Structural health monitoring of Canton Tower using Bayesian framework

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Abstract. This paper reports the structural health monitoring benchmark study results for the Canton Tower using Bayesian methods. In this study, output-only modal identification and finite element model updating are considered using a given set of structural acceleration measurements and the corresponding ambient conditions of 24 hours. In the first stage, the Bayesian spectral density approach is used for output-only modal identification with the acceleration time histories as the excitation to the tower is unknown. The modal parameters and the associated uncertainty can be estimated through Bayesian inference. Uncertainty quantification is important for determination of statistically significant change of the modal parameters and for weighting assignment in the subsequent stage of model updating. In the second stage, a Bayesian model updating approach is utilized to update the finite element model of the tower. The uncertain stiffness parameters can be obtained by minimizing an objective function that is a weighted sum of the square of the differences (residuals) between the identified modal parameters and the corresponding values of the model. The weightings distinguish the contribution of different residuals with different uncertain levels. They are obtained using the Bayesian spectral density approach in the first stage. Again, uncertainty of the stiffness parameters can be quantified with Bayesian inference. Finally, this Bayesian framework is applied to the 24-hour field measurements to investigate the variation of the modal and stiffness parameters under changing ambient conditions. Results show that the Bayesian framework successfully achieves the goal of the first task of this benchmark study.

Keywords: ambient vibration; Bayesian analysis; high-rise structures; model updating; structural health monitoring; system identification

1. Introduction

Structural health monitoring has received extensive attention in last decades (Doebling *et al.* 1996, Farrar and Doebling 1997, Sohn *et al.* 2003, Brownjohn 2007). The objective is to evaluate the structural integrity and diagnose possible damages in a structure. Modal and stiffness parameters are widely adopted indicators for diagnosis of the structural health conditions (Farrar and Doebling 1997, Brownjohn 2007). In order to provide reliable estimates of these indicators from response measurements, tremendous research effort has been devoted to the development of effective identification methodologies. Meanwhile, various applications were presented to demonstrate the appropriateness and limitations of different approaches (Doebling *et al.* 1996, Sohn *et al.* 2003). In

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recent years, the Asian-Pacific Network of Centers for Research in Smart Structures Technology (ANCRiSST) established a structural health monitoring benchmark study to provide an international platform for comparison among different structural health monitoring algorithms and strategies. The comprehensive structural health monitoring project was developed on the 610 m height Canton Tower (formerly named Guangzhou New Television Tower) (Ni *et al*. 2009, Ni and Zhou 2010, Ni *et al.* 2012). The structural health monitoring system consists of over 700 sensors to capture the field measurements of the structural response as well as the ambient conditions of the operating environment. Some noteworthy references about this structure and the structural health monitoring system include the finite element analysis (Ni *et al.* 2012), modal analysis (Chen *et al.* 2011, Niu *et al.* 2011, Ye *et al.* 2011), vibration control (Ni and Zhou 2010, He *et al.* 2011), and wireless sensing technology (Ni *et al.* 2011).

This paper tackles with the first task of the benchmark study using Bayesian inference. Regarding this task, a set of 24-hour structural acceleration time histories and the corresponding ambient conditions were provided for output-only system identification and model updating. In addition, the mass and stiffness matrices of a 3D reduced model with 185 DOFs were provided as a reference model. This reduced model was obtained by model reduction from a fine 3D finite element model with 122,476 elements with 505,164 degrees of freedom. Detailed descriptions on the modular design of the structural health monitoring system, the task requirements of the benchmark problem and the finite element model can be found in Ni *et al.* (2009), Ni *et al*. (2012) and the official website (http://www.cse.polyu.edu.hk/benchmark/).

Bayesian inference provides a promising and feasible identification solution for the purpose of structural health monitoring (Beck and Katafygiotis 1998, Vanik *et al*. 2000, Ching and Beck 2004, Gaitanaros *et al.* 2010, Yuen 2010a). An attracting feature of Bayesian inference is that not only the optimal estimates can be determined but also their associated uncertainties can be quantified in the form of probability distributions. It provides useful information for assessment of structural health conditions. In this study, the Bayesian spectral density approach (Katafygiotis and Yuen 2001) is applied for output-only modal identification. This frequency-domain method employs the statistical characteristics of the discrete Fourier transform to construct the likelihood function of the modal parameters. Based on the modal identification results, a Bayesian model updating approach is formulated to update the finite element model of the tower. In contrast to the weighted least squares method which usually requires subjective decision on the weightings of the residuals, the Bayesian approach provides rational assignment of such weightings. The structural response of the tower is utilized to demonstrate the effectiveness of the Bayesian framework on modal identification and model updating.

Thereafter, the ambient influence on the modal and stiffness parameters of the tower is examined. Previous long-term structural health monitoring studies revealed that operating conditions may induce substantial effects on the diagnostic parameters (Askegaard and Mossing 1988, Clinton *et al.* 2006, Liu and DeWolf 2007, Zhou *et al*. 2008, Xia *et al*. 2011). In order to conduct reliable assessment on the structural health condition, it is necessary to understand the ambient influence on the structural health indicators (Tamura and Suganuma 1996). Taking the advantage of Bayesian inference, the statistical uncertainty can be quantified and, therefore, can be distinguish from the actual change of the parameters. The result shows that the Bayesian framework successfully achieves the task goals of the benchmark study.

2. Formulation

The Bayesian structural health monitoring framework used in this study is a two-stage approach. It was investigated by Lam *et al*. (2004) for the IASC-ASCE structural health monitoring benchmark problem (Johnson *et al*. 2004). In the first stage, the modal frequencies, damping ratios and mode shapes are identified using the Bayesian spectral density approach (Katafygiotis and Yuen 2001). In the second stage, the modal identification results, including the estimated values and the associated uncertainty, will be utilized to update the finite element model of the structure. In the following sections, these modal identification and model updating methods will be introduced.

2.1 Bayesian spectral density approach for output-only modal identification

Consider a linear dynamical system with N_d degrees of freedom and equation of motion:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{T}_0\boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrix, respectively; and $\boldsymbol{T}_0$ is a force distributing matrix. The external excitation $\boldsymbol{F}$ can modeled as zero-mean Gaussian white noise with spectral intensity matrix $\boldsymbol{S}_F(\omega) = \boldsymbol{S}_{F0}$. Assume that the measurement $\boldsymbol{Y}_N = \{\boldsymbol{y}(n), n = 1, 2, \ldots, N\}$ contains N_o channels of structural response, corrupted by the measurement noise $\boldsymbol{\varepsilon}$

$$\boldsymbol{Y}(n) = \boldsymbol{L}_0\boldsymbol{q}(n) + \boldsymbol{\varepsilon}(n) \tag{2}$$

where $\boldsymbol{Y}(n) \in \mathbb{R}^{N_o}$ is the measurement at the n^{th} time step; $\boldsymbol{q}(n) \in \mathbb{R}^{N_d}$ is the concerned structural response (e.g., displacement or acceleration) at the same time step; and $\boldsymbol{L}_0 \in \mathbb{R}^{N_o \times N_d}$; is the observation matrix comprised of zeros and ones. The measurement noise $\boldsymbol{\varepsilon}$ is modeled as zero-mean discrete Gaussian independent and identical distributed (*i.i.d.*) process with covariance matrix Σ_ε.

