STRUCTURAL MODEL TUNING CAPABILITY IN AN OBJECT-ORIENTED MULTIDISCIPLINARY DESIGN, ANALYSIS, AND OPTIMIZATION TOOL

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Abstract

Updating the finite element model using measured data is a challenging problem in the area of structural dynamics. The model updating process requires not only satisfactory correlations between analytical and experimental results, but also the retention of dynamic properties of structures. Accurate rigid body dynamics are important for flight control system design and aeroelastic trim analysis. Minimizing the difference between analytical and experimental results is a type of optimization problem. In this research, a multidisciplinary design, analysis, and optimization [MDAO] tool is introduced to optimize the objective function and constraints such that the mass properties, the natural frequencies, and the mode shapes are matched to the target data as well as the mass matrix being orthogonalized.

Nomenclature

CG	center of gravity
$\mathbf{F}$	original objective function
FE	finite element
GVT	ground vibration test
$\mathbf{g}_i$	inequality constraints
$\mathbf{h}_j$	equality constraints
I_{XX}	computed x moment of inertia about the center of gravity
I_{XXG}	target x moment of inertia about the center of gravity
I_{YY}	computed y moment of inertia about the center of gravity
I_{YYG}	target y moment of inertia about the center of gravity
I_{ZZ}	computed z moment of inertia about the center of gravity
I_{ZZG}	target z moment of inertia about the center of gravity
$\mathbf{J}_i$	objective functions (optimization problem statement number $i = 1, 2, \ldots , 13$)
$\mathbf{K}$	stiffness matrix
$\overline{\mathbf{K}}$	orthonormalized stiffness matrix
$\mathbf{L}$	new objective function
$\mathbf{M}$	mass matrix
$\overline{\mathbf{M}}$	orthonormalized mass matrix
MAC	modal assurance criteria
MDAO	multidisciplinary design, analysis, and optimization
m	number of equality constraints
n	number of modes
q	number of inequality constraints
$\mathbf{T}$	transformation matrix
$\mathbf{W}$	Computed total mass
$\mathbf{W}_G$	Target total mass
X	x-coordinate of the computed center of gravity
$\overline{\mathbf{X}}$	design variables vector
X_G	x-coordinate of target center of gravity
Y_G	y-coordinate of target center of gravity
Y	y-coordinate of the computed center of gravity
Z	z-coordinate of the computed center of gravity
Z_G	z-coordinate of target center of gravity
ε	small tolerance value for inequality constraints
λ	Lagrange multiplier
$\boldsymbol{\Phi}$	computed eigen-matrix
$\boldsymbol{\Phi}_G$	target eigen-matrix

Ω_j j-th computed frequency

Ω_{jG} j-th target frequency

1 Introduction

One of the top-level challenges of multidisciplinary design, analysis, and optimization [MDAO] tool development for modern aircraft is synergistic design, analysis, simulation, and testing. This challenge puts a clear emphasis on synchronizing all phases of experimental testing (ground and flight), analytical model updating, high- and low-fidelity simulations for model validation, and complementary design. Compatible information flow between these procedures will result in a coherent feedback process for data-to-modeling-to-design continuity using systematic and competent vertically integrated design tools and ensure that the unique benefits of data gained from flight research are integrated into the vehicle development process.

One of the basic inputs into aeroservoelastic analysis is the underlying structural dynamics model, usually a finite element [FE] model. Generally created during aircraft development by the builders, the accuracy and fidelity of this with respect to the actual modal frequencies and shapes is critical. Models are often inaccurate, due to many factors such as joint stiffness, free play, unmodeled structural elements, or non-linear structural behavior. Thus, flight-test missions often require 'tuning' of the original FE model, for aeroservoelastic envelope clearance, to match experimentally observed structural characteristics.

Accurate modeling of rigid body dynamics is important for flight control system design and aeroelastic trim analysis. In general, structural dynamics FE models for production aircraft need to be correlated to measured data to ensure the accuracy of the numerical models. Small modeling errors in the FE model will cause errors in the calculated structural flexibility and mass, thus propagating into unpredictable errors in the calculated aeroelastic and aeroservoelastic responses. If measured mode shapes will be associated with an FE model of the structure, they should be adjusted to reduce the structural

dynamic modeling errors in the flutter analysis, thus also improving confidence of flight safety.

The primary objective of the current study is to add model tuning capabilities in an MDAO tool. This model tuning technique is essentially based on a non-linear optimization problem, with the variables to be minimized being the differences between the model and the experimental values, including the dynamics variables and the static loading deflections, the total mass, and center of gravity [CG] of the test article.

Model tuning is a common method to improve the correlation between analytical and experimental modal data, and many techniques have been proposed [1, 2]. These techniques can be divided into two categories: direct methods (adjust the mass and stiffness matrices directly) and parametric methods (correct the models by changing the structural parameters). The direct methods correct mass and stiffness matrices without taking into account the physical characteristics of the structures and may not be appropriate for use in model updating processes. In this paper, the updating method used in the optimization process is the parametric method. In the optimization process, structural parameters are selected as design variables: structural sizing information (thickness, cross-sectional area, area moment of inertia, torsional constant, etc.); point properties (lumped mass, spring constants, etc.); and materials properties (density, Young's modulus, etc.). Objective function and constraint equations include mass properties, mass matrix orthogonality, frequencies, and mode shapes. The use of these equations minimizes the difference between analytical results and target data.

2 Optimization Background

Discrepancies between ground vibration test [GVT] data and numerical results are common. Discrepancies in frequencies and mode shapes are minimized using a series of optimization procedures [3]. Recently, the National Aeronautics and Space Administration [NASA] Dryden Flight Research Center [DFRC] began developing an MDAO tool [4]. This MDAO tool is object-oriented: users can either use the built-in pre- and post-processor to

convert design variables to structural parameters and generate objective functions, or easily plug in their own analyzer for the optimization analysis. The heart of this tool is the central executive module. Users will utilize this module to select input files, solution modules, and output files; and monitor the status of current jobs. There are two optimization algorithms adopted in this MDAO tool: the traditional gradient-based algorithm [5], and the genetic algorithm [6]. Gradient-based algorithms work well for continuous design variable problems, whereas genetic algorithms can handle continuous as well as discrete design variable problems easily. When there are multiple local minima, genetic algorithms are able to find the global optimum results, whereas gradient-based methods may converge to a locally minimum value. In this research work, the genetic algorithm is used for the solution of the optimization problem.

The genetic algorithm is directly applicable only to unconstrained optimization; it is necessary to use some additional methods to solve the constrained optimization problem. The most popular approach is to add penalty functions in proportion to the magnitude of constraint violation to the objective function [7]. The general form of the penalty function is

$$\mathbf{L}(\overline{\mathbf{X}}) = \mathbf{F}(\overline{\mathbf{X}}) + \sum_{i=1}^{q} \lambda_i \mathbf{g}_i(\overline{\mathbf{X}}) + \sum_{j=1}^{m} \lambda_{j+q} \mathbf{h}_j(\overline{\mathbf{X}})$$

where $\mathbf{L}(\overline{\mathbf{X}})$ indicates the new objective function to be optimized, $\mathbf{F}(\overline{\mathbf{X}})$ is the original objective function, $\mathbf{g}_i(\overline{\mathbf{X}})$ is the inequality constraint, $\mathbf{h}_j(\overline{\mathbf{X}})$ is the equality constraint, λ_i are the Lagrange multipliers, $\overline{\mathbf{X}}$ is the design variables vector, and q and m are the number of inequality and equality constraints, respectively.

Matching the mass properties, the mass matrix orthogonality, and the natural frequencies and mode shapes to target value at the same time is a multiple objective functions problem. The easy way to minimize multiple objective functions is to convert the problem into one with only a single objective function and optimize in the usual fashion, however, this is time-consuming. One of the solution methods

for a multi-objective optimization problem is to minimize one objective while constraining the remaining objectives to be less than given target values. This method is employed in this paper, since our main goal is to match the frequencies and mode shapes while minimizing the error in the rigid body dynamics and mass properties.

2.1 Mass Properties

The difference in the analytical and target values of the total mass, the CG, and the mass moment of inertias at the CG location are minimized to have the identical rigid body dynamics.

$$\mathbf{J}_1 = (\mathbf{W} - \mathbf{W}_G)^2 / \mathbf{W}_G^2$$

$$\mathbf{J}_2 = (\mathbf{X} - \mathbf{X}_G)^2 / \mathbf{X}_G^2$$

$$\mathbf{J}_3 = (\mathbf{Y} - \mathbf{Y}_G)^2 / \mathbf{Y}_G^2$$

$$\mathbf{J}_4 = (\mathbf{Z} - \mathbf{Z}_G)^2 / \mathbf{Z}_G^2$$

$$\mathbf{J}_5 = (\mathbf{I}_{\mathrm{XX}} - \mathbf{I}_{\mathrm{XX}G})^2 / \mathbf{I}_{\mathrm{XX}G}^2$$

$$\mathbf{J}_6 = (\mathbf{I}_{\mathrm{YY}} - \mathbf{I}_{\mathrm{YY}G})^2 / \mathbf{I}_{\mathrm{YY}G}^2$$

$$\mathbf{J}_7 = (\mathbf{I}_{\mathrm{ZZ}} - \mathbf{I}_{\mathrm{ZZ}G})^2 / \mathbf{I}_{\mathrm{ZZ}G}^2$$

$$\mathbf{J}_8 = (\mathbf{I}_{\mathrm{XY}} - \mathbf{I}_{\mathrm{XY}G})^2 / \mathbf{I}_{\mathrm{XY}G}^2$$

$$\mathbf{J}_9 = (\mathbf{I}_{\mathrm{YZ}} - \mathbf{I}_{\mathrm{YZ}G})^2 / \mathbf{I}_{\mathrm{YZ}G}^2$$

$$\mathbf{J}_{10} = (\mathbf{I}_{\mathrm{ZX}} - \mathbf{I}_{\mathrm{ZX}G})^2 / \mathbf{I}_{\mathrm{ZX}G}^2$$

2.2 Mass Matrix

The off-diagonal terms of the orthonormalized mass matrix are reduced to improve the mass orthogonality:

$$\mathbf{J}_{11} = \sum_{i=1, j=1, i \neq j}^{n} (\overline{\mathbf{M}}_{ij})^2$$

where n is the number of modes to be matched and $\overline{\mathbf{M}}$ is defined as

$$\bar{\mathbf{M}} = \mathbf{\Phi}_G^T \mathbf{T}^T \mathbf{M} \mathbf{T} \mathbf{\Phi}_G$$

In the above equation the mass matrix, $\mathbf{M}$, is calculated from the FE model, while the target eigen-matrix $\mathbf{\Phi}_G$ is measured from the GVT. The eigen-matrix $\mathbf{\Phi}_G$ remains constant during the optimization procedure. A transformation matrix $\mathbf{T}$ in the above equation is based on Guyan reduction [8] or improved reduction system [9]. This reduction is required due to the limited number of available sensor locations.

2.3 Frequencies and Mode Shapes

Two different types of error norm can be used. In the first option, the objective function considered combines an index which identifies normalized errors from the GVT and computed frequencies with another index which defines the total error associated with the off-diagonal terms of the orthonormalized stiffness matrix.

$$\mathbf{J}_{12} = \sum_{i=1}^{n} \left(\frac{\mathbf{\Omega}_i - \mathbf{\Omega}_{iG}}{\mathbf{\Omega}_i} \right)^2$$

$$\mathbf{J}_{13} = \sum_{i=1, j=1, i \neq j}^{n} \left(\bar{\mathbf{K}}_{ij} \right)^2$$

The matrix $\bar{\mathbf{K}}$ are obtained from the following matrix products,

$$\bar{\mathbf{K}} = \mathbf{\Phi}_G^T \mathbf{T}^T \mathbf{K} \mathbf{T} \mathbf{\Phi}_G$$

where the stiffness matrix, $\mathbf{K}$, is calculated from the FE model.

In the second option, the error norm combines the same index used above (which defines the normalized error in the GVT and computed frequencies) with another index which defines the total error between the GVT and computed mode shapes at given sensor points.

$$\mathbf{J}_{12} = \sum_{i=1}^{n} \left(\frac{\mathbf{\Omega}_i - \mathbf{\Omega}_{iG}}{\mathbf{\Omega}_i} \right)^2$$

$$\mathbf{J}_{13} = \sum_{i=1, j=1, i \neq j}^{n} \left(\mathbf{\Phi}_i - \mathbf{\Phi}_{iG} \right)^2$$

In this research, the second optimization option is employed since the definition of the objective function is much simpler than in the first option. Any errors in both the modal frequencies and the mode shapes are minimized by including an index for each of these in the objective function. For this optimization, a small number of sensor locations can be used at which errors between the GVT and computed mode shapes are obtained. Any one of $\mathbf{J}_1$ thru $\mathbf{J}_{13}$ can be used as the objective function with the others treated as constraints. This gives the flexibility to achieve the particular optimization goal while maintaining the other properties at as close to the target value as possible. The optimization problem statement can be written as

Minimize: $\mathbf{J}_i$
Such that: $\mathbf{J}_k \leq \varepsilon_k$, for k = 1 thru 13 and $k \neq i$

where ε_k is a small value which can be adjusted according to the tolerance of each constraint condition.

3 Applications

3.1 Square Cantilever Plate

A cantilever plate shown in Fig. 1 is used to demonstrate how to set up design variables, the objective function, and the constraints for the optimization process. The target configuration of the plate is 10 in. by 10 in. and 0.1 in. thick, containing 16 quadrilateral elements and 100 (20 × 5) degrees of freedom [DOFs]. Only 12 DOFs as shown in Fig.1 are used to simulate sensor output. The modulus of elasticity and Poisson's ratio are 1.0×10^7 psi and 0.3, respectively. The mass density is 2.39×10^{-4} slug/in^3.

The FE analysis results based on the target configuration are used as target values. The optimization process starts by selecting thickness and mass density to be the design variables. Total mass, CG, moment of inertia, and mass orthogonality are selected as constrained equations. Frequencies and mode shape errors are selected as the objective

function. Initial design variables of 0.5 in. thick and a mass density of 5.0×10^{-4} slug/in^3 are modeled such that a discrepancy between the two models is generated. Twenty populations and 100 generations are used for the genetic algorithm. Mass properties, modal characteristics, and design variables before and after optimizations are given in Tables 1 and 2. The thickness and mass density have converged to the target values and the frequencies and mode shapes have minimal errors. The optimization history of the objective function is shown in Fig. 2.

3.2 Aerostructures Test Wing

A second example is an experiment known as the aerostructures test wing [ATW] which was designed by NASA DFRC to research aeroelastic instabilities. Specifically, this experiment was used to study an instability known as flutter. Flight flutter testing is the process of determining a flight envelope within which an aircraft will not experience flutter. Flight flutter testing is very dangerous and expensive because predictions of the instability are often unreliable due to uncertainties in the structural dynamic and aerodynamic models.

The ATW was a small-scale airplane wing comprised of an airfoil and wing tip boom as shown in Fig. 3. This wing was formulated based on a NACA-65A004 airfoil shape with a 3.28 aspect ratio. The wing had a span of 18 in. with root chord length of 13.2 in. and tip chord length of 8.7 in. The total area of this wing was 197 in^2. The wing tip boom was a 1-in. diameter hollow tube of 21.5 in. length. The total weight of the wing was 2.66 lb.

Ground vibration tests have been performed to determine the dynamic modal characteristics of the ATW [10]. It is shown in Table 3 that the first bending and torsion modes were at 13.76 and 20.76 Hz, respectively. Corresponding frequencies and mode shapes computed using MSC/NASTRAN (MSC.Software Corporation, Santa Ana, California, USA) [11] are also listed in Table 3 and given in Fig. 4, respectively.

