivates the definition of a particular matrix model, called 'microcanonical matrix model', as a tentative nonperturbative formulation, by introducing a matrix representation of the commutation constraint (2·5). This model is quite akin to the type IIB matrix model. [17]

From the point of view of conformal invariance, the equivalence of this action with that of the ordinary formulation is exhibited by the presence of the same Virasoro condition as the usual one. We can easily derive it in the form of constraints in the Hamiltonian formalism:

$$\mathcal{P}^2 + \frac{1}{4\pi\alpha'}\acute{X}^2 = 0, \quad \mathcal{P}\cdot\acute{X} = 0. \tag{5·2}$$

In deriving this relation, it is essential to use the condition coming from the variation of the auxiliary field e,

$$\frac{1}{e}\sqrt{-\frac{1}{2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2} = \lambda \tag{5·3}$$

which we proposed to refer to as a 'conformal constraint' in Ref. 25). Under these circumstances, we can proceed to the ordinary quantization with the Virasoro constraint as a first class constraint. In this case, there is apparently no place where the noncommutativity of space-time coordinates appears. The space-time uncertainty relation is embodied in conformal invariance which is typically represented by the Virasoro condition.

Now let us change to another possible representation of the Schild action by introducing a new auxiliary field $b_{\mu\nu}(\xi)$, which is a space-time antisymmetric tensor of second rank but is also a world-sheet density:

$$S_b = -\frac{1}{2}\int d^2\xi\, e\Big\{\frac{1}{e^2}\Big[\frac{1}{\lambda^2}\Big(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu} + \frac{1}{2}b_{\mu\nu}^2\Big)\Big] + 1\Big\}. \tag{5·4}$$

This can further be rewritten by making the rescaling $b_{\mu\nu} \to e b_{\mu\nu}$ of the b field:

$$S_{b2} = -\int d^2\xi\,\Big\{\frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu} + \frac{1}{2}e\Big(\frac{1}{2\lambda^2}b_{\mu\nu}^2 + 1\Big)\Big\}. \tag{5·5}$$

Note that the b field is then a world-sheet scalar. Usually, this Lagrangian is not convenient for quantization, since it contains only first derivatives with respect to

*) The original action proposed in Ref. 60) did not contain the auxiliary field e. However, an equivalent condition was imposed by hand.

the world-sheet (proper) time, leading to second class constraints, and there is no kinetic term and no Hamiltonian. From the viewpoint of noncommutative space-time coordinates, on the other hand, the second class constraints making identifications between some components of momenta and coordinates, could be the origin of the noncommutativity. If we assume for the moment that the external b field is independent of the world-sheet time, the Dirac bracket taking account the second class constraint is

$$\{X^\mu(\sigma_1), X^\nu(\sigma_2)\}_D = \lambda^2((\partial_\sigma b(\sigma_1)^{-1})^{\mu\nu}\delta(\sigma_1 - \sigma_2).$$

To see that this conforms to the space-time uncertainty relation, it is more appropriate to rewrite it as

$$\left\{X^\mu(\sigma_1), \frac{1}{\lambda}\partial_\sigma b^\mu_\nu(\sigma_2)X^\nu(\sigma_2)\right\}_D = \lambda\delta(\sigma_1 - \sigma_2). \tag{5·6}$$

Since the b field satisfies the constraint equation

$$\frac{1}{2\lambda^2}b^2_{\mu\nu} = -1, \tag{5·7}$$

assuming that the auxiliary field e is first integrated over, we must have nonvanishing time-like components b_{0i} of order λ:

$$b^2_{0i} = \lambda^2 + \frac{1}{2}b^2_{ij} \geq \lambda^2.$$

Then (5·6) is characteristic of the noncommutativity between the target time and the space-like extension of strings.

In the general case of a time dependent auxiliary field b, it is not straightforward to interpret the above action within the ordinary framework of canonical quantization, since the system is no longer a conserved system, with explicit time dependence in the action. However, the essence of the noncommutativity lies in the presence of the phase factor itself,

$$\exp\left[i\int d^2\xi\,\frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu\partial_b X^\nu b_{\mu\nu}\right],$$

rather than a formal interpretation in terms of operator algebra. The path integral in principle contains all the information of both the operator algebra and its representation. Let us assume the appearance of this phase factor is an indispensable part of any quantization based on the action (5·5). Then, we can qualitatively see a characteristic noncommutativity between time and space directions directly in this phase factor for the general case. To avoid a complication associated with the boundary we restrict ourselves to closed strings in the following discussion.

