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EFFECTS OF CRACK GROWTH RATE AND CREEP IN STATIC FRACTURE OF CONCRETE

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ABSTRACT

The paper deals with the effect of crack growth rate and creep in static fracture of concrete. The available experimental data and various modeling approaches, including a rate-dependent generalization of the cohesive crack model based on the activation energy concept are reviewed. Attention is then focused on the description of these effects by means of a generalization of the equivalent linear elastic fracture model based on the R-curve concept, and a new model of this type is presented in detail. The crack propagation velocity is assumed to depend on the ratio of the stress intensity factor to its critical value from the R-curve. This dependence can be assumed as a power function with an exponent much larger than 1. The shape of the R-curve is determined as the envelope of the fracture equilibrium curves corresponding to the maximum load values for geometrically similar specimens of different sizes. The creep in the bulk of a concrete specimen must be taken into account in the case of static loading, which is done by replacing the elastic constants with a linear viscoelastic operator in time. The model fits the existing data on concrete (as well as rock) reasonably well. It exhibits not only the effects of size and rate, but for concrete it also exhibits an increase of brittleness with a decrease of loading rate, manifested as a shift of the maximum load points in the size effect plot toward linear elastic fracture mechanics (LEFM).

1. INTRODUCTION

The effect of loading rate on concrete strength and fracture properties has been studied intensely (e.g., [1]–[8]), however, only in the context of dynamic fracture. The problem is equally interesting for static fracture with loading durations ranging from 1 s to many years. The question is whether the trends known in dynamic fracture continue, and a further unknown is the effect of creep, which may be expected to be pronounced and must obviously be taken into account for long durations. A complicating feature for concrete is that the fracture model must be nonlinear and must reflect the experimentally observed size effect, which exists at widely different rates of loading but differs from one loading rate magnitude to another. Experimental results of increasing scope have been obtained ([10]–[13]).

The conference presentation begins by a discussion of the available experimental data, including some recent results at Northwestern University on the size effect in fracture specimens loaded at rates differing by over five orders of magnitude [14] and on the reversal of softening to hardening caused by a sudden large increase of loading rate [15]. Subsequently, various modeling approaches are reviewed in the conference presentation, and in particular a recent rate-dependent generalization of the cohesive crack model based on the activation energy concept for bond ruptures is described in some detail.

Attention is then focussed on the generalization of the R-curve model to rate effect and creep, which will be presented in this paper. The present R-curve model is an extension and refinement of an R-curve model with rate effect and creep presented at a recent conference [16]. A more extensive experimental support will also be demonstrated in this paper. In full detail, the model is presented in a separate article [17]. The basic idea of generalizing the R-curve approach to take into account the size effect and rate effect including creep, was briefly outlined in [11].

2. FORMULATION OF THE MODEL

To analyze the response of specimens under a controlled rate r of the crack mouth opening displacement (CMOD) Δ, one needs the following relations from linear elastic fracture mechanics (LEFM): $\Delta = P\delta(\alpha)/Eb, K = Pk(\alpha)/b\sqrt{d}$, in which P = applied load, E = elastic modulus, b = specimen thickness, d = characteristic dimension (taken here as the beam depth), $\alpha = a/d, a$ = crack length and δ, k = functions that are known from LEFM. The classical rate-independent R-curve concept of fracture propagation is based on the assumption that the critical energy release rate R depends on the crack propagation length $c = a - a_0$, where a_0 = initial crack (notch) length. The corresponding critical stress intensity factor is $K_R(c) = \sqrt{ER(c)}$.

It has been shown that the R-curve can be completely determined from the size effect on the measured maximum loads of geometrically similar specimens of different sizes. The procedure given in [18] yields a function ρ such that $R(c) = G_f\rho(c/c_f), K_R(c) = K_f\sqrt{\rho(c/c_f)}$, where G_f, K_f and c_f is fracture energy, fracture toughness and effective process zone length for an infinitely large specimen, which are geometry (shape) independent constants.

One process causing the rate effect is the fact that the crack growth rate is always finite and depends on the stress intensity factor K, as may be justified by considering the rate process theory for interatomic or intermolecular bond ruptures based on the concept of activation energy and Maxwell-Boltzmann distribution of thermal energies. Such considerations suggest that the crack growth rate should be expressed as $\dot{a} = \kappa_0 K^n \exp(-Q/RT)$, in which κ_0, n = constants, Q = activation energy, R = gas constant and T = absolute temperature. Since we do not study the temperature effect, this reduces to $\dot{a} = \kappa_0 K^n$. This relation may be applicable only to materials with a very small fracture process zone, and must therefore be generalized, which can be done using the R-curve concept. A simple generalization is given by the power law

$$\dot{a} = \kappa(K/K_R)^n, \tag{1}$$

in which κ and n are constants to be found empirically. The exponent n is expected to be very large so that for small K/K_R, the crack growth rates are negligible.