The generalized eigenvalue problem of the structure is given by

$$\boldsymbol{K\Phi} = \boldsymbol{M\Phi\Lambda} \tag{3}$$

where the spectral matrix $\Lambda = diag(\omega_1^2, \ldots, \omega_{N_d}^2)$, $\omega_1^2 \le \omega_2^2 \le \ldots \le \omega_{N_d}^2$ contains the squared modal frequencies on its diagonal. The modal matrix $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_{N_d}]$ contains the mode shape vectors.

Assume that there are N_t modes significantly contributing to the structural response and the modal parameters of these modes are to be identified. Use $\boldsymbol{\alpha}$ to denote the uncertain modal parameter vector for identification. It consists of: (i) the modal frequencies ω_m and damping ratios ζ_m of the concerned modes; (ii) the partial mode shapes $\boldsymbol{\Psi}_m$ including only the components at the observed degrees of freedom of the concerned modes. Since the mass matrix is unknown and only some of the degrees of freedom are observed in practical situation, mass normalization cannot be used. Here, each mode shape is normalized such that its component with largest absolute value is unity. Therefore, these normalizing components will be excluded from the uncertain modal parameter vector; and (iii) the upper triangles (diagonal inclusive) of $\boldsymbol{S}_{F0}$ and Σ_ε as symmetry defines the lower triangular part.

To identify the uncertain modal parameter vector α, a discrete estimator of the power spectral density matrix is utilized (Katafygiotis and Yuen 2001)

$$\boldsymbol{S}_{y,N}(\omega_k) = \frac{\Delta t}{2\pi N}\sum_{n,n'=0}^{N-1}\boldsymbol{y}(n)\boldsymbol{y}(n')^T\exp[-i\omega_k(n-n')\Delta t] \tag{4}$$

where Δt is the sampling time step; $\Delta\omega = (2\pi)/N\Delta t$ is the frequency precision in the discrete Fourier transform; and $\omega_k = k\Delta\omega$, $k = 0, 1, \ldots, \mathrm{INT}(N/2)$.

With N_s independent sets of discrete time histories $\boldsymbol{Y} = \{\boldsymbol{Y}_N^{(s)}, s = 1, 2, \ldots, N_s\}$, the averaged spectral density estimator can be obtained

$$\boldsymbol{S}_{y,N}^{avg}(\omega_k) = \frac{1}{N_s}\sum_{s=1}^{N_s}\boldsymbol{S}_{y,N}^{(s)}(\omega_k) \tag{5}$$

where $\boldsymbol{S}_{y,N}^{(s)}(\omega_k)$ can be calculated using Eq. (4) with measurement $\boldsymbol{Y}_N^{(s)}$. Given that $N_s \geq N_o$, the averaged spectral density matrix estimator follows the central complex Wishart distribution with N_s dimension and N_o degrees of freedom (Krishnaiah 1976). With a properly selected frequency range Ξ, the averaged spectral density matrix estimators in $\boldsymbol{S}_{\Xi}^{avg} = \{\boldsymbol{S}_{y,N}^{avg}(\omega_k), \omega_k \in \Xi\}$ are approximately independent (Yuen *et al.* 2002).

Using the Bayes' theorem, the posterior probability density function (PDF) of α given $\boldsymbol{S}_{\Xi}^{avg}$ is (Beck and Katafygiotis 1998)

$$p(\alpha|\boldsymbol{S}_{\Xi}^{avg}) = c_1 p(\alpha)p(\boldsymbol{S}_{\Xi}^{avg}|\alpha) \tag{6}$$

where c_1 is a normalizing constant. The prior PDF $p(\alpha)$ quantifies the prior information of the modal parameters in α. Throughout this study, it is taken as a uniform distribution over the entire possible range of α. Therefore, it can be absorbed into the normalizing constant. The likelihood function $p(\boldsymbol{S}_{\Xi}^{avg}|\alpha)$ is given by product of Wishart distributions (Yuen and Beck 2003)

$$p(\boldsymbol{S}_{\Xi}^{avg}|\alpha) = c_2\prod_{\omega_k\in\Xi}\frac{1}{|E[\boldsymbol{S}_{y,N}(\omega_k)|\alpha]|^{N_s}}exp[-N_s tr(\{E[\boldsymbol{S}_{y,N}(\omega_k)|\alpha]\}^{-1}\boldsymbol{S}_{y,N}^{avg}(\omega_k))] \tag{7}$$

where c_2 is a constant that does not depend on the modal parameters; $E[.|\alpha]$ is the conditional expectation given a particular modal parameter vector α; $|.|$ and $tr(.)$ are the determinant and trace of a matrix, respectively. The optimal modal parameter vector $\hat{\alpha}$ can be determined by maximizing its posterior PDF. However, to provide better computational condition, the optimal modal parameters can be obtained equivalently by minimizing the objective function defined as $J_1(\alpha) = -\ln p(\boldsymbol{S}_{\Xi}^{avg}|\alpha)$. This can be done by using the function "fminsearch" in MATLAB (Matlab 1994). Consequently, the covariance matrix of the modal parameters is given by the inverse of the Hessian of $J_1(\alpha)$ evaluated at $\alpha = \hat{\alpha}$, i.e., $\Sigma_\alpha = [\boldsymbol{H}(\hat{\alpha})]^{-1} \equiv [\nabla J_1(\alpha)\nabla^T|_{\alpha=\hat{\alpha}}]^{-1}$ (Yuen 2010a). With the uniform prior PDF, the elements of $\boldsymbol{H}(\hat{\alpha})$ can be expressed as

$$H^{(r,r')}(\hat{\alpha}) = N_s\left[\sum_{\omega_k\in\Xi}\frac{\partial^2}{\partial\alpha_r\alpha_{r'}}\{\ln|E[\boldsymbol{S}_{y,N}(\omega_k)|\alpha]| + tr(\{E[\boldsymbol{S}_{y,N}(\omega_k)|\alpha]\}^{-1}\boldsymbol{S}_{y,N}^{avg}(\omega_k))\}\right]_{\alpha=\hat{\alpha}} \tag{8}$$

and it can be computed using finite difference. Based on the modal identification results, a Bayesian probabilistic technique will be introduced next for finite element model updating.