First, in the presence of this phase factor, the most dominant configurations for the b field for a generic world-sheet configuration of the string coordinates are those with the smallest possible absolute values allowed under the constraint (5·7). This is because the cancellation of the path integral over the world-sheet coordinate becomes stronger as the absolute value of b increases. So let us first consider the case where the spatial components are zero: $b_{ij} = 0$, leading $b_{0i}^2 = \lambda^2$. The effect of the spatial components b_{ij}, corresponding to the noncommutativity among spatial coordinates, will be briefly described later. Under this approximation, dependence on the world-sheet coordinate in the b field satisfying the constraint can be expressed as a local $O(D-1)$ rotation belonging to a coset $O(D-1)/O(D-2)$:

$$b_{0i}(\tau, \sigma) = \lambda S_{ri}(\tau, \sigma). \tag{5·8}$$

Here we represent the coset element by the matrix elements S_{ri}, with r being the radial direction for definiteness.

Let us now choose the time-like gauge

$$\partial_\sigma X^0 = 0$$

and treat the target time as a globally defined dynamical variable on the world-sheet as a function of the world-sheet time parameter τ. Then the phase factor reduces to

$$\exp\Big[i\int d\tau \frac{1}{\lambda^2}\dot{X}^0 \int d\sigma b_{0i}(\xi)\partial_b X^i\Big].$$

We can interpret this phase factor as arising from the product of the short-time (with respect to world-sheet time) matrix elements

$$\begin{aligned}&\Big\langle X^0\Big(\tau + \frac{1}{2}\epsilon\Big)\Big| X^0\Big(\tau - \frac{1}{2}\epsilon\Big)\Big\rangle \\ &\quad = \int [d\vec{X} dS(\tau,\sigma)] \Big\langle X^0\Big(\tau + \frac{1}{2}\epsilon\Big)\Big|\vec{X}, \partial_\sigma S\Big\rangle\Big\langle \vec{X}, \partial_\sigma S\Big| X^0\Big(\tau - \frac{1}{2}\epsilon\Big)\Big\rangle,\end{aligned} \tag{5·9}$$

where the intermediate state to be integrated over is inserted at the mid-point and the matrix elements are

$$\Big\langle X^0\Big(\tau + \frac{1}{2}\epsilon\Big)\Big|\vec{X}, \partial_\sigma S\Big\rangle = \exp\Big[i\frac{1}{\lambda^2}X^0\Big(\tau + \frac{1}{2}\epsilon\Big)\int d\sigma b_{0i}(\xi)\partial_b X^i\Big], \tag{5·10}$$

$$\Big\langle \vec{X}, \partial_\sigma S\Big| X^0\Big(\tau - \frac{1}{2}\epsilon\Big)\Big\rangle = \exp\Big[-i\frac{1}{\lambda^2}X^0\Big(\tau - \frac{1}{2}\epsilon\Big)\int d\sigma b_{0i}(\xi)\partial_b X^i\Big]. \tag{5·11}$$

In a more familiar operator form, this would correspond to the commutator

$$\Big[X^0, \int d\sigma\, S_{ri}\partial_\sigma X^i\Big] = i\lambda$$

at each instant of the world-sheet time. But the phase factors, as exhibited in (5·10) and (5·11), lead directly to an uncertainty relation of the form

$$|\Delta X^0||\Delta\vec{X}| \gtrsim \lambda, \tag{5·12}$$

$$|\Delta\vec{X}| = \sqrt{\Big\langle\Big(\Delta\int d\sigma S_{ri}\partial_\sigma X^i(\sigma)\Big)^2\Big\rangle} \tag{5·13}$$

with respect to the orders of magnitude of uncertainties in the path integral, by the same mechanism as the ordinary Fourier transformation. We note that (5·13) is invariant under reparametrization with respect to σ. Furthermore, the latter is acceptable as a measure for the spatial uncertainty, since it locally measures the length along the tangent of the profile of closed strings at a fixed world-sheet time, including the possibility of multiple winding, provided it does not vanish. In particular, when $\partial_\sigma X^i(\sigma)$ and $S_{ri}(\sigma)$ as two vectors in the target space are parallel to each other along the string, it precisely agrees with the proper length measured along the string. For general random configurations of the orientation of these vectors, (5·13) is a possible general definition of the length of a string in a coarse-grained form.

