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Stronger C-odd color charge correlations in the proton at higher energy

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The non-forward eikonal scattering matrix for dipole-proton scattering at high energy obtains an imaginary part due to a C-odd three gluon exchange. We present numerical estimates for the perturbative Odderon amplitude as a function of dipole size, impact parameter, their relative azimuthal angle, and light-cone momentum cutoff x. The proton is approximated as $\psi_{\rm qqq}|qqq\rangle + \psi_{\rm qqqg}|qqqg\rangle$, where $\psi_{\rm qqq}$ is a non-perturbative three quark model wave function while the gluon emission is computed in light-cone perturbation theory. We find that the Odderon amplitude increases as x decreases from 0.1 to 0.01. At yet lower x, the reversal of this energy dependence would reflect the onset of universal small-x renormalization group evolution.

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I. INTRODUCTION

The S-matrix for high-energy eikonal scattering of a quark - antiquark dipole off the proton is [1–4]

$$\mathcal{S}(\vec{x},\vec{y}) = \frac{1}{N_c}\,\left\langle \mathrm{tr}\, U\left(\vec{x}\right)\, U^\dagger\left(\vec{y}\right)\right\rangle\,. \tag{1}$$

Below we shall also use the impact parameter $\vec{b}=(\vec{x}+\vec{y})/2$ and dipole (transverse) vectors $\vec{r}=\vec{y}-\vec{x}$ where $\vec{r}$ points from the anti-quark to the quark. The $\langle\cdots\rangle$ brackets denote the matrix element between the incoming proton state $|P^+,\vec{P}=0\rangle$ and the outgoing state $\langle P^+,\vec{K}|$, where $\vec{K}$ denotes the proton transverse momentum. Our sign convention for the coupling in the covariant derivative, $D_\mu = \partial_\mu + igA^a_\mu t^a$, follows Ref. [5]. Hence, the path ordered exponential of the field in covariant gauge (Wilson line) which represents the eikonal scattering of the quark is

$$U(\vec{x}) = \mathcal{P} e^{-ig\int \mathrm{d}x^- A^{+a}(x^-,\vec{x})\, t^a}\,. \tag{2}$$

Our convention for the Wilson line and for the dipole S-matrix agrees with Ref. [6]. Others such as Ref. [7] define $\mathcal{S}(\vec{x},\vec{y})$ with $U\leftrightarrow U^\dagger$; however, they also take $\vec{r}=\vec{x}-\vec{y}$, so in all, the sign for the imaginary part of the S-matrix is the same.

Indeed, our focus here is on the imaginary part $O(\vec{r},\vec{b})$ of the S-matrix, the so-called "b-dependent Odderon", which starts out in perturbation theory as C-odd three gluon exchange. This amplitude is odd under C-conjugation, i.e. exchange of quark and anti-quark. The relation of various Odderon amplitudes to Generalized Transverse Momentum Dependent parton distributions (GTMDs) has been elucidated in refs. [7–12].

The C-odd three gluon exchange couples to cubic color charge fluctuations in the proton [13],

$$\mathrm{Im}\, S(\vec{r},\vec{b}) = O(\vec{r},\vec{b}) = -\frac{5}{18}\, g^6\, \frac{1}{2}\,\frac{1}{3} \int\limits_{q_1,q_2,q_3>q_{\min}} \frac{1}{q_1^2}\frac{1}{q_2^2}\frac{1}{q_3^2}\, \sin(\vec{b}\cdot\vec{K})\, G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$
$$\left[\sum_{i=1,2,3}\left(\sin\left(\vec{r}\cdot\vec{q}_i+\frac{1}{2}\vec{r}\cdot\vec{K}\right)-\sin\left(\vec{r}\cdot\vec{q}_i^{\,\prime}+\frac{1}{2}\vec{r}\cdot\vec{K}'\right)\right)-\sin\left(\frac{1}{2}\vec{r}\cdot\vec{K}\right)+\sin\left(\frac{1}{2}\vec{r}\cdot\vec{K}'\right)\right]\,, \tag{3}$$