2.2 Bayesian model updating using identified modal parameters

Note that only the modal frequencies and mode shapes will be used for finite element model updating. Therefore, the identified target vector is defined to include only the identified modal parameters to be used for model updating: $\hat{\chi} = [\hat{\chi}_1^T, \ldots, \hat{\chi}_{N_m}^T]^T$ where $\hat{\chi}_m = [\hat{\omega}_m, \hat{\Psi}_m^T]^T$, $m = 1,2,\ldots,N_m$. In other words, the identified target vector includes some of the elements in $\hat{\alpha}$. Specifically, it consists of the identified modal frequencies and N_n ($\le N_o$) components of the identified mode shape vector for N_m ($\le N_t$) modes. The covariance matrix of the identified target vector is denoted as Σ_χ and it is a submatrix of Σ_α (with appropriate rearrangement of the rows and columns) obtained from the previous stage of modal identification.

The structural model includes a prescribed mass matrix $\boldsymbol{M}$ and an uncertain stiffness matrix $\boldsymbol{K}(\boldsymbol{\theta})$ governed by the uncertain stiffness parameter vector $\boldsymbol{\theta} = [\theta_1, \theta_1, \ldots, \theta_{N_\theta}]^T$ (Yuen and Katafygiotis 2006). By solving the generalized eigenvalue problem of the dynamical model $(\boldsymbol{M}, \boldsymbol{K}(\boldsymbol{\theta}))$ in Eq. (3), the modal frequencies and mode shape vectors can be computed for a given stiffness parameter vector $\boldsymbol{\theta}$. Then, the target vector $\chi(\boldsymbol{\theta})$ of a given structural model can be obtained. The residual vector $\hat{\chi} - \chi(\boldsymbol{\theta})$ follows the zero-mean Gaussian distribution with covariance matrix Σ_χ.

Using the Bayes' theorem, the posterior PDF $\boldsymbol{\theta}$ of given the identified target vector can be readily obtained (Beck and Katafygiotis 1998)

$$p(\boldsymbol{\theta}|\hat{\chi}) = c_3 p(\boldsymbol{\theta}) p(\hat{\chi}|\boldsymbol{\theta}) \tag{9}$$

where c_3 is a normalizing constant. The prior PDF $p(\boldsymbol{\theta})$ quantifies the prior information of the stiffness parameters in $\boldsymbol{\theta}$ and a uniform distribution is used throughout this study. The likelihood function $p(\hat{\chi}|\boldsymbol{\theta})$ can be expressed as

$$p(\hat{\chi}|\boldsymbol{\theta}) = (2\pi)^{-\frac{N_m N_n}{2}} |\Sigma_\chi|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}[\hat{\chi} - \chi(\boldsymbol{\theta})]^T \Sigma_\chi^{-1} [\hat{\chi} - \chi(\boldsymbol{\theta})]\right\} \tag{10}$$

Again, instead of maximizing the posterior PDF, one can equivalently minimize the objective function $J_2(\boldsymbol{\theta}) = -\ln p\,(\hat{\chi}|\boldsymbol{\theta})$ to obtain the optimal stiffness parameter vector $\hat{\boldsymbol{\theta}}$. This can be done by using the function "fminsearch" in MATLAB (Matlab 1994). The associated covariance matrix is given by the inverse of the Hessian of $J_2(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$, i.e., $\Sigma_\theta = [\boldsymbol{H}(\hat{\boldsymbol{\theta}})]^{-1} \equiv [\nabla J_2(\boldsymbol{\theta}) \nabla^T|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}}]^{-1}$. It is worth noting that minimization of the objective function $J_2(\boldsymbol{\theta})$ is equivalent to the weighted least-squares solution for a diagonal Σ_χ. The Bayesian inference provides a rational assignment of the weightings. The weightings are inversely proportional to the posterior variance obtained from the first stage. Therefore, residuals (differences between the identified target vector and its corresponding value of a structural model) will be given higher weightings if they are associated with smaller posterior uncertainty. Moreover, by using the Bayesian influence (Box and Tiao 1992), the parametric uncertainty can be quantified. This provides valuable information for further judgment of the structural health condition (Vanik *et al.* 2000, Yuen and Kuok 2011) and it will be further discussed in the subsequent sections.

3. Application to the structural health monitoring benchmark study of Canton Tower

3.1 Background information

A structural health monitoring benchmark study was established and it was based on the full-scale field measurement of the 610 m tall Canton Tower (Ni *et al*. 2009, Ni *et al.* 2012). Twenty uni-axial accelerometers were installed in various locations of the tower to measure the structural response. In the first task of this benchmark study, 24-hour field measurements of the structural acceleration time histories and the corresponding ambient conditions (temperature and wind properties) were provided. In addition, the mass and stiffness matrix of a reduced 3D beam model with 185 degrees of freedom were given as a reference model for finite element model updating. This reduced model was constructed using model reduction from a full model of 122,476 elements with 505,164 degrees of freedom obtained by using ANSYS. Detailed descriptions on the modular design of the structural health monitoring system, the task requirements of the benchmark problem and the finite element models can be found in Ni *et al.* (2009), Ni *et al.* (2012) and the official website (http://www.cse.polyu.edu.hk/benchmark/). In this section, we first present detailed results of modal identification and model updating using the acceleration measurements of the first hour (obtained from 18:00 to 19:00 on 20th January 2010). Then, correlation between the modal/stiffness parameters and the ambient conditions throughout the entire duration of 24 hours will be presented in Section 4.