The effect of spatial components b_{ij} can be taken into account if we generalize the local rotation to the local Lorentz group $O(D-1,1)$ in (5·8). This is due to the fact that we can restrict the components of the auxiliary field b to those which have nonvanishing product $b_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu$. Since the antisymmetric tensor $\sigma_{\mu\nu} = \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu/2$ can be locally transformed to one corresponding to a time-like two-dimensional plane,*) we can assume a parametrization, say $b_{\mu\nu} = \lambda S_{0\mu} S_{r\nu}$, using the rotation matrix of $O(D-1,1)$. This leads to a correction to the definition of the spatial uncertainty as

$$|\varDelta \vec{X}| = \sqrt{\left\langle \left(\varDelta \int d\sigma (S_{00}S_{ri} - S_{0i}S_{r0}) \partial_\sigma X^i(\sigma) \right)^2 \right\rangle}.$$

Also, there arises an induced noncommutativity among the spatial components, corresponding to the phase factor

$$\exp\left[i\frac{1}{\lambda^2} \int d\tau d\sigma\, \dot{X}^i X'^j (S_{0i}S_{rj} - S_{0j}S_{ri}) \right].$$

This should be interpreted as residual noncommutativity, which is necessary to preserve Lorentz invariance in the presence of the primary noncommutativity between time and space.

Although a more rigorous formulation is desirable, our discussion seems to already suggest the quite remarkable possibility that space-time noncommutativity alone governs the essential features of the dynamics. This would not be so surprising if we recall that the space-time uncertainty relation can be regarded as a reinterpretation of the time-energy uncertainty relation. As such, its proper formulation would necessarily amount to formulating the Hamiltonian appropriately, as should have been clear from our foregoing discussions.

Of course, this particular formalism does not seem convenient for performing concrete computations of string amplitudes, at least with the technical tools presently available to us. Also, our discussion, being based on the world-sheet picture, is still perturbative in its nature. As we have stressed, the space-time uncertainty relation should be valid nonperturbatively, and hence must be ultimately reformulated without relying on the world-sheet picture on the basis of some framework which

*) In terms of invariants, this corresponds to the following property. If $\sigma^2_{\mu\nu} = \varSigma_{\mu\nu}$, then $\varSigma^2_{\mu\nu} = \mathrm{Tr}(\varSigma)\varSigma_{\mu\nu}/2$. Thus there is only one independent Lorentz invariant.

is second-quantized from the outset. The connection with matrix models discussed in a previous work[25] is certainly suggestive of a nonperturbative formulation, but unfortunately it seems to be yet lacking some key ingredients for a definitive formulation. We hope, however, that the above argument gives some impetus for further investigation aimed at constructing truly nonperturbative and calculable formulations in the future. For example, from the viewpoint of an analogy between classical phase space and space-time that we mentioned in discussing the generalized conformal transformation, study of the most general transformations which preserve the form $i\int d^2\xi\,\frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu}$ might be a direction to be pursued.

In connection with this, it might be possible to reinterpret directly the action (5·5) as a generalized deformation quantization of space-time geometry itself. This expectation also suggests a formulation from the viewpoint of M-theory by interpreting the world sheet of strings as a section of a membrane, and using a sort of formalism related to the Nambu bracket.[61] We also mention that to make the comparison with local field theory, the approach suggested in Ref. 62) might be of some relevance in the case of open strings. We have left all these possibilities as challenging and promising problems for the future.

The reader may have noticed the similarity regarding the appearance of noncommutativity in the present discussion with that in the recent discussions of noncommutative Yang-Mills theory based on D-branes. An obvious difference is that our b field is a world-sheet field which always exist even without the presence of the external space-time B field. Note that we obtained noncommutativity in the sense of the target space-time from the world sheet b field in the bulk of the string world sheet. But this noncommutativity is simply another representation of the space-time uncertainty relation already exhibited in the usual formulation with manifest conformal symmetry. Also, in our case, the dominant components of the b field are the time-like components b_{0i}, in contrast to the space-like components of the B field in Refs. 63) and 64). If we had treated D-branes using the above formulation based on the Schild action for open strings attached to D-branes by adding the constant space-time B_{ij} field, we would have obtained the noncommutativiy between time and space directions as above along the D-brane world volume, in addition to the noncommutativity among spatial directions along D-branes in association with B_{ij}.

Our approach to noncommutativity is also quite different from that of Ref. 65) in type-IIB matrix models. However, since the Schild action is intimately connected to the type-IIB matrix model, it would be very interesting to seek some possible relation with it.

We emphasize again that the noncommutativity discussed in the present section between time and space is a property which is intrinsic to the dynamics of fundamental strings, and it has nothing to do with the presence or absence of the external B field. Of course, the space-time B field is automatically contained as a state of closed strings in any valid formulation of (orientable) string theory. In quantum theory, we have to take into account its vacuum fluctuations. In this broad sense, these two different origins of space-time noncommutativity might be united in some nonperturbative framework, by identifying the fluctuations of the space-time B field and the world sheet b field self-consistently.

§6. Further remarks

In this final section, we discuss some miscellaneous points which have not been treated in the preceding sections and may become the source of confusion. We also comment on some future possibilities.

Frame dependence, (p, q) strings, and S-duality

Since the space-time uncertainty relation is a statement which contains a dimensionful parameter ℓ_s, we have to specify the frame for the metric in the sense of the Weyl transformation, with respect to which the string length parameter is defined. In the foregoing discussions, we always tacitly assumed that the string length ℓ_s is the proportional constant in front of the world-sheet string action, say $(1/\ell_s^2) \int d^2\xi g_{\mu\nu}\partial_z X^\mu \partial_{\bar{z}} X^\nu + \cdots$, using the standard conformal gauge. Therefore the frame of the space-time metric $g_{\mu\nu}(X)$ which should be used for the space-time uncertainty relation is the so-called string frame metric. This is important when we consider the S-duality transformation, under which the string-frame metric is not invariant.

Suppose we start with the fundamental string [$(1, 0)$ string] in type IIB theory and make a S-duality transformation which send $(1, 0)$ strings to (p, q) strings. In the original (1,0) picture, the other (p, q) strings are soliton excitations. Therefore, their interaction and motion are governed by the fundamental strings. In this sense, the space-time uncertainty relation must be satisfied using the original string frame metric at least in the weak coupling regime, where the tension of the (1,0) string is smaller than the (p, q) strings, provided we correctly identify the uncertainties. Note that the same can also be said for other higher dimensional D-branes.*) As long as we consider them in the weak string coupling regime with respect to the original fundamental string, all of the dynamics are basically expressible in terms of the fundamental strings. Although we now know that string theory is full of objects of various dimensions, they cannot be treated in a completely democratic way from the point of view of their real dynamics.

However, if we wish to use the picture in which the (p, q) string is now treated as the fundamental string in the regime where the transformed string coupling $g_s^{(p,q)} = \exp \phi_{(p,q)}$ is weak, and hence the original string coupling is in general in a strong-coupling regime, we have to use the world-sheet action of the (p, q) string to describe the dynamics. Then it is essential to shift our space-time Weyl frame correspondingly. Namely, the space-time string metric must also be transformed by the same S-duality transformation. This precisely cancels the difference of tensions for $(1, 0)$ and (p, q). This is, of course, as it should be as long as the S-duality transformation is a *symmetry* of the type IIB superstring theory. The space-time uncertainty relation is therefore invariant under the S-duality transformation. Thus, at least in S-duality symmetric theories, the space-time uncertainty relation must be valid for arbitrary string coupling, provided the appropriate change of the Weyl frame is made according to the transformation law of S-duality and the uncertainties

*) For a discussion of some uncertainty relations along the D-brane world volume, see Ref. 66).

are redefined correspondingly.

In formulas, this can be expressed as follows. The world-sheet bosonic action for the (p,q) string is, using the ordinary string metric of the target space-time as the fundamental $(1,0)$ string,

$$T_{(p,q)} \int d^2\xi \, g_{\mu\nu}(X)\partial_z X^\mu \partial_{\bar{z}} X^\nu,$$

where the tension of the (p,q) string in the original string frame units is given by [67]

$$T_{(p,q)} = \bar{\triangle}^{1/2}_{(p,q)} \frac{1}{\ell_s^2}, \tag{6·1}$$

and

$$\bar{\triangle}_{(p,q)} = |p - q\rho|^2 = \exp(\phi_{(p,q)} - \phi_{(1,0)}), \tag{6·2}$$

with $\rho = \frac{\chi}{2\pi} + ie^{-\phi}$. On the other hand, the space-time string metrics are related by

$$g^{(p,q)}_{\mu\nu}(X)\exp(-\phi_{(p,q)}/2) = g^{(1,0)}_{\mu\nu}(X)\exp(-\phi_{(1,0)}/2),$$

with $g_{\mu\nu} = g^{(1,0)}_{\mu\nu}$, corresponding to the S-duality invariance of the Einstein frame metric. Combining these relations, we confirm that the world-sheet action of the (p,q) string is equal to

$$\frac{1}{\ell_s^2} \int d^2\xi \, g^{(p,q)}_{\mu\nu}(X)\partial_z X^\mu \partial_{\bar{z}} X^\nu.$$

Thus we have a space-time uncertainty relation with the same string length ℓ_s as that before making the transformation.

