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Research Article

Structural Reanalysis Based on FRFs Using Sherman–Morrison–Woodbury Formula

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Structural dynamic modification is a popular approach to obtain desire frequencies and dynamic characteristics. It has been observed that reanalyzing the modified structure usually involves complicated calculations when modifications are concerned with numerous degrees of freedom (DOFs), especially adding substructures to these DOFs. This paper proposed a method to reanalyze the frequency response functions (FRFs) of structures with multiple co-ordinates modifications. Two different cases are taken into consideration in the modifications, including adding (or decreasing) masses, stiffness, and damping, as well as adding spring-mass substructures, which makes the method more practical. This method is developed by employing Sherman–Morrison and Woodbury (SMW) formula based on the FRFs related to the modifications coordinates of the original system. The advantage of this method is that neither a physical model nor a modal model is required; instead, it needs only the FRFs, which can be directly measured by experimental modal testing. Another salient feature of this proposed strategy is that the FRFs of the modified structure can be calculated in only one step. Validation of this proposed method is demonstrated using various numerical examples. It is shown that the method is very effective and can be considered for real applications.

1. Introduction

Structural dynamic modification has been applied widely in practical engineering. In many engineering cases, structural dynamic modifications are used to obtain better dynamic characteristics of structures. Generally speaking, structural dynamic modification refers to a method to get certain structural dynamic characteristics by changing the local physical parameters (masses, stiffness, and damping), such as the need to avoid resonance or assignment of certain frequencies on desired locations. Structural dynamic modification is an economical and effective means of improving the dynamic characteristics of mechanical structures. This approach is widely used in aerospace, marine, automotive, civil engineering, bridge, and machinery industries.

The problems involved in structural dynamic modification can be divided into two categories: "forward problem" and "inverse problem." