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Strong γ-Syncronization in Fuzzy Automata

V. Karthikeyan and M. Rajasekar

Mathematics Section
Faculty of Engineering and Technology
Annamalai University, Annamalainagar
Chidambaram, Tamil Nadu, India-608 002
vkarthikau@gmail.com
mrajdiv@yahoo.com

Abstract

In this paper strongγ- synchronized automata is introduced and algorithm is given to find the strong γ- synchronized word for a fuzzy automata.

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Keywords: stability relation, γ-synchronized automata, strong γ-synchronized automata

1 Introduction

The concept of fuzzy automata has been discussed by J. N. Mordeson, D. S. Malik [1]. He has proposed fuzzy automata as a model of pattern recognition. Synchronization of automata was discussed by karel culik, Juhani Karhumaki, Jarkko kari [2]. Reducibility and stability relation in crisp case are defined and discussed in [2]. R. L. Adler L. W. Goodwyn and B. Weiss [3] shown that aperiodicity was necessary for existence of synchronized word in an automata. The synchronization in fuzzy automata was studied by Rm. Somasundaram and M. Rajaskar [4]. Stability relation in fuzzy deterministic automata is used to find the γ- synchronized word in fuzzy automata, if it exist [5]. Here the stability relation is an equivalence relation on the states set of a fuzzy automata.

In this paper, we introduced a new concept called strong γ-synchronized automata, that is we find a word that brings each state of a fuzzy automata to a single state with minimal weight in fuzzy automata μ, $0 < \mu \leq 1$.

In the second section, we give a basic concepts of fuzzy automata,γ-synchronized automata and stability relation.

In the third section, we introduced a new definition strong γ- synchronized automata with example. Further, we prove that every γ- synchronized automata is strong γ- synchronized automata.

In the fourth section, we establish an algorithm to find strong γ-synchronized word for a fuzzy automata. Throughout this paper we consider aperiodic fuzzy deterministic automata.

2 Basic concepts

2.1 Fuzzy automata

A finite fuzzy automata is a system of 5 tuples, $M = (\Sigma,\ Q,\ \pi,\ \eta,\ f_M)$ where
Q-set of states $\{q_1,\ q_2, ...,\ q_n\}$
Σ-alphabets (or) input symbols
π-$Q \to [0, 1]$ initial state designator
η-$Q \to [0, 1]$ final state designator
f_M-function from $Q \times \Sigma \times Q \to [0, 1]$
$f_M(q_i, \sigma, q_j) = \mu$ $0 < \mu \leq 1$ means when M is in state q_i and reads the input σ will move to the state q_j with weight function μ. For each $\sigma \in \Sigma$ we can form a $n \times n$ matrix $F(\sigma)$ whose (i, j) the element is $f_M(q_i, \sigma, q_j)$
For $x \in \Sigma^*$ and if $x = \sigma_1,\ \sigma_2,\ ....,\ \sigma_m$
$F(x) = F(\sigma_1) \circ F(\sigma_2) \circ ..... \circ F(\sigma_m)$
In otherwords $F(x)$ is the fuzzy sum of fuzzy products of weights taken over the paths in the automata.

Note

$f_M(i, x, j)$ is the (i, j) the element of $F(x)$
$f_M(s, x, t) \quad =\text{Max}\{\text{Min}\{f_M(s, \sigma_1, q_1), f_M(q_1, \sigma_2, q_2), ..., f_M(q_{m-1}, \sigma_m, t)\}\}$
where Max is taken over all the paths from s to t.

Note

$F_{pq}(w)$ denotes p^{th} row and q^{th} column of a matrix $F(w)$.

2.2 Fuzzy deterministic automata

A fuzzy automata M is called deterministic if for each $a \in \Sigma$ there exists a unique state $q_a$such that $f_M(q,\ a,\ q_a) > 0$ for $q \in Q$ otherwise it is called non-deterministic.

2.3 Aperiodic fuzzy automata

Let $\pi = p_1,\ p_2,\ ...p_t$ be a partition of the states set Q such that if $f_M(q_i, a, q_j) > 0$ for some $a \in \Sigma$ then $q_i \in p_r$ and $q_j \in p_{r+1}$. Then π will be called periodic partition of order $t \geq 2$.
An automata M is periodic of period $t \geq 2$ if and only if $t = Maxcard(\pi)$ where this maximum is taken over all periodic partitions π of M. If M has no periodic partition, then M is called aperiodic.