Curved or compactified space-time and a remark on T-duality

Another point related to that discussed above is that the space-time uncertainty relation must be valid qualitatively in general curved space-times allowed as backgrounds of string theory, as long as the world-sheet conformal invariance is not violated. In this case too, it is essential to use the string frame metric to measure the invariant (or proper) length appropriately with respect to time and space directions.*)

A somewhat related, but different point involves the interpretation of T-duality from the viewpoint of the space-time uncertainty relation. T-duality asserts that under the compactification of a spatial direction along a circle, the theory with a radius R is equivalent to that with ℓ_s^2/R. This is due to the mapping $n \to m, R \to \ell_s^2/R$ between the momentum modes whose mass spectrum is n/R and the winding modes whose spectrum is mR/ℓ_s^2. From the viewpoint of the space-time uncertainty relation, the uncertainty with respect to the former, referring only to the center-of-mass momentum, must be translated into an uncertainty with respect to energy by

$$\Delta T_1 \sim R_1/\Delta n_1$$

*) For example, the discrepancy claimed in Ref. 68) can easily be corrected by using the proper length appropriately.

which implies the lower bound for the spatial uncertainty $\Delta X_1 \sim \ell_s^2 \Delta n_1/R_1$. Here we have used the label 1 to denote the uncertainty relation in theory 1. Suppose that theory 1 is mapped into theory 2, which is compactified with a radius R_2, by identifying the spatial uncertainty $\Delta X_1 \to \Delta X_2 = R_2 \ell_s \Delta m_2$ originating from the uncertainty with respect to the winding number, giving $\Delta T_2 \sim \ell_s^2/R_2 \Delta m_2$. Thus the uncertainty relations of the two theories are related to each other by making the mapping

$$n_1 \to m_2, \quad m_1 \to n_2, \quad R_1 \to \ell_s^2/R_2.$$

This is precisely the mapping of the T-duality transformation. Thus T-duality is consistent with the space-time uncertainty relation, as it should be. In connection with this, it must be kept in mind that for the uncertainty with respect to spatial directions, we have to take into account windings. For example, the definition of the spatial uncertainty suggested from the Schild action, as discussed in the last section, indeed naturally contains the winding effect. Another remark is that, in our interpretation, T-duality is a statement regarding duality between small and large distances in time and space directions, rather than regarding the existence of a minimal distance as is often expressed in the literature.

The role of supersymmetry ?

In our discussions, the space-time supersymmetry has not played a fundamental role. The reason is that the supersymmetry is not directly responsible for the short distance structure of string theory. It rather plays a central role in ensuring the theory be well defined, at least perturbatively, in the long distance regime. However, the space-time uncertainty relation essentially demands dual roles between ultraviolet and infrared regimes by interchanging the temporal and spatial directions. In this sense, the space-time supersymmetry actually plays an important subsidiary role in order to make the theories well-defined in both ultraviolet and infrared regimes. Such an instance was already explained for the case of D-particle dynamics.

In connection with this, a question arises whether we have to impose, in future nonperturbative formulations of string theory, supersymmetry as an additional assumption which is not automatically guaranteed from fundamental principles alone. Although we do not know the answer, recent developments [69)] on unstable D-brane systems indicate that the mere appearance of tachyons should no more be regarded as the criterion of unacceptable theories. This only signifies that the perturbative vacuum we have chosen to start with is wrong. Indeed, it was recently shown by the present author [70)] that the 10-dimensional (orientable) open string theory with both bosons and fermions, either its Neveu-Schwarz-Ramond or Green-Schwarz formulation, has a hidden $N = 2$ space-time supersymmetry automatically without making the standard GSO projection. It is an important question whether a similar interpretation is possible for closed string theories as well.

M-theory interpretation of the space-time uncertainty relation ?