The FE model has been tuned to match the experimental data, but still the frequency error of 9.9% is observed for the second mode. This amount of frequency error violates the 3% limit for the primary modes described in military specifications [12, 13]. The 4% error in the total weight is also listed in Table 3. Therefore, the FE model needs to be further updated for a more reliable flutter analysis. The original FE model used rigid body elements to connect the wing tip to the boom which could produce the so-called 'idealization error.' Therefore, we used scalar springs to replace rigid body elements so that stiffness could be adjusted in this area. Point masses and scalar springs are selected for the design variables to minimize the frequencies and total weight errors. Two runs have been performed to demonstrate the sensitivity of the optimization solution to the constraint equations: (1) $\mathbf{J}_{12}$ was used as the objective function and $\mathbf{J}_1$ as a constraint equation; (2) $\mathbf{J}_{12}$ was used as the objective function and $\mathbf{J}_1$ thru $\mathbf{J}_{11}$ and $\mathbf{J}_{13}$ as constraint equations. With 50 populations and 100 generations of genetic algorithm optimization parameters, the final frequencies and total weight for case (1) are listed in Table 4. A summary of the center of gravity, moment of inertia, mass orthogonality, and MAC for the ATW for case (1) are shown in Table 5. Table 6 shows the final frequencies and total weight for case (2); a summary of the center of gravity, moment of inertia, mass orthogonality, and MAC are shown in Table 7. The optimization histories for the objective function of case (1) and case (2) are shown in Figs. 5 and 6 respectively. In case (1), there is a great reduction in the total weight and frequency errors but no improvement for the mass orthogonality. In case (2), total mass, mass orthogonality and frequencies are improved but not as much in case (1).

4 Conclusions

Simple and efficient model tuning capabilities based on a non-linear optimization problem are successfully integrated with the multidisciplinary design, analysis, and optimization [MDAO] tool developed at the NASA Dryden Flight Research Center. Instead of modifying the stiffness and mass matrices

directly, we updated the structural parameters such that the mass properties, mass matrix, frequencies, and mode shapes were matched to the target data, maintaining some similarity with the actual structure. The computer program has been coded in such a way that each J_1 thru J_{13} can be used as a constraint or objective function. When J_i is selected as the objective function, all or part of the J_k ($k \neq i$) can be selected as a set of constraints. This gives the flexibility to achieve a particular optimization goal.

Two examples were used to demonstrate the application of this model updating process. These examples showed that the number of constraint equations that is adequate to be used in the optimization process depends on the complexity of the model. For a simple model, the number of constraint equations may not have much effect on the solution, but for a complex model this effect could be significant. In either case, the approach investigated in this work proved to be a robust method of improving the accuracy of structural dynamics finite element models.

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Tables

	Target	Initial	Error, %
Thickness	0.1	0.5	400.0
Mass density	0.000239	0.0005	109.2
Total mass	0.00239	0.025	946.0
CG	5.0, 5.0, 0.0	5.0, 5.0, 0.0	0
Ixx Iyy Izz Ixy, Iyz, Izx	0.0224 0.0224 0.0448 0.0, 0.0, 0.0	0.234 0.234 0.468 0.0, 0.0, 0.0	944.6
Mode 1, Hz	33.27	114.84	245.2
Mode 2, Hz	77.84	265.00	240.4
Mode 3, Hz	187.91	650.70	246.3
Modal assurance criteria		0.999 0.999 0.996	

Table 1. Errors between the target and the initial configuration of the cantilever plate.

	Target	Final	Error, %
Thickness	0.1	0.0977	-2.3
Mass density	0.000239	0.000236	-1.2
Total mass	0.00239	0.00231	-3.3
CG	5.0, 5.0, 0.0	5.0, 5.0, 0.0	0
Ixx Iyy Izz Ixy, Iyz, Izx	0.0224 0.0224 0.0448 0.0, 0.0, 0.0	0.02162 0.02162 0.04320 0.0, 0.0, 0.0	-3.5
Mode 1, Hz	33.27	32.74	-1.6
Mode 2, Hz	77.84	77.01	-1.1
Mode 3, Hz	187.91	186.54	-0.7
Modal assurance criteria		0.999 0.999 0.999	

Table 2. Errors between the target and the final configuration of the cantilever plate.

	GVT, Hz	NASTRAN, Hz (Guyan/Full)	Error, %
Mode 1	13.763	13.354/13.354	-2.97/-2.97
Mode 2	20.763	22.819/22.819	9.90/9.90
Mode 3	77.833	79.062/78.771	1.58/1.21
Total weight, lb	2.66	2.77	4.13

Table 3. Frequencies and total weight of the aerostructures test wing before optimization.

	GVT, Hz	NASTRAN, Hz (Guyan/Full)	Error, %
Mode 1	13.763	13.406/13.405	-2.59/-2.60
Mode 2	20.763	21.014/21.013	1.21/1.2
Mode 3	77.833	77.871/77.502	0.04/-0.40
Total weight, lb	2.66	2.698	1.43

Table 6. Frequencies and total weight of the aerostructures test wing after optimization (with J_{11} constraint).

	GVT, Hz	NASTRAN, Hz (Guyan/Full)	Error, %
Mode 1	13.763	13.753/13.753	-0.07/-0.07
Mode 2	20.763	20.764/20.763	0.00/0.00
Mode 3	77.833	77.817/77.842	-0.02/-0.45
Total weight, lb	2.66	2.67	0.37

Table 4. Frequencies and total weight of the aerostructures test wing after optimization (without J_{11} constraint).

	Before optimization	After optimization
CG (X, Y, Z)	12.94, 9.16, 0.0	12.88, 8.8, 0.0
Ixx	161.22	152.06
Iyy	113.08	112.83
Izz	268.20	258.79
Ixy	95.27	93.75
Ixz	0.011	0.00996
Iyz	-0.028	-0.0349
M11	1.0	1.0
M12	0.089	0.157
M13	0.177	0.148
M22	1.0	1.0
M23	0.093	0.109
M33	1.0	1.0
MAC	0.99	0.98
	0.99	0.99
	0.95	0.95

Table 5. Summary of center of gravity, moment of inertia, mass orthogonality, and modal assurance criteria for the aerostructures test wing before and after optimization (without J_{11} constraint).

	Before optimization	After optimization
CG (X, Y, Z)	12.94, 9.16, 0.0	12.72, 8.91, 0.0
Ix	161.22	154.78
Iy	113.08	102.57
Iz	268.20	251.26
Ixy	95.27	89.45
Ixz	0.011	0.0068
Iyz	-0.028	-0.033
M11	1.0	1.0
M12	0.089	0.0297
M13	0.177	0.119
M22	1.0	1.0
M23	0.093	0.028
M33	1.0	1.0
MAC	0.99	0.95
	0.99	0.97
	0.95	0.95

