NASA Technical Memorandum 78687

(NASA-TM-78687) STRUCTURAL EFFICIENCY OF LONG LIGHTLY LOADED TRUSS AND ISOGRID COLUMNS FOR SPACE APPLICATIONS (NASA) 54 p HC A04/MF A01 CSCL 20K

N78-33480

Unclas

G3/39 33829

STRUCTURAL EFFICIENCY OF LONG LIGHTLY LOADED TRUSS AND ISOGRID COLUMNS FOR SPACE APPLICATIONS

Martin M. Mikulas, Jr.

July 1978

National Aeronautics and Space Administration

Langley Research Center
Hampton, Virginia 23665

STRUCTURAL EFFICIENCY OF LONG LIGHTLY LOADED TRUSS AND ISOGRID COLUMNS FOR SPACE APPLICATIONS

BY

Martin M. Mikulas, Jr.
Langley Research Center

INTRODUCTION

Large space structures built up from a series of long columns to form a stiff skeletal framework are being considered for a number of NASA's future space missions. A number of papers have been written on the general characteristics of such structures (see references 1 and 2) including as many of the design conditions as are presently conceived. One general conclusion from these and other studies is that individual member loads in space structures are quite low compared with earth-based structures.

There are a number of concepts being considered for placing long columns into orbit. In reference 3, tapered tubular columns which are nested to form high density payloads are investigated. In reference 4 a discussion is presented of structures which are manufactured in space and in reference 5 deployable Astro Mast columns are studied. These approaches result in a number of different structural arrangements which are tailored specifically to accommodate the construction technique being used. The present paper is concerned primarily with design methods for long, lightly loaded columns and with a technique for comparing the masses of different column concepts.

To understand the mass characteristics of long lightly loaded columns, the four column concepts shown in figure 1 are investigated. The first is a tubular column which has a mass close to that of a tapered nestable tubular column. The second is a three longeron truss column constructed from tubular members. Such a configuration is representative of a class of columns which could be manufactured or assembled in orbit and was selected because the tubular construction is extremely efficient and represents a low mass baseline against which other constructions can be compared. The third concept considered is a three longeron truss column constructed from solid rod members. This configuration is attractive for very lightly loaded columns where member dimensions become very small and it also is representative of the mass of Astro Mast type columns. The fourth concept is a tubular column with open gridwork "isogrid" walls. This configuration is quite efficient in the lightly loaded range because the open gridwork can provide an average wall thickness which is very small. This type of construction may also be amenable to manufacturing in space.

The design procedures in the present paper are based primarily on designing for an initial imperfection of the column. Column imperfections can result from a number of causes such as manufacturing, thermal gradients, and lateral accelerations. The effect of imperfections on the column performance is presented and the selection of the magnitude of the imperfection is discussed.

In selecting a column concept for a given application a number of factors such as cost, reliability, integrity, and structural efficiency must be considered. A procedure for comparing the structural efficiency of different concepts for heavily loaded aircraft structures is presented in reference 6. This procedure is discussed in the present paper and a new set of structural efficiency parameters which are more suitable for lightly loaded space structures are presented. These new parameters are used to demonstrate the relative efficiencies of the four column concepts considered.

SYMBOLS

a	Maximum amplitude of sinusoidal imperfection
$\bar{a}$	Acceleration
A	Cross sectional area
b	Width and thickness of square isogrid rib
c	Knockdown factor defined in equation (4)
C	Extensional stiffness for the isogrid wall ($C = \frac{Et}{1-\nu^2}$ for an isotropic, monocoque wall)
c_1	Rib end fixity factor defined in equation (C-13)
c_2	Knockdown factor defined in equation (C-2)
D	Bending stiffness for the isogrid wall ($D = \frac{Et^3}{12(1-\nu^2)}$ for an isotropic, monocoque wall)
E	Young's Modulus
f	Lowest natural frequency of column
F_{cr}	Critical buckling load for a single 0^0 isogrid rib
g	Acceleration of gravity
g_ξ	A constant defined by Newtons' second low ($\text{force} = \frac{\text{mass} \times \text{acceleration}}{g_\xi}$)
h	Rib spacing in isogrid wall
I	Moment of inertia
ℓ	Column length

M_{max}	Maximum moment in column induced by an imperfection
M	Column mass
m	Number of 0^0 (axial) ribs in the isogrid column
n	Number of bays in a column
P	Column design load
P_E	Euler buckling load of column
P_L	Load at which local buckling occurs in a tube
r	Radius of column
r_ℓ	Radius of longeron
R	Radius of truss cross section (see Sketch A in Appendix A)
t	Wall thickness
t_m	Minimum practical wall thickness
V	Shear load in column induced by an imperfection
V_{max}	Maximum shear load in column induced by an imperfection
ρ	Mass density
σ_{all}	Maximum allowable stress for the material
σ_L	Local buckling stress of a tube
θ	Diagonal angle
ν	Poisson's ratio

STRUCTURAL-EFFICIENCY PARAMETERS

To compare the relative masses of various column concepts over a wide range of loadings and lengths, it is necessary to make use of structural efficiency parameters which are obtained by properly relating the column mass to the loadings and geometric proportions considered. For many earth based structures the accepted principle for obtaining least mass proportions of columns is to design for simultaneous occurrence of the local and general modes of buckling under the applied loading. For very heavily loaded short columns it may be necessary to consider material yielding or strength as a failure mechanism. This approach to column design as well as the standard structural efficiency parameters are presented in reference 6. In the present paper the standard structural efficiency parameters for columns are developed using the tubular column as an example and new efficiency parameters for lightly loaded or long columns are proposed.

Column Mass Equations

The three general regions of loading and/or lengths for columns are (1) columns subjected to high loads and/or are short, (2) columns subjected to intermediate values of load and/or are of intermediate length, and (3) columns subjected to low loadings and/or are long. For heavily loaded or short columns wall crippling and material yielding or failure is a dominant design consideration. The limiting heavily loaded condition is when the mass is governed by material allowables only. The column mass M for such a condition is independent of cross-section geometry and may be written as

$$M = \frac{\rho P \ell}{\sigma_{all}} \qquad (1)$$

where ρ is the density of the material, σ_{all} is the allowable working stress of the material, P is the axial compressive load on the column and ℓ is the column length.

For columns of intermediate load or length, local and general buckling dominate the design. As an example, consider a thin walled tubular column. The Euler buckling load, P_E, for such a column is

$$P_E = \frac{\pi^2 EI}{\ell^2} = \frac{\pi^3 Er^3 t}{\ell^2} \qquad (2)$$

where r is the tube radius and t is the wall thickness. The load P_L at which local wall buckling occurs is

$$P_L = \sigma_L A \qquad (3)$$

where A is the tube cross-sectional area and σ_L is the local buckling stress given by

$$\sigma_L = c\left(\frac{.6Et}{r}\right) \qquad (4)$$

and c is an imperfection knockdown which may be found in reference 7. Substituting for σ_L into equation (3) and using $A = 2\pi rt$ results in the local buckling load for a thin walled tubular column as

$$P_L = 1.2\, c\, \pi E t^2 \qquad (5)$$

The mass M of a tubular column is written as

$$M = \rho A \ell = 2\pi r t \rho \ell \qquad (6)$$

By equating the buckling loads given by equations (2) and (5) to the applied loading P as

$$P_E = P_L = P$$

and eliminating the wall thickness t and the tube radius r from equations (2), (5), and (6) results in the following mass expression for a tubular column in the intermediate load range.

$$M = \left(\frac{4}{.6c\pi}\right)^{1/3} \frac{\rho}{E^{2/3}} P^{2/3} \ell^{5/3} \qquad (7)$$

For lightly loaded or long columns the wall thickness reaches a practical minimum and for lower loadings local wall buckling does not occur. For such columns only Euler buckling is critical and the wall thickness t_m is a constant. The mass equation for such a condition is determined by eliminating r from equations (2) and (6), and is given by

$$M = \frac{2\rho}{E^{1/3}} t_m^{2/3} P^{1/3} \ell^{5/3} \qquad (8)$$

Standard Structural Efficiency Parameters

The standard structural efficiency expressions for the first two conditions are obtained by dividing equations (1) and (7) by ℓ^3. For the heavily loaded region the structural efficiency expression becomes

$$\underbrace{\frac{M}{\ell^3}}_{\text{mass parameter}} = \underbrace{\frac{\rho}{\sigma_{all}}}_{\text{material parameter}} \underbrace{\frac{P}{\ell^2}}_{\text{loading index}} \qquad (9)$$

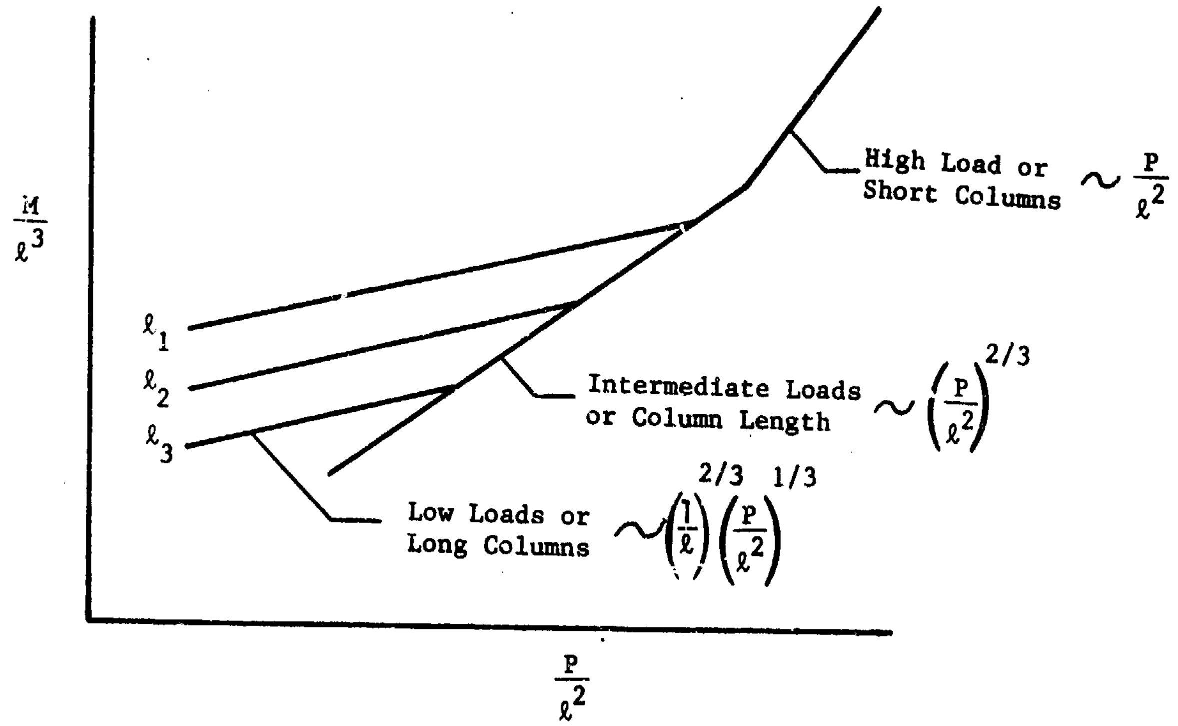
and for the intermediate load range the structural efficiency expression becomes

$$\underbrace{\frac{M}{\ell^3}}_{\text{mass parameter}} = \underbrace{\left(\frac{4}{.6c\pi}\right)^{1/3}}_{\text{shape factor}} \underbrace{\frac{\rho}{E^{2/3}}}_{\text{material parameter}} \underbrace{\left(\frac{P}{\ell^2}\right)^{2/3}}_{\text{loading index}} \qquad (10)$$

In both of these expressions the mass parameter is related to the loading index with no other dependence on load or length. For column cross-sections other than tubular the only factor that changes is the shape factor in equation (10). Thus, the relative efficiency of different column concepts can be determined for a wide range of loads and lengths by comparing their shape factors. For long columns or for low loadings dividing equation (8) by ℓ^3 results in

$$\frac{M}{\ell^3} = \frac{2\rho}{E^{1/3}} \left(\frac{t_m}{\ell}\right)^{2/3} \left(\frac{P}{\ell^2}\right)^{1/3} \qquad (11)$$

In this expression it can be seen that the mass parameter M/ℓ^3 is a function of the length ℓ as well as the loading index P/ℓ^2. A logarithmic plot of these structural efficiency expressions are shown in sketch a for the three general loadings ranges.



Sketch a. Standard Structural Efficiency Chart for Columns

Columns in most earth based applications are in the intermediate or high load regions, thus the standard parameters as shown are appropriate for comparing various concepts because of their independence of length or loading. However, studies indicate that columns for space applications will be in the long and/or lightly loaded region. The dependence of the mass parameter on column length for such conditions significantly reduces the effectiveness of these parameters in comparing various column concepts.

