Strong convergence of an explicit numerical method for SDEs with non-globally Lipschitz continuous coefficients

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Overview

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Consider

- $d, m \in \mathbb{N}$, $T \in (0, \infty)$ and a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a normal filtration $(\mathcal{F}_t)_{t\in[0,T]}$,
- an $(\mathcal{F}_t)_{t\in[0,T]}$-Brownian motion $W\colon [0, T] \times \Omega \to \mathbb{R}^m$,
- continuous functions $\mu\colon \mathbb{R}^d \to \mathbb{R}^d$, $\sigma\colon \mathbb{R}^d \to \mathbb{R}^{d\times m}$ and
- an $\mathcal{F}_0/\mathcal{B}(\mathbb{R}^d)$-measurable $\xi\colon \Omega \to \mathbb{R}$ with $\mathbb{E}\|\xi\|^p < \infty \,\forall\, p \in [1, \infty)$.

Let $X\colon [0, T] \times \Omega \to \mathbb{R}$ be an up to modifications unique adapted stochastic process with continuous sample paths satisfying

$$X_t = \xi + \int_0^t \mu(X_s)\, ds + \int_0^t \sigma(X_s)\, dW_s$$

$\mathbb{P}$-a.s. for all $t \in [0, T]$. Short form:

$$dX_t = \mu(X_t)\, dt + \sigma(X_t)\, dW_t, \qquad X_0 = \xi, \qquad t \in [0, T].$$

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with $X_0 = \xi$ and $t \in [0, T]$.

- The **goal** of this talk is to solve (1).
- A central motivation for solving (1) comes from **financial engineering**, see, e.g., Lewis (2000), Glasserman (2004) and Higham (2004).
- Since explicit solutions are typically not available, we want to solve (1) approximatively: **Computational Stochastics**.
- Problem (1) is not contained in the standard literature in computational stochastics, e.g.,
  - Kloeden & Platen (1992) and
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Examples of SDEs

Black-Scholes model; $\bar{\mu}, \bar{\sigma}, x_0 \in (0, \infty)$:

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Lewis stochastic volatility model; $\bar{\mu}, \hat{\mu}, \tilde{\mu}, \tilde{\sigma} \in (0, \infty)$ appropriate:

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with $X_0 = x_0 \in (0, \infty)^2$ and $t \in [0, T]$.

An SDE with a cubic drift and additive noise:

$$dX_t = -X_t^3\, dt + dW_t, \qquad X_0 = 0, \qquad t \in [0, 1].$$

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Overview

The **explicit Euler scheme** $Y_n^N \colon \Omega \to \mathbb{R}^d$, $n \in \{0, 1, \dots, N\}$, $N \in \mathbb{N}$, is given by $Y_0^N = \xi$ and

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for all $n \in \{0, 1, \dots, N-1\}$, $N \in \mathbb{N}$.

Theorem (Maruyama 1955; Kloeden and Platen 1992)

Let μ and σ be globally Lipschitz continuous. Then there exists a real number $C \in [0, \infty)$ such that

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The **explicit Euler scheme** $Y_n^N \colon \Omega \to \mathbb{R}^d$, $n \in \{0, 1, \ldots, N\}$, $N \in \mathbb{N}$, is given by $Y_0^N = \xi$ and

$$Y_{n+1}^N = Y_n^N + \frac{T}{N} \cdot \mu\big(Y_n^N\big) + \sigma\big(Y_n^N\big) \left(W_{\frac{(n+1)T}{N}} - W_{\frac{nT}{N}}\right)$$

for all $n \in \{0, 1, \ldots, N-1\}$, $N \in \mathbb{N}$.