Table 7. Summary of center of gravity, moment of inertia, mass orthogonality, and modal assurance criteria for the aerostructures test wing before and after optimization (with J_{11} constraint).
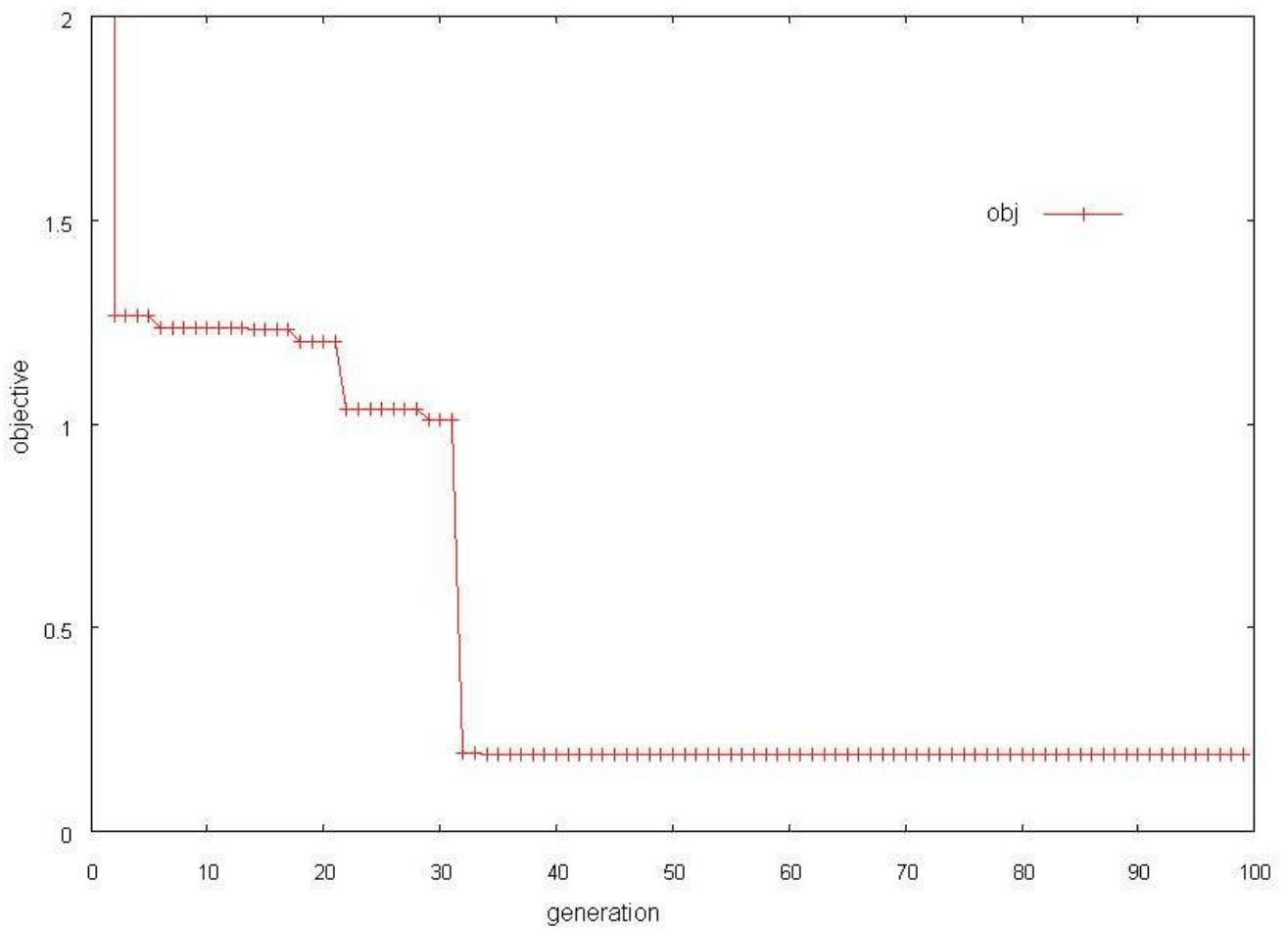
Figures
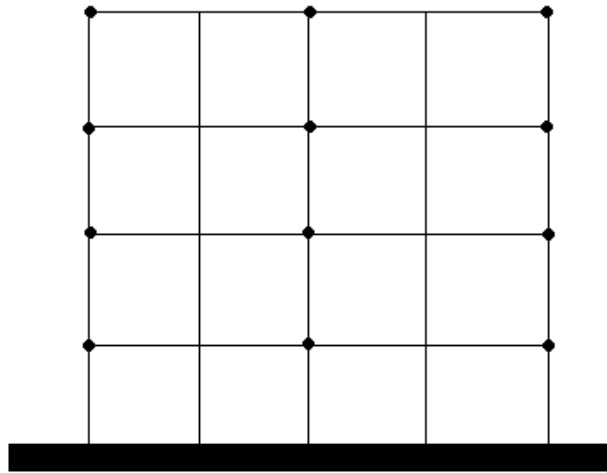
Fig. 1. Cantilever plate.

Fig. 2. Objective function history of the cantilever plate.

Fig. 3. Aerostructures test wing.

(a) MSC/NASTRAN model.

(b) Mode 1: 13.354 Hz.

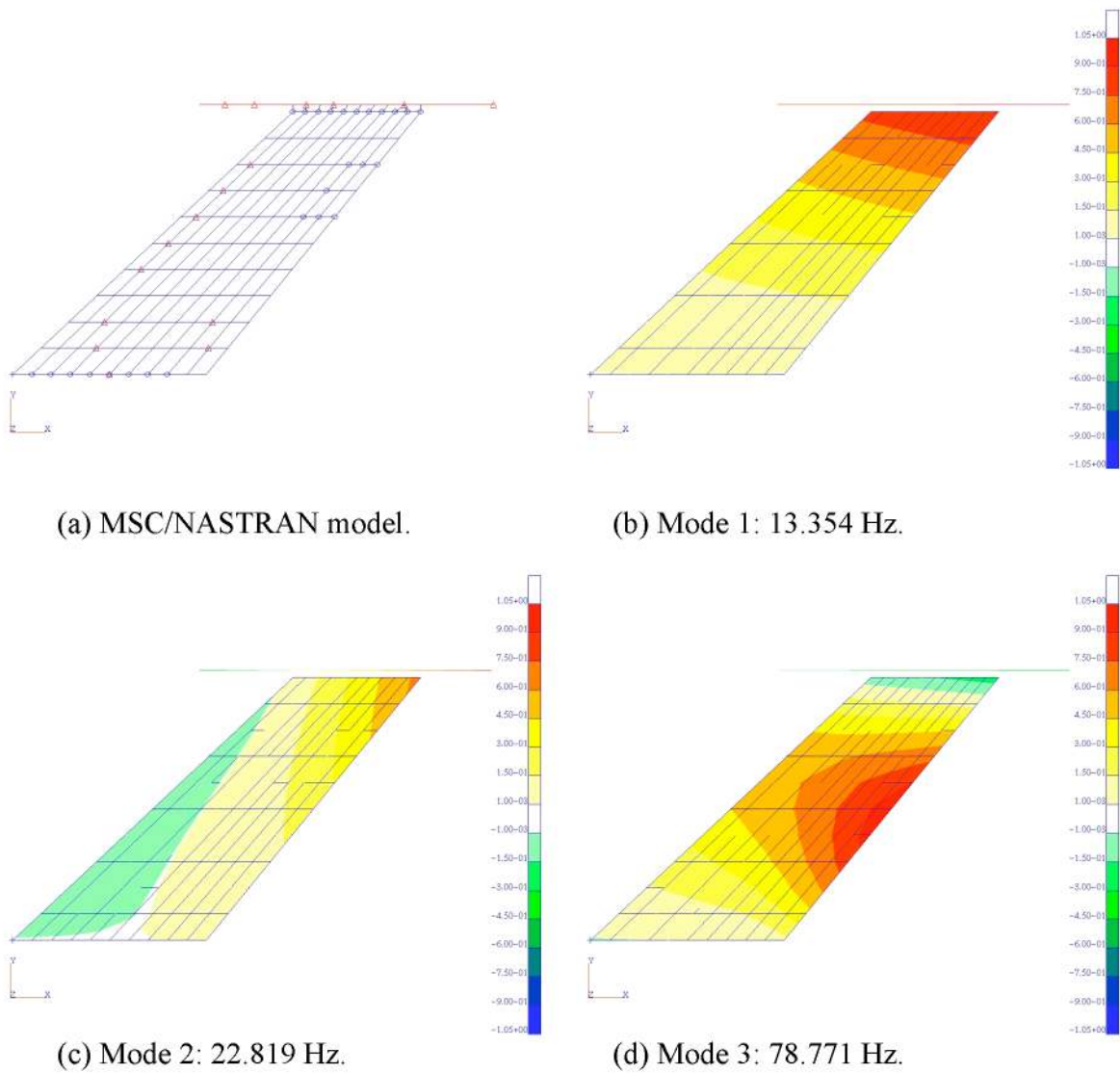(c) Mode 2: 22.819 Hz.

(d) Mode 3: 78.771 Hz.