2.4 Stability relation

We say that two states p and q are stable and denoted by $p \equiv q$. If for anyword $u \in \Sigma^*$ there exists a word $w \in \Sigma^*$ and $r \in Q$ such that $F_{pr}(uw) > 0 \Leftrightarrow F_{qr}(uw) > 0$.
Example

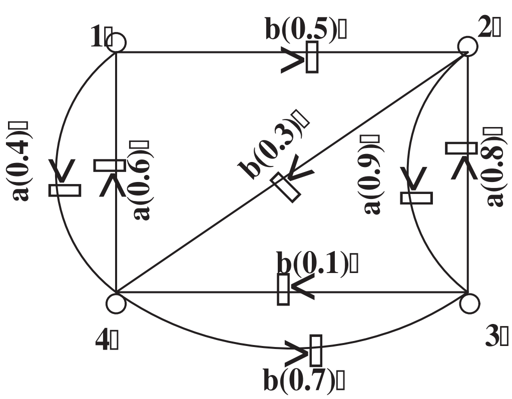

Fig-2.1

In the above automata, for any word u, there exist a word $w = abb$ such that $F_{1p}(uw) > 0$ and $F_{4p}(uw) > 0$,$p \in Q$. Hence 1 and 4 are stability related. Also $F_{2q}(uw) > 0$and$F_{3q}(uw) > 0$,$q \in Q$. Hence 2 and 3 are stability related.

2.5 γ-synchronized automata

Let $M = (\Sigma,\ Q,\ f_M)$ be a finite fuzzy automata without final and initial state designator. We say that the automata is γ-synchronized at the state s,$s \in Q$ if there exist a real number γ with $0 < \gamma \leq 1$ and a word $w \in \Sigma^*$ that takes each state q of Q into s such that $f_M(q, w, s) \geq \gamma$.
Example

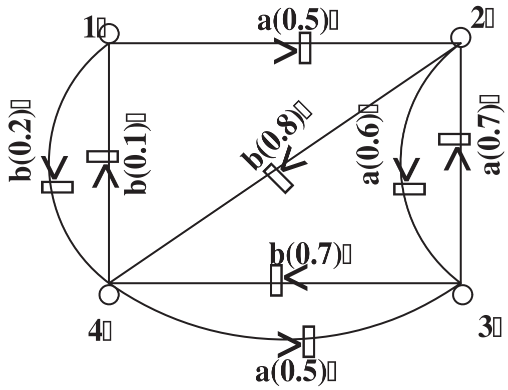

Fig-2.2

In the above automata,

there exist a word $ab \in \Sigma^*$ such that $F(ab) = \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$

Hence the automata is a γ- synchronized automata.

3 strong γ- synchronized automata

Let $M = (\Sigma,\ Q,\ f_M)$ be a fuzzy deterministic automata. We say that the automata is strong γ-synchronized at the state s,$s \in Q$ if there exist a minimal real number γ in fuzzy automata with $0 < \gamma \leq 1$and a word $v \in \Sigma^*$ that takes each state $q \in Q$ into s such that $f_M(q, v, s) = \gamma$.

Example

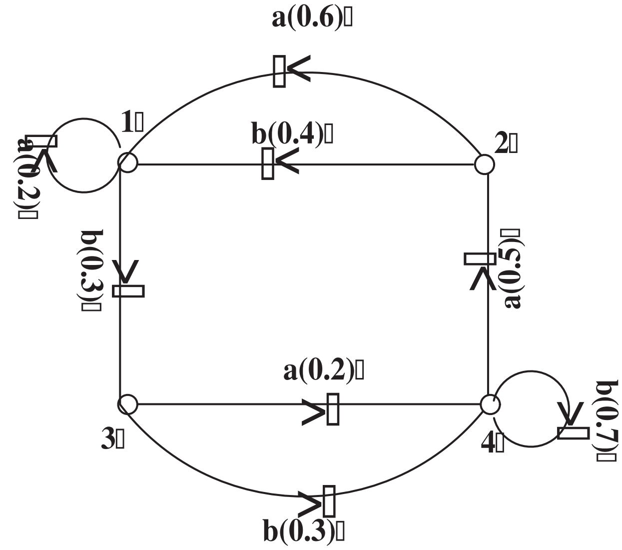

Fig-3.1

In the above automata,

there exist a word $aaa \in \Sigma^*$ such that $F(aaa) = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \end{bmatrix}$

Hence the automata is a strong γ- synchronized automata.