Structural Efficiency Parameters For Lightly Loaded And/Or Long Columns

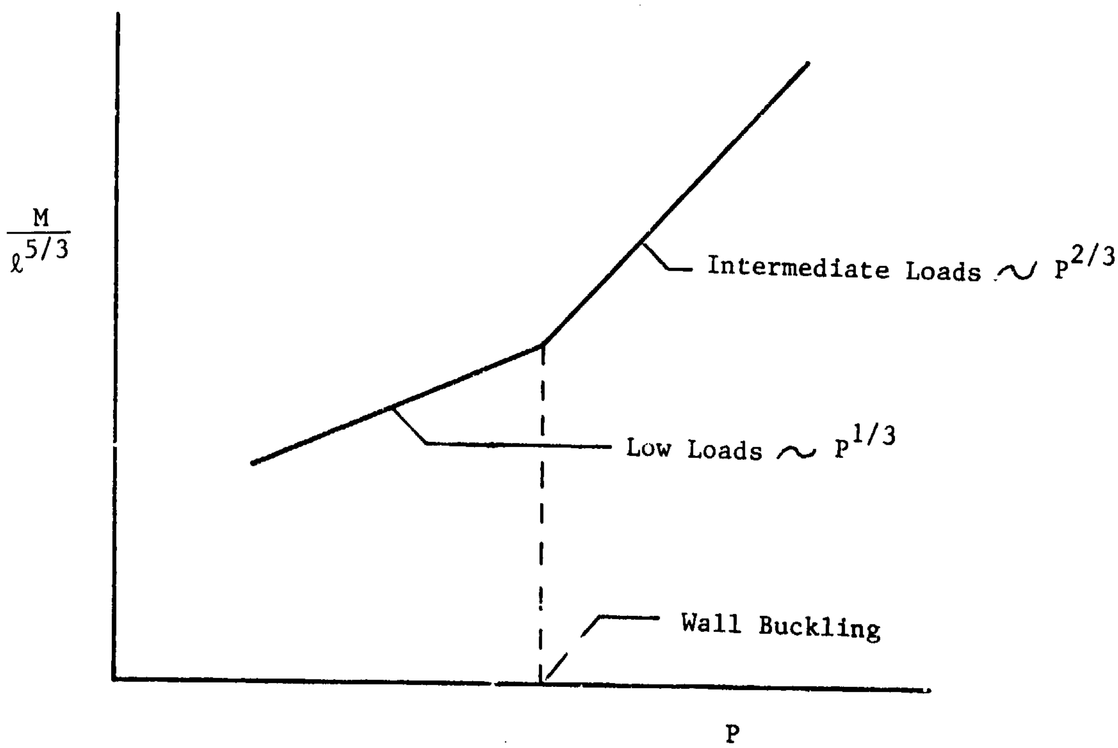
For columns in the low and intermediate load range it is proposed that the structural efficiency parameters $M/\ell^{5/3}$ and P be used to minimize the effect of changes in column length. Using these parameters the structural efficiency expression for tubular columns in the intermediate load range is obtained from equation (7) as

$$\frac{M}{\ell^{5/3}} = \left(\frac{4}{.6c\pi}\right)^{1/3} \frac{\rho}{E^{2/3}} P^{2/3} \qquad (12)$$

and from equation (8) the structural efficiency expression for lightly loaded tubular columns is

$$\frac{M}{\ell^{5/3}} = \frac{2\rho}{E^{1/3}} t_m^{2/3} P^{1/3} \qquad (13)$$

Logarithmic plots of these structural efficiency expressions are shown in Sketch b.



Sketch b. Structural Efficiency Chart for Long or Lightly Loaded Columns

It is shown in Sketch b that the proposed structural efficiency curves for tubular columns are independent of length for the low and intermediate load ranges considered. Although length independence is not exhibited for all cross sections, the utility of these structural efficiency parameters will be demonstrated for other column concepts in the next section.

COLUMN CONFIGURATIONS AND MASSES

In this section numerical results in the form of the structural efficiency parameters developed in the previous section are presented for the masses of several column configurations for relatively low loadings. The tubular column is used as the basic configuration against which other column constructions are compared. Results are presented for tubular columns made of graphite/epoxy and aluminum while results for the other configurations are for graphite/epoxy only.

Tubular Column

A structural efficiency comparison of aluminum and graphite/epoxy tubular columns is presented in figure 2. The plots are made from equations (12) and (13) and the material properties and parameters used are as follows:

$$\left.\begin{aligned} E &= 68.9\ \text{GN/m}^2\ (10 \times 10^6\ \text{psi}) \\ \rho &= 2767\ \text{kg/m}^3\ (.1\ \text{lbm/in}^3) \\ t_m &= .381\ \text{mm}\ (.015\ \text{in.}) \end{aligned}\right\}\ \text{Aluminum Columns}$$

$$\nu = .6$$

$$\left.\begin{aligned} E &= 110.2\ \text{GN/m}^2\ (16 \times 10^6\ \text{psi}) \\ \rho &= 1522\ \text{kg/m}^3\ (0.055\ \text{lbm/in}^3) \\ t_m &= .381\ \text{mm}\ (.015\ \text{in.}) \end{aligned}\right\}\ \text{Graphite/Epoxy Columns}$$

The value of E chosen for the graphite/epoxy columns is a nominal value based on a wall composed primarily of axial direction material which is sandwiched by a small amount of circumferential material to prevent local buckling and to improve its toughness for handling purposes.

Local buckling of the aluminum column occurs at a load of 22,637N (5089 lbf) as given by equation (5). For higher values of load the column wall is local buckling critical and the column mass is given by equation (12). For lower values of load the column wall is constrained by the minimum thickness chosen of .381 mm and the mass is given by equation (13). An orthotropic cylinder buckling analysis was made for a graphite/epoxy column with a wall construction of $(90_{.1}/0_{.8}/90_{.1})$ to determine the load at which local buckling would occur. For the chosen minimum wall thickness of .381 mm with 80%

axial direction material and the remaining 20% sandwiching the wall circumferentially, a local buckling load of 14,680 N (3300 lbf) was obtained. For lower values of load the column wall is constrained by the minimum thickness and the mass is given by equation (13). For higher values of the load, optimum proportions of axial and circumferential material would have to be determined by a procedure which includes an analysis for the buckling of orthotropic cylinders. The higher load range is not investigated in this paper and the dashed line in figure 2 is an estimate to indicate how the mass would vary with load. From figure 2 it can be seen that graphite/epoxy tubular columns have a mass which is about 48% of the mass of aluminum columns in the lightly loaded minimum thickness range.

Tubular Truss Columns

A procedure for obtaining the mass of a double laced, three longeron truss column constructed of tubular members is developed in Appendix A. In that development it was assumed that the applied loadings are low enough that all members are minimum thickness designed. The numerical results presented here are for a graphite/epoxy tubular truss using the same properties as in the previous section for the tubular column and the minimum thickness is again chosen as .381 mm (.015 in.).

Initial imperfection parameter.- In reference 8 it is recommended that a parameter a/ℓ (where a is the maximum amplitude of the imperfection) be used to specify column imperfections for design purposes. In that reference a study is made of imperfections in columns designed for civil engineering type structures and a design value for a/ℓ is proposed. Because there are a number of factors that can contribute to an imperfection in a column and because of the importance of reducing mass in space structures it is not likely that specifying a single value for a simple parameter such as a/ℓ will suffice for design purposes of large space structures. In the present study the importance of including the effects of initial imperfections in design is emphasized. The parameter a/ℓ is used only as a mechanism for discussing imperfections and it is cautioned that assessments of the imperfection will have to be made for individual applications.

Potential load reductions.- To investigate the effect of imperfections on reducing the buckling load of truss columns, a column designed without an imperfection is considered. For a trus column with no imperfection and consequently no bending the buckling load of the longerons P_L may be taken as one-third of the Euler buckling load P_E of the column, which is written as

$$P_L = \frac{P_E}{3} \tag{14}$$

By substituting for P_L from equation (14) into equation (A-5) and eliminating I and M_{max} using equations (A-1) and (A-4), an expression for the reduction in the buckling load P of a truss column with $\theta=45^o$ is obtained as

$$\frac{1}{P/P_E} = 1 + 2\sqrt{3}\, n \frac{a}{\ell} \frac{1}{1 - P/P_E} \tag{15}$$

A plot of $\frac{P}{P_E}$ as a function of $\frac{a}{\ell}$ is presented in figure 3 for several values of the number of bays, n. The range of the imperfection parameter was chosen to include the value of .0025 which is recommended for design in reference 8. The values of n chosen are typical of minimum mass truss columns as will be discussed subsequently. The main point to be made from this figure is that the buckling load of a truss column diminishes rapidly as a function of imperfection and that such effects should be considered in the initial design.

Mass sensitivity to imperfections.- A parametric study was made of truss columns for a range of loadings from 50 N to 25,000 N and a range of column lengths from 5 m to 500 m using the design procedure developed in Appendix A. The imperfections considered covered a range of values of a/ℓ from .0001 to .004. Details of the results of this study are presented in Table I. In figure 4 the masses of columns 5m, 50m, and 500m long designed for a compressive load of 500 N are plotted as a function of the imperfection parameter a/ℓ to show the sensitivity of column mass to assumed imperfection. In figure 4 the column masses are normalized with respect to the masses of the columns which were designed for a value of the imperfection parameter of .001. The .001 values were chosen as a reference since subsequent figures on column mass use that value of the imperfection parameter as a basis for comparison of different column concepts.

It can be seen in figure 4 that the change in mass that occurs as a result of designing for different values of the imperfection is fairly similar for the range of lengths shown and, in fact, this same trend exists for all loads and lengths considered. Because of the large change in column mass as a function of assumed initial imperfection it is important that these effects be considered early in the design process. As can be seen in Table I, the critical buckling load, P, of a minimum mass (imperfection designed) truss column is on the order of one-half of the Euler buckling capability of the column.

Imperfection magnitude.- Imperfections in a column can be due to a number of reasons such as manufacturing, thermal gradients, and lateral accelerations. To obtain some insight into the possible magnitudes of imperfections the lateral acceleration of a column is considered. The specific case analyzed is a column accelerated laterally by loads at each end. Such a situation could arise if the column was a member in a large structure which was performing an orbital maneuver caused by thrusters located at each end of the member. The acceleration $\bar{a}$ required to cause an imperfection a can be found from the solution for a lateral load on a beam simple supported at both ends as

$$\frac{\bar{a}}{g} = \frac{384}{5\pi^2} \left(\frac{a}{\ell}\right) \frac{P_E g_\xi}{Mg} \tag{16}$$

where $P_E = \frac{\pi^2 EI}{\ell^2}$ and g is the acceleration of gravity. A plot of the acceleration as a function of the imperfection parameter a/ℓ is presented in figure 5 for several values of design loads and lengths. The quantities used in equation (16) to make this plot were obtained from Table I. Figure 5 shows that for 500 m columns the lateral accelerations required to cause substantial imperfections in the column straightness are quite low. Such imperfections can cause significant reductions in the column load carrying capability as shown in Figure 3 as well as cause direct bending and shear loads which must be accounted for. For a proper column design, other possible causes of imperfections such as thermal gradients and manufacturing must also be considered.

Tubular truss masses.- The numerical results discussed in this section and tabulated in Table I were obtained from the design procedure developed in Appendix A for a tubular truss with an imperfection. The column loadings considered ranged from 50 N to 25,000 N, the column lengths were varied from 5 m to 500 m, and the imperfection parameter a/ℓ was varied from .0001 to .004. Although all of the results presented in Table I are for a diagonal angle θ equal to 45° a study was made to determine the effect on column mass of changes in the angle θ. Typical results from this study are shown in figure 6 for 500 m columns designed for the loading shown on the figure. As can be seen from figure 6, slight mass savings are possible by considering diagonal angles other than 45°. This mass savings is accompanied by a significant increase in the number of bays which may not be desirable considering costs, fabrication complexity, or other factors.

The masses of tubular truss columns are plotted on figure 7 in the form of a structural efficiency plot as discussed previously for a value of the imperfection parameter a/ℓ equal to .001. The mass of a graphite/epoxy tubular column is also shown for comparison. Although the masses of tubular truss columns presented in this form are not independent of length as is the case for tubular columns, the variation is not large. The variation of the mass curves with length would be much larger if the standard structural efficiency parameters $\frac{M}{\ell^3}$ vs. $\frac{P}{\ell^2}$ were used. It is shown in figure 7 that, compared with a tubular column, the tubular truss column has increasing mass advantages for the longer columns and lower loads. However, it should be cautioned that for very low loads the member dimensions become unreasonably small, as can be seen in Table I and for these cases it would be necessary to modify the column configuration. One possibility is to consider the truss column to be made up of solid rod members instead of tubular members; a concept which is evaluated in the next section. Although heavily loaded column designs (where local buckling of the longeron wall would be critical) are not investigated herein, the dashed lines that start around 18,000 N indicate approximately where this effect would have to be considered.

A quantity which is of importance in the design of large space structures is the lowest natural frequency of the whole structure or any of its elements. If a column is considered as a simply supported element of a larger structure its lowest natural frequency is

$$f = \frac{1}{\ell}\left(\frac{g_{\xi}P_E}{M\ell}\right)^{1/2} \tag{17}$$

Using this equation and data from Table I, the lowest natural frequencies of 50 m and 500 m truss columns are plotted as a function of design load P in figure 8. It can be seen that, although the column frequency changes significantly with length, only small increases in frequency can be obtained by increasing the design load.

Solid Rod Truss Columns

A procedure for obtaining the mass of a double laced, three longeron truss column constructed of solid rod members is presented in Appendix B. The procedure is the same as was developed for tubular trusses in Appendix A with appropriate changes made to account for the differences in member geometry. The numerical results discussed in this section and tabulated in Table II were obtained from the design procedure presented in Appendix B. The column loadings considered ranged from 5 N to 5,000 N, the column lengths were varied from 5 m to 500 m, and the imperfection parameter a/ℓ was varied from .0001 to .004. All results are obtained for a diagonal angle of 45° and the rods are assumed to be made of unidirectional graphite/epoxy material with a modulus of 124 GN/m².

Solid rod truss masses.- The masses of solid rod truss columns are plotted in figure 9 in the form of a structural efficiency plot as discussed previously for a value of the imperfection parameter a/ℓ equal to .001. The mass of a graphite/epoxy tubular column is also shown for comparison. It can be seen from figure 9 that a solid rod truss has significant mass advantages over a single tubular column for low loadings and it is in this range that such a construction should be considered for applications.