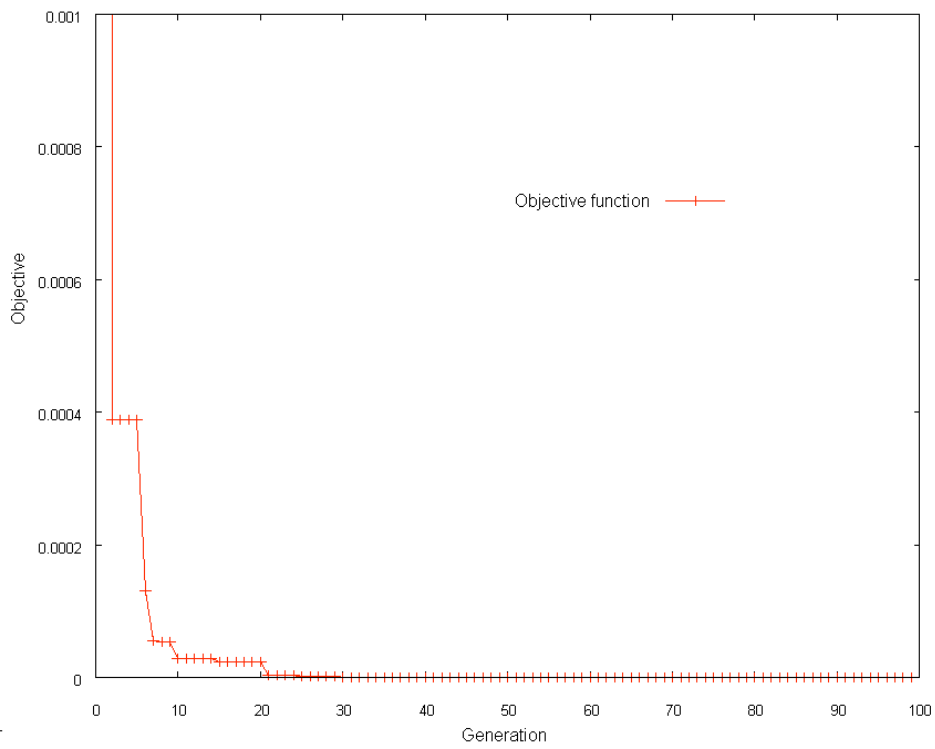Fig. 4. Finite element model and mode shapes.

Fig. 5. Objective function history of the aerostructures test wing (without $\mathbf{J}_{11}$ constraint).
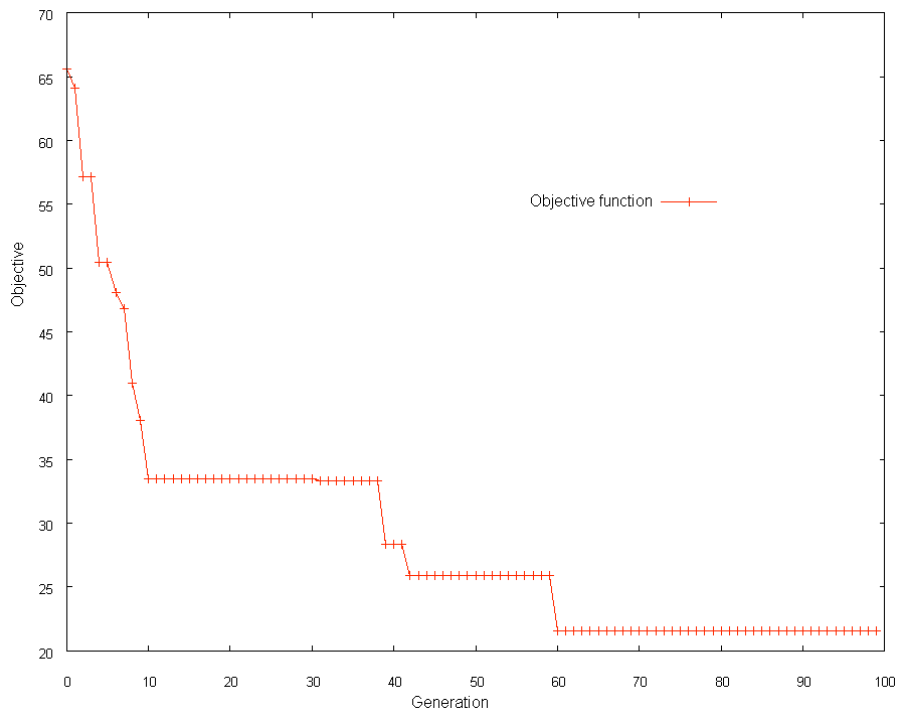

Fig. 6. Objective function history of the aerostructures test wing (with $\mathbf{J}_{11}$ constraint).

Copyright Statement

Structural Model Tuning Capability in an Object-Oriented Multidisciplinary Design, Analysis, and Optimization Tool

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Objectives

☐ To develop a simple and efficient in-house code for updating finite element model using experimental test data

☐ Be part of the MDAO tool development at NASA Dryden Flight Research Center

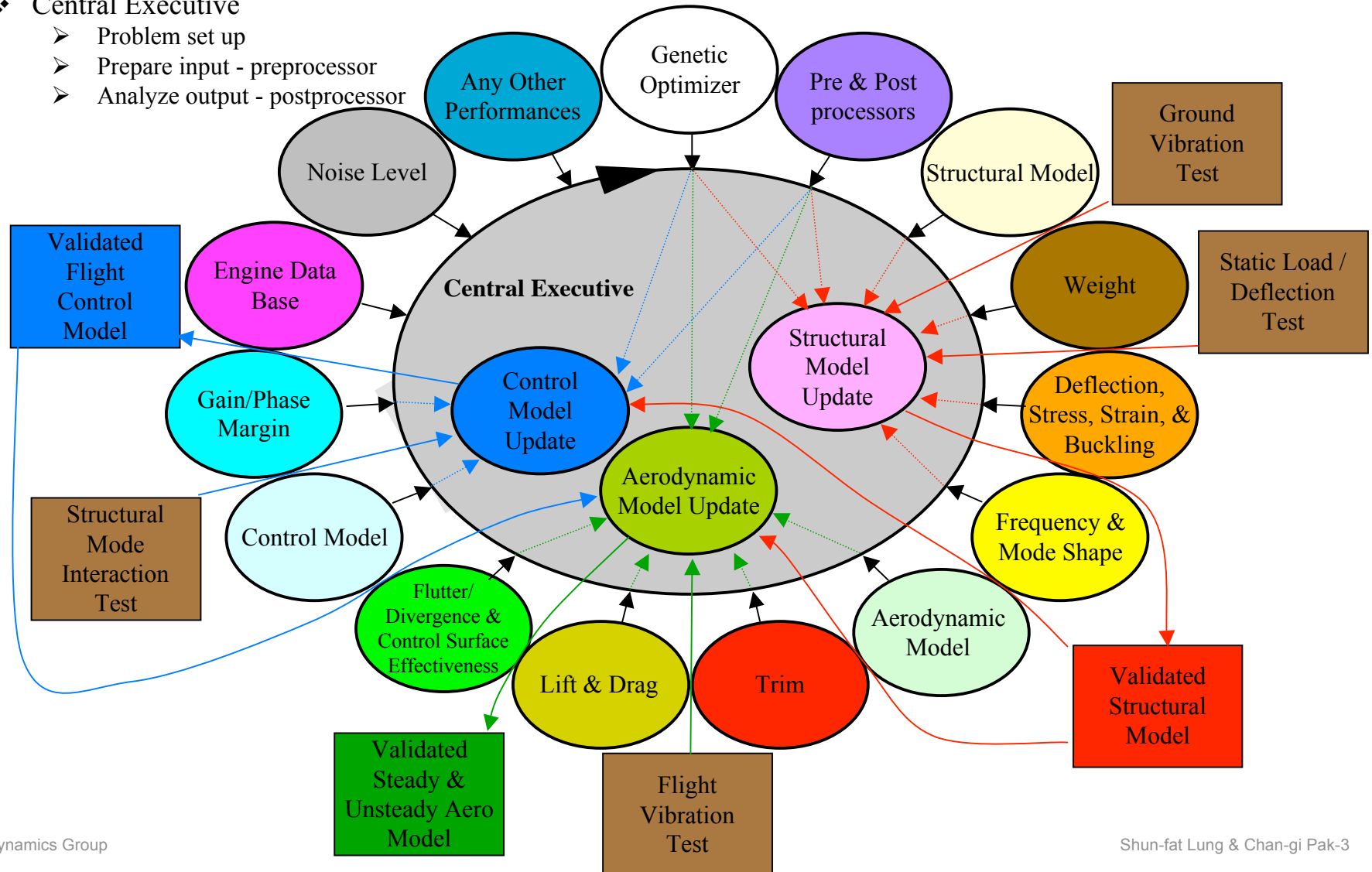☐ Support flight test by providing accurate flutter prediction based on reliable mass and stiffness matrices

Introduction

- ❏ **MDAO Tool**
  - ❖ Central Executive
    - ➢ Problem set up
    - ➢ Prepare input - preprocessor
    - ➢ Analyze output - postprocessor

Introduction (Cont.)