Theorem 3.1. *Every γ-synchronized automata is a strong γ-synchronized automata.*

Proof. Let $M = (\Sigma,\ Q,\ f_M)$ be a γ-synchronized automata. Since it is a γ-synchronized there exist a word $w \in \Sigma^*$ that takes each state $q \in Q$ into $s,\ s \in Q$ and a real number γ with $0 < \gamma \leq 1$ such that $f_M(q, w, s) \geq \gamma$. In M, there exist two states q_i, q_j with a minimal real number μ where $\mu \in (0, 1]$ such that $f_M(q_i, a, q_j) = \mu$ for some $a \in \Sigma$. For proving it is a strong γ-synchronized it is enough to show that there exist a word $v \in \Sigma^*$ that takes each state $q \in Q$ into $s,\ s \in Q$ such that $f_M(q, v, s) = \mu$ where μ is a minimal weight and $\mu \in (0, 1]$. Assume that $f_M(q, w, s) \geq \gamma\ \forall q \in Q$. Since it is aperiodic and deterministic automata there exist a word $u \in \Sigma^*$ such that $f_M(q, wu, q_i) = \mu_k\ \forall q \in Q, \mu_k \in (0, 1]$. Now, $f_M(q, wua, q_j) = \mu$. Now, Let $wua = v$ such that $f_M(q, v, q_j) = \mu\ \forall\ q \in Q$. Hence the γ- synchronized automata is a strong γ-synchronized automata for the word $v \in \Sigma^*$.

4 ALGORITHM

1) Consider the Non-synchronized fuzzy automata M.

2) Using stability relation, find the equivalence classes of the states of M.

3) Construct the Quotient automaton (F) by considering each equivalence class as a state.

4) Relabel the Quotient automaton (F) into F', preserving the stability classes.

5) Obtain M' from F' which is relabeling of M.

6) M' will give the γ-synchronized word (say w) at the state $s, s \in Q$.

7) In M', there exist two states q_i, q_j such that $f_{M'}(q_i, a, q_j) = \mu$ where μ is a minimal real number for some $a \in \Sigma$ and $\mu \in (0, 1]$.

8) Choose the suitable word (say u) such that the word u reaches the state q_i from the state s.

9) The word wua takes each state $q \in Q$ into the state q_j such that $f_{M'}(q, wua, q_j) = \mu$. Hence the word wua gives the strong γ-synchronized automata.

Example

In Fig-2.1, the corresponding Quotient automata F is

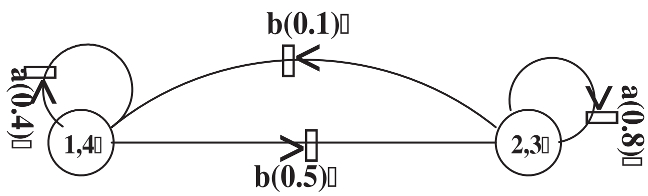

Fig-4.1

Relabeled Quotient automata F' is

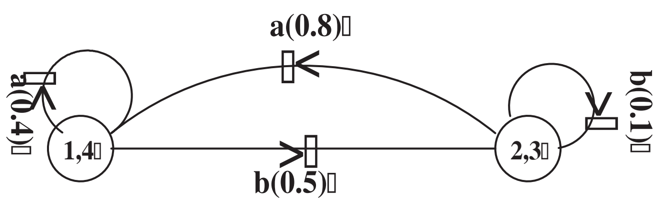

Fig-4.2

Relabeled automata M' is

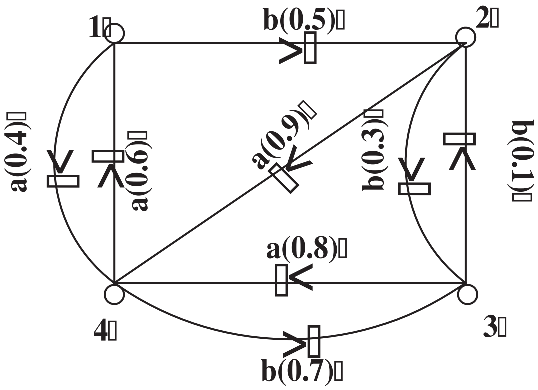

Fig-4.3

The above automata is γ-synchronized for the word $ba \in \Sigma^*$ such that $F(ba) = \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0.7 \end{bmatrix}$

In $M^{'}$, there exist two states 3 *and* 2 such that $f_{M'}(3, b, 2) = 0.1$.
The word $babb \in \Sigma^*$ is strong γ-synchronized for the automata $M^{'}$ such that

$$F(babb) = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \end{bmatrix}$$

Hence the automata is a strong γ-synchronized automata.

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