3.2 Modal identification with the field measurements of Canton Tower

In this section, we report the results using the first hour of acceleration response measurement. It is partitioned into six subsets of 10 minutes (i.e., $N_s = 6$) in order to obtain the averaged spectral density matrix estimator. With a 50 Hz sampling frequency, the number of data point N in each subset is equal to 30,000. The diagonal elements of the averaged spectral density matrix estimators

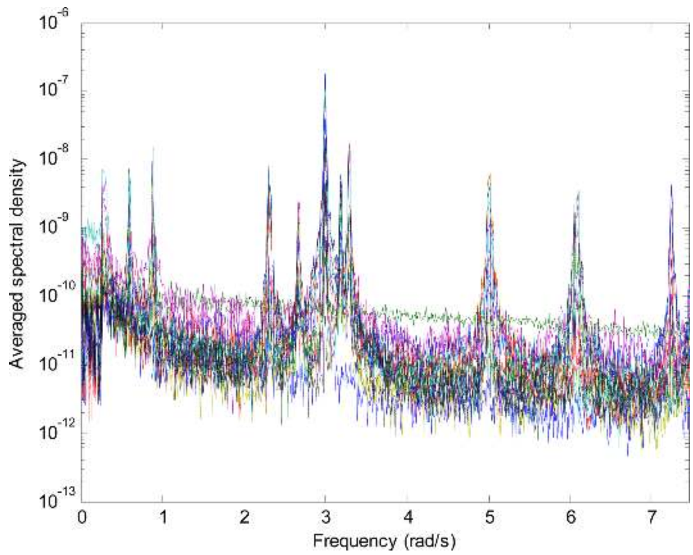


Fig. 1 Averaged response spectra of the 20 channels acceleration measurements at the first hour (from 18:00 to 19:00 on 20th January, 2010)

Table 1 Identification result of the modal frequencies and damping ratios

Para. \ Mode	1	2	3	4	5	6	7	8	9	10
$\hat{\omega}_m$ (rad/s)	0.5984	0.8796	2.3110	2.6735	2.9946	3.1875	3.2918	5.0071	6.0765	7.2395
σ_{ω_m} (rad/s)	0.0014	0.0011	0.0015	0.0015	0.0013	0.0017	0.0014	0.0018	0.0019	0.0017
COV	0.0024	0.0012	0.0007	0.0006	0.0004	0.0005	0.0005	0.0004	0.0003	0.0002
$\hat{\zeta}_m$ (%)	1.0323	0.5015	0.3519	0.2457	0.1939	0.1587	0.1418	0.1966	0.1933	0.1080
σ_{ζ_m}	0.2338	0.1058	0.0710	0.0597	0.0394	0.0610	0.0438	0.0359	0.0315	0.0233
COV	0.2265	0.2110	0.2018	0.2430	0.2032	0.3844	0.3089	0.1826	0.1630	0.2157

of the 20-channel of acceleration response are shown in Fig. 1. Since the first peak is not stable, it is not considered as a mode (Chen *et al*. 2011).

The identification results of the modal frequencies and damping ratios of the first ten modes (i.e., $N_t = 10$) are shown in Table 1. It lists the identified values, the posterior standard derivations and the coefficients of variation (COV). It is found that the posterior standard derivations are in acceptable range. The COVs of all the identified modal frequencies are less than 0.25%. They are sufficiently for detection of any notable changes of the modal frequencies. On the other hand, it is not surprising to obtain large posterior uncertainty for the damping ratios and the COV values are around 20% in most cases. However, this will not affect the model updating results as the damping ratios will not participate in this process.

The structural health monitoring benchmark study of Canton Tower serves as a platform to compare the performance of different methods. Herein, the identified modal frequencies are compared with the results of three references including Chen *et al*. (2011), Niu *et al*. (2011), and the 3D finite element full model with ANSYS (http://www.cse.polyu.edu.hk/benchmark/). Chen *et al*. (2011) considered the structural behavior of the tower under different excitation conditions. The modal parameters obtained with the enhanced frequency domain decomposition method under the ambient excitation conditions are taken for the comparison in this paper. Niu *et al*. (2011) presented the averaged modal parameters calculated from the entire set of 24-hour measurement with two modal identification methods. Here, we will compare our results with the ones obtained from the vector auto-regressive method. Finally, the modal frequencies of the ANSYS full model provided in the benchmark study are also considered. The comparison is summarized in Fig. 2. For the identification results of the Bayesian approach, error bars with three posterior standard derivations ($\pm 3\sigma$) confidence intervals are also provided in the figure. However, since the standard derivations of the estimated modal frequencies are very small (as shown in Table 1), the error bars almost overlap with the optimal values. The identification results show good agreement with Chen *et al*. (2011) and Niu *et al*. (2011). Nevertheless, it is not surprising to observe discrepancies with the ANSYS finite element model because the latter does not incorporate information from the measurements.

In the same fashion as Fig. 2, Fig. 3 shows the identified damping ratios by the three aforementioned modal identification methods. The error bars indicate substantially larger posterior uncertainty than those with the modal frequencies. The comparison shows that the identification result of the Bayesian approach is relatively closer to the result in Niu *et al*. (2011). It is found that the damping ratios obtained in Chen *et al*. (2011) are the largest in general.

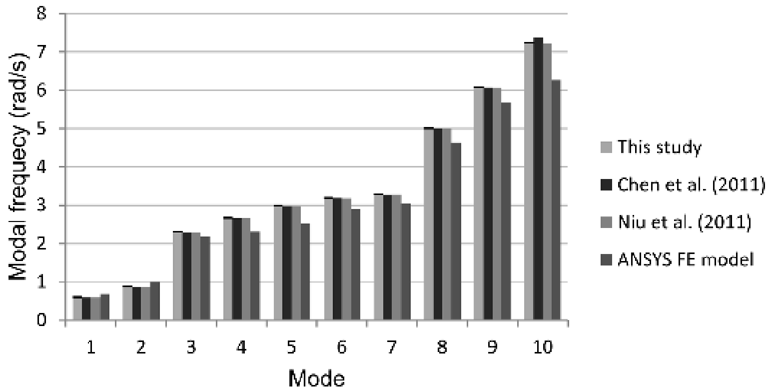


Fig. 2 Comparison of the identified modal frequencies

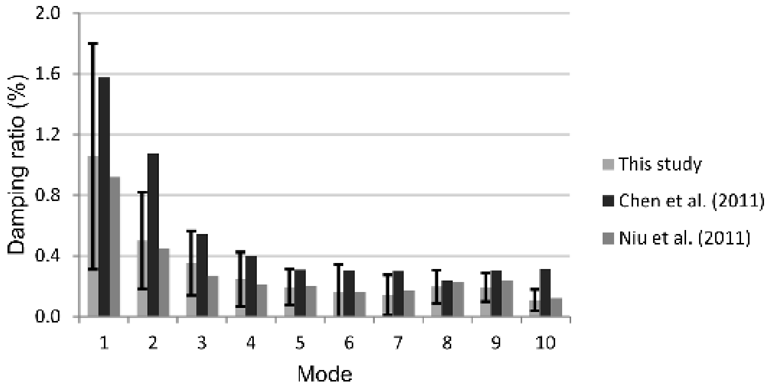
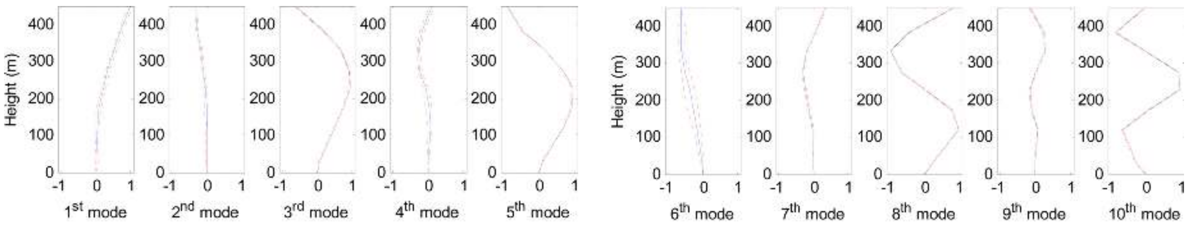