Let us next reconsider the relevance of the space-time uncertainty principle to the M-theory conjecture. In §4, we derived the M-theory scale from two different points of view, namely those of microscopic black holes in 10-dimensional space-times and of

D-particle dynamics. In particular, the former argument shows that the appearance of the M-theory scale can be a quite general phenomenon, not necessarily associated with D-branes.

One of the basic elements of the M-theory conjecture is that in 11 dimensions the role of fundamental strings is replaced by that of membranes, which are wrapped around the compactified circle of radius $R_{11} = g_s \ell_s$. From this point of view, it seems natural [39), 72)] to further reinterpret the space-time uncertainty relation as

$$\Delta T \Delta X \gtrsim \ell_s^2 \sim \ell_M^3/R_{11} \ \rightarrow \Delta T \Delta \vec{X} \Delta X_{11} \gtrsim \ell_M^3 \sim G_{11} \tag{6·3}$$

by setting $\Delta X_{11} \sim R_{11}$ as the uncertainty along the 11-th direction and taking $\Delta X \rightarrow \Delta \vec{X}$, which is identified as the spatial uncertainty in the 9 dimensional transverse directions. This is in accord with the membrane action which has two space-like directions along the world volume of membrane. In Ref. 39), we have discussed the affinity of this relation with AdS/CFT correspondence in 11 dimensions. This also motivated the study of the Nambu bracket in Ref. 71). The original stringy space-time uncertainty relation would then be an approximation of this relation in the limit of small compactification radius. Once we move to this viewpoint, the fundamental scale is now $\ell_M = \ell_{11} \sim g_s^{1/3} \ell_s$. Of course, any genuinely 11 dimensional effects only appear for large compactification radius, $R_{11} \gg \ell_M$. In this regime, all the characteristic scales of the theory are governed by the order ℓ_M. The appearance of different scales for time and space scales in 10 dimensions controlled by the string coupling is obviously an effect of the small compactification scale $R_{11} \ll \ell_M$.

For example, we can apply the same argument for microscopic black holes to derive the criterion determining where truly M-theory effects take place. The black hole uncertainty relation places a restriction in 11 dimensions as

$$\Delta T (\Delta X)^8 \gtrsim \ell_M^9. \tag{6·4}$$

Comparing with the M-theory uncertainty relation (6·3), we find that the critical point is of the same order,

$$\Delta T_c \propto \Delta X_c \propto \ell_M,$$

if we treat all the spatial directions equivalently. This is more or less evident from the outset since there can be no other scales than ℓ_M unless one puts them in by hand. Therefore in this case, the dimensionless proportionality coefficients are very important in order to ascertain various characteristic scales. In this sense, in M-theory, understanding the real nonperturbative mechanisms for generating the low-energy scales becomes completely nonperturbative at a much higher level than in 10 dimensional string theory.

We also note that it is straightforward to extend the Schild action approach introduced in §5 to a noncommutative geometric formulation for the quantization of a membrane. In this case, the role of the world-sheet auxiliary field $b_{\mu\nu}$ is played by a world-volume 3 rank tensor field $c_{\alpha\beta\gamma}(\xi)$. We can easily derive an analog of the stringy uncertainty (5·13) for membranes.

Quite recently, it has been argued [73)] that the relation (6·3) is compatible with the so-called 'stringy exclusion principle' [74)] on AdS space-times, by reinterpreting an observation made in Ref. 75). Also, an approach proposed in Ref. 76) to the stringy exclusion principle suggests a connection with the quantum group interpretation, another possible manifestation of noncommutativity, of these phenomena.

A fundamental question

In the beginning of this paper, we repeatedly stressed the importance of reinterpreting the role of world-sheet conformal symmetry in terms of some new language, which is not in principle dependent upon perturbation theory, as a motivation of our proposal of the space-time uncertainty principle. There, however, remains still one of the most mysterious questions in string theory. Why does string theory contain gravity?*) Of course, we have checked the consistency of the space-time uncertainty relation with the presence of gravity from various viewpoints. In spite of these many checks, it is still unclear what ensures the appearance of general relativity in the long distance regime. The main reason for this deficiency is that we have not gained an appropriate understanding of the symmetries associated with the space-time uncertainty principle in terms of the target space-time. The generalized conformal symmetry we mentioned in §4 might contain some ideas which might form a germ for investigation in such directions.