❑ Everyone believes the test data except for the experimentalist, and no one believes the finite element model except for the analyst.

  ❖ Some of the discrepancies come from analytical Finite Element modeling uncertainties, noise in the test results, and/or inadequate sensor and actuator locations.

❑ MIL-STD-1540C Section 6.2.10

  ❖ Test Requirements for Launch, Upper-Stage, & Space Vehicles

  ❖ Less than *3%* and *10%* frequency errors for the *primary* and *secondary* modes, respectively

  ❖ Less than *10%* off-diagonal terms in orthonormalized mass matrix

❑ AFFTC-TIH-90-001 (Structures Flight Test Handbook)

  ❖ If measured mode shapes are going to be associated with a finite element model of the structure, *it will probably need to be adjusted to match the lumped mass modeling of the analysis.*

  ❖ Based on the measured mode shape matrix [Φ] and the analytical mass matrix [M] , the following operation is performed.

$$\Phi^T M \Phi$$

The results is near diagonalization of the resulting matrix with values close to 1 on the diagonal and values close to zero in the off-diagonal terms. Experimental reality dictates that the data will not produce exact unity or null values, so *10 percent* of these targets are accepted as *good orthogonality* and the data can be confidently correlated with the finite element model.

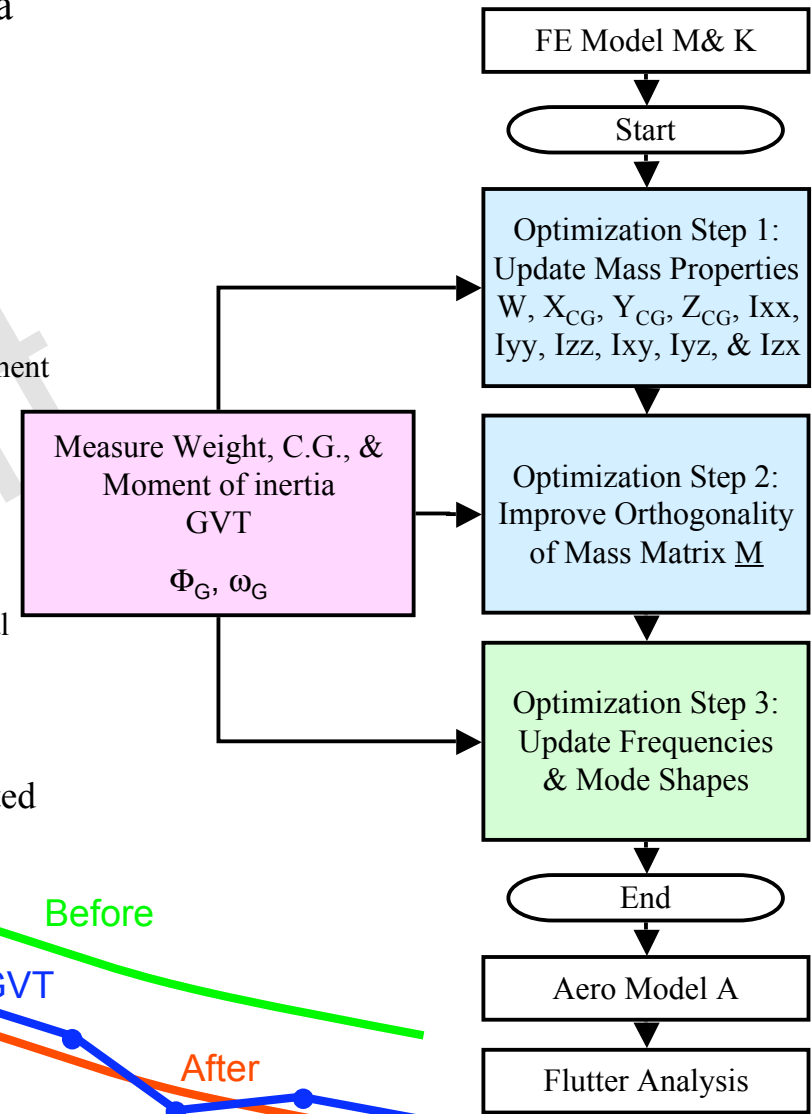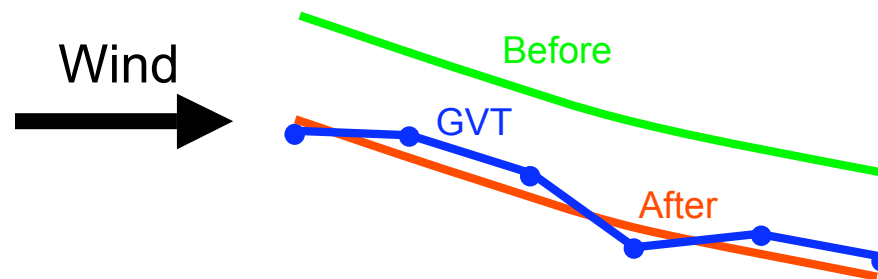# Flutter Analysis @ NASA Dryden Flight Research Center

❑ Update Finite Element Structural Model using Test Data

   ❖ Recall MIL-STD-1540C Section 6.2.10

      ➢ Less than 3% frequency error: primary modes

      ➢ Less than 10% frequency error: secondary modes

      ➢ Less than 10% off-diagonal terms in mass matrix

   ❖ Update Mass

      ➢ Minimize errors in total weight, C.G. location, and mass moment of inertia

      ➢ Minimize off-diagonal terms in orthogonal mass matrix

   ❖ Update Stiffness

      ➢ Minimize errors in frequencies

      ➢ Minimize errors in mode shapes and/or minimize off diagonal terms in orthogonal stiffness matrix

❑ Flutter Analysis

   ❖ Uncertainties in the structural dynamic model are eliminated through the use of "model update technique"

   ❖ Based on analytical modes

FE Model M & K

Start

Optimization Step 1: Update Mass Properties W, X_{CG}, Y_{CG}, Z_{CG}, Ixx, Iyy, Izz, Ixy, Iyz, & Izx

Optimization Step 2: Improve Orthogonality of Mass Matrix $\underline{M}$

Optimization Step 3: Update Frequencies & Mode Shapes

End

Aero Model A

Flutter Analysis

Measure Weight, C.G., & Moment of inertia GVT

Φ_G, ω_G

Wind

Before
GVT
After

Mathematical Background of the Model Tuning Technique

❑ Optimization Problem Statement

    ❖ Minimize J_i

    ❖ Such that $|J_k| \leq \varepsilon_k$ $k = 1...13$ & $k \neq i$

❑ Step 1: Improve Rigid Body Mass Properties

    ❖ Errors in Total Mass

    ❖ Errors in CG Locations

    ❖ Errors in Mass Moment of Inertias

Mass Properties	Objective Functions & Constraints
Total Mass	$J_1 = (W - W_G)^2 / W_G^2$
CG Locations	$J_2 = (X - X_G)^2 / X_G^2$
	$J_3 = (Y - Y_G)^2 / Y_G^2$
	$J_4 = (Z - Z_G)^2 / Z_G^2$
Mass Moment of Inertias	$J_5 = (I_{XX} - I_{XXG})^2 / I_{XXG}^2$
	$J_6 = (I_{YY} - I_{YYG})^2 / I_{YYG}^2$
	$J_7 = (I_{ZZ} - I_{ZZG})^2 / I_{ZZG}^2$
	$J_8 = (I_{XY} - I_{XYG})^2 / I_{XYG}^2$
	$J_9 = (I_{YZ} - I_{YZG})^2 / I_{YZG}^2$
	$J_{10} = (I_{ZX} - I_{ZXG})^2 / I_{ZXG}^2$

❑ Step 2: Improve Mass Matrix

❖ Off-diagonal terms of Orthonormalized Mass Matrix: $\underline{\mathbf{M}} = \mathbf{\Phi}_G^T\, \mathbf{T}^T\mathbf{M}\, \mathbf{T}\mathbf{\Phi}_G$