Fig. 3 Comparison of the identified damping ratios

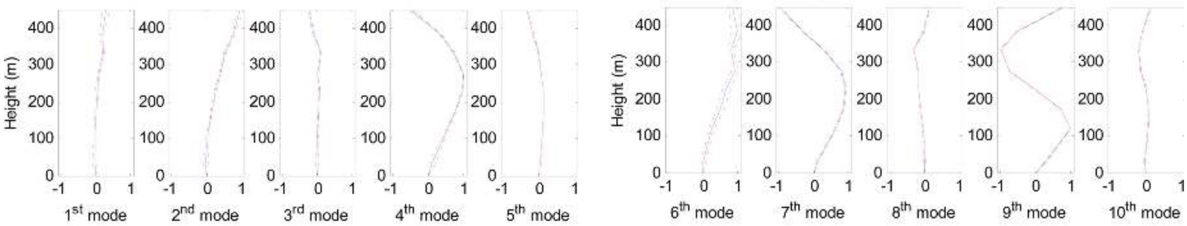
Figs. 4(a) and (b) shows the identified mode shapes of the direction along the short-axis and the long-axis, respectively. Again, the dashed lines indicate the associated $\pm 3\sigma$ confidence intervals. The confidence intervals are sufficiently narrow so that the identified mode shapes are reliable for model updating. Based on the modal identification results, the model updating can be proceeded and results will be presented next.

3.3 Model updating using the identified modal parameters

In this benchmark study, a 3D finite element reduced model was provided as a reference model for model updating (http://www.cse.polyu.edu.hk/benchmark/, Chen *et al*. 2011). This model was obtained by idealizing the tower as a cantilever beam with 37 beam elements and 38 nodes (including a fixed node). For each node, there are 5 degrees of freedom, in which two are translational in the horizontal directions and three are rotational (two flexural and one torsional). As a result, there are 185 degrees of freedom for the entire structure. The mass matrix $\boldsymbol{M} \in \mathbb{R}^{185 \times 185}$ is assumed fixed and it will not be updated. The stiffness matrix $\boldsymbol{K}(\theta) \in \mathbb{R}^{185 \times 185}$ is parameterized with two stiffness parameters in this study, i.e., $N_\theta = 2$. The first refers to translational effect and the second refers to rotational effects. The elements of the stiffness matrix can be expressed as follows



(a) Short-axis direction



(b) Long-axis direction

Fig. 4 Identified mode shape with $\pm 3\sigma$ confidence intervals (a) short-axis direction and (b) long-axis direction

$$K_{ij}(\boldsymbol{\theta}) = \begin{cases} \theta_1 K_{ij}^{\eta}, & i,j = 1,2(\mathrm{mod}\ \ 5) \\ \theta_2 K_{ij}^{\eta}, & i,j = 3,4,5(\mathrm{mod}\ \ 5) \\ \sqrt{\theta_1\theta_2} K_{ij}^{\eta}, & \text{otherwise} \end{cases} \tag{11}$$

where K_{ij}^{η} is the (i,j) element of the nominal stiffness matrix. The first group associates with the translational degrees of freedom while the second group associates with the rotational degrees of freedom. The cross terms are multiplied with $\sqrt{\theta_1\theta_2}$ to ensure positive definiteness of the stiffness matrix.

The Bayesian model updating approach is applied to estimate the stiffness parameters. For this purpose, the modal identification results of the first four modes are utilized. As shown in the pilot study by Ni *et al.* (2012), the first four modes correspond to the first bending mode of the short axis, the first bending mode of the long axis, the second bending mode of the short axis and the second bending mode of the long axis, respectively. The identified target vector for model updating is $\hat{\chi} = [\hat{\chi}_1^T, \hat{\chi}_2^T, \hat{\chi}_3^T, \hat{\chi}_4^T]^T$ where $\hat{\chi}_i = [\hat{\omega}_m, \hat{\psi}_m^T]^T$, $m = 1,\ldots,4$. The partial mode shape vectors of the first and third modes ($\hat{\psi}_1$ and $\hat{\psi}_3$) consist of the mode shape components of channel 1, 3, 8, 13, 15 and 18. These degrees of freedom are chosen because their direction aligns with the short axis. By the same argument, $\hat{\psi}_2$ and $\hat{\psi}_4$ consist of the mode shape components of channel 2, 4, 9, 14, 16 and 20.

Table 2 shows the identified stiffness parameters, the posterior standard derivations and the corresponding coefficients of variation (COV). The identified stiffness parameter vector is $\hat{\boldsymbol{\theta}}$ =

Table 2 Estimation result of the stiffness parameters

	$\hat{\theta}$	σ_θ	COV
θ_1	0.9908	0.0027	0.0027
θ_2	0.5920	0.0029	0.0050

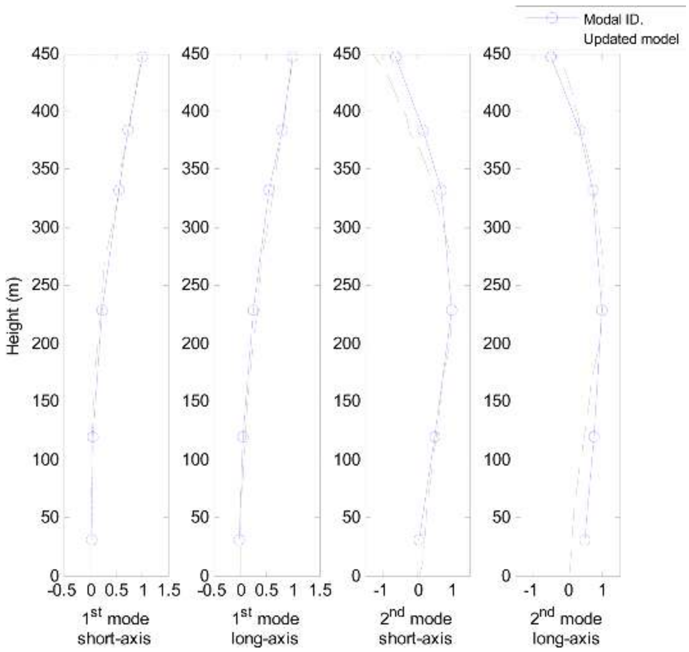


Fig. 5 Comparison of the identified and estimated mode shape components

$[0.9908, 0.5920]^T$. Fig. 5 shows the comparison of the identified mode shapes and the corresponding values from the updated model. It is found that the identified mode shapes can be well fitted. The accuracy can be evaluated by the partial modal assurance criterion (*pMAC*) (Heylen and Janter 1990)