Replacing the elastic modulus E with a corresponding compliance operator for creep, the CMOD is expressed as

$$\Delta(t) = b^{-1}\int_{t_0}^{t} J(t,t')\,d[P(t')\delta(t')] \tag{2}$$

in which t = current time, t_0 = time at the first loading and $J(t,t')$ = compliance function for creep in the bulk of the specimen, which is well known.

One very interesting observation from testing of the size effect in concrete at various rates ([10],[12]) is that, in the plot of the logarithm of nominal stress versus log d, the response shifts to the right (i.e. toward the LEFM) as loading rate is getting slower. This suggests that c_f should decrease with a decreasing rate of loading, which must be described in terms of $\dot{a}$ because the material properties cannot directly depend on the loading rate. Therefore, it has been further assumed that

$$c_f = c_{f0}(\dot{a}/\dot{a}_0)^{1/m}, \tag{3}$$

in which $\dot{a}_0$ is a constant chosen for convenience and c_{f0}, m are constants to be determined empirically. This effect is, however, not seen in the tests of limestone [14], which is probably related to the fact that limestone does not exhibit creep.

The problem can be reduced to the solution of the following two coupled nonlinear integral and differential equations:

$$\dot{\alpha}(t) = \frac{\kappa c_{f0}^{n/2}}{b^n d^{n+1-n/2m} K_f^n \dot{a}_0^{n/2m}} \left[\frac{P(t)k(\alpha(t))\dot{\alpha}^{1/2m}(t)}{\rho^{1/2}(\alpha(t)-\alpha_0)}\right]^n, \tag{4}$$

$$\int_{t_0}^{t} J(t,t')\,d[P(t')\delta(t')] = br(t-t_0), \tag{5}$$

which are to be solved for the proper initial conditions.

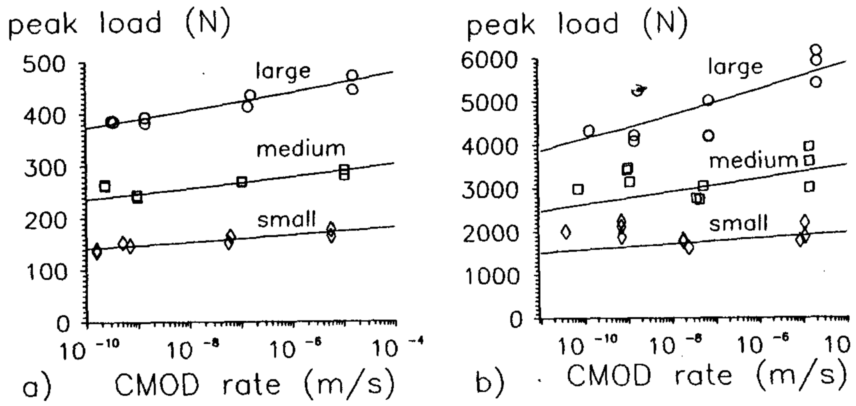

Figure 1. Rate effect for a) limestone, b) concrete.

3. COMPARISON TO EXPERIMENTS

The solution of (4), (5) has been carried out numerically in small time steps. The values of the material parameters $\kappa, n, K_f, c_{f0}, m$ have been varied in a trial and error fashion and ultimately optimized. Fig. 1a shows the best fit of experimental data for limestone reported in [14], which has been obtained using the rate equation (2) with $\kappa = 60.14\,\mathrm{m\,s^{-1}}$, $n = 6$, $K_f = 818.5\,\mathrm{kN\,m^{-3/2}}$ and a constant value of $c_f = 6.4\,\mathrm{mm}$. As one can see, reasonably good fits can be obtained, which validates the present formulation.

To model the rate and size effect for concrete as reported in [12] (Fig. 1b), the rate equation (1) has been used with $\kappa = 8 \times 10^{-6}\,\mathrm{m\,s^{-1}}$, $n = 29$ and $K_f = 900\,\mathrm{kN\,m^{-3/2}}$. The parameter c_f has been allowed to vary with varying crack propagation rate according to (5) with $c_{f0} = 14\,\mathrm{mm}$, $\dot{a}_0 = 0.01\,\mathrm{m\,s^{-1}}$, $m = 17$, in order to get a shift of the size effect plot towards LEFM for slow loading rates (Fig. 2b). However, the experimentally observed shift of brittleness (Fig. 2a) seems to be too strong to be exactly reproduced by the present numerical model. A further experimental study on the physical nature of this interesting phenomenon is needed.