Guyan reduction

$$T = T_G = \begin{bmatrix} I \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \qquad M = \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \qquad K = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix}$$
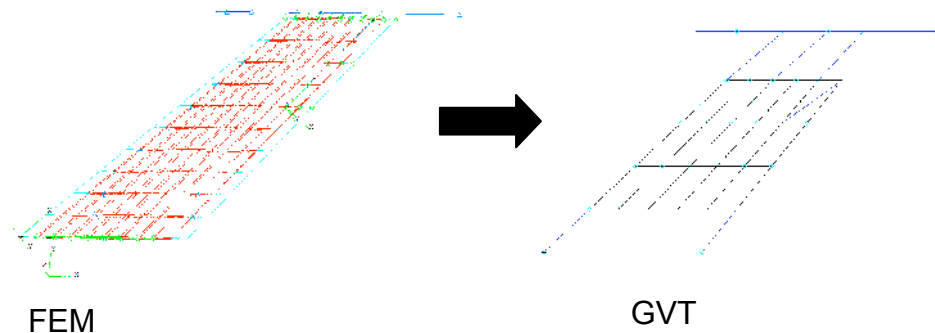
Improved reduction system

$$T = T_{IRS} = \begin{bmatrix} I \\ -K_{ss}^{-1}K_{sm} + (K_{ss}^{-1}M_{ss} - K_{ss}^{-1}M_{ss}K_{ss}^{-1}K_{sm})M_G^{-1}K_G \end{bmatrix}$$

$$M_G = T_G^T M T_G$$

$$K_G = T_G^T K T_G$$

$$J_{11} = \sum_{i=1, j=1, i\neq j}^{n} \underline{M}_{ij}^2$$

FEM

GVT

❑ Step 3: Frequencies and Mode Shapes

❖ Errors in Frequencies

$$J_{12} = \sum_{i=1}^{n} \left(\frac{\Omega_i - \omega_i}{\Omega_i} \right)^2$$

❖ Option 1: Off-diagonal terms of Orthonormalized Stiffness Matrix: $\underline{\mathbf{K}} = \mathbf{\Phi}_G{}^{\mathrm{T}} \mathbf{T}^{\mathrm{T}} \mathbf{K} \mathbf{T} \mathbf{\Phi}_G$

$$J_{13} = \sum_{i=1, j=1, i \neq j}^{n} \underline{K}_{ij}^2$$

❖ Option 2: Errors in Mode Shapes

$$J_{13} = \sum_{i=1}^{m} \left(\mathbf{\Phi}_i - \mathbf{\Phi}_{iG} \right)^2$$

n: number of modes   m: number of sensors

Sample 1: Cantilevered Square Plate

❑ Isotropic Plate
  ❖ 10" Chord, 10" Span, & 0.1" Thickness
  ❖ Young's modulus = 10^7 psi
  ❖ Poisson's ratio = 0.3
  ❖ Mass density = 2.39×10^{-4} slug/in^3

❑ Finite Element Model
  ❖ 16 CQUAD (quadrilateral) elements
  ❖ 100 (= 20 x 5) DOF
  ❖ 12 sensors

❑ Design Variables
  ❖ Starting Values
    ➢ Thickness: 0.5"
    ➢ Mass density: 5.00×10^{-4} slug/in^3

❑ Object Function & Constraints
  ❖ Object Function: J_{12}
  ❖ Constraints: J_1 thru J_{11} & J_{13}

❑ Optimizer
  ❖ Based on "Genetic Algorithm"
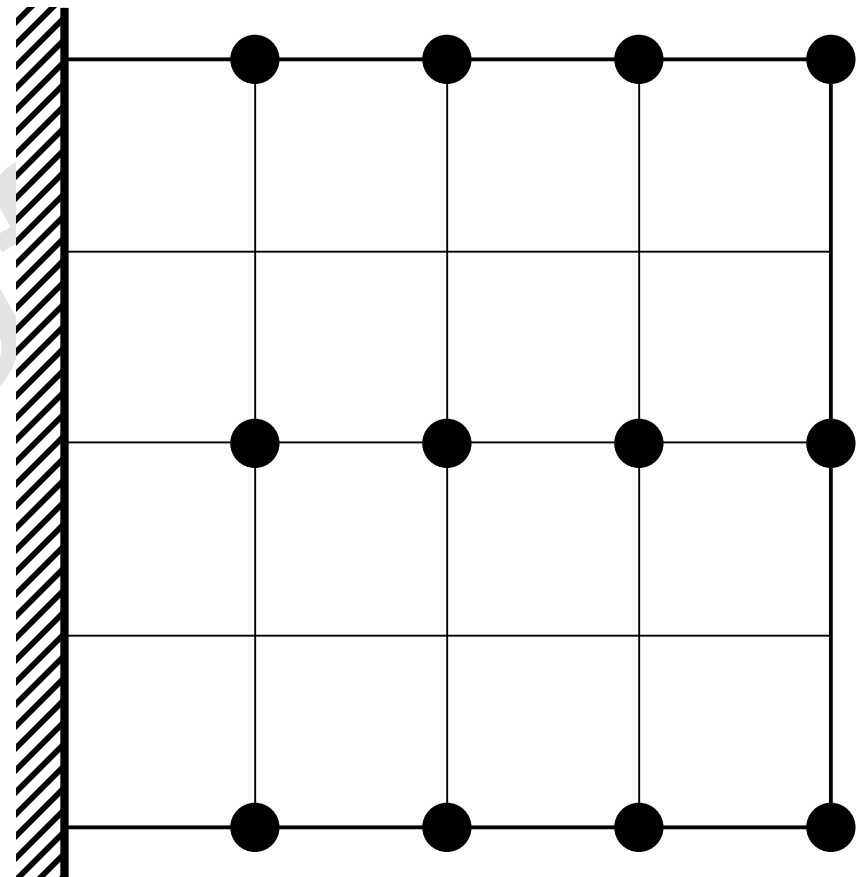
Cantilevered Square Plate : Results

		Target	Initial (error %)	Final (error %)
Design Variables	Thickness	.100	0.5 (400)	.0997 (-0.3)
	Mass Density	.000239	.0005 (109)	.000243 (1.67)
Rigid Mass Properties	Total Mass	.00239	.025 (946)	.00239 (0.0)
	x CG	5.0	5.0 (0.0)	5.0 (0.0)
	y CG	5.0	5.0 (0.0)	5.0 (0.0)
	z CG	0.0	0.0 (0.0)	0.0 (0.0)
	Ixx	.0224	.234 (947)	.0224 (0.0)
	Iyy	.0224	.234 (947)	.0224 (0.0)
	Izz	.0448	.468 (947)	.0448 (0.0)
	Ixy, Iyz, Izx	0.0	0.0 (0.0)	0.0 (0.0)
Frequencies	Mode 1, Hz	33.27	114.8 (245)	33.12 **(-0.47)**
	Mode 2, Hz	77.84	265.0 (240)	77.89 **(-0.07)**
	Mode 3, Hz	187.9	650.7 (246)	188.71 **(-0.42)**
MAC Values	Mode 1	1.000	.999	.999
	Mode 2	1.000	.999	.999
	Mode 3	1.000	.996	.999