$$pMAC_m = \frac{(\Sigma_{i=1}^{N_n} \hat{\psi}_{m,i} \psi_{m,i})^2}{\Sigma_{i=1}^{N_n} \hat{\psi}_{m,i}^2 \Sigma_{i=1}^{N_n} \psi_{m,i}^2} \tag{12}$$

where $m = 1, \ldots, 4$; $\hat{\psi}_{m,i}$ and $\psi_{m,i}$ are the identified mode shape component of the i^{th} channel of the m^{th} mode and its corresponding *pMAC* value of the updated model, respectively. If the identified mode shape is perfectly fitted, the corresponding value will be equal to unity. It turns out that the values of the first four modes are 0.9987, 0.9921, 0.7643 and 0.8517. It is found that all *pMAC* values are larger than 0.75 and the values of the first two modes are very close to one.

Table 3 shows the comparison of the identified modal frequencies and the corresponding values of the updated finite element model. It is found that the percentage of difference between the identified and estimated modal frequencies of the first four modes is less than 15% while that of the latter six

Table 3 Identified modal frequencies and the corresponding values of the model

Modal freq.	1	2	3	4	5	6	7	8	9	10
Identified (rad/s)	0.5984	0.8796	2.3110	2.6735	2.9946	3.1875	3.2918	5.0071	6.0765	7.2395
Estimated (rad/s)	0.6787	0.9910	2.0743	2.2914	2.3471	2.5229	2.9694	4.4006	5.4749	5.7523
Difference (%)	13.4	12.7	10.2	14.3	21.6	20.9	9.8	12.1	9.9	20.5

modes is less than 22%. In this study, a simple model with two stiffness parameters is utilized for the model updating. In order to improve the fitting capability, a model with adjustable stiffness parameters is needed.

It is realized that understanding the physical behavior of the underlying system is a key for construction of a proper model. A preferable model should possess good balance between the data fitting capability and robustness. Selection of a realistic and applicable model to describe the structural behavior is a challenging topic (Beck and Yuen 2004, Beck 2010, Yuen 2010b). Furthermore, an appropriate dynamical model can serve as a baseline model and it is essential for structural health monitoring.

4. Ambient influence on the modal and stiffness parameters

The Bayesian modal identification and model updating framework is applied to the 24-hour field measurements of the tower to study the ambient influence on the structural behavior. This issue was first addressed in Chen *et al*. (2011). In particular, we will investigate the effects of ambient temperature and wind speed on the modal frequencies and damping ratios of the first ten modes and the stiffness parameters of the finite element model. It is worth emphasizing again that Bayesian inference allows for uncertainty quantification of the identified modal and stiffness parameters (Box and Tiao 1992). This is important to distinguish statistical uncertainty from any actual change of the parameters.

4.1 Ambient effects on modal frequencies and damping ratios

Fig. 6 depicts the time histories of the ambient temperature, the 10-minute averaged wind speed, and wind direction. The ambient temperature varied in the range from 14.7^{o}C to 18.3^{o}C. Throughout the 24-hour monitoring period, the wind excitation remained in calm condition. The 10-minute averaged wind speed was mostly between 1 m/s and 4 m/s, and the wind direction covered a range