We have written $O(\vec{r},\vec{b})$ in a form which is more suitable for numerical integration, in particular the amplitude vanishes already at the integrand level when $\vec{r} \perp \vec{b}$ and different momenta $\vec{q}_i$ appear in a symmetric form. The sign of $O(\vec{r}, \vec{b})$ differs from Ref. [13] because here we employ the more common convention $\vec{r} = \vec{y} - \vec{x}$ rather than $\vec{r} = \vec{x} - \vec{y}$. Here the parameter $g = \sqrt{4\pi\alpha_s}$ is the strong coupling constant, $\vec{K} = -(\vec{q}_1 + \vec{q}_2 + \vec{q}_3)$ is the transverse momentum transfer given $\vec{P} = 0$ for the incoming proton, and $\int_q$ is shorthand for $\int \mathrm{d}^2q/(2\pi)^2$. In addition, the transverse momentum vectors $\vec{q}_i'$ correspond to sign-flipped components along $\vec{b}$. We have also introduced a low momentum cutoff $q_{\min}$ for numerical stability; no significant dependence on this cutoff was observed when $q_{\min} < 0.1\,\mathrm{GeV}$, except in regions where $O(\vec{r}, \vec{b})$ has a very small magnitude. The actual numerical results shown in this paper are obtained using $q_{\min} = 0.03\,\mathrm{GeV}$.

We denote the C-odd part of the light-cone gauge correlator of three color charge operators as

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\right\rangle_{C=-} \equiv \frac{1}{4} d^{abc}\, g^3\, G_3^-(\vec{q}_1, \vec{q}_2, \vec{q}_3) \;. \tag{4}$$

Ref. [14] evaluated $G_3^-(\vec{q}_1, \vec{q}_2, \vec{q}_3)$ for a non-perturbative three quark light-cone constituent quark model [15, 16]. This model provides realistic one-particle longitudinal and transverse momentum distributions, and also encodes momentum correlations. We refer to this three-quark light-cone wave function as the leading-order (LO) approximation.

The diagrams corresponding to corrections to the impact factor due to the perturbative emission of a gluon have been computed in Ref. [17]; they are too numerous to be listed again here. This will be referred to as the next-to-leading order (NLO) approximation. The purpose of this paper is to present numerical results for $O(\vec{r}, \vec{b})$ from this approach, which together with analogous results for the real part of $S(\vec{r}, \vec{b})$ [18, 19] provide a complete set of initial conditions for small-x evolution of the dipole S-matrix. The questions we address here are about the overall magnitude of the three gluon exchange amplitude, and its dependence on $r = |\vec{r}|$, $b = |\vec{b}|$, their relative angle θ, and on the cutoff x on the parton light-cone momentum which appears in G_3^-.

The non-vanishing imaginary part of the S-matrix can be probed, for example, via charge asymmetries in diffractive electroproduction of a $\pi^+\pi^-$ pair [20, 21], exclusive production of a pseudo-scalar meson [22–26] in deeply-inelastic scattering (DIS) or ultra-peripheral proton-nucleus collisions, lepton-meson azimuthal angle correlations in exclusive processes [27] as well as in exclusive production of a vector meson in $p+p$ scattering [28] via "pomeron-odderon fusion".

Finally, it is also our goal to provide numerical estimates for initial conditions for small-x QCD evolution of the (hard) Odderon $O(\vec{r}, \vec{b})$ [29–31]. Their crude knowledge, see e.g. Refs. [7, 31], is a key limitation for quantitative predictions of the observables mentioned above in the energy regime of the Electron-Ion Collider (EIC) [32–34].