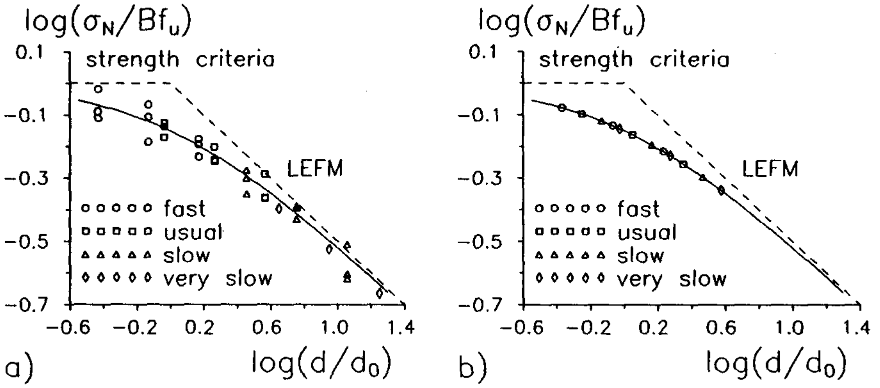

Figure 2: Size effect for concrete: a) experiments, b) theory.

The effect of a sudden change of loading rate was experimentally studied in [15]. The initial CMOD rate was held constant until the load reached the peak value P_1 and dropped to some lower value P_c; then the rate was suddenly increased or decreased by several orders of magnitude. For a sufficiently large increase of the loading rate, the load started increasing again and a second peak P_2 could be observed (Fig. 3a). On the other hand, a decrease of the loading rate was followed by a fast drop of the load-CMOD curve (Fig. 3b). The rate-dependent R-curve model exhibits qualitatively the same behavior (Fig. 3c). The tests suggest that, after a rate change, the curve for the new rate asymptotically approach the curve for a constant rate test with a rate equal to the new rate. The theory agrees with this behavior also (Fig. 3c).

Quantitative agreement between theory and experiments can be verified by plotting the ratio P_2/P_1 versus P_c/P_1 for all results available for the relative rate change 10^3. Instead of trying to adjust the parameters so as to fit the experimental data, their values were taken from the best fit of constant-CMOD tests [12]. It is gratifying that these parameter values lead to a satisfactory agreement (Fig. 3d).

It may be concluded that the R-curve concept, if generalized for the crack propagation rate effect and creep in the specimen bulk, can give a good description of both the observed size effects and rate effects in concrete (as well as limestone). It is a simple model of this complex phenomenon, which would lend itself easily to practical applications.

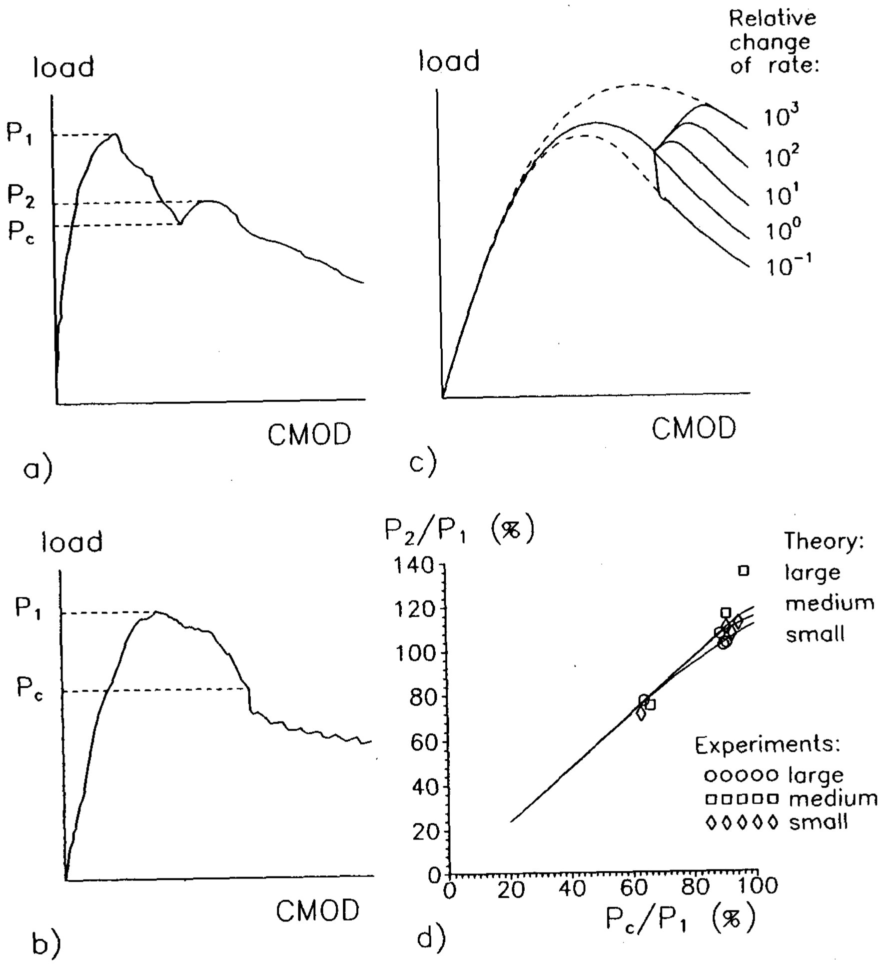

Figure 3: Tests with a sudden rate change: a) experimental load-CMOD curve (rate increased), b) experimental load-CMOD curve (rate decreased), c) theoretical load-CMOD curves, d) second peak versus load at rate change.

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