Isogrid Wall Tubular Columns

The motivation for selecting an open isogrid wall for tubular columns is that it allows a small effective minimum wall thickness to be obtained. For these very lightly loaded columns, a significant mass decrease results. In addition, the manufacturing processes in constructing the column could be simple. Relatively thin gage, unidirectional graphite strips can be cold-formed into the isogrid configuration and then bonded or welded at the joints. The efficiency of the column and the ease of fabrication are the important considerations for a space-based beam building machine. The structural efficiency of the column is considered here.

The procedures used for designing minimum mass, isogrid wall columns are developed in Appendix C. Since including a lateral imperfection in the column makes the problem significantly harder to solve, the design of straight columns is considered first. The design approach for the imperfect or bowed isogrid column is very similar to that for the truss columns and is considered in detail in Appendix C.

Numerical results were obtained for straight isogrid columns using the first approach in Appendix C, and for columns with an imperfection, a/ℓ = .0010 using the second approach in Appendix C. Three column axial loads (500, 5000, and 25000 N) and three column lengths (5, 50, and 500 m) were considered. The properties taken for the unidirectional graphite/epoxy are $E = 124.0$ GN/m^2 (18×10^6 psi) and $\rho = 1522$ kg/m^3. The end fixity factor, c_1, in equation (C-1) for buckling of an individual rib was taken to be 1.0 which assumes that the rib is simply supported at the ends.

Detailed information on the straight and bowed column designs is presented in Table III. By comparing the straight and bowed column designs in Table III it is possible to make observations about the effect of the imperfection on the column design. Because additional stress is applied to the isogrid wall due to the imperfection, the rib dimension b is increased to prevent rib buckling and the tube radius r is increased to prevent local wall buckling. The number of axial ribs, m, for the bowed columns is, in most cases, equal to that for the straight column. An increase in the mass parameter, $M/\ell^{5/3}$, of approximately 20 to 60 percent results from considering the imperfection.

A structural efficiency plot of the isogrid column with $a/\ell = .001$ compared with the tubular column is presented in figure 10. The tubular column data is independent of column length when shown on such a plot and the data for the isogrid wall tubular columns is nearly length independent. For low loadings it can be seen from figure 10 that the isogrid column is much more efficient than the tubular column. However, at higher loadings where the tubular column becomes local buckling critical, the lines for the two columns intersect.

It should be mentioned that the local buckling of the isogrid wall as predicted by equations (C-3) and (C-15) is based on obtaining equivalent isotropic, monocoque properties for the isogrid. This procedure has been used in the past (reference 9) but has not been applied to open isogrid wall cylinders with the type of wall construction envisioned here or with r/t values as high as those in the designs here. The high r/t values (i.e., 1000) cause the cylinder to be very sensitive to imperfections in the wall and cause the buckling wavelength in the circumferential direction to be quite small. When the buckle wavelength is of the same order as the rib spacing in the isogrid wall, the approximation of the discrete isogrid by an isotropic material is inaccurate Because of the high r/t values in the cylinders designed here, the continuum analysis for local wall buckling may be somewhat inaccurate.

Comparison Of Structural Efficiencies

The masses of the tubular column, the tubular truss column, the solid rod truss column, and the isogrid-wall tubular column are compared in the form of a structural efficiency chart in figure 11. All the masses shown are for columns manufactured from graphite/epoxy and further details are presented in Tables I, II and III. Although the tubular truss construction has the least mass for high loads, its usefulness is limited for low loads because of minimum practical wall thickness considerations. For design loads around 500 N or less a transition to a different construction such as the solid rod truss would have to be made for manufacturing reasons. The isogrid-wall tubular column is quite attractive from a mass consideration, however, considerable work is required to validate the buckling prediction techniques for open grid-work, composite material walls.

The mass and load parameters used in figure 11 facilitate the comparison of different column concepts for the lightly loaded range. This chart can be used for the rapid selection of a configuration for a given application or it can be used to evaluate the merits of other column concepts and material concepts.

CONCLUDING REMARKS

The mass characteristics of long, lightly loaded columns for space applications are investigated by analyzing four column configurations. The configurations investigated are, a three-longeron truss column with tubular members, a three-longeron truss column with solid rod members, a tubular column, and an isogrid-wall tubular column. Design procedures including the effects of an initial imperfection are developed for the four configurations and numerically demonstrated.

A new set of structural efficiency parameters are developed specifically for lightly loaded columns such as are expected to find application in large space structures. These parameters permit a comparison of the masses of different column configurations over a wide range of design loads and lengths. The use of these parameters is demonstrated in the form of a structural efficiency chart using the four previously discussed column configurations as examples.

To understand the characteristics of long, lightly loaded columns, the design procedures developed herein were numerically exercised on graphite/epoxy columns 5 m to 500 m long, and subjected to a range of loadings from 50 N to 25,000 N. Results presented include the effects of imperfections on column load carrying capability, the increase in column mass which results from designing for an imperfection, the masses and member sizes of the four column configurations investigated, and a study of the lowest natural frequency of long columns. Specific conclusions from these studies are as follows:

1. Imperfections that result from a number of causes such as manufacturing, thermal gradients, or lateral accelerations could reduce column load carrying capability by 50% if not taken into account.

2. Designing for an imperfection in truss columns enables the diagonals to be sized in a rational fashion.

3. Designing for an imperfection causes significant mass penalties which should be accounted for early in the design process.

4. Relatively low lateral accelerations (on the order of .001 g) can cause a substantial out-of-straightness of 500 m long columns which must be accounted for in their design.

5. The lowest natural frequency of a column changes quite rapidly with length as expected. The frequency of lightly loaded 500 m columns is on the order of .05 Hz and this value does not change markedly with changes in design load.

6. Tubular truss columns are very efficient except for very low loads where tube dimensions become impractically small.

7. For very low loads a solid rod truss structure is quite efficient and is not compromised by manufacturing considerations.

8. Isogrid-wall tubular columns are also quite efficient for low design loads, however, more work is needed in developing accurate buckling prediction techniques for open gridwork, composite material walls.

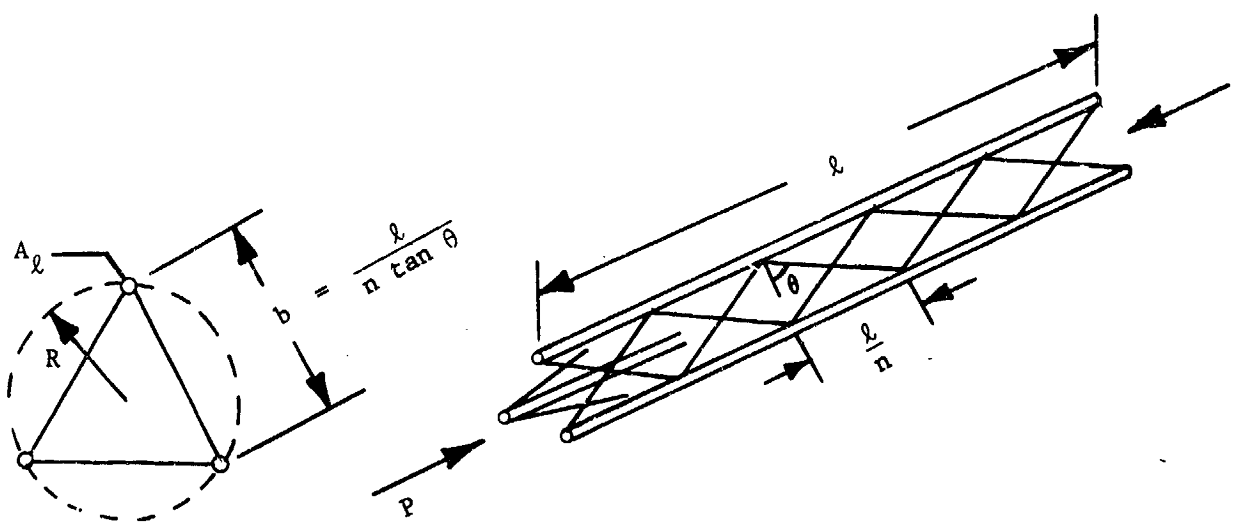
APPENDIX A

MASS OF TUBULAR TRUSS COLUMNS

In this appendix a procedure is presented for calculating the mass of a tubular truss column such as shown in figure 1-b. Although the diagonals of this truss structure represent a significant portion of the total mass, they are difficult to size since for an ideal truss column, the diagonals carry no load. Sizing for a transverse shear stiffness requirement leads to unreasonably small diagonals so that another criterion is needed. For civil engineering structures a "rule-of-thumb" criterion which has been used in the past (see reference 10) is to design the diagonals to carry a shear load equal to 2% of the axial compression load. In the present study the diagonals are sized by an induced shear load from an assumed lateral imperfection of the column. This approach provides a mechanism by which the imperfection can rationally be related to either a fabrication process or deformations due to thermal distortions or to lateral accelerations.

Truss Geometry

The configuration considered is a three longeron truss with double laced diagonals such as shown in sketch A.



Sketch A. Geometry of a Three Longeron Tubular Truss

The moment of inertia for the truss is given by

$$I = 2A_\ell\left(\frac{b}{2}\right)^2 = \frac{A_\ell\,\ell^2}{2n^2\tan^2\theta} \qquad \text{(A-1)}$$

where $A_\ell = 2\pi r_\ell t_\ell$ is the cross-sectional area of each longeron, n is the number of bays in the column, and θ is the angle of the diagonals as defined in sketch A.

Truss Analysis

The consideration of an imperfection in the straightness of a column has two primary implications. The first is that an additional load is imposed on the longerons due to an induced moment and the second is that an induced shear load results that must be carried by the diagonals.

Longeron design consideration.- The moment which is induced in a compressively loaded column with a sinusoidal imperfection is taken from reference 8 as

$$M = \frac{P}{1 - P/P_E} a \sin\left(\frac{\pi x}{\ell}\right) \tag{A-2}$$

where a is the maximum amplitude of the initial imperfection, x is an axial coordinate referenced from the end of the column, and P_E is the Euler buckling load of a perfect column which is defined as

$$P_E = \frac{\pi^2 EI}{\ell^2} \tag{A-3}$$

The maximum moment occurs at the center of the column ($x = \frac{\ell}{2}$) and is obtained from equation (A-2) as

$$M_{max} = \frac{Pa}{1 - P/P_E} \tag{A-4}$$

The maximum load in one of the longerons (P_L) is

$$P_L = \frac{P}{3} + \frac{M_{max} R A_\ell}{I} \tag{A-5}$$

where $A_\ell = 2\pi r_\ell t_\ell$, R is defined in Sketch A and can be written as

$R = \dfrac{\ell}{n\sqrt{3}\tan\theta}$, I is given by equation (A-1), and $P_L = \dfrac{\pi^2 E(\pi r_\ell^3 t_\ell)}{(\ell/n^2)}$.

Substituting these quantities into equation (A-5) yields

$$\frac{\pi^3 E r_\ell^3 t_\ell n^2}{\ell^2} = \frac{P}{3} + \frac{2Pa\sqrt{3}\tan\theta}{3\ell}\;\frac{n}{1 - \dfrac{Pn^2\tan^2\theta}{\pi^3 E r_\ell t_\ell}} \qquad \text{(A-6)}$$

This equation can be used to determine the radius of the longeron r_ℓ which is required to carry an applied load P for a given imperfection a, and an assumed number of bays n. The number of bays is then determined by minimizing the total column mass as will be discussed later.

Diagonal design consideration.- The shear load V which is induced in a compressively loaded column is determined by taking the derivative of the moment in equation (A-2) with respect to the axial coordinate x. This results in

$$V = \frac{P\frac{\pi a}{\ell}}{1 - P/P_E}\cos\left(\frac{\pi x}{\ell}\right) \qquad \text{(A-7)}$$

The maximum shear load occurs at the end of the column $(x = 0)$ and is obtained from equation (A-5) as

$$V_{max} = \frac{P\frac{\pi a}{\ell}}{1 - P/P_E} \qquad \text{(A-8)}$$

From equilibrium considerations the load in a diagonal P_d due to a shear load is

$$P_d = \frac{V_{max}}{4\cos\theta\cos 30^\circ} \qquad \text{(A-9)}$$

Substituting for V_{max} from equation (A-8) into (A-9) yields the induced diagonal load in an imperfect column as

$$P_d = \frac{P\frac{\pi a}{\ell}}{2\sqrt{3}\cos\theta(1 - P/P_E)} \qquad \text{(A-10)}$$

This load can then be used to determine the required diagonal mass which is done in the next section.

Truss Mass

The total mass of the truss column is taken as the sum of the mass of the 3 longerons M_ℓ and the mass of the 6 n diagonals M_d with the mass of the joints being assumed to be a constant percentage of the total sum. The truss mass M is written as

$$M = 1.15(M_\ell + M_d) \tag{A-11}$$

where it is assumed that the joints have a mass that is 15% of the mass of the longerons and diagonals. The mass of the longerons may be written as

$$M_\ell = 3\rho_\ell A_\ell \ell = 6\rho_\ell \pi r_\ell t_\ell \ell \tag{A-12}$$

and the mass of the diagonals may be written as

$$M_d = 6n\left[\frac{2\rho_d}{E_d^{1/3}} t_d^{2/3} P_d^{1/3} \ell_d^{5/3}\right] \tag{A-13}$$

where the mass of each diagonal has been taken as that given by equation (8). The length of each diagonal ℓ_d is

$$\ell_d = \frac{\ell}{n \sin \theta} \tag{A-14}$$

Substituting these quantities into equation (A-11) gives the following expression for the mass of a tubular truss column

$$M = 1.15\left[6\rho_\ell \pi r_\ell t_\ell \ell + \frac{12\rho_d}{E_d^{1/3}} t_d^{2/3} \frac{P_d^{1/3} \ell^{5/3}}{\sin \theta^{5/3} n^{2/3}}\right] \tag{A-15}$$

where the diagonal load P_d is given by equation (A-10) and the tube radius r_ℓ is obtained from equation (A-6). Equation (A-15) can be minimized with respect to the number of bays n to determine minimum mass proportions for a tubular truss column. The procedure for obtaining minimum mass truss columns for a given value of θ is as follows:

1. Select an integer value of n and solve for r_ℓ from equation (A-6).

2. Substitute the values for n and r_ℓ into equation (A-10) and solve for P_d.

3. Substitute the values for n, r_ℓ, and P_d into equation (A-15) to obtain the column mass for the selected value of n.

4. Repeat this procedure for different values of n until a minimum mass column is obtained.

Numerical results were obtained for a range of loadings using this procedure and the results are presented in the text of the paper.

