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Publication Date
1979-05-09

Peer reviewed

Conf. 790327--113

Presented at the Particle Accelerator
Conference, San Francisco, CA,
March 12-14, 1979

LBL-8906

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Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

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Abstract

General properties as well as specific configurations of a new family of strong Rare Earth Cobalt quadrupoles are discussed. *Work prepared by Lawrence Berkeley Laboratory and funded by Los Alamos Scientific Laboratory for the Department.

Introduction

For some applications, one of the performance limitations of conventional quadrupoles is caused by the power dissipation in the coils: It puts an upper limit on the current density that can be used, which in turn limits for small aperture quadrupoles the achievable pole tip field far below the field that the properties of steel would allow. The subject of this paper is a new design of Rare Earth Cobalt (REC) quadrupoles that allows construction of compact quadrupoles with magnet aperture fields of at least 1.2T with presently available materials.

Because of space limitations, I describe here only the basic ideas and most important properties of REC quadrupoles. The details, derivations of formulas etc., will be contained in a separate paper.[1] While that paper will have general expressions for 2N-pole magnets, this paper is intentionally restricted to the discussion of quadrupoles. The magnetic properties of REC are described in some detail, even though this description does not contain anything that has not been known for more than ten years. The motivation is my feeling that the astounding simplicity of this material is not sufficiently well known; and it is this simplicity that leads to a good understanding of REC systems, which in turn leads to improvements in design.

REC Properties

The development of REC materials started in 1966 with Strnat's[2] work. The make up of most currently available materials can be summarized as follows: It is a sintered block of small, oriented, highly anisotropic crystals (composed roughly of one part Rare Earth metal per five parts Co) strongly magnetized in the preferred crystaline direction, customarily called the easy axis.

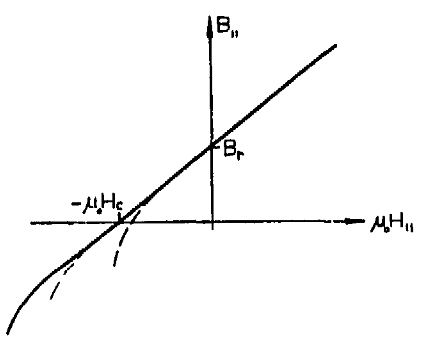

Figure 1: $B_{\parallel}(H_{\parallel})$ for REC

Figure 1 shows the relationship between B and H in the direction parallel to the easy axis. Presently available materials have a remanent field B_r in the range .8T to .95T, and materials with even larger B_r will probably be available in the not too distant future. Over a wide range of field values, the $B_{\parallel}(H_{\parallel})$ curve is for all intents and purposes a straight line, with $dB_{\parallel}/d\mu_0 H_{\parallel} = \mu_{\parallel} = 1.04$. The point on the curve where the $B_{\parallel}(H_{\parallel})$ curve starts to deviate from the straight line depends on the material composition and manufacturing process.

While some materials start to break off in the lower part of the second quadrant, others have a straight line to $B_{\parallel} \lesssim -.5\, B_r$. As long as one stays on the straight line, one can move up and down on the $B_{\parallel}(H_{\parallel})$ curve without any significant change of the curve. That means in particular that one can assemble a system from magnetized pieces. In the direction perpendicular to the easy axis, the relationship between $B_{\perp}$ and $H_{\perp}$ is given by $B_{\perp} = \mu_0 \mu_{\perp} H_{\perp}$; $\mu_{\perp} \approx 1.03$ and holds over a range of several T.

A convenient way to express these properties in the magnetostatic equations is the following: The material behaves like a weakly, and slightly anisotropic, permeable material, with either an impressed current density

$$\vec{j} = \text{curl}\, \vec{H}_c$$

or an impressed charge density

$$\rho = -\text{div}\, \vec{B}_r \quad ,$$

with $\vec{H}_c$ and $\vec{B}_r$ equaling vectors of magnitude H_c and B_r in the (local) direction of the easy axis.

For homogeniously magnetized pieces of material, $\vec{j}$ and ρ are zero everywhere except at the surface of the pieces, where one finds a current sheet $\vec{I}' = \vec{n} \times \vec{H}_c$ ($\vec{n}$ = unit vector normal to surface) or a surface charge density $\sigma = \vec{n}\, \vec{B}_r$

Since $\mu_{\parallel}-1$ and $\mu_{\perp}-1$ are so small, it is for most purposes sufficiently accurate to say that a piece of this material behaves like vacuum with a field independent surface current or surface charge.[3] One of the most significant aspects of this statement is the fact that in the absence of saturating materials, linear superposition of fields is valid. Unless explicitly stated otherwise, in the rest of the paper it is alway assumed that $\mu_{\parallel} = \mu_{\perp} = 1$.