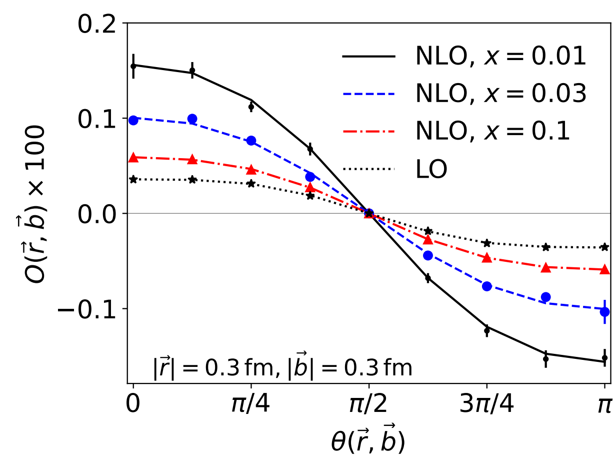

FIG. 1: Angular dependence of $O(\vec{r}, \vec{b})$ at various x and $r = b = 0.3$ fm, which is predominantly $\sim \hat{r}\cdot\hat{b}$. The coefficients (scaled by 100) are $a_1 = 0.16, a_3 = -0.0063$ at $x = 0.01$, $a_1 = 0.10, a_3 = -0.0030$ at $x = 0.03$ and $a_1 = 0.063, a_3 = -0.0035$ at $x = 0.1$. For comparison: at leading order the fitted coefficients are $a_1 = 0.040$ and $a_3 = -0.0040$. The error bars show the estimated uncertainty of the numerical Monte Carlo integration.

II. RESULTS

The results presented here apply when the C-odd exchange can be described by the exchange of three gluons, i.e. in the perturbative regime. This should be the case when the scattered dipole is small and/or when the momentum transfer (conjugate to the impact parameter) is large. Furthermore, since we only consider the $|qqq\rangle$ and $|qqqg\rangle$ Fock states of the proton, we restrict to $x \gtrsim 0.01$. The results shown below have been obtained with $\alpha_s = 0.2$; note that aside from the overall α_s^3 prefactor in Eq. (3), the NLO contribution to G_3^-, too, depends on the coupling, see Ref. [17]. Note also that the coupling does not run at this order as the perturbative one gluon emission corrections are $\mathcal{O}(\alpha_s)$.

The non-perturbative three-quark wave function for the proton used in the numerical analysis is the "harmonic oscillator" wave function of Ref. [16]. It has been used previously in Refs. [17, 19] for estimates of the real part of the S-matrix. The parameters of the wave function are constrained by the proton radius, the anomalous magnetic moment and the axial coupling of the proton and the neutron. Given these constraints, color charge correlators are not very sensitive to the particular model of the three-quark wave function [19]. Also, following Ref. [19], here we evaluate all diagrams for the three gluon exchange with a collinear regulator of $m_{\mathrm{col}} = 0.2\,\mathrm{GeV}$; this is consistent with the typical quark transverse momentum in the wave function of Refs. [15, 16].

At the level of accuracy that we achieved in evaluating

Eq. (3) we found that the angular dependence of the odderon amplitude is well approximated by

$$O(\vec{r},\vec{b}) = a_1(r,b)\,\cos\theta + a_3(r,b)\,\cos 3\theta\ , \qquad (5)$$

where θ is the azimuthal angle made by $\vec{b}$ and $\vec{r}$. We typically find that the magnitude of a_3 is much smaller than that of a_1 except in the vicinity of a sign change of $a_1(r,b)$ where $O(\vec{r},\vec{b})$ is small. The angular dependence of the odderon amplitude at $r = b = 0.3\,\text{fm}$ is shown in Fig. 1. The amplitude obtained from the leading order calculation, where the dependence on the parton momentum fraction cutoff x is negligible, is compared to the result of the NLO computation at $x = 0.1, x = 0.03$ and $x = 0.01$.

These results show the correction due to the perturbative gluon for different values of x. At $x = 0.1$ this correction is moderate, visible mostly for (anti-)parallel $\vec{r}$ and $\vec{b}$, as the phase space for gluon emission is restricted. Note that the Odderon amplitude vanishes exactly when $\theta = 0$ as can be seen from Eq. (3). For smaller x, although the qualitative angular dependence remains the same, we observe a considerable *increase* of the Odderon amplitude $|O(\vec{r},\vec{b})|$.

To further demonstrate the role of the NLO corrections on the Odderon amplitude, we show in Figs. 2 and 3 the dominant a_1 coefficient as a function of impact parameter (Fig. 2) and dipole size (Fig. 3). The next-to-leading order amplitudes computed at different longitudinal momentum fraction cutoffs x are compared with the leading order result. The Odderon amplitude is parity odd and so it vanishes at $b = 0$. It increases with impact parameter and peaks at b slightly less than 0.2 fm, for a dipole size $r = 0.3$ fm, followed by a smooth fall-off towards large b. The peak at $b \lesssim 0.2$ fm is seen at much smaller scales than the transverse size $\sqrt{\langle b^2 \rangle} \simeq 0.6$ fm associated with the real part of the S-matrix extracted from fits to HERA data on exclusive J/Ψ production in DIS [35]. The peak position depends weakly on r but remains at $b \lesssim 0.3$ fm for all dipole sizes $r \lesssim 0.8$ fm considered here. Again we notice that the qualitative shape of $a_1(b)$ is preserved by the NLO correction. However, while this correction is moderate at $x = 0.1$, it increases strongly with decreasing x.