APPENDIX B

MASS OF SOLID ROD TRUSS COLUMNS

The procedure for designing three-longeron truss columns with solid rod members is essentially the same as in the previous Appendix for tubular truss columns with appropriate changes made for member cross-section. By making these changes in equation (A-5), an equation for determining the radius of the longerons r_ℓ for a solid rod truss is

$$\frac{\pi^3 E n^2 r_\ell^4}{4\ell^2} = \frac{P}{3} + \frac{2}{3}\frac{Pa\sqrt{3}\tan\theta}{\ell}\frac{n}{\left(1 - \frac{2Pn^2\tan^2\theta}{\pi^3 E r_\ell^2}\right)} \tag{B-1}$$

The mass of the diagonals in a truss made of solid rod members is

$$M_d = 6n\left[\frac{2\rho_d}{\pi^{1/2}}\frac{P_d^{1/2}\ell_d^2}{E_d^{1/2}}\right] \tag{B-2}$$

where the load in the diagonals P_d is given by equation (A-10).

The total mass of the solid rod truss may be obtained from equation (A-11) by substituting the appropriate expression for the solid longeron mass and substituting for the diagonal mass from equation (B-2); the following equation is obtained:

$$M = 1.15\left[3\rho_\ell \pi r_\ell^2 \ell + \frac{12\rho_d P_d^{1/2}\ell^2}{\pi^{1/2}E_d^{1/2}\sin^2\theta\; n}\right] \tag{B-3}$$

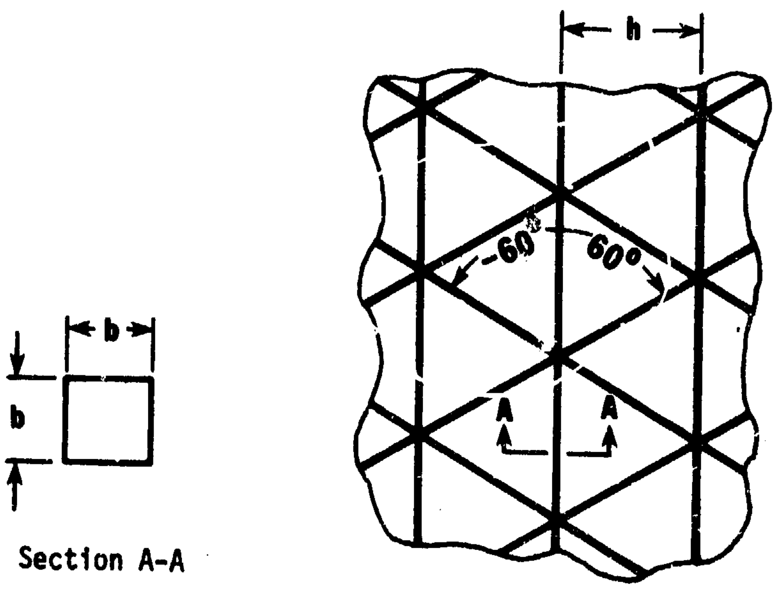
Equation (B-3) can be minimized with respect to the number of bays n as was done for tubular truss columns in Appendix A to determine minimum mass proportions for a solid-rod truss column. This is done in the text of the paper and results are presented for a range of loadings.

APPENDIX C

MASS OF ISOGRID WALL TUBULAR COLUMNS

A procedure is presented here for designing minimum mass isogrid wall tubular columns. Although there are an infinite number of orientations for the isogrid wall ribs with respect to the tube axis, one particular orientation, the ($60^\circ/0^\circ/-60^\circ$) configuration where the angles denote the orientation of the square ribs with respect to the tube axis (see figure 1-d and sketch C) was chosen for analysis. Design procedures for both the straight column and one with a lateral imperfection are derived.

In the analysis of the straight, isogrid wall column, three failure mechanisms are considered: (1) overall Euler buckling of the column, (2) local buckling of an individual rib member in the wall, and (3) general instability of the isogrid wall.



Sketch C.- Isogrid geometry.

The expression for general instability of the isogrid wall and overall Euler buckling are derived by substituting the isogrid wall stiffnesses into the analogous expressions for the isotropic monocoque structure. The equivalent isotropic Poisson's ratio, extensional stiffness, and bending stiffness for the isogrid wall have been derived previously (see for example, reference 9) and are as follows:

$$\nu = \frac{1}{3} \tag{C-1}$$

$$C = \frac{9Eb^2}{8h} \tag{C-2}$$

$$D = \frac{3Eb^4}{32h} \tag{C-3}$$

where b is the thickness and width of the square rib cross section and h is the spacing between ribs.

For the $60^0/0^0_0/-60^0$ column, h can be conveniently expressed in terms of the number of 0^0 members, m , around the circumference and the tube radius r. Equations (C-2) and (C-3) can be rewritten as

$$C = \frac{9Eb^2m}{16\pi r} \tag{C-4}$$

$$D = \frac{3Eb^4m}{64\pi r} \tag{C-5}$$

The equation for overall Euler buckling of a thin walled tube can be written as

$$P_{cr} = \frac{\pi^3 r^3 C(1-\nu^2)}{\ell^2} \tag{C-6}$$

which reduces to $P_{cr} = \pi^3 r^3 Et/\ell^2$ for the monocoque case. The equation for Euler buckling of the isogrid column can be obtained by substituting equations (C-1) and (C-4) into equation (C-6) and is given by

$$P_{cr} = \frac{E\pi^2 r^2 b^2 m}{2\ell^2} \tag{C-7}$$

The equation for general instability can be derived in a similar manner. For an isotropic shell, general instability is predicted by

$$P_{cr} = c_2 4\pi \sqrt{CD(1-\nu^2)} \qquad \text{(C-8)}$$

which reduces to the familiar equation $P_{cr} = c_2 2\pi E t^2 / \sqrt{3(1-\nu^2)}$ for the monocoque shell. The coefficient c_2 is a knockdown factor included to account for the effects of initial imperfections. Equation (C-8) can be rewritten for the isogrid wall by substitution for ν, C, and D from equations (C-1), (C-4) and (C-5). The result is

$$P_{cr} = \frac{.612\ c_2 m E b^3}{r} \qquad \text{(C-9)}$$

Appropriate selection of the knockdown factor, c_2, is somewhat difficult because very little data are available for open isogrid cylinders. It is well-known, however, that for isotropic, monocoque shells, considerable reduction in the theoretical buckling load can be observed, making the inclusion of c_2 important. Therefore, due to lack of additional information, an empirical equation for c_2 for isotropic shells from reference 7 was used. This equation is

$$c_2 = 1 - .901\left(1 - e^{-\frac{1}{16}\sqrt{\frac{r}{t}}}\right) \qquad \text{(C-10)}$$

For the open isogrid, the monocoque thickness, t, is replaced by the rib thickness, b. The result is the equation for knockdown factor used in this study

$$c_2 = 1 - .901\left(1 - e^{-\frac{1}{16}\sqrt{\frac{r}{b}}}\right) \qquad \text{(C-11)}$$

The third failure mode is buckling of a 0^o rib as a small Euler column. If the rib properties are substituted into the familiar equation for Euler buckling and the rib length expressed in terms of the column radius, r, and the number of 0^o ribs, m, the critical load for the rib can be written

$$F_{cr} = \frac{c_1\ E b^4 m^2}{64 r^2} \qquad \text{(C-12)}$$

where c_1 is an end fixity factor associated with the support of the 0^o ribs by 60^o ribs. Multiplying equation (C-12) by the number of 0^o members m gives the critical column load to cause rib buckling.

$$P_{cr} = \frac{c_1 \; Eb^4 m^3}{64 r^2} \qquad \text{(C-13)}$$

Since there are three failure modes (Euler buckling, rib buckling, local wall buckling) and three free design parameters (r, b, m), the minimum mass design can be obtained by selecting the three design parameters so that all three failures occur at the same load.

If equations (C-7) and (C-13) are equated and the design load, P, substituted for P_{cr} in each case, the following two equations in terms of m and P can be obtained.

$$b = \left(\frac{8\sqrt{2} \; P\ell}{\pi E m^2 \sqrt{c_1}} \right)^{1/3} \qquad \text{(C-14)}$$

$$r = \frac{\ell}{\pi b} \left(\frac{2P}{Em} \right)^{1/2} \qquad \text{(C-15)}$$

The minimum mass isogrid column is determined as follows:

1. Select a given value of m which must be even to allow continuity of the isogrid mesh in the column wall and a value of P.

2. Solve equations (C-14) and (C-15) for b and r.

3. Equation (C-9) may now be solved for P_{cr}. When P_{cr} is approximately equal to P, all three failure modes occur simultaneously at the design load and the minimum mass column results.

The mass of the isogrid column is given by

$$M = 3\rho b^2 m\ell \qquad \text{(C-16)}$$

In this study the frequency of vibration of the simply supported column is also calculated. This is found from the frequency equation for a simply supported beam

$$f = \frac{\pi}{2} \left(\frac{g_\xi EI}{M\ell^3} \right)^{1/2} \qquad \text{(C-17)}$$

by substituting $I = \frac{r^2 b^2 m}{2}$ for the isogrid wall tube.

The design of isogrid wall columns with an assumed lateral imperfection is more difficult than straight columns because (1) the governing expressions are slightly more complicated, and (2) the nature of the problem changes such that the minimum mass design can no longer be found by simple, direct solution of the buckling equations. As a result of the imperfection, the column wall is loaded asymmetrically which increases the susceptibility of the walls to local buckling and the ribs to local Euler buckling. A second result is the elimination of overall Euler buckling as a failure criteria; the design load will now be lower than the Euler load for the column. There are still three free design parameters (r, b, m) but only two failure equations (rib buckling and local wall buckling). The procedure used for finding the minimum mass column will be discussed later.

The approach to including the imperfection in the analysis is similar to that in Appendix A. One difference is that buckling of the "diagonal" members due to the induced transverse shear force is not considered here. The $\pm 60^{\circ}$ ribs are the equivalent of the diagonals in the truss column. However, for the isogrid, these 60° ribs have the same cross sectional area and length as the 0° ribs. Therefore, buckling due to an imperfection induced transverse shear is never a critical failure mode. An induced bending moment is assumed to be the only resultant effect of the initial imperfection.

The maximum bending moment in the column, M_{max}, with an imperfection of amplitude, a, is given by equation (A-12). The Euler load for the isogrid column, P_E, in equation (A-12) is given by equation (C-7). The force in a 0^{o} rib can now be expressed in terms of M_{max} and P as

$$F_{rib} = \frac{P}{m} + \frac{2M_{max}}{rm} \qquad \text{(C-18)}$$

If F_{rib} is equation (C-18) is equated to the rib buckling load F_{cr} (equation C-12), an equation for buckling of the rib in terms of the column axial load, P, and the maximum moment, M_{max}, can be obtained. Substituting the expressions for M_{max}, equation (A-12), and P_E, equation (C-7), into equation (C-18) results in the final equation governing rib buckling which is

$$\frac{c_1 E b^4 m^3}{64 r^2} = P + \frac{2Pa}{r\left(1 - \frac{2\ell^2 P}{\pi^2 E r^2 b^2 m}\right)} \qquad \text{(C-19)}$$

The expression for buckling of the isogrid wall can be obtained by modifying equation (C-9) to account for the additional axial stress caused by M_{max} and is given by

$$\frac{c_2 \; .612m \; Eb^3}{r} = P + \frac{2Pa}{r\left(1 - \frac{2\ell^2 P}{\pi^2 E r^2 b^2 m}\right)} \qquad \text{(C-20)}$$

The techniques previously used for determining the minimum mass column are not applicable here because solution of equation (C-19) or equation (C-20) for any of the design variables (r, b, m) is not easily done. Therefore, a numerical algorithm for constrained minimization was used to find the minimum weight design subject to the constraints, equations (C-19) and (C-20). Since the problem has only three design variables and two constraints, very simple minimization algorithms can be used. However, because of its proven reliability a more sophisticated optimization computer code, CONMIN (Ref. 11), was used to minimize the mass, equation (C-16), subject to equations (C-19) and (C-20).

REFERENCES

1. Mikulas, M. M., Jr.; Bush, H. G.; and Card, M. F.: Structural Stiffness, Strength and Dynamic Characteristics of Large Tetrahedral Space Truss Structures. NASA TMX-74001, March 1977.

2. Bush, H. G.; Mikulas, M. M., Jr.; and Heard, Walter L., Jr.: Some Design Considerations for Large Space Structures. AIAA J. Vol. 16, No. 4, April 1978, pp. 352-359.

3. Bush, H. G.; and Mikulas, M. M., Jr.: A Nestable Tapered Column Concept for Large Space Structures. NASA TMX-73927. July 1976.

4. Hagler, Thomas; Patterson, Herbert G.; and Nathan, C. Allan: Learning to Build Large Structures in Space. Astronautics and Aeronautics, December 1977.

5. Crawford, R. F.: Strength and Efficiency of Deployable Booms for Space Applications. AIAA Paper No. 71-396, April 1971.