Theorem (Maruyama 1955; Kloeden and Platen 1992)

Let μ and σ be globally Lipschitz continuous. Then there exists a real number $C \in [0, \infty)$ such that

$$\left(\mathbb{E}\Big[\big\|X_T - Y_N^N\big\|^2\Big]\right)^{\frac{1}{2}} \leq C \cdot N^{-\frac{1}{2}}$$

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Overview

Convergence of Euler's method

$$\lim_{N\to\infty} \mathbb{E}\Big[\|X_T - Y_N^N\|^2 \Big] = 0, \qquad \lim_{N\to\infty} \Big| \mathbb{E}\Big[\|X_T\|^2 \Big] - \mathbb{E}\Big[\|Y_N^N\|^2 \Big] \Big| = 0$$

for SDEs with superlinearly growing coefficients such as

an SDE with a cubic drift and additive noise:

$$dX_t = -X_t^3\, dt + dW_t, \qquad X_0 = 0, \qquad t \in [0,1]$$

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Examples of SDEs

Divergence of Euler's method

$$\lim_{N\to\infty} \mathbb{E}\Big[\big| X_T - Y_N^N \big|^2 \Big] = \infty, \quad \lim_{N\to\infty} \Big| \mathbb{E}\Big[(X_T)^2 \Big] - \mathbb{E}\Big[(Y_N^N)^2 \Big] \Big| = \infty$$

holds for:

A SDE with a cubic drift and additive noise:

$$dX_t = -X_t^3 \, dt + dW_t, \qquad X_0 = 0, \qquad t \in [0, 1].$$

Variance process in the **Lewis stochastic volatility model**:

$$dX_t = X_t \left(\hat{\mu} - \tilde{\mu} \, X_t \right) dt + \tilde{\sigma} \left(X_t \right)^{\frac{3}{2}} dW_t, \qquad X_0 = x_0, \qquad t \in [0, T].$$

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Some ideas in the divergence proof of Euler's method

Fix large $N \in \mathbb{N}$ and consider

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The well known instability of Euler's method then gives

$$Y_0^N = N \qquad \text{positive},$$

$$Y_1^N = Y_0^N - \tfrac{1}{N}\left(Y_0^N\right)^3 = N - N^2 \approx -N^2 \qquad \text{negative},$$

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$$|Y_k^N| \gtrapprox N^{(2^k)} \quad \forall\ k \in \{0, 1, 2, \ldots, N\}$$

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Now consider the SDE

$$dX_t = -X_t^3 \, dt + dW_t, \qquad X_0 = 0, \qquad t \in [0, 1]$$

and define "**events of instabilities**"

$$\Omega_N := \left\{ \omega \in \Omega \colon \sup_{1 \le k \le N-1} \left| W_{\frac{k+1}{N}}(\omega) - W_{\frac{k}{N}}(\omega) \right| \le 1, \; W_{\frac{1}{N}}(\omega) - W_0(\omega) \ge 3N \right\}$$

for all $N \in \mathbb{N}$. Estimates on the previous slide then indicate that

$$\left| \mathbf{Y_N^N}(\omega) \right| \ge \mathbf{N}^{\left(\mathbf{2}^{(\mathbf{N}-\mathbf{1})} \right)} \tag{2}$$

for all $\omega \in \Omega_N$, $N \in \mathbb{N}$. Moreover,

$$e^{-cN^3} \le \mathbb{P}\left[\Omega_N \right] \le e^{-\tilde{c}N^3} \tag{3}$$

for all $N \in \mathbb{N}$ with $c, \tilde{c} \in (0, \infty)$ appropriate. Combining (2) and (3) shows

$$\mathbb{E}\left[\left| Y_N^N \right| \right] \ge \mathbb{P}\left[\Omega_N \right] \cdot N^{\left(2^{(N-1)} \right)} \ge e^{-cN^3} \cdot N^{\left(2^{(N-1)} \right)} \xrightarrow{N \to \infty} \infty.$$

This gives $\lim_{N \to \infty} \mathbb{E}\left[|X_T - Y_N^N|^2 \right] = \infty$, $\lim_{N \to \infty} \left| \mathbb{E}\left[|X_T|^2 \right] - \mathbb{E}\left[|Y_N^N|^2 \right] \right| = \infty$.

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$$\mathbb{E}\Big[|Y_N^N|\Big] \ge \mathbb{P}\big[\Omega_N\big] \cdot N^{\left(2^{(N-1)}\right)} \ge e^{-cN^3} \cdot N^{\left(2^{(N-1)}\right)} \xrightarrow{N \to \infty} \infty.$$

This gives $\lim_{N \to \infty} \mathbb{E}\Big[|X_T - Y_N^N|^2\Big] = \infty$, $\lim_{N \to \infty} \Big| \mathbb{E}\Big[|X_T|^2\Big] - \mathbb{E}\Big[|Y_N^N|^2\Big] \Big| = \infty$.