Although many questions still remain, summarizing all that we have discussed in the present paper, it seems not unreasonable to assert that the space-time uncertainty principle may be one of the possible general underlying principles governing the main qualitative features of string/M theory. Of course, the scope of qualitative principles, such as our space-time uncertainty principle, is much too limited to make any concrete predictions without having definite mathematical formulations. In this paper, we have tried to clarify its meaning and implications as far as we can at the present stage of development. It would be extremely interesting to arrange the various aspects discussed here into a unified mathematical scheme.

Acknowledgements

The present paper was essentially completed during the author's visit to Brown University in March, 2000. He would like to thank the Department of Physics there for the hospitality and also discussions with D. Lowe, A. Jevicki and S. Ramgoolam during preparation of the manuscript. The present work is supported in part by a Grant-in-Aid for Scientific Research (No. 09640337) and a Grant-in-Aid for International Scientific Research (Joint Research, No. 10044061) from the Ministry of Education, Science and Culture.

References

1) For a convenient review of string theory including some recent development, see, e.g., J. Polchinski, *String Theory* Vols. I & II (Cambridge Univ. Press, 1998).
2) T. Yoneya, Lett. Nuovo. Cim. **8** (1973), 951; Prog. Theor. Phys. **51**(1974), 1907; **56** (1976), 1310.
3) J. Scherk and J. Schwarz, Nucl. Phys. **B81** (1974), 118; Phys. Lett. **57B** (1975), 463.
4) M. Kaku and K. Kikkawa, Phys. Rev. **D10** (1974), 1110, 1823.

*) For a recent general review on this question, see Ref. 77).

5) E. Witten, Nucl. Phys. **B268** (1986), 79; **B276** (1986), 291.
6) H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Phys. Rev. **D34** (1986), 2360.
7) A. Neveu and P. C. West, Phys. Lett. **168B** (1986), 192.
8) D. Friedan and S. Shenker, Nucl. Phys. **B281** (1987), 509.
9) T. Banks and E. Martinec, Nucl. Phys. **B294** (1987), 733.
10) N. Nakanishi, Prog. Theor. Phys. **45** (1971), 436.
11) T. Yoneya, Prog. Theor. Phys. **48** (1972), 2044.
12) B. Giddings and E. Martinec, Nucl. Phys. **B278** (1986), 91.
13) For a review, see, e.g., J. Polchinski, Les Houshes lecture, hep-th/9411028 (1994).
14) J. Polchinski, Phys. Rev. Lett. **75** (1995), 4724.
15) E. Witten, Nucl. Phys. **B460** (1995), 330.
16) T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. **D55** (1997), 5112.
17) N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys **B498** (1997), 467.
18) T. Yoneya, Duality and Indeterminacy Principle in String Theory in *Wandering in the Fields*, ed. K. Kawarabayashi and A. Ukawa (World Scientific, 1987), p. 419.
See also, String Theory and Quantum Gravity in *Quantum String Theory*, ed. N. Kawamoto and T. Kugo (Springer, 1988), p. 23.
19) T. Yoneya, Mod. Phys. Lett. **A4** (1989), 1587.
20) See, e.g., D. Gross, *Proc. XXIV Int. Conf. High Energy Physics, Munich*, ed. R. Lotthaus and J. Kühn (Springer Verlag, 1989).
21) S. Shenker, hep-th/9509132.
22) D. Gross and P. Mende, Nucl. Phys. **B303** (1988), 407.
23) For the present author, it is not clear what is the original reference for this particular proposal. As an example of works in which the relation has been stressed, see, e.g.,
R. Guida, K. Konishi and P. Provero, Mod. Phys. Lett. **A6** (1991), 1487 and earlier references therein.
24) E. Witten, Nucl. Phys. **B443** (1995), 85.
25) T. Yoneya, Prog. Theor. Phys. **97** (1997), 949.
26) S. Doplicher, K. Fredenhagen and J. E. Roberts, Commun. Math. Phys. **172** (1995), 187; Phys. Lett. **B331** (1994), 39.
27) See, e.g., L. V. Alfors, *Conformal Invariants : Topics in Geometric Function Theory* (McGraw-Hill, New York, 1973), Chapter 5.
28) P. Mende and H. Ooguri, Nucl. Phys. **B303** (1988), 407.
29) D. Lowe, Nucl. Phys. **B456** (1995), 257.
30) M. Soldate, Phys. Lett. **186B** (1987), 321.
31) D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. **197B** (1987), 81.
See also an interesting paper by Veneziano prior to this work, G. Veneziano, Europhys. Lett. **2** (1986), 199.
32) A. H. Mueller, Nucl. Phys. **B118** (1977), 253.
See also a related discussion in Ref. 51) based on parton picture.
33) F. Celcius and A. Martin, Phys. Lett. **8** (1964), 80.
34) E. Martinec, Class. Quan. Grav. **10** (1993), L187.
35) D. Lowe, J. Polchinski, L. Susskind, L. Thorlacius and J. Uglam, Phys. Rev. **D52** (1995), 6997.
36) For a recent related discussion and references for earlier works, see, N. Sasakura, hep-th/0001161; Prog. Theor. Phys. **102** (1999), 169.
37) G. 't Hooft, gr-qc/9608037.
38) M. Li and T. Yoneya, Phys. Rev. Lett. **78** (1997), 1219.
39) M. Li and T. Yoneya, Chaos, Solitons and Fractals **10** (1999), 423; hep-th/9806240.
40) M. Douglas, D. Kabat, P. Pouliot and S. Shenker, Nucl. Phys. **B485** (1997), 85.
41) K. Becker, M. Becker, J. Polchinski and A. Tseytlin, Phys. Rev. **D56** (1997), 3174.
42) Y. Okawa and T. Yoneya, Nucl. Phys. **B538** (1998), 67; **B541** (1999), 163.
43) Y. Kazama and T. Muramatsu, hep-th/0003161.
44) A. Jevicki and T. Yoneya, Nucl. Phys. **B535** (1998), 335.
45) A. Jevicki, Y. Kazama and T. Yoneya, Phys. Rev. Lett. **81** (1998), 5072 ; Phys. Rev. **D59** (1999), 066001.
46) Y. Sekino and T. Yoneya, Nucl. Phys. in press; hep-th/hep-th/9907029.
47) T. Yoneya, Class. Quan. Gravity **17** (2000), 1307.