6. Shanley, F. R.: Weight-Strength Analysis of Aircraft Structures. 2nd Edition, Dover Publications, Inc., 1960.

7. NASA Space Vehicle Design Criteria: Buckling of Thin-Walled Circular Cylinders. NASA SP-8007, August 1968.

8. Timoshenko, S. P.; and Gere, J. M.: Theory of Elastic Stability. 2nd Edition, McGraw Hill Book Company, 1961.

9. Isogrid Design Handbook - MDC G4295 A (Contract NAS 8-28619), McDonnell Douglas Astronautics Co., Feb. 1973. (Available as NASA CR-124075).

10. Structural Engineering Handbook, Edited by Gaylord, Edwin H., Jr.; and Gaylord, Charles N. McGraw-Hill Book Co., 1968.

11. Vanderplaats, Garret N.: CONMIN - A Fortran Program for Constrained Function Minimization User's Manual, NASA TMX-62282, August, 1973.

Table I. Details of Minimum Mass Graphite/Epoxy Tubular Truss Columns ($\theta = 45°$)

Parameter	P	$M/\ell^{5/3}$	r_ℓ	r_d	f
Units	N	$kg/m^{5/3}$	mm	mm	Hz

$\ell = 5$ m $\qquad \frac{a}{\ell} = .0001$

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	*					
5000.	2.19×10^{-2}	25	3.73	.48	.641	34.9
25000.	5.08×10^{-2}	15	8.94	1.02	.482	59.1

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	4.75×10^{-3}	80	3.78	.48	.650	1.09
5000.	1.49×10^{-2}	45	11.81	1.52	.657	1.94
25000.	3.33×10^{-2}	30	26.34	3.40	.655	2.90

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	3.27×10^{-3}	145	12.07	1.52	.669	.065
5000.	1.03×10^{-2}	80	37.85	4.80	.650	.109
25000.	2.27×10^{-2}	55	82.55	11.10	.703	.158

*Diagonals reduced to solid rods.

Table I. (continued)

$a/\ell = .0005$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	8.41×10^{-3}	45	1.24	.25	.626	18.0
5000.	2.62×10^{-2}	25	3.86	.81	.623	32.3
25000.	6.01×10^{-2}	15	9.07	1.73	.476	54.7

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	5.83×10^{-3}	80	4.06	.79	.603	1.02
5000.	1.81×10^{-2}	45	12.40	2.54	.626	1.80
25000.	4.03×10^{-2}	30	27.18	5.72	.633	2.69

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	4.07×10^{-3}	140	13.59	2.41	.553	.059
5000.	1.26×10^{-2}	80	40.64	7.85	.603	.102
25000.	2.77×10^{-2}	55	87.88	18.16	.661	.147

Table I. (continued)

$a/\ell = .0010$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	9.38×10^{-3}	45	1.30	.30	.598	17.5
5000.	2.91×10^{-2}	25	3.96	.99	.605	31.1
25000.	6.61×10^{-2}	15	9.19	2.18	.469	52.6

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	6.53×10^{-3}	80	4.34	.97	.566	1.0
5000.	2.02×10^{-2}	45	12.98	3.12	.598	1.8
25000.	4.46×10^{-2}	30	28.19	7.06	.611	2.6

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	4.61×10^{-2}	140	14.81	2.95	.508	.06
5000.	1.41×10^{-2}	80	43.43	9.60	.566	.10
25000.	3.10×10^{-2}	55	92.71	22.15	.626	.14

Table I. (continued)

$a/\ell = .0015$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	1.01×10^{-2}	45	1.35	.36	.576	17.2
5000.	3.12×10^{-2}	25	4.06	1.12	.589	30.5
25000.	7.03×10^{-2}	15	9.35	2.49	.462	51.3

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	7.05×10^{-3}	80	4.57	1.07	.538	.986
5000.	2.17×10^{-2}	45	13.49	3.51	.576	1.72
25000.	4.77×10^{-2}	30	29.08	7.95	.594	2.55

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	5.00×10^{-3}	140	15.75	3.30	.478	.058
5000.	1.52×10^{-2}	80	45.62	10.77	.538	.10
25000.	3.33×10^{-2}	55	96.52	24.79	.600	.141

Table I. (continued)

$a/\ell = .0040$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	1.22×10^{-2}	45	1.52	.46	.508	16.6
5000.	3.74×10^{-2}	25	4.47	1.50	.536	29.2
25000.	8.37×10^{-2}	15	9.91	3.38	.436	48.4

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	8.66×10^{-3}	85	5.33	1.42	.519	.906
5000.	2.62×10^{-2}	45	15.29	4.62	.508	1.66
25000.	5.75×10^{-2}	30	32.26	10.54	.535	2.45

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	6.22×10^{-3}	150	18.97	4.32	.455	.053
5000.	1.86×10^{-2}	85	53.34	14.15	.519	.091
25000.	4.05×10^{-2}	50	112.27	32.51	.522	.137

Table I. (continued)

$a/\ell = .0025$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	1.11×10^{-2}	45	1.42	.41	.543	16.9
5000.	3.43×10^{-2}	25	4.24	1.32	.565	29.8
25000.	7.67×10^{-2}	15	9.58	2.92	.451	49.8

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	7.81×10^{-3}	80	4.93	1.24	.499	.972
5000.	2.38×10^{-2}	45	14.30	4.04	.543	1.69
25000.	5.25×10^{-2}	30	30.48	9.25	.566	2.49

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
500.	5.58×10^{-3}	145	17.22	3.81	.468	.055
5000.	1.68×10^{-2}	80	49.20	12.42	.499	.097
25000.	3.68×10^{-2}	55	103.30	28.45	.562	.139

Table II. Details of Minimum Mass Graphite/Epoxy Solid Rod Truss Columns ($\theta = 45°$)

Parameter	P	$M/\ell^{5/3}$	r_ℓ	r_d	f
Units	N	kg/m$^{5/3}$	mm	mm	Hz

$\frac{a}{\ell} = .0001$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	3.64×10^{-4}	130	.23	.05	.788	7.73
50.	1.66×10^{-3}	85	.51	.10	.723	11.9
500.	7.63×10^{-3}	60	1.07	.25	.810	16.6
5000.	3.51×10^{-2}	40	2.31	.56	.776	25.1

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	3.78×10^{-4}	275	.53	.10	.712	.37
50.	1.71×10^{-3}	185	1.12	.25	.759	.55
500.	7.81×10^{-3}	125	2.39	.53	.718	.81
5000.	3.58×10^{-2}	85	5.08	1.24	.833	1.19

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	4.05×10^{-4}	580	1.19	.23	.623	.018
50.	1.81×10^{-3}	400	2.49	.51	.670	.026
500.	8.14×10^{-3}	275	5.26	1.14	.740	.037
5000.	3.70×10^{-2}	185	11.15	2.41	.714	.055

Table II. (continued)

$a/\ell = .0005$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	4.55×10^{-4}	125	.25	.08	.617	7.8
50.	2.05×10^{-3}	85	.53	.15	.645	11.4
500.	9.34×10^{-3}	60	1.14	.36	.726	15.9
5000.	4.24×10^{-2}	40	2.41	.79	.716	23.7

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	4.98×10^{-4}	275	.58	.15	.567	.361
50.	2.21×10^{-3}	180	1.22	.33	.558	.547
500.	9.82×10^{-3}	125	2.57	.74	.617	.780
5000.	4.42×10^{-2}	85	5.38	1.60	.645	1.14

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	5.70×10^{-4}	595	1.37	.30	.130	.017
50.	2.46×10^{-3}	405	2.84	.69	.381	.025
500.	1.07×10^{-2}	275	5.89	1.50	.567	.036
5000.	4.75×10^{-2}	185	12.27	3.30	.592	.053

Table II. - (continued)

$a/\ell = .0010$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	5.27×10^{-4}	125	.28	.08	.551	7.69
50.	2.34×10^{-3}	85	.56	.18	.658	11.2
500.	1.06×10^{-2}	55	1.19	.41	.551	17.2
5000.	4.77×10^{-2}	40	2.49	.89	.671	23.1

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	5.91×10^{-4}	280	.64	.18	.504	.351
50.	2.58×10^{-3}	185	1.32	.38	.547	.526
500.	1.13×10^{-2}	125	2.72	.84	.550	.769
5000.	5.04×10^{-2}	85	5.66	1.83	.586	1.12

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	6.99×10^{-4}	610	1.52	.36	.417	.017
50.	2.98×10^{-3}	420	3.10	.79	.476	.024
500.	1.28×10^{-2}	275	6.35	1.70	.486	.036
5000.	5.56×10^{-2}	180	13.13	3.76	.489	.054

Table II. - (continued)

$a/\ell = .0015$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	5.81×10^{-4}	130	.28	.10	.549	7.33
50.	2.56×10^{-3}	85	.58	.20	.546	11.1
500.	1.15×10^{-2}	55	1.22	.43	.521	16.9
5000.	5.17×10^{-2}	40	2.57	.97	.637	22.8

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	6.63×10^{-4}	285	.66	.18	.469	.344
50.	2.87×10^{-3}	190	1.37	.41	.497	.509
500.	1.25×10^{-2}	125	2.82	.89	.507	.763
5000.	5.52×10^{-2}	85	5.87	1.98	.546	1.11

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	7.96×10^{-4}	640	1.63	.38	.404	.016
50.	3.35×10^{-3}	435	3.30	.84	.433	.023
500.	1.43×10^{-2}	285	6.71	1.85	.486	.034
5000.	6.18×10^{-2}	190	13.74	4.06	.497	.051

Table II. (continued)

$a/\ell = .0025$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	6.65×10^{-4}	130	.30	.10	.486	7.27
50.	2.91×10^{-3}	85	.61	.23	.49	10.9
500.	1.29×10^{-2}	60	1.27	.48	.573	15.2
5000.	5.79×10^{-2}	40	2.57	1.07	.589	22.4

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	7.77×10^{-4}	300	.71	.20	.447	.326
50.	3.33×10^{-3}	195	1.47	.46	.457	.493
500.	1.44×10^{-2}	130	3.00	.99	.523	.727
5000.	6.28×10^{-2}	85	6.20	2.18	.490	1.09

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	9.49×10^{-4}	680	1.78	.41	.382	.015
50.	3.96×10^{-3}	455	3.58	.91	.421	.022
500.	1.67×10^{-2}	300	7.24	2.03	.447	.033
5000.	7.15×10^{-2}	195	14.71	4.47	.457	049

Table II.- (continued)

$a/\ell = .004$

$\ell = 5$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	7.67×10^{-4}	135	.33	.10	.460	6.97
50.	3.33×10^{-3}	90	.66	.23	.490	10.2
500.	1.46×10^{-2}	60	1.35	.53	.517	15.0
5000.	6.51×10^{-2}	40	2.77	1.17	.539	22.1

$\ell = 50$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	9.09×10^{-4}	310	.79	.23	.409	.315
50.	3.84×10^{-3}	210	1.57	.48	.458	.457
500.	1.65×10^{-2}	135	3.20	1.07	.460	.697
5000.	7.17×10^{-2}	90	6.55	2.39	.543	1.02

$\ell = 500$ m

P	$M/\ell^{5/3}$	n	r_ℓ	r_d	P/P_E	f
5.	1.12×10^{-3}	715	1.93	.46	.356	.014
50.	4.67×10^{-3}	485	3.89	.99	.406	.021
500.	1.96×10^{-2}	315	7.82	2.18	.435	.031
5000.	8.31×10^{-2}	205	15.80	4.88	.437	.047

Table III. Details of Minimum Mass Graphite/Epoxy Isogrid Columns

Parameter	P	$M/\ell^{5/3}$	r_ℓ	r_d	f
Units	N	$kg/m^{5/3}$	mm	mm	Hz

$\frac{a}{\ell} = 0.0$

$\ell = 5$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	6.72×10^{-3}	66	6.88	.025	1.00	16.0
5000.	3.29×10^{-2}	56	9.83	.061	1.00	22.8
25000.	1.01×10^{-1}	48	12.5	.117	1.00	29.0

$\ell = 50$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	6.16×10^{-3}	86	33.5	.046	1.00	.774
5000.	2.98×10^{-2}	76	48.0	.107	1.00	1.11
25000.	8.95×10^{-2}	70	62.0	.196	1.00	1.44

$\ell = 500$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	5.70×10^{-3}	108	161.	.086	1.00	.037
5000.	2.75×10^{-2}	96	232.	.198	1.00	.054
25000.	8.23×10^{-2}	90	300.	.356	1.00	.070

Table III. (Continued)

$a/\ell = .0010$

$\ell = 5$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	8.60×10^{-3}	66	7.32	.028	.692	17.0
5000.	4.07×10^{-2}	56	10.6	.069	.698	24.5
25000.	1.22×10^{-1}	48	13.0	.127	.770	30.2

$\ell = 50$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	8.63×10^{-3}	88	36.8	.053	.587	.853
5000.	3.97×10^{-2}	78	52.1	.122	.636	1.21
25000.	1.15×10^{-1}	72	66.5	.218	.678	1.54

$\ell = 500$ m

P	$M/\ell^{5/3}$	m	r	b	P/P_E	f
500.	9.30×10^{-3}	106	175.	.109	.521	.040
5000.	4.15×10^{-2}	94	244.	.246	.596	.057
25000.	1.18×10^{-1}	90	325.	.427	.591	.075