For completeness, it should be pointed out that some oriented ferrites behave qualitatively similar to REC, but are quantiatively different: $\mu = 1.1$ or larger; and $B_r \approx .2-.35$T.

Properties of a Multipiece Quadrupole

The crossection of a quadrupole consisting of 16 trapezoidal REC pieces is shown in Figure 2. The arrows in each piece indicate the direction of the easy axis throughout that piece. If the radial symmetry line of a piece forms the angle φ with the x axis, then the angle α between the direction of the easy axis and the symmetry line follows for this design the relationship

$$\alpha = 2\varphi \qquad (1)$$

For most of this section, we will discuss the two dimensional fields produced by structures such as shown in Figure 2.

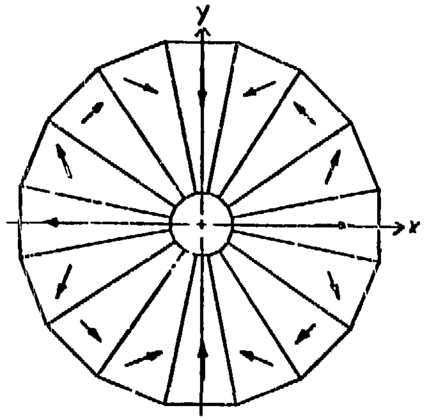

Figure 2: Schematic Crossection of 16 Piece REC Quadrupole

A plot of the B-lines in a 45° slice of the magnet shown in Figure 2 is shown in Figure 3.

If one has a quadrupole that consists of M trapezoidal REC pieces with their easy axes oriented according to equ.(1), and if one piece is bisected by the +x axis, with intersection coordinates $x = r_1$, and $x = r_2$, then the quadrupole strength at the aperture, i.e. $x = r_1$, is given by

$$B(r_1)=2B_r\cos^2\pi/M \cdot \frac{\sin 2\pi/M}{2\pi/M} \cdot (1-r_1/r_2) \qquad (2)$$

For $M \to \infty$, i.e. a quadrupole with continuously varying easy axes, equ.(2) reduces to

$$B(r_1)=2B_r\ (1-r_1/r_2) \qquad (3)$$

In an unpublished internal report, J.P. Blewett[4] gives this expression, but fails to discuss the anisotropy of the material that is necessary to produce these fields or that the easy axis of the material has to be arranged according to equ.(1)

If one formulates the variational problem: how should the continously variable easy axes between two concentric circles be oriented in order to get the largest possible quadrupole field inside the smaller circle, one gets equ. (1) as an answer, i.e. equ. (3) expresses the strongest quadrupole field achievable with REC material between two such circles. The 16-piece quadrupole shown in Fig. 2 represents a compromise between ease of manufacturing and assembling the pieces; and coming close to fulfilling equ.(3). The loss in strength due to the finite number of pieces is only 6.3%, compared to losses of 10.9% and 23.2% for 12 and 8 piece quadrupoles.

The harmonics possible in a structure consisting of M identical pieces with their easy axes oriented according to equ. (1) are given by

$$n=2+\mu M;\ \mu=0,1,2... \qquad (4)$$

Equ. (4) expresses a high degree of symmetry. That symmetry is a direct consequence of a general theorem: If in an assembly of REC pieces, all easy axes are rotated by the angle β, then the direction of the magnetic field everywhere outside the REC is rotated by the angle $-\beta$ without any change in magnitude. This theorem is valid only in two dimensions, and then only if there are no materials with $\mu \gg 1$ present.

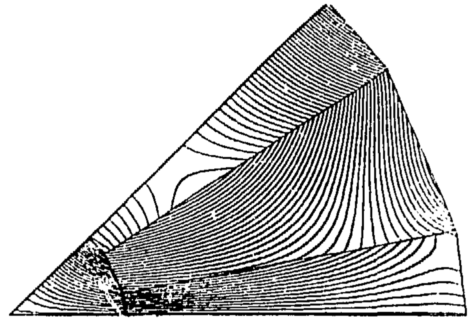

Figure 3: Field Lines in 45° Slice of 16 Piece REC Quadrupole

The first undesirable harmonic for the 16 piece quadrupole is rather large, but can be made zero by having thin non-magnetic sheets between adjacent pieces. The first undesirable harmonic ($n = 34$) is then of such high order to be of no concern.

The field radially outside a multipiece quadrupole is very weak and decays like r^{1-M}. In the unlikely event that strayfield should be of concern, it can be eliminated with a thin soft iron shield, without having a significant effect on the fields in the aperture region.