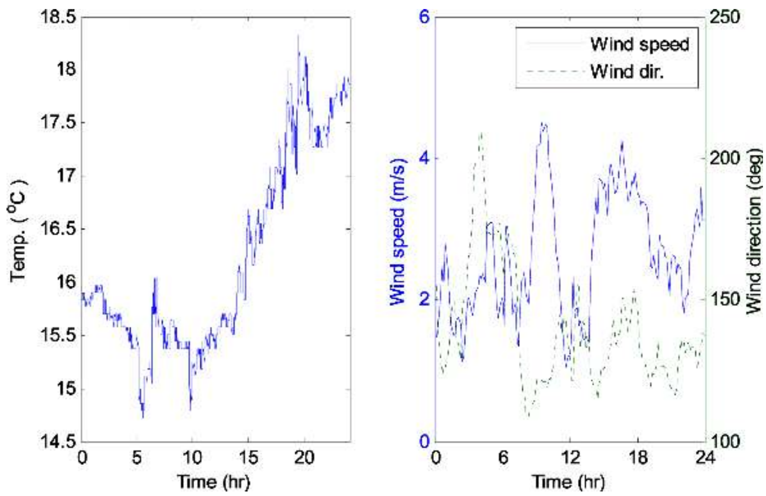


Fig. 6 Ambient temperature and wind characteristics

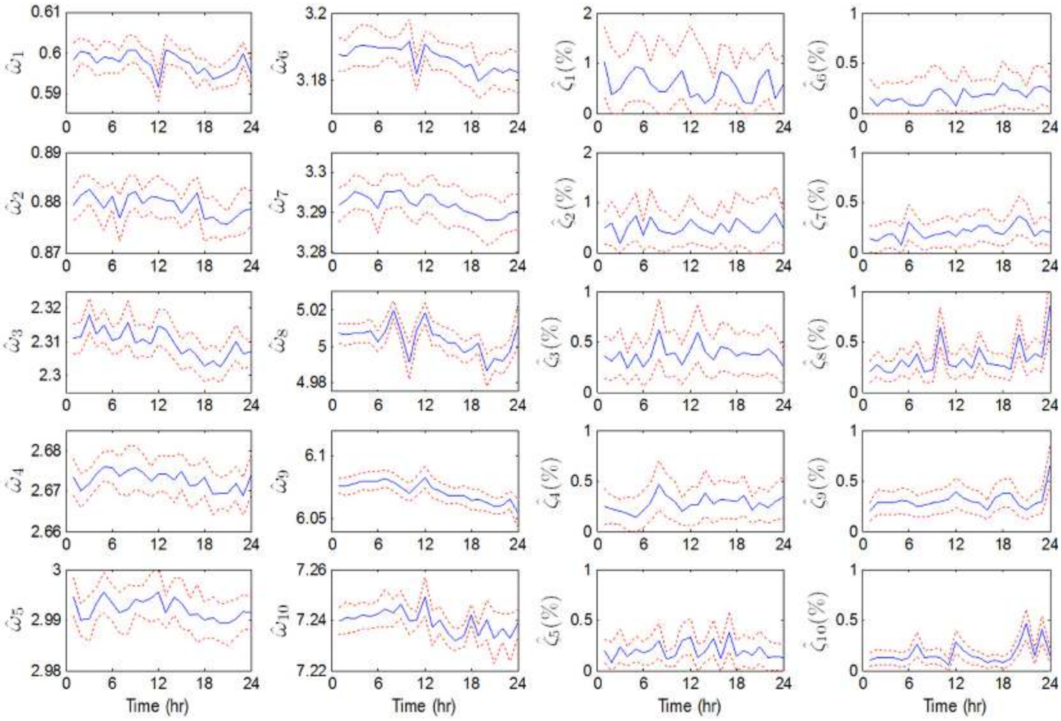


Fig. 7 Variation of the identified modal frequencies and damping ratios

of 100°. Fig. 7 shows the variation of the identified modal frequencies and damping ratios with their $\pm 3\sigma$ confidence intervals. For the modal frequencies, fluctuation can be observed for all ten modes. However, the largest change, associated with the first mode, was only 1.6% between its maximum and minimum. Comparing with the uncertainty level indicated by the confidence intervals, there is no evidence to conclude any actual change of the modal frequencies. On the other hand, the fluctuation of the damping ratios was much more severe. The range of the identified values was between 0.05% and 1.03%. Again, it is not surprising to observe that the COVs of the damping ratios were much larger than those of the modal frequencies. The considerable estimation uncertainty of the damping ratios can be caused by complicated energy dissipation mechanisms of structures (Jeary 1986, Kareem and Gurley 1996). Therefore, it is difficult to draw reliable conclusion on the relationship between the identified damping ratios and the ambient conditions.

Fig. 8 illustrates the relationship between the ambient temperature and the modal parameters. It reveals a gently decreasing trend of the modal frequencies versus the ambient temperature. This is generally observed from cantilever beam-like structures. When the temperature increases, the length of the structure increases and the Young's modulus of the materials decreases. As a result, the modal frequencies will decrease. Nevertheless, the resultant changes of modal frequencies are very small because the monitoring period covered a range of only 3.6°C. For the damping ratios, the identified values are highly scattering and no distinct trend can be observed.

In the same fashion as Fig. 8, Fig. 9 shows the relationship between the wind speed and the modal parameters. For both the modal frequencies and damping ratios, no significant trend can be concluded with the wind speed. This is due to the fact that the wind was calm throughout the