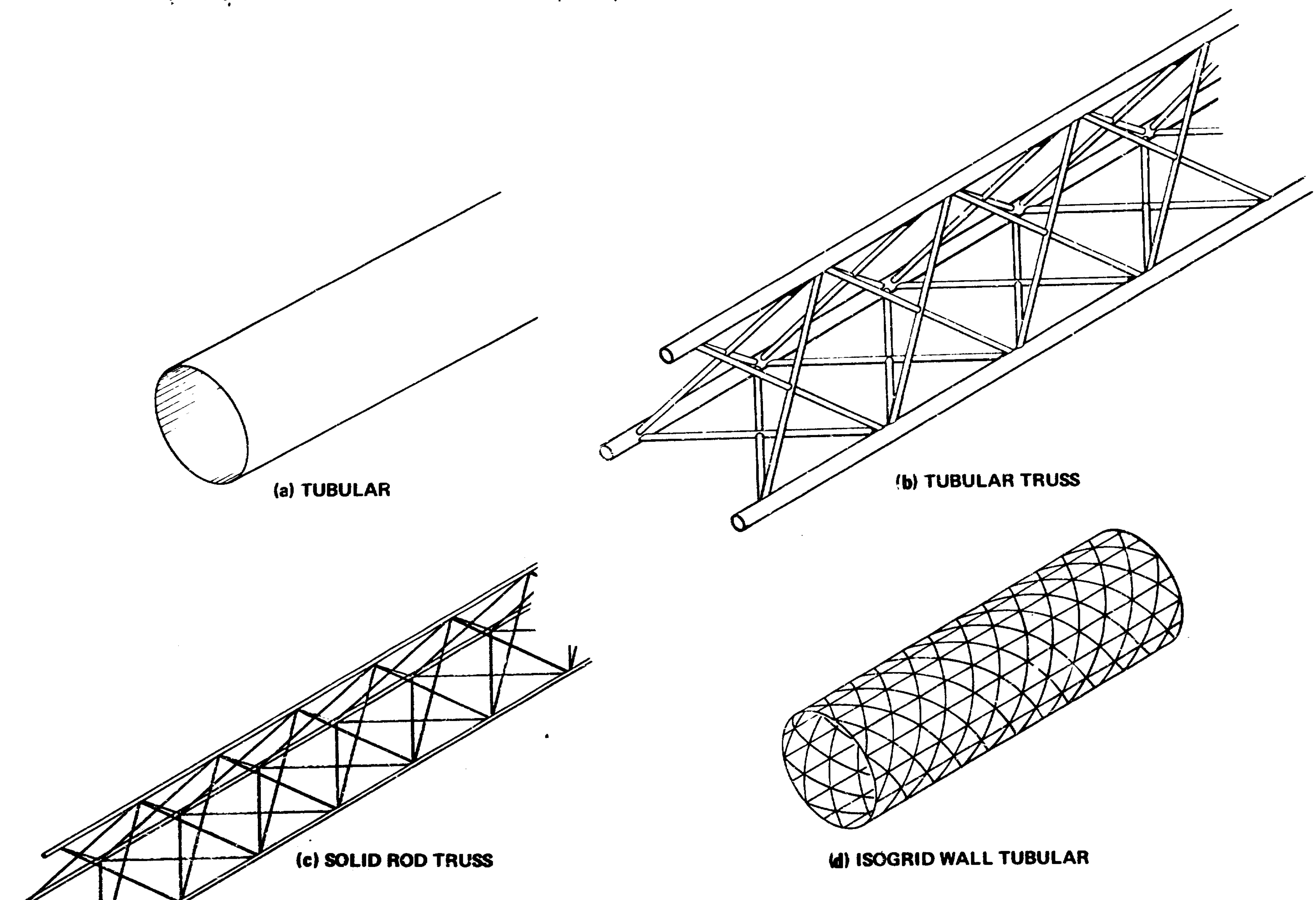


Figure 1.- Column concepts considered.

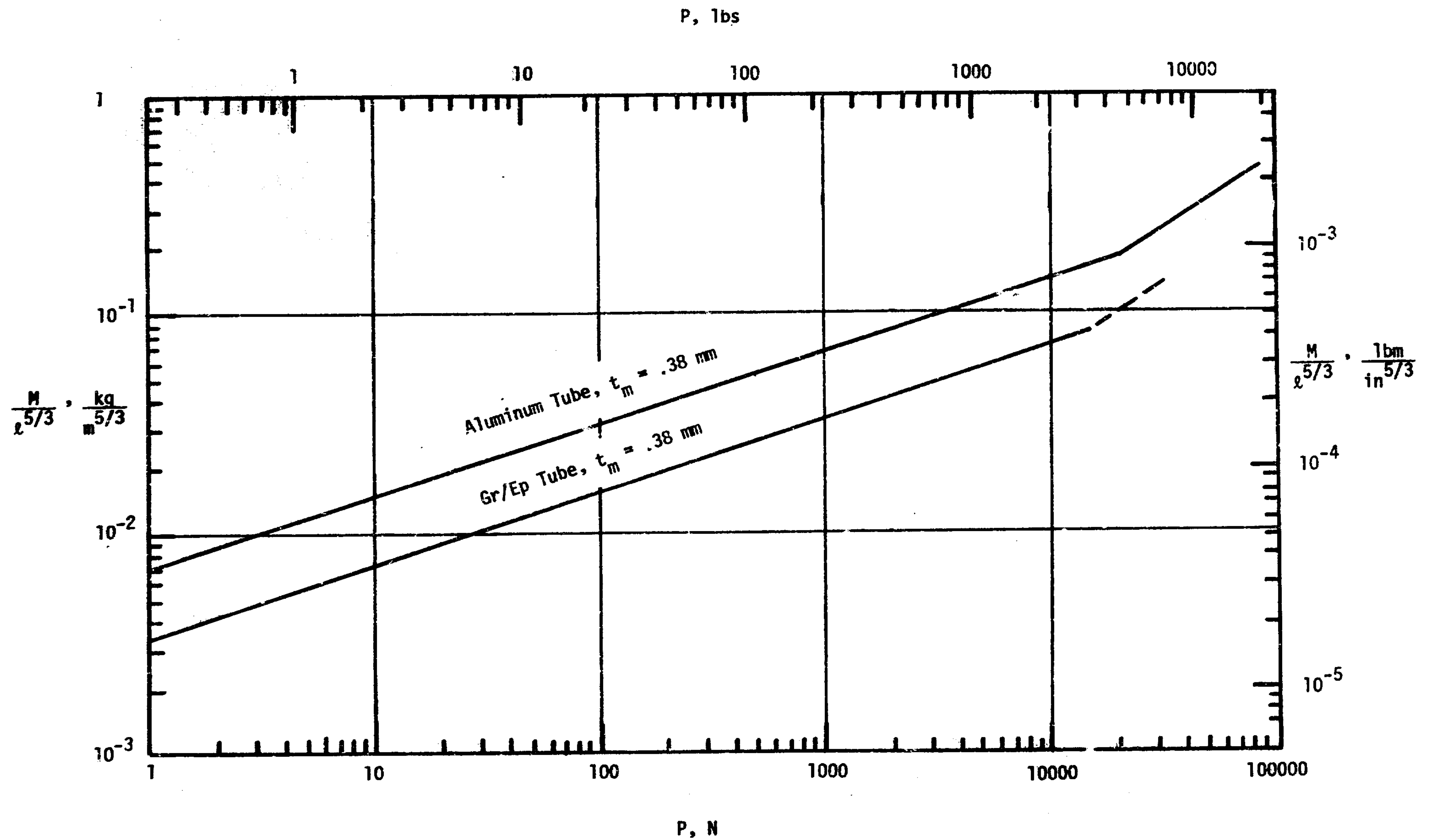


Figure 2.- Mass of lightly loaded, thin walled tubular columns.

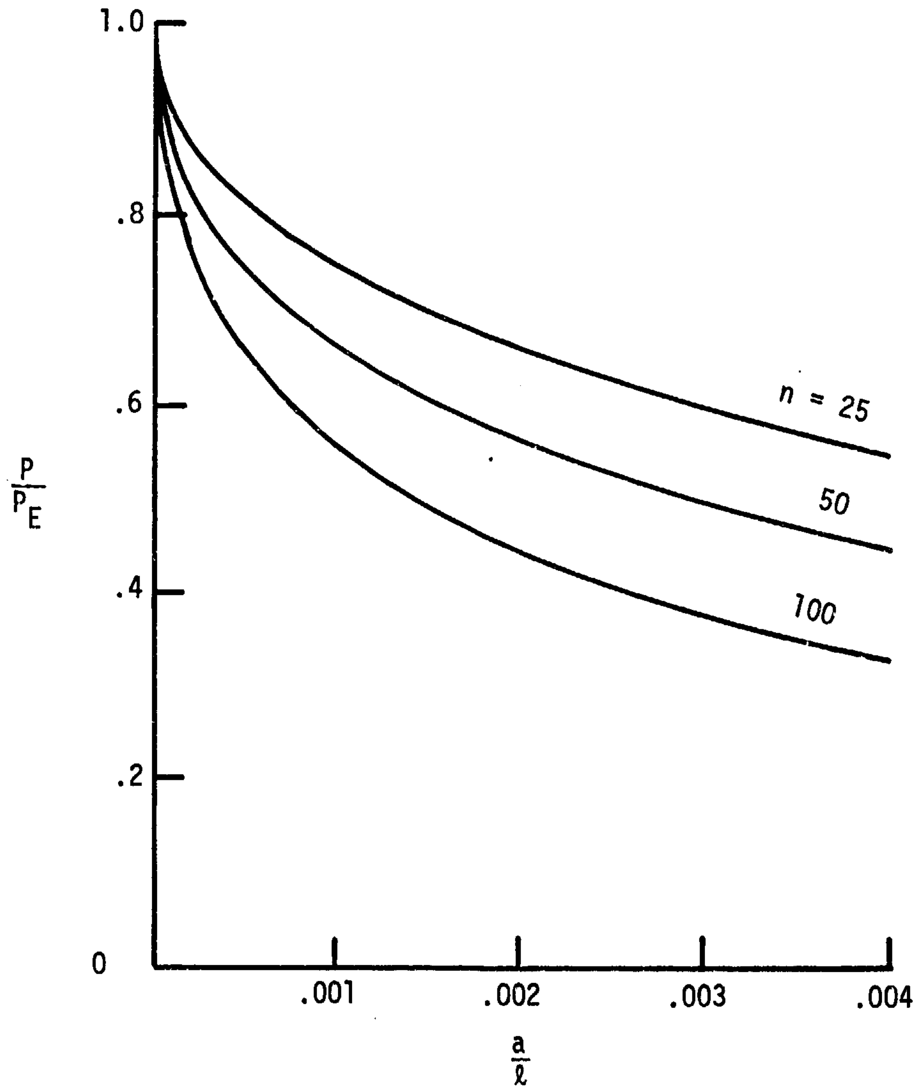


Figure 3.- Reduction in buckling load of tubular longeron truss column as a function of initial imperfection.

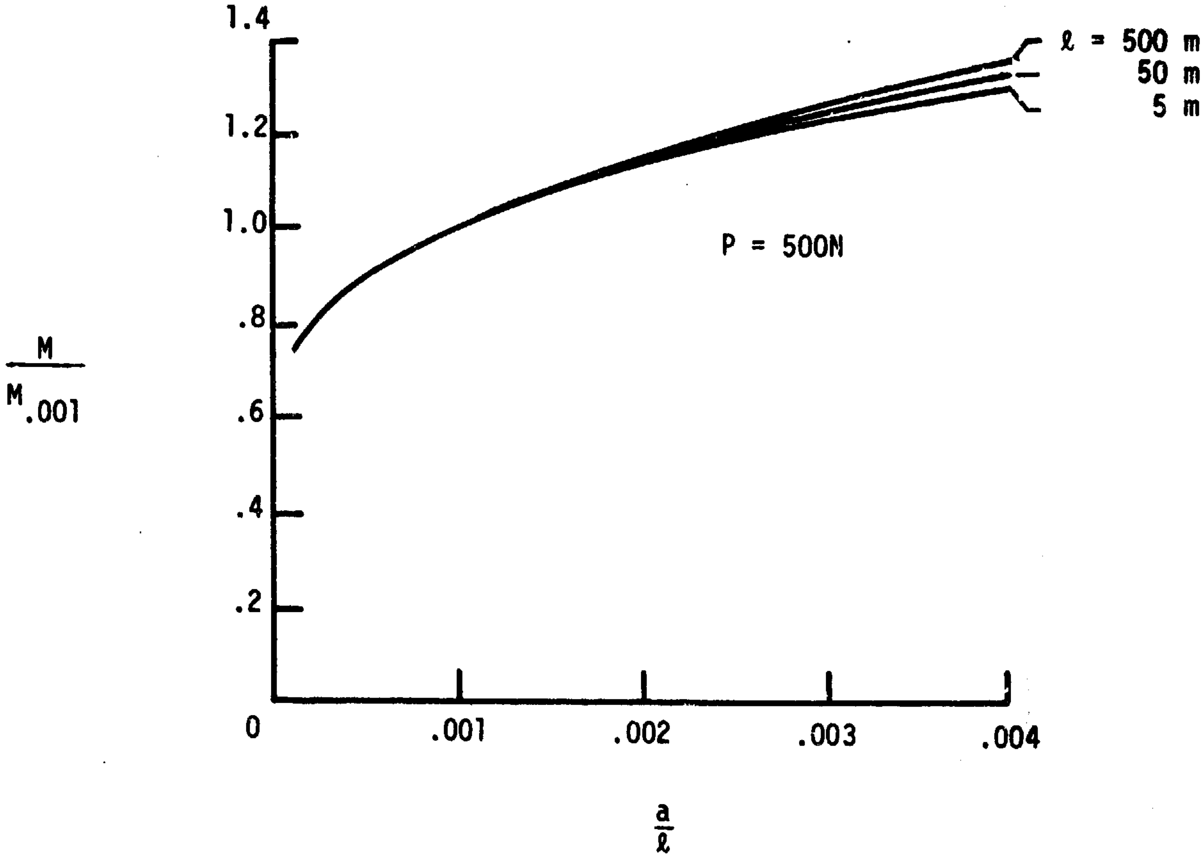


Figure 4.- Change in the mass of a tubular longeron truss column as a function of initial imperfection.