Fig. 3 shows the expected rapid increase of a_1 with dipole size r at fixed b. It levels off at about $r \simeq 0.7$ fm and then decreases again towards larger r where the dipole grows as large as the proton and a perturbative calculation looses validity. This behavior is qualitatively similar to the one obtained for the real part of the S-matrix in a similar calculation in Ref. [19]. These results are not particularly sensitive to the collinear cutoff: using $m_{\rm col} = 0.3\,\text{GeV}$ instead of $0.2\,\text{GeV}$ results in 5% (20%) larger scattering amplitude at small (large) r.

It is interesting to compare the typical magnitude of the Odderon exchange amplitude obtained here to parameterizations commonly employed in the literature as initial conditions at $x \simeq 0.01$ for small-x evolution. Fig. 4

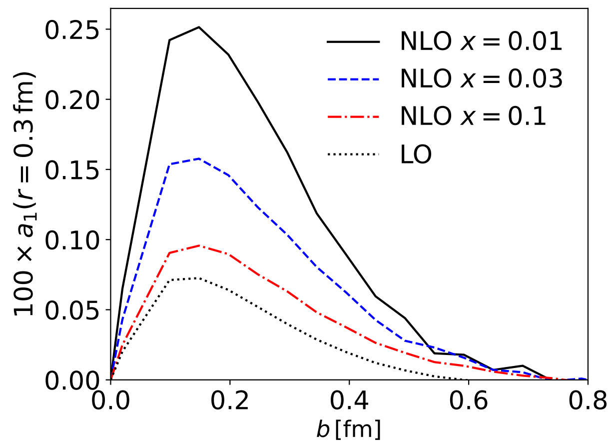

FIG. 2: Impact parameter dependence of the odderon amplitude modulation coefficient a_1 defined in Eq. (5).

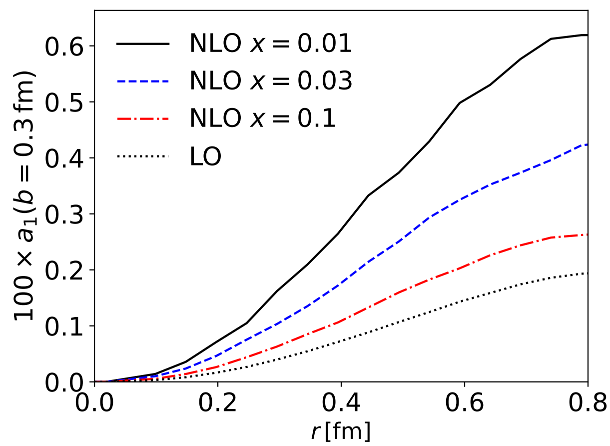

FIG. 3: Dipole size dependence of a_1 at $b = 0.3$ fm, and various x.

of Ref. [31], for example, depicts Odderon amplitudes which reach maximum values of ≈ 0.15 and 0.4, respectively. The initial "spin dependent Odderon" amplitude of Refs. [7, 11] coincides with the first model of Ref. [31]. The maximal (over angle θ and dipole size r) value for the Odderon that we obtain at $x \gtrsim 0.01$ is about $5 \cdot 10^{-3}$ for $\alpha_{\rm s} = 0.2$ used in this work. On the other hand, the quasi-classical Odderon amplitude derived for a large nucleus, Eq. (56) of Ref. [36] (also see [8, 29, 37]), if applied to a proton (at $r = 2b = 0.7$ fm) with Gaussian transverse "profile function" [35], is smaller than our result by about one order of magnitude.