Because of the linear superposition of fields produced by REC assemblies, a quadrupole with adjustable gradient can be made by placing a small REC quadrupole into the aperture region or a larger REC quad, and rotating one relative to the other. Similarly, one can place a REC magnet into a conventional magnet and obtain linear superposition of fields provided the REC is not driven out of the range where the $B_{\parallel}(H_{\parallel})$ is a straight line.

Another direct consequence of the validity of the linear superposition principle is the property that the effective length of a REC quadrupole is the same as its physical length. Computing the three dimensional fringe fields of the quadrupole shown in Figure 2 involves only elementary transcendental functions. However, a fair amount of bookkeeping and computation of coordinate transformations is involved, since one has to compute the fields produced by 48 different rectangular charge sheets.

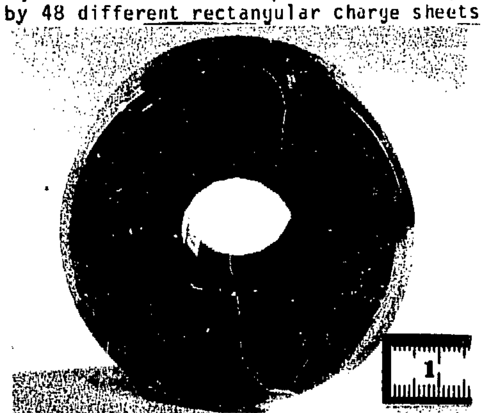

Figure 4: Photograph of 16 Piece Quadrupole Built by NEN

A photograph of a 16-piece quadrupole built by New England Nuclear[5] is shown in Fig.(4). Measurements on that quadrupole, as well as analysis with the computer code PANDIRA[6], confirm predictions from the idealized theory, with the exception of small effects due to non zero values of $\mu_{\parallel}-1$ and $\mu_{\perp}-1$. The only such effect worthy to note is the presence of harmonies other than those given by equ.(4). They are, however, very small: The field error at the magnet aperture due to n=6 is .2%; n=14:.1%; n=10: too small to measure. While amplitudes are sufficiently small to be of any concern for most applications, they could be tuned away if it were necessary.

As long as equ.(1) is observed and a reasonable volume filling factor is achieved, any arrangement of REC material will give a quadrupole with a strength close to that given by equ.(3). Depending on circumstances, availability of material etc., one might for instance want to use tightly packed circular rods that are diagonally magnetized. The quadrupole shown in Figure 1 serves only as an example of a system that is fairly easily manufactured and comes close to the strength given by equ.(3).

The quadrupole shown in Fig.(1) requires pieces with five different orientations of the easy axis relative to the symmetry line of the pieces. That number can be reduced to four by rotating the easy axis in every piece of the magnet by 22.5°. However, the benefits from that reduction of the number of different pieces are not really significant.

The advantages of these REC quadrupoles are obvious: For small apertures they produce quadrupole fields at the magnet aperture that are much larger than those achievable with any other method, they are compact, and they need neither power supplies nor cooling. The disadvantages are mainly cost: When no machining is necessary the price of ready to use REC is of the order of 1-2\$$cm^{-3}$. When accurate grinding is necessary, the price goes up to 10\$$cm^{-3}$ for large quantities, and 30\$$cm^{-3}$ for small quantities. It is possible, at least in principle, to replace part of the REC in a quadrupole with soft steel without changing the field in the aperture region. However, the savings in REC material are probably not sufficient to offset the additional grinding costs.

The magnetic properties of REC are fairly insensitive to temperature up to 150°C, and with proper precautions, one can operate at temperatures of 200-250°C. REC is brittle, and the magnetic forces between pieces are substantial, requiring rather careful assembly procedures to protect the material (and fingertips!) from damage.

Generalizations

Multipole magnets of order N will also be described in Ref.(1). Equ's (1) and (4) become in the general case of a multipiece 2N pole:

$$\alpha = N\varphi \qquad (5)$$

$$n = N + \mu M\ ;\ \mu = 0, 1, 2 \cdots \qquad (6)$$

If the individual pieces are trapezoids, equ.(2) becomes

$$B(r_1) = B_r \cdot \frac{N}{N-1} \cdot \left(\cos \pi/M\right)^N \frac{\sin(N\pi/M)}{N\pi/M} \cdot \left(1 - \left(\frac{r_1}{r_2}\right)^{N-1}\right) \qquad (7)$$

For a dipole (N=1), equ.7 becomes (8)

$$B = B_r \cos(\pi/M) \cdot \ln(r_1/r_1)$$

Linear arrays of REC magnets are used to contain plasmas, and will probably be used in the future as wigglers for the production of synchrotron radiation in electron storage rings. The application of the concepts developed for construction of multipole magnets to these linear arrays will also be treated in a forthcoming publication.

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