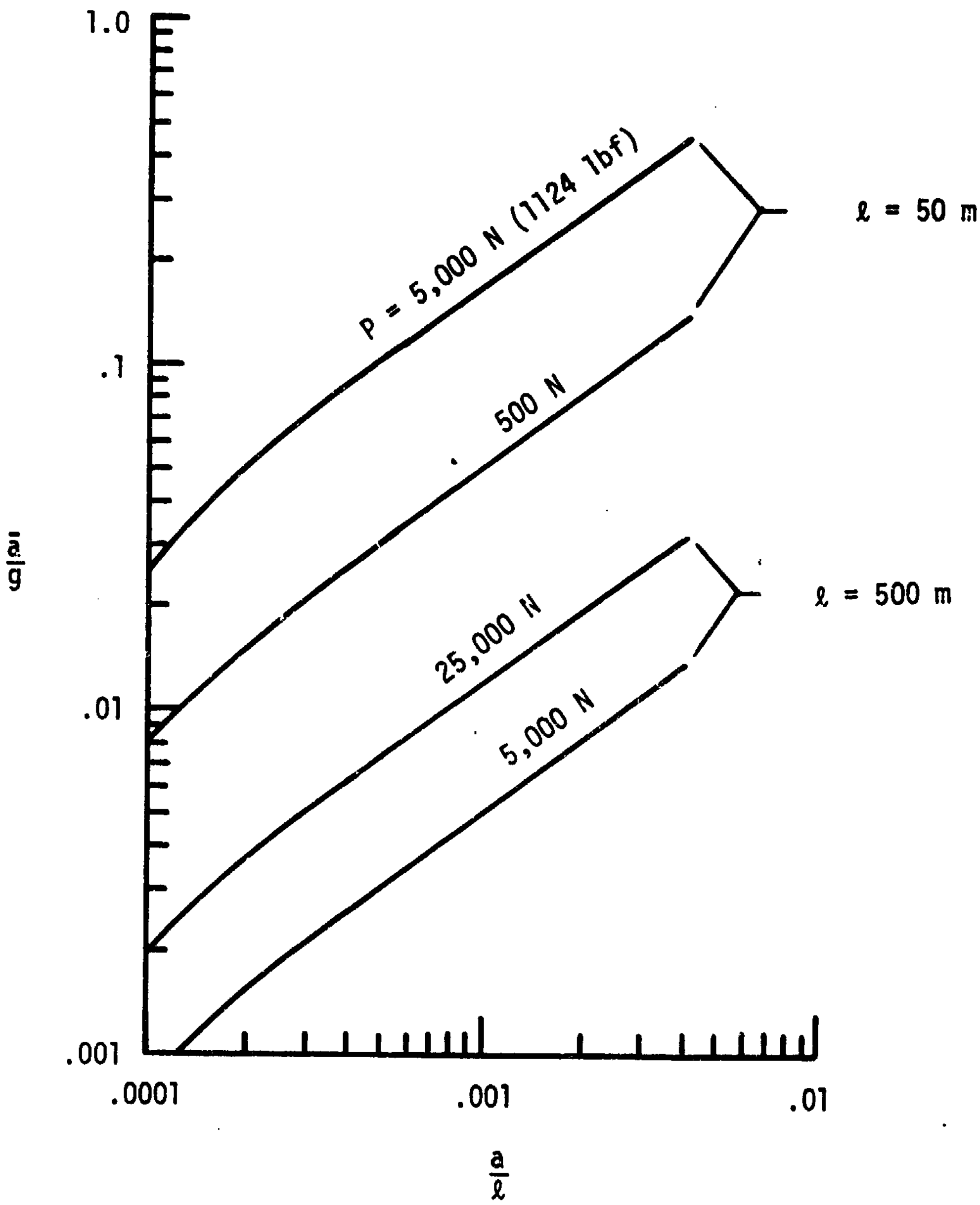


Figure 5.- Lateral acceleration of a tubular longeron truss column required to cause an initial imperfection.

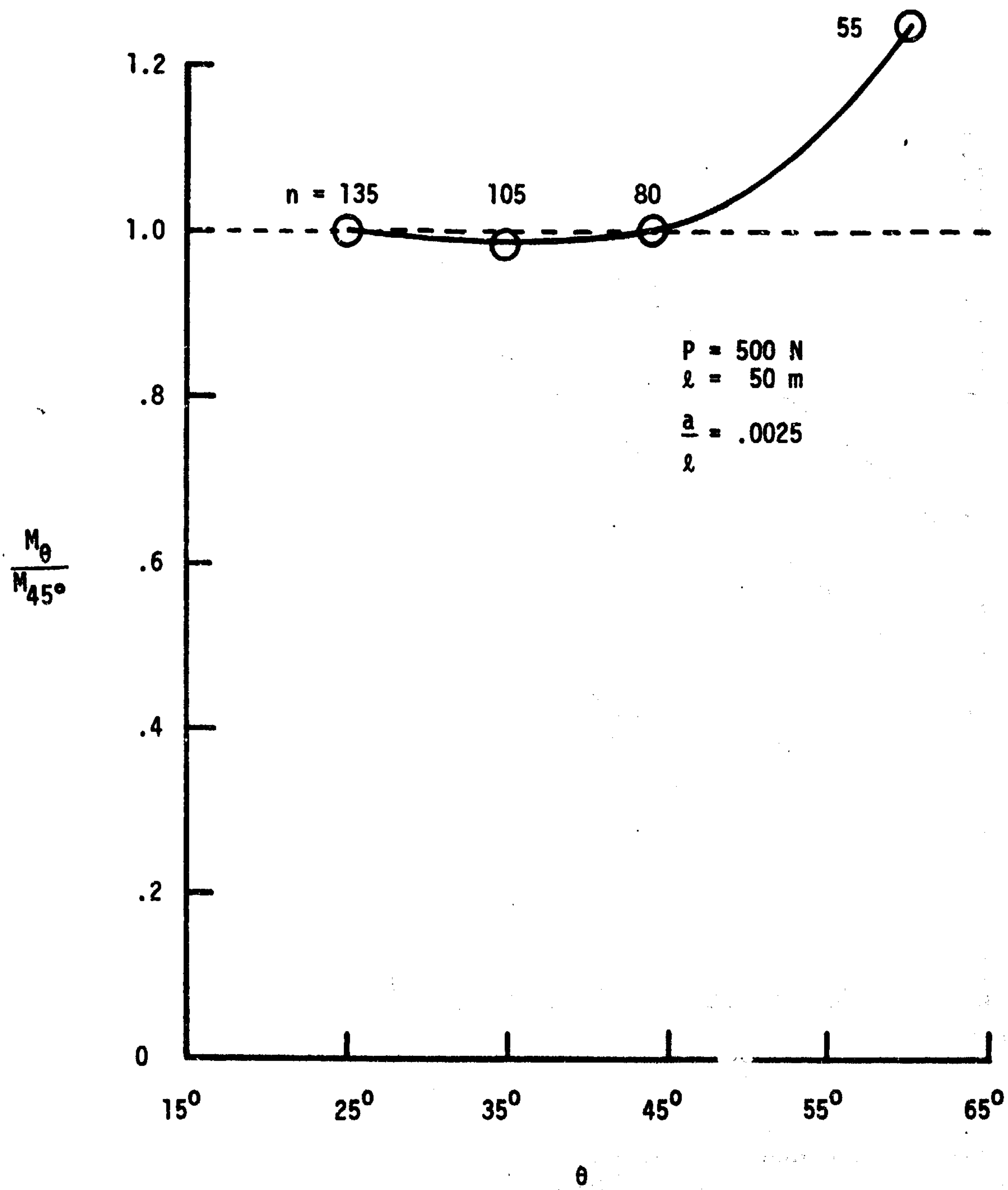


Figure 6.- Variation in the mass of a tubular longeron truss column as a function of diagonal angle for a typical set of design parameters.

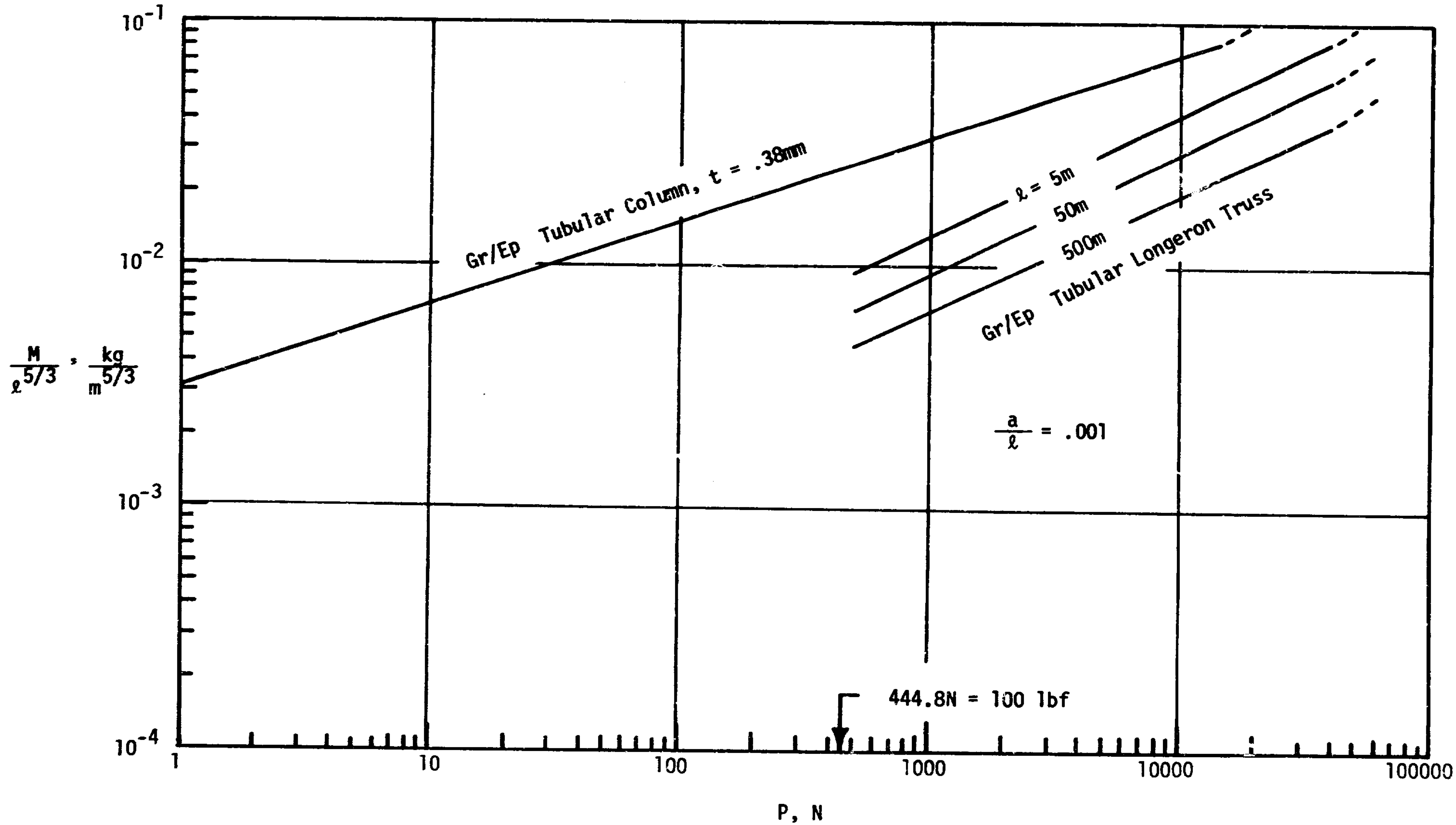


Figure 7.- Mass of graphite/epoxy tubular longeron truss columns as a function of design load.