Finally, we illustrate the dominant a_1 modulation coefficient at NLO as a function of both r and b in Fig. 4 for $x = 0.1$ and in Fig. 5 for $x = 0.03$. Aside from the increasing magnitude, there is no clear qualitative change in the shape of the Odderon amplitude. At large b the

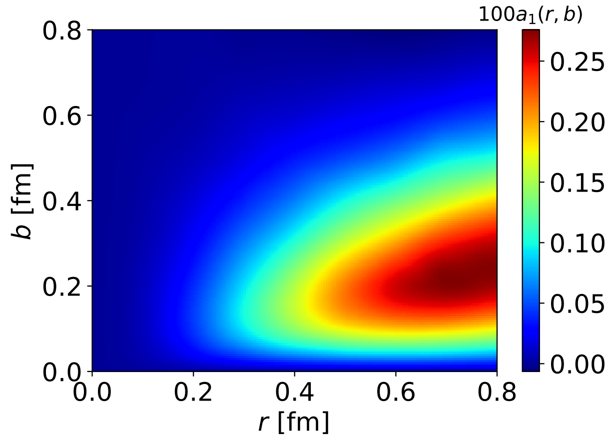

FIG. 4: Odderon modulation coefficient a_1 as a function of r and b at $x = 0.1$ calculated at NLO accuracy.

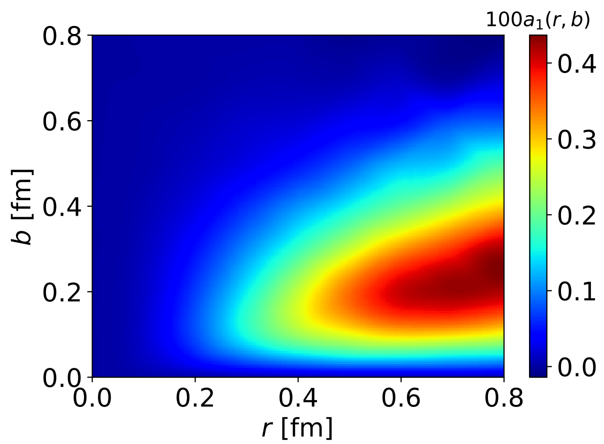

FIG. 5: Odderon modulation coefficient a_1 as a function of r and b at $x = 0.03$ calculated at NLO accuracy. Note that the color scheme is different than in Fig. 4.

a_1 coefficient also changes sign which is visible in these figures. In the supplementary material we provide tables for the a_1 and a_3 coefficients (which are interpolated when generating figures 4 and 5) as functions of r and b at $x = 0.1, 0.03$ and $x = 0.01$, and for comparison also for the LO three quark proton wave function.

III. DISCUSSION

We have presented for the first time an estimate for the perturbative, C-odd, dipole-proton three gluon exchange amplitude $O(\vec{r}, \vec{b})$ at moderately small longitudinal momentum fraction x where the target proton includes a perturbative gluon on top of a non-perturbative three-quark Fock state. This is a necessary input for the perturbative small-x evolution of the Odderon. We find that $O(\vec{r}, \vec{b})$ increases when the $|qqqg\rangle$ Fock state is added as the number of diagrams increases by an order of magnitude. Once the proton contains a sufficient number of color charges, the average dipole S-matrix at rapidity $Y = \log x_0/x$ will be given by an average over the configurations of A^+ in the proton:

$$S_Y(\vec{x}, \vec{y}) = \int \mathrm{D}A^+ \, W_Y[A^+] \, \frac{1}{N_c} \mathrm{tr}\, U(\vec{x}) U^\dagger(\vec{y}) \; . \tag{6}$$

Here $W_Y[A^+]$ is the weight functional at evolution rapidity Y, and x_0 is the longitudinal momentum fraction at the initial condition. A small step towards lower x allows for the emission of an additional soft gluon, resulting in a small change of $W_Y[A^+]$, i.e. the small-x renormalization group (RG) flow [38–50].

For weak scattering the average value of $1 - S$ is small and the evolution of the imaginary part O is given by [29–31]

$$\begin{aligned} \partial_Y O(\vec{x}, \vec{y}) \;&=\; \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2 \vec{z} \, \frac{(\vec{x} - \vec{y})^2}{(\vec{x} - \vec{z})^2 \, (\vec{z} - \vec{y})^2} \\ &\qquad [O(\vec{x}, \vec{z}) + O(\vec{z}, \vec{y}) - O(\vec{x}, \vec{y})] \; . \end{aligned} \tag{7}$$

For small r the first two terms largely cancel, leaving the negative virtual correction and a *decreasing* Odderon amplitude with decreasing x. (For asymptotically small x the above evolution equation leads to [29] the energy independent Bartels-Lipatov-Vacca Odderon [51].) The observation of such behavior would indicate the onset of the universal flow predicted by the small-x RG. Our analysis provides a lower bound on the number of pre-populated Fock states.