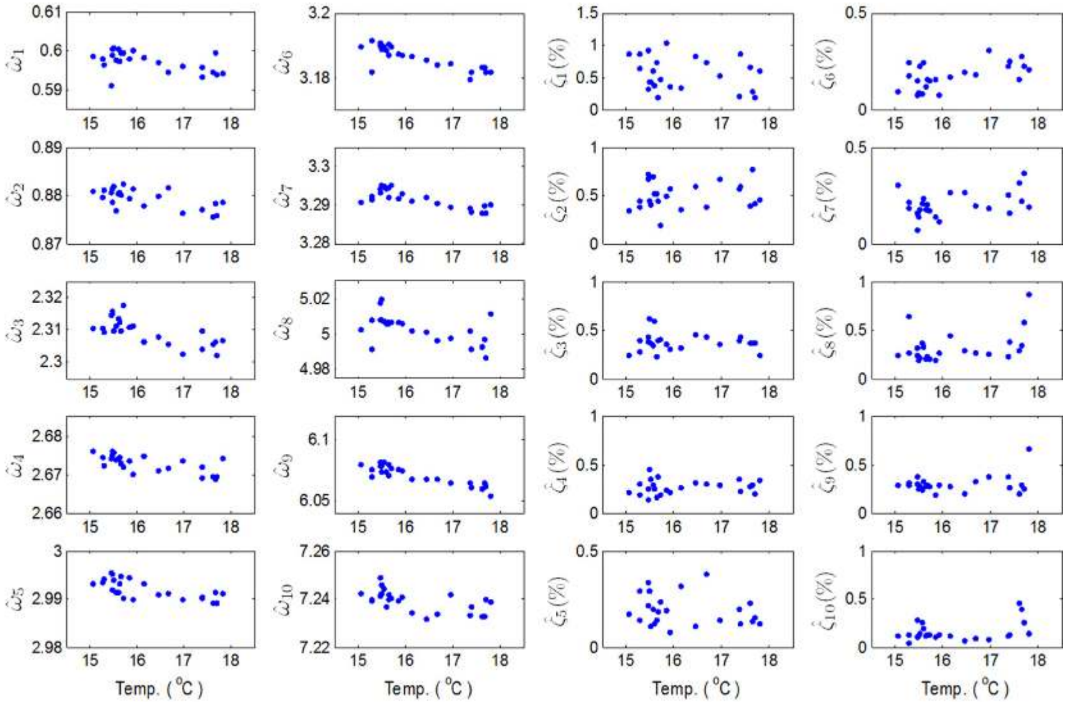


Fig. 8 Identified modal parameters versus ambient temperature

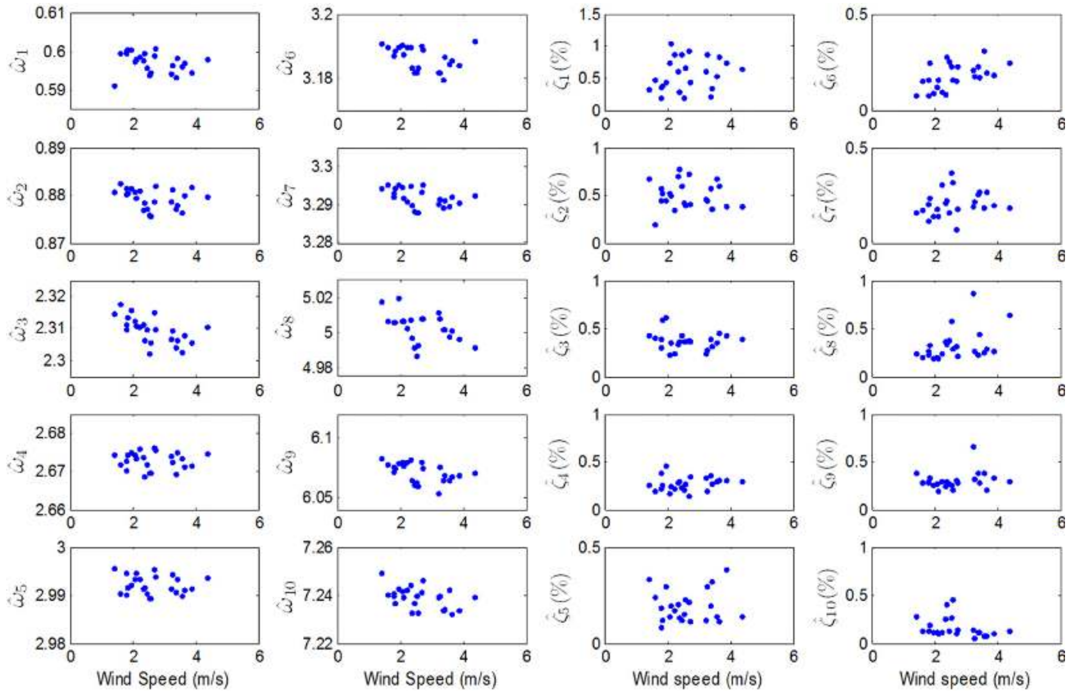


Fig. 9 Identified modal parameters versus wind speed

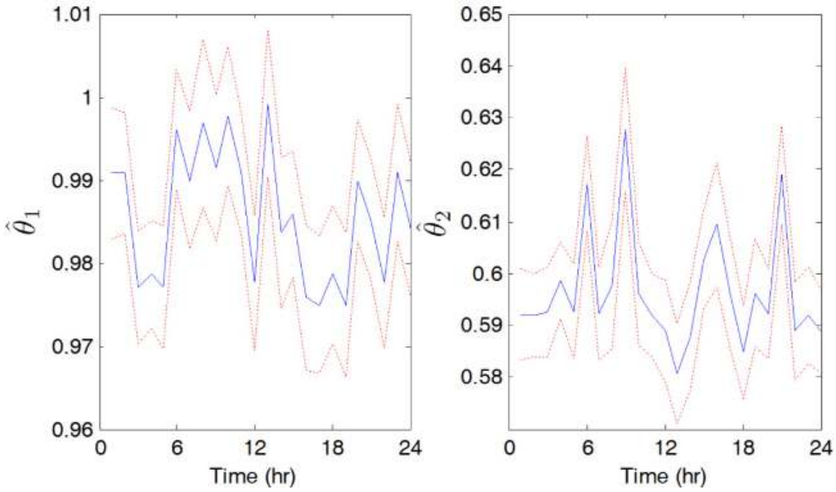


Fig. 10 Variation of the stiffness parameters

monitoring period. Therefore, the wind loading remained at a low level and the structural behavior was virtually linear. Of course, as found in previous studies (Jeary 1986, Tamura and Suganuma 1996), severe wind excitation, such as typhoon and hurricane, can induce nonlinear response of structures. This causes considerable reduction of the modal frequencies and increase of the damping ratios of the equivalent linear system for modal identification. However, it is necessary to obtain measurements of the tower in such excitation level for further investigation.

4.2 Ambient effects on stiffness parameters

Finally, the ambient influence on the stiffness parameters is discussed. Fig. 10 shows the identified stiffness parameters with $\pm 3\sigma$ confidence intervals versus time. The stiffness parameter θ_1 varied between 0.9750 and 0.9992. The difference was 2.4%, which is in the same order of the fluctuation of the modal frequencies. On the other hand, the stiffness parameter θ_2 varies from 0.5807 to 0.6277, that is, a range of 7.5% of difference. Regarding the uncertainty of the estimation, the average of the COVs of θ_1 and θ_2 are equal to 0.003 and 0.005, respectively. It indicates that the estimation of the stiffness parameters is precise. However, considering the confidence intervals, there is no evidence to conclude any significant change for both stiffness parameters in the monitoring period.

Fig. 11 shows the identified stiffness parameters versus the ambient temperature and wind speed. The stiffness parameter θ_1 shows a gently decreasing trend with the temperature but θ_2 is highly scattering. Again, the variation of temperature was too small to induce significant change of the stiffness of the structure. Similarly, the relationship between the wind speed and the stiffness parameters is not statistically significant under the calm wind condition. This study shows the ambient influence on the modal and stiffness parameters under the 24-hour monitoring period. In order to achieve thorough investigation on this issue, long-term measurements are necessary to cover larger range of temperature and wind speed.

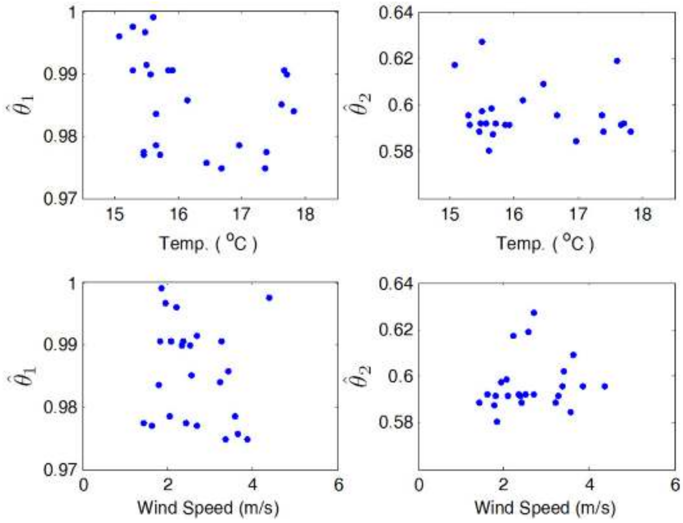


Fig. 11 Stiffness parameters versus ambient conditions

5. Conclusions

This paper presents the results for the structural health monitoring benchmark study of Canton Tower. A two-stage Bayesian structural health monitoring framework was used for output-only modal identification and finite element model updating using the given set of structural acceleration time histories. In the first stage, the Bayesian spectral density approach was used for output-only modal identification. The modal parameters and the associated uncertainty were estimated. In the second stage, a Bayesian model updating approach was utilized to update the finite element model of the tower. The uncertainty of the modal and stiffness parameters were quantified with Bayesian inference. Ambient influence on the modal and stiffness parameters of the tower was also investigated. It turned out that gently decreasing trend could be observed of the modal frequencies/ translational stiffness parameter against the ambient temperature. However, no other trend of the modal /stiffness parameters could be observed against the ambient conditions. The reason includes the large posterior uncertainty of the damping ratios and the narrow range of the ambient temperature and wind speed. Nevertheless, results show that the Bayesian framework can successfully achieve the goal of the first task of the Canton Tower benchmark study.

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