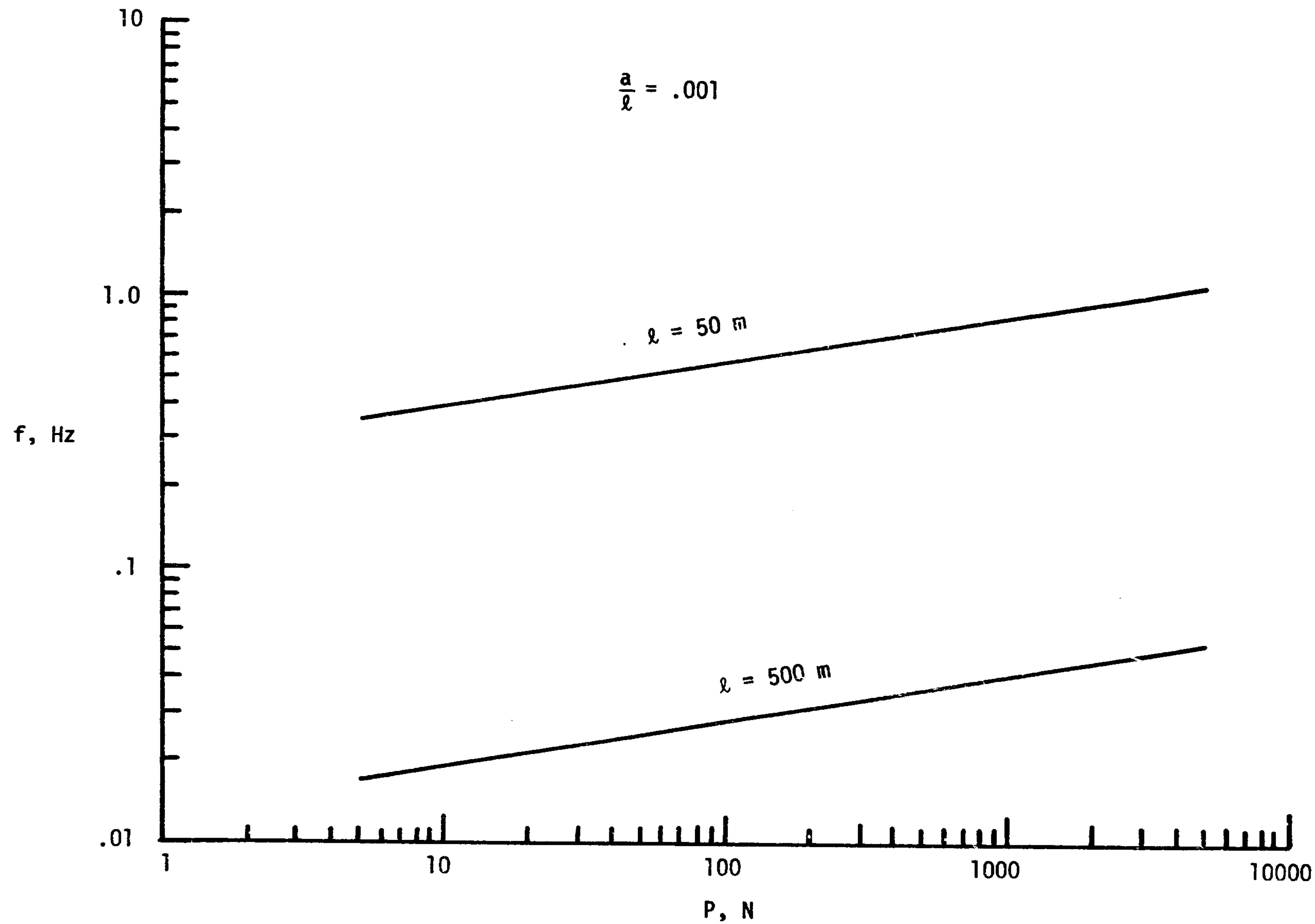


Figure 8.- Lowest natural frequency of tubular longeron truss columns as a function of design load.

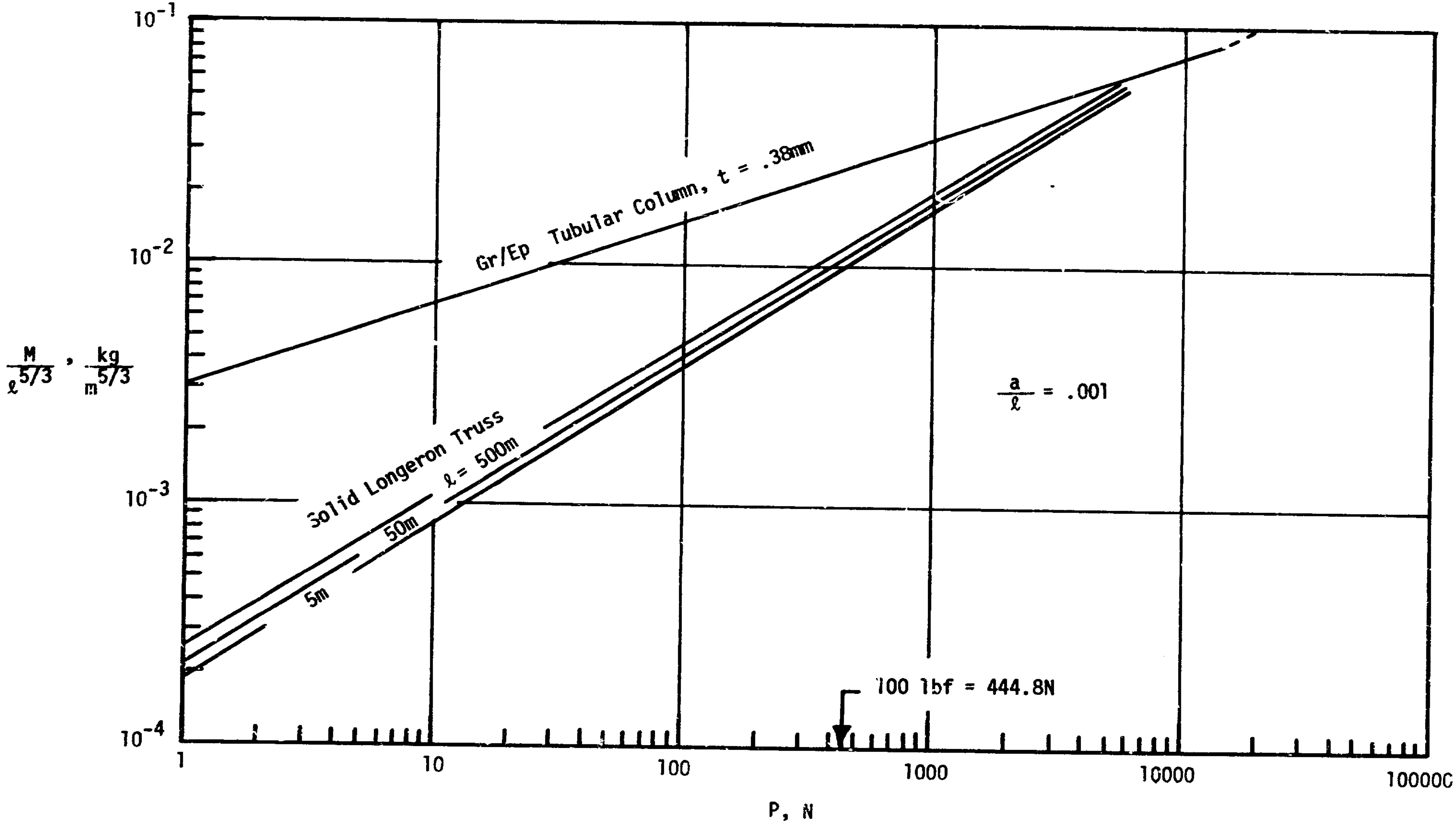


Figure 9.- Mass of graphite/epoxy solid longeron truss columns as a function of design load.

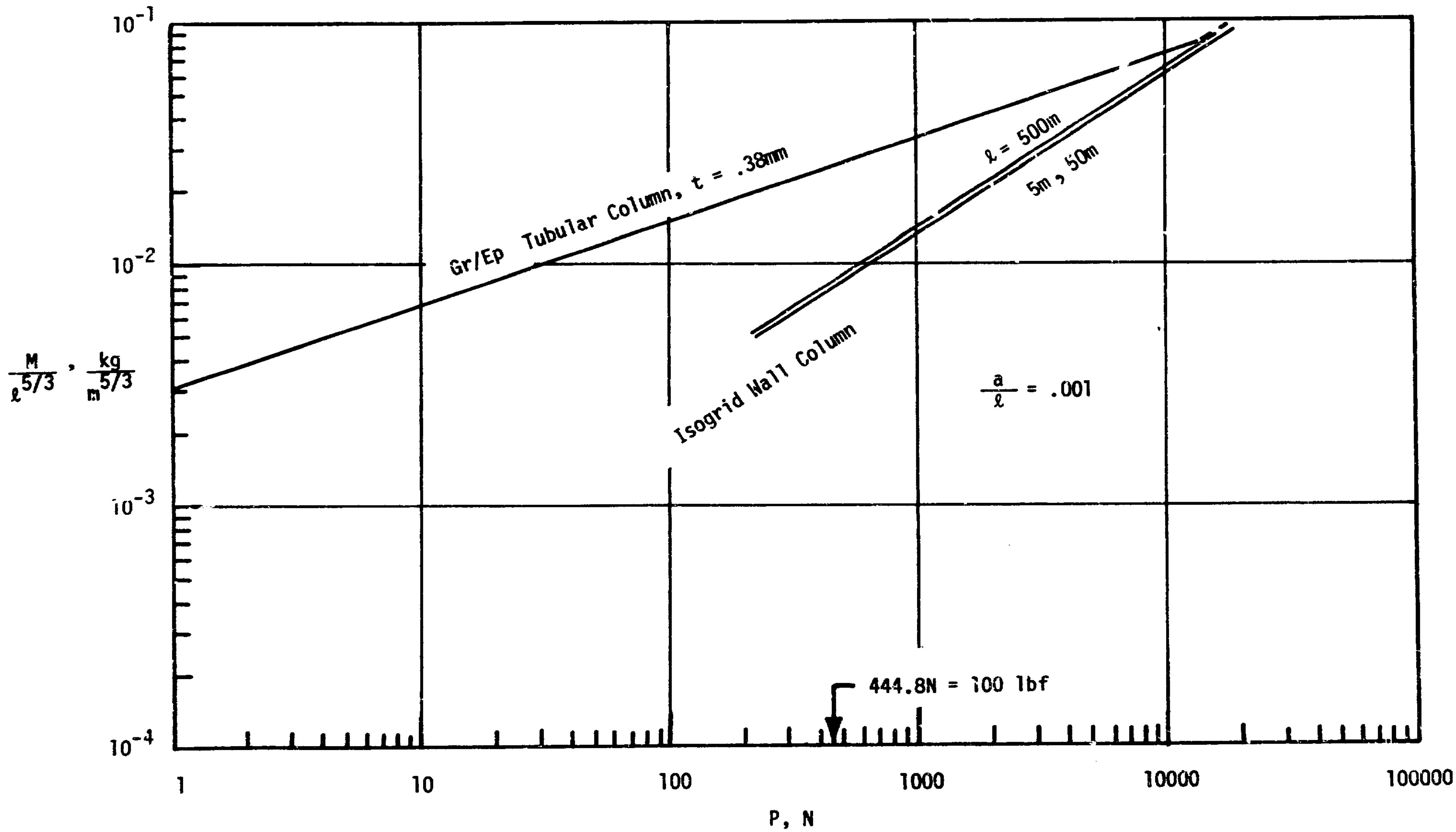


Figure 10.- Mass of graphite/epoxy isogrid wall tubular column as a function of design load.

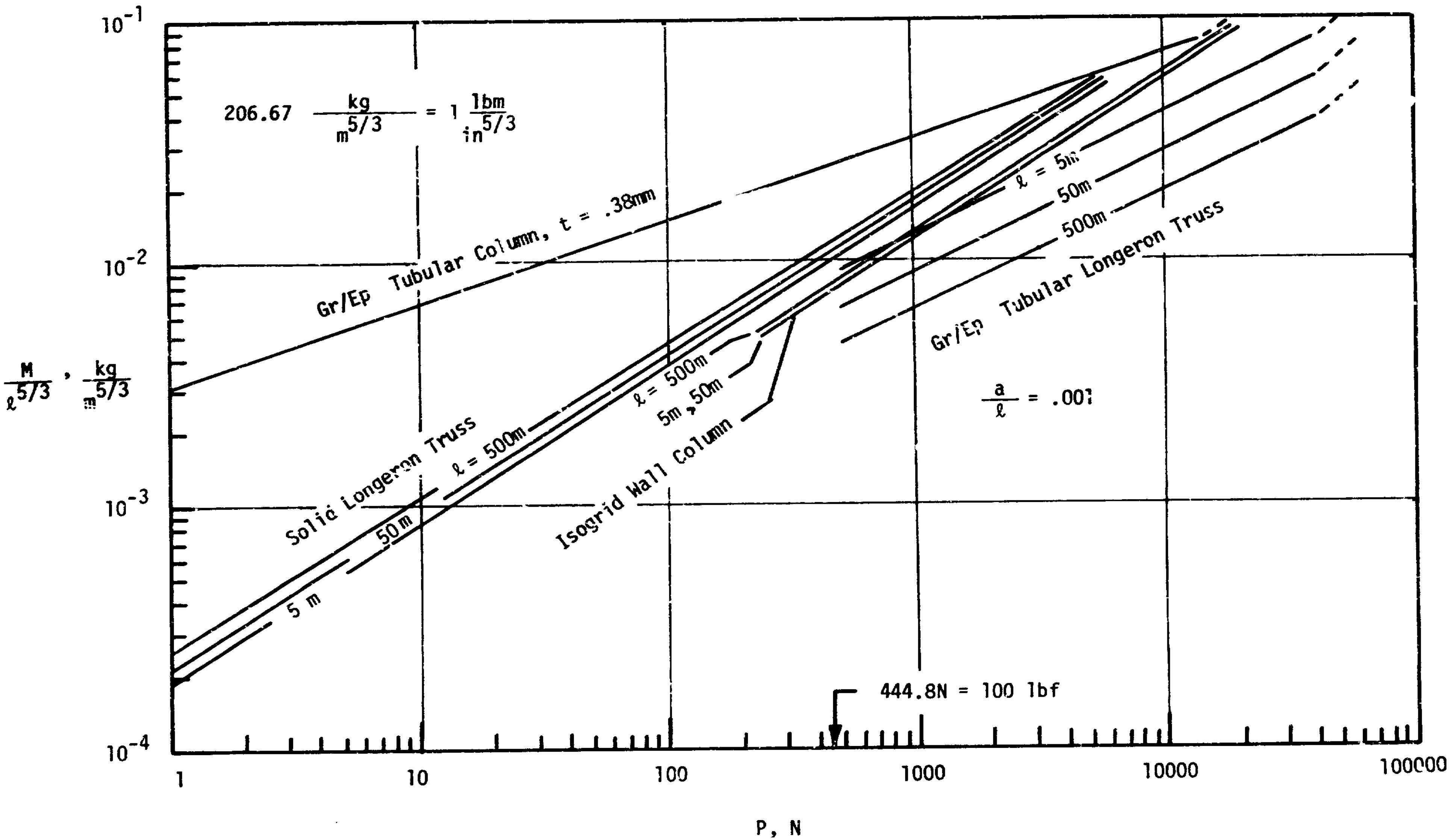


Figure 11.- Masses of several lightly loaded column concepts as a function of design load.