The angular dependence of the Odderon amplitude is found to be well described by $\cos \phi_{\vec{r}\vec{b}}$, with a small correction proportional to $\cos 3\phi_{\vec{r}\vec{b}}$ which is significant only in the region where $O(\vec{r}, \vec{b})$ is very small. The small magnitude of the perturbative Odderon amplitude obtained here indicates that high luminosities available e.g. at the EIC are necessary to access the Odderon experimentally. For example, Ref. [26] obtained $\mathrm{d}\sigma/\mathrm{d}t \simeq 40$ fb/GeV^2 for exclusive η_c production in DIS at low Q^2, $|t| = 1.5$ GeV^2, $x = 0.1$, in the LO approximation with $\alpha_\mathrm{s} = 0.35$. We intend to compute cross sections for various physical processes from our dipole S-matrix in the future.

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[1] N. N. Nikolaev and B. G. Zakharov, *Colour transparency and scaling properties of nuclear shadowing in deep inelastic scattering*, Z. Phys. **C49** (1991) 607.
[2] A. H. Mueller, *Unitarity and the BFKL pomeron*, Nucl. Phys. B **437** (1995) 107 [arXiv:hep-ph/9408245].
[3] A. H. Mueller in *Cargese 2001, QCD perspectives on hot and dense matter*, pp. 45–72. 2001. arXiv:hep-ph/0111244.
[4] Y. V. Kovchegov and E. Levin, *Quantum chromodynamics at high energy*, vol. 33. Cambridge University Press, 8, 2012.
[5] S. J. Brodsky, H.-C. Pauli and S. S. Pinsky, *Quantum chromodynamics and other field theories on the light cone*, Phys. Rept. **301** (1998) 299 [arXiv:hep-ph/9705477 [hep-ph]].
[6] Y. V. Kovchegov and M. D. Sievert, *Sivers function in the quasiclassical approximation*, Phys. Rev. **D89** (2014) 054035 [arXiv:1310.5028 [hep-ph]].
[7] X. Yao, Y. Hagiwara and Y. Hatta, *Computing the gluon Sivers function at small-x*, Phys. Lett. B **790** (2019) 361 [arXiv:1812.03959 [hep-ph]].
[8] J. Zhou, *Transverse single spin asymmetries at small x and the anomalous magnetic moment*, Phys. Rev. D **89** (2014) no. 7 074050 [arXiv:1308.5912 [hep-ph]].
[9] D. Boer, M. G. Echevarria, P. Mulders and J. Zhou, *Single spin asymmetries from a single Wilson loop*, Phys. Rev. Lett. **116** (2016) no. 12 122001 [arXiv:1511.03485 [hep-ph]].
[10] R. Boussarie, Y. Hatta, L. Szymanowski and S. Wallon, *Probing the Gluon Sivers Function with an Unpolarized Target: GTMD Distributions and the Odderons*, Phys. Rev. Lett. **124** (2020) no. 17 172501 [arXiv:1912.08182 [hep-ph]].
[11] Y. Hagiwara, Y. Hatta, R. Pasechnik and J. Zhou, *Spin-dependent Pomeron and Odderon in elastic proton-proton scattering*, Eur. Phys. J. C **80** (2020) no. 5 427 [arXiv:2003.03680 [hep-ph]].
[12] D. Boer, Y. Hagiwara, J. Zhou and Y.-j. Zhou, *Scale evolution of T-odd gluon TMDs at small x*, Phys. Rev. D **105** (2022) no. 9 096017 [arXiv:2203.00267 [hep-ph]].
[13] A. Dumitru, G. A. Miller and R. Venugopalan, *Extracting many-body color charge correlators in the proton from exclusive DIS at large Bjorken x*, Phys. Rev. D **98** (2018) no. 9 094004 [arXiv:1808.02501 [hep-ph]].
[14] A. Dumitru, V. Skokov and T. Stebel, *Subfemtometer scale color charge correlations in the proton*, Phys. Rev. D **101** (2020) no. 5 054004 [arXiv:2001.04516 [hep-ph]].