Cantilevered Square Plate : Results (continue)
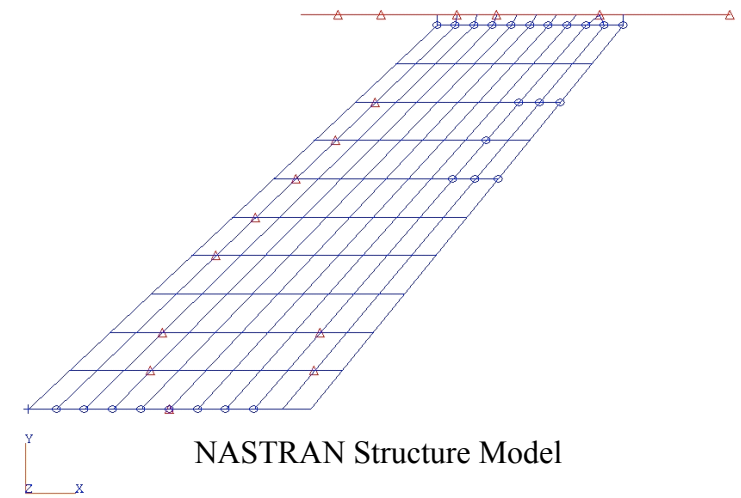
Objective Function Histories
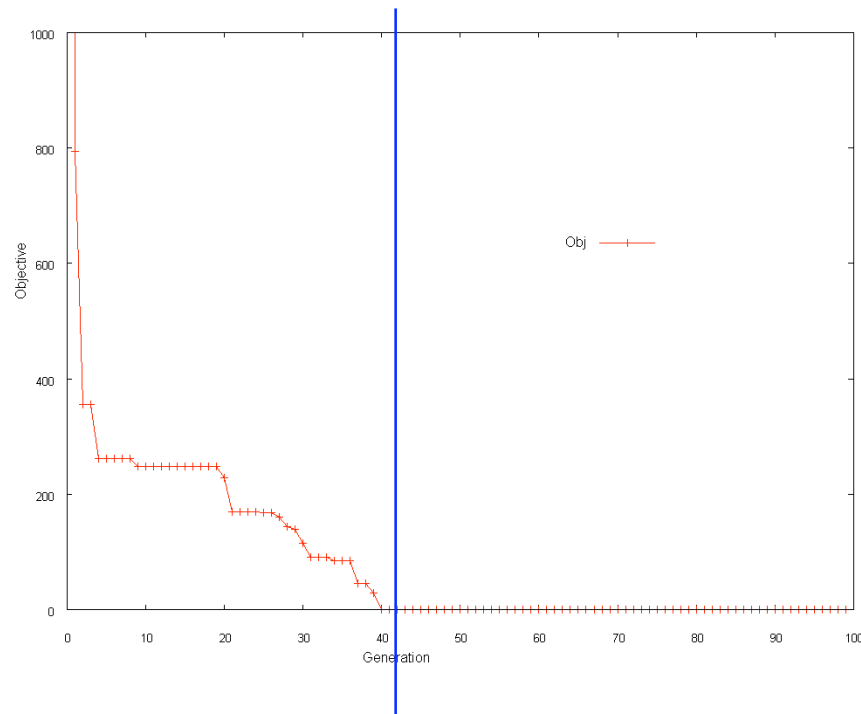
❑ Objective Function

   ❖ $J_{12} = 0.40978 \times 10^{-4}$

❑ Converges after 40 generations

Sample 2: Aerostructures Test Wing

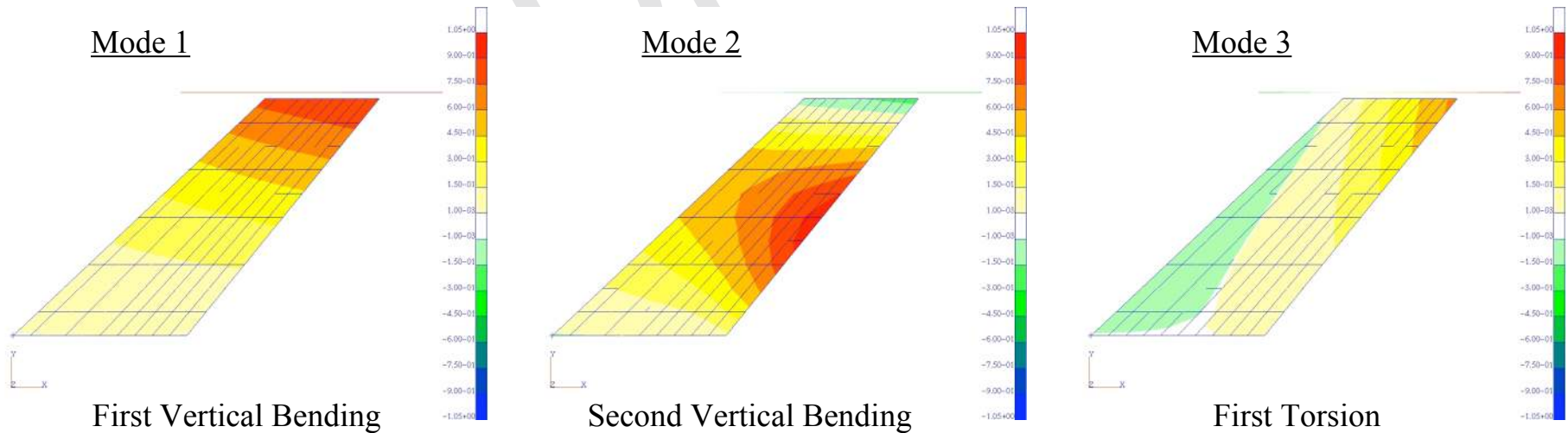| Number of DOFs in FE Model	1311
Number of Accelerometers for GVT	35

NASTRAN Structure Model

Aerostructures Test Wing: Results

Frequency Comparisons

	GVT	Before Optimization			After Optimization without J_{11}			After Optimization with J_{11}		
	Frequency (Hz)	Frequency/Error (Hz/%)		MAC Value	Frequency/Error (Hz/%)		MAC Value	Frequency/Error (Hz/%)		MAC Value
		Guyan	Full order		Guyan	Full order		Guyan	Full order	
Mode 1	13.76	13.35/-3.0	13.35/-3.0	99	13.75/-0.1	13.75/-0.1	98	13.41/-2.5	13.41/-2.5	95
Mode 2	20.76	22.82/9.9	22.82/9.9	99	20.76/0.0	20.76/0.0	99	21.01/1.2	21.01/1.2	97
Mode 3	77.83	79.06/1.6	78.77/1.2	95	77.82/-0.0	77.84/0.0	95	77.87/0.1	77.50/-0.4	95

Mode 1

First Vertical Bending

Mode 2

Second Vertical Bending

Mode 3

First Torsion

Mass Properties

	Measured	Before Optimization			After Optimization without J_{11}			After Optimization with J_{11}		
Weight	2.66 lb	2.77 lb (error 4.1%)			2.67 lb (error 0.4%)			2.70 lb (error 1.5%)		
X_{CG}	N/A	12.94 inch			12.88 inch			12.72 inch		
Y_{CG}	N/A	9.16 inch			8.80 inch			8.91 inch		
Z_{CG}	N/A	0.0 inch			0.0 inch			0.0 inch		
I_{XX}	N/A	161.22 lb-inch^2			152.06 lb-inch^2			154.78 lb-inch^2		
I_{YX}	N/A	95.27 lb-inch^2			93.75 lb-inch^2			89.45 lb-inch^2		
I_{YY}	N/A	113.08 lb-inch^2			112.83 lb-inch^2			102.57 lb-inch^2		
I_{ZX}	N/A	0.011 lb-inch^2			0.010 lb-inch^2			0.010 lb-inch^2		
I_{ZY}	N/A	-0.028 lb-inch^2			-0.035 lb-inch^2			-0.033 lb-inch^2		
I_{ZZ}	N/A	268.2 lb-inch^2			258.79 lb-inch^2			251.26 lb-inch^2		
Orthonormalized Mass Matrix		1	8.9 %	17.7 %	1	15.7 %	14.8 %	1	3.0 %	11.9 %
		.089	1	9.3 %	.157	1	10.9 %	.030	1	2.8 %
		.177	.093	1	.148	.109	1	.119	.028	1

Objective Function Histories

❑ Objective function

    ❖ Without J_{11} constraint equation, objective function converges after 21 generations

    ❖ With J_{11} constraint equation, objective function converges after 60 generations

Conclusions

❑ Simple and efficient model tuning capabilities based on optimization problem are successfully integrated with the MDAO tool developed at NASA Dryden Flight Research Center.

    ❖ Structural properties are matched to the measured target data.

        ➢ Mass properties, Orthonormalized mass & stiffness matrices, Frequencies, & mode shapes

    ❖ Performance Indices: J_1 through J_{13}

        ➢ If J_i is selected as objective function, then

        ➢ All or part of J_k (k≠i) can be selected as constraints.

❑ Two examples are used to demonstrate the application of this model tuning process.

    ❖ Cantilever plate – Idealized problem

        ➢ Use J_{12} as objective function

        ➢ Use J_1 thru J_{11}, J_{13} as constraint equations

    ❖ Aerostructures Test Wing – Practical problem

        ➢ Based on available experimental data

        ➢ Use J_{12} as objective function

            ✓ Use J_1 and J_{13} as constraint equations.

            ✓ Use J_1, J_{11} and J_{13} as constraint equations

Questions ?