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Theorem (Higham, Mao & Stuart 2002)

Let σ be globally Lipschitz continuous and let μ be **globally one-sided Lipschitz continuous** *with an at most polynomially growing continuous derivative. Then there exists a real number $C \in [0, \infty)$ such that*

$$\left(\mathbb{E}\left[\left\|X_T - \tilde{Y}_N^N\right\|^2\right]\right)^{\frac{1}{2}} \leq C \cdot N^{-\frac{1}{2}}$$

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The implicit Euler scheme requires **additional computational effort** for computing the zero of a nonlinear equation in each time step.

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Nonlinear SDEs: The implicit Euler method converges but simulations for it on a computer require **additional computational effort**. The explicit Euler method is **explicit** and **easy to simulate** but

$$\lim_{N\to\infty} \mathbb{E}\Big[\big| X_T - Y_N^N \big|^2 \Big] = \infty \qquad (4)$$

Higham (2010) reviews this divergence and a long time divergence result and states "... it is clear that **any other explicit numerical method can suffer the same fate.** This brings us to a key point. Unlike in the deterministic ODE case, for non-linear SDEs, we **introduce implicitness** not in the hope of improving efficiency by allowing larger stepsize, but in the hope of obtaining a method that satisfies **the fundamental requirements of accuracy and stability**." This motivated us to ask:

Is there any explicit numerical method which **does not suffer** from (4)?

Is there any **explicit and easily simulatable** numerical method which **converges strongly** for SDEs with superlinearly growing coefficients?

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for all $n \in \{0, 1, \ldots, N-1\}$, $N \in \mathbb{N}$.

Theorem (Hutzenthaler, J & Kloeden 2010)

Let σ be globally Lipschitz continuous and let μ be **globally one-sided Lipschitz continuous** *with an at most polynomially growing continuous derivative. Then there exists a real number $C \in [0, \infty)$ such that*

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In the case $\sigma(x) = 0$ for all $x \in \mathbb{R}$ we have

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for all $n \in \{0, 1, \ldots, N-1\}$ and therefore $\left\|\bar{\mathbf{Y}}^{\mathbf{N}}_{\mathbf{N}}\right\| \leq \|\xi\| + \mathbf{N}$ for all $N \in \mathbb{N}$ (**at most linear growth** in N).

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$$dX_t = -X_t^3 \, dt, \qquad X_0 = \xi = N, \qquad t \in [0, 1].$$

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In the case $\sigma(x) = 0$ for all $x \in \mathbb{R}$ we have

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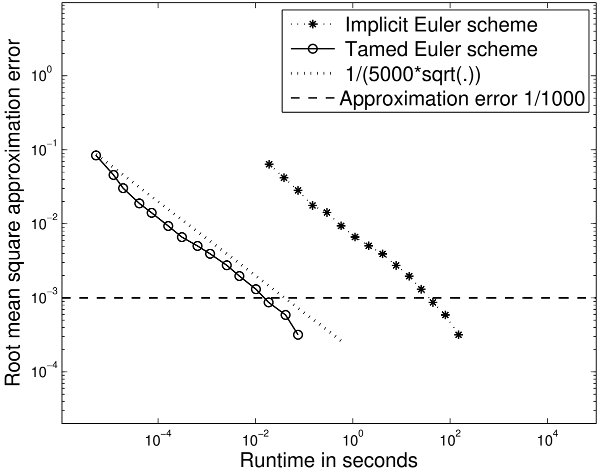
$dX_t = -X_t^5\,dt + X_t\,dW_t,\ X_0 = 1,\ t \in [0,1].$



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- The explicit Euler scheme, does, in general, **not converge** strongly to the exact solution of the SDE (see Hutzenthaler, J & Kloeden 2009).
- This is in fundamentral constrast to the convergence of the explicit Euler method to the exact solution in the **deterministic case**.
- There **exist explicit numerical approximation methods** which overcome the lack of convergence of the explicit Euler method and which **converge strongly to the exact solution of the SDE** (see Hutzenthaler, J & Kloeden 2010). For convergence, there is thus **no need of implicitness**.

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Many thanks for your attention!