[15] F. Schlumpf, *Relativistic constituent quark model of electroweak properties of baryons*, Phys. Rev. D **47** (1993) 4114 [arXiv:hep-ph/9212250]. [Erratum: Phys.Rev.D 49, 6246 (1994)].
[16] S. J. Brodsky and F. Schlumpf, *Wave function independent relations between the nucleon axial coupling g_A and the nucleon magnetic moments*, Phys. Lett. B **329** (1994) 111 [arXiv:hep-ph/9402214].
[17] A. Dumitru, H. Mäntysaari and R. Paatelainen, *Cubic color charge correlator in a proton made of three quarks and a gluon*, Phys. Rev. D **105** (2022) no. 3 036007 [arXiv:2106.12623 [hep-ph]].
[18] A. Dumitru and R. Paatelainen, *Sub-femtometer scale color charge fluctuations in a proton made of three quarks and a gluon*, Phys. Rev. D **103** (2021) no. 3 034026 [arXiv:2010.11245 [hep-ph]].
[19] A. Dumitru, H. Mäntysaari and R. Paatelainen, *Color charge correlations in the proton at NLO: Beyond geometry based intuition*, Phys. Lett. B **820** (2021) 136560 [arXiv:2103.11682 [hep-ph]].
[20] P. Hägler, B. Pire, L. Szymanowski and O. Teryaev, *Hunting the QCD-Odderon in hard diffractive electroproduction of two pions*, Phys. Lett. B **535** (2002) 117 [arXiv:hep-ph/0202231]. [Erratum: Phys.Lett.B 540, 324–325 (2002)].
[21] P. Hägler, B. Pire, L. Szymanowski and O. Teryaev, *Pomeron - odderon interference effects in electroproduction of two pions*, Eur. Phys. J. C **26** (2002) 261 [arXiv:hep-ph/0207224].
[22] J. Czyzewski, J. Kwiecinski, L. Motyka and M. Sadzikowski, *Exclusive η_c photoproduction and electroproduction at HERA as a possible probe of the odderon singularity in QCD*, Phys. Lett. B **398** (1997) 400 [arXiv:hep-ph/9611225]. [Erratum: Phys.Lett.B 411, 402 (1997)].
[23] R. Engel, D. Ivanov, R. Kirschner and L. Szymanowski, *Diffractive meson production from virtual photons with odd charge - parity exchange*, Eur. Phys. J. C **4** (1998) 93 [arXiv:hep-ph/9707362].
[24] W. Kilian and O. Nachtmann, *Single pseudoscalar meson production in diffractive e p scattering*, Eur. Phys. J. C **5** (1998) 317 [arXiv:hep-ph/9712371].
[25] M. Rueter, H. G. Dosch and O. Nachtmann, *Odd CP contributions to diffractive processes*, Phys. Rev. D **59** (1999) 014018 [arXiv:hep-ph/9806342].
[26] A. Dumitru and T. Stebel, *Multiquark matrix elements in the proton and three gluon exchange for exclusive η_c production in photon-proton diffractive scattering*, Phys. Rev. D **99** (2019) no. 9 094038 [arXiv:1903.07660 [hep-ph]].
[27] H. Mäntysaari, K. Roy, F. Salazar and B. Schenke, *Gluon imaging using azimuthal correlations in diffractive scattering at the Electron-Ion Collider*, Phys. Rev. D **103** (2021) no. 9 094026 [arXiv:2011.02464 [hep-ph]].

[28] A. Bzdak, L. Motyka, L. Szymanowski and J. R. Cudell, *Exclusive J/ψ and Upsilon hadroproduction and the QCD odderon*, Phys. Rev. D **75** (2007) 094023 [arXiv:hep-ph/0702134].

[29] Y. V. Kovchegov, L. Szymanowski and S. Wallon, *Perturbative odderon in the dipole model*, Phys. Lett. B **586** (2004) 267 [arXiv:hep-ph/0309281].

[30] Y. Hatta, E. Iancu, K. Itakura and L. McLerran, *Odderon in the color glass condensate*, Nucl. Phys. A **760** (2005) 172 [arXiv:hep-ph/0501171].

[31] T. Lappi, A. Ramnath, K. Rummukainen and H. Weigert, *JIMWLK evolution of the odderon*, Phys. Rev. D **94** (2016) no. 5 054014 [arXiv:1606.00551 [hep-ph]].