48) T. Banks and L. Susskind, hep-th/9511491.
49) N. Seiberg, Phys. Rev. Lett. **79** (1997), 3577.
A. Sen, hep-th/9709200.
50) For a review, see, e.g., L. Susskind and J. Uglam, hep-th/9511257.
51) L. Susskind, Phys. Rev. **D49** (1994), 6606.
52) G. Horowitz and J. Polchinski, Phys. Rev. **D55** (1977), 6189.
53) G. 't Hooft, gr-qc/9310026.
54) L. Susskind, J. Math. Phys. **36** (1995), 6377.
55) L. Susskind, hep-th/9901079.
56) A. W. Peet and J. Polchinski, hep-th/9809022.
57) L. Susskind and E. Witten, hep-th/9805114.
58) J. Maldacena, Adv. Theor. Math. Phys. **2** (1998), 231.
59) J. Beckenstein, hep-th/0003058.
60) A. Schild, Phys. Rev. **D16** (1977), 1722.
61) Y. Nambu, Phys. Rev. **D7** (1973), 2405.
62) M. Kato, Phys. Lett. **B245** (1990), 43.
63) A. Conne, M. R. Douglas and A. Schwarz, J. High Energy Phys. **02** (1998), 003; hep-th/9711162.
64) N. Seiberg and E. Witten, hep-th/9908142.
65) H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, hep-th/9908141.
66) C. S. Chu, P. M. Ho and Y. C. Kao, Phys. Rev. **D60** (1999), 126003.
67) J. H. Schwarz, Phys. Lett. **B360** (1995), 13.
68) D. Bak and S. J. Rey, hep-th/9902101.
69) A. Sen, J. High Energy Phys. **08** (1998), 010, 012.
70) T. Yoneya, Nucl. Phys., in press; hep-th/9912255.
71) H. Awata, M. Li, D. Minic and T. Yoneya, hep-th/9906248.
72) D. Minic, Phys. Lett. **B442** (1998), 102.
73) M. Li, hep-th/0003173.
74) J. Maldacena and A. Strominger, J. High Energy Phys. **12** (1998), 005.
75) J. McGreevy, L. Susskind and N. Toumbas, hep-th/0003075.
76) A. Jevicki and S. Ramgoolam, J. High Energy Phys. **04** (1999), 032.
77) T. Yoneya, hep-th/0004075, *Proceedings of Frontiers of Theoretical Physics, ITP, Beijing, November, 1999.*