[32] A. Accardi *et. al.*, *Electron Ion Collider: The Next QCD Frontier: Understanding the glue that binds us all*, Eur. Phys. J. A **52** (2016) no. 9 268 [arXiv:1212.1701 [nucl-ex]].

[33] E. Aschenauer, S. Fazio, J. Lee, H. Mäntysaari, B. Page, B. Schenke, T. Ullrich, R. Venugopalan and P. Zurita, *The electron–ion collider: assessing the energy dependence of key measurements*, Rept. Prog. Phys. **82** (2019) no. 2 024301 [arXiv:1708.01527 [nucl-ex]].

[34] R. Abdul Khalek *et. al.*, *Science Requirements and Detector Concepts for the Electron-Ion Collider: EIC Yellow Report*, arXiv:2103.05419 [physics.ins-det].

[35] H. Kowalski, L. Motyka and G. Watt, *Exclusive diffractive processes at HERA within the dipole picture*, Phys. Rev. **D74** (2006) 074016 [arXiv:hep-ph/0606272].

[36] Y. V. Kovchegov and M. D. Sievert, *A New Mechanism for Generating a Single Transverse Spin Asymmetry*, Phys. Rev. D **86** (2012) 034028 [arXiv:1201.5890 [hep-ph]]. [Erratum: Phys.Rev.D 86, 079906 (2012)].

[37] S. Jeon and R. Venugopalan, *A classical odderon in QCD at high energies*, Phys. Rev. **D71** (2005) 125003 [arXiv:hep-ph/0503219 [hep-ph]].

[38] I. Balitsky, *Operator expansion for high-energy scattering*, Nucl. Phys. **B463** (1996) 99 [arXiv:hep-ph/9509348].

[39] I. Balitsky, *Factorization and high-energy effective action*, Phys. Rev. **D60** (1999) 014020 [arXiv:hep-ph/9812311].

[40] I. Balitsky, *Factorization for high-energy scattering*, Phys. Rev. Lett. **81** (1998) 2024 [arXiv:hep-ph/9807434].

[41] Y. V. Kovchegov, *Small x F_2 structure function of a nucleus including multiple pomeron exchanges*, Phys. Rev. D **60** (1999) 034008 [arXiv:hep-ph/9901281].

[42] Y. V. Kovchegov, *Unitarization of the BFKL pomeron on a nucleus*, Phys. Rev. **D61** (2000) 074018 [arXiv:hep-ph/9905214].

[43] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The BFKL equation from the Wilson renormalization group*, Nucl. Phys. **B504** (1997) 415 [arXiv:hep-ph/9701284].

[44] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The Wilson renormalization group for low x physics: Towards the high density regime*, Phys. Rev. **D59** (1999) 014014 [arXiv:hep-ph/9706377].

[45] J. Jalilian-Marian, A. Kovner and H. Weigert, *The Wilson renormalization group for low x physics: Gluon evolution at finite parton density*, Phys. Rev. **D59** (1999) 014015 [arXiv:hep-ph/9709432].

[46] H. Weigert, *Unitarity at small Bjorken x*, Nucl. Phys. **A703** (2002) 823 [arXiv:hep-ph/0004044 [hep-ph]].

[47] E. Iancu, A. Leonidov and L. D. McLerran, *Nonlinear gluon evolution in the color glass condensate. i*, Nucl. Phys. **A692** (2001) 583 [arXiv:hep-ph/0011241].

[48] E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, *Nonlinear gluon evolution in the color glass condensate. ii*, Nucl. Phys. **A703** (2002) 489 [arXiv:hep-ph/0109115].

[49] E. Iancu, A. Leonidov and L. D. McLerran, *The renormalization group equation for the color glass condensate*, Phys. Lett. **B510** (2001) 133 [arXiv:hep-ph/0102009].

[50] J.-P. Blaizot, E. Iancu and H. Weigert, *Nonlinear gluon evolution in path integral form*, Nucl. Phys. **A713** (2003) 441 [arXiv:hep-ph/0206279 [hep-ph]].

[51] J. Bartels, L. N. Lipatov and G. P. Vacca, *A New odderon solution in perturbative QCD*, Phys. Lett. B **477** (2000) 178 [arXiv:hep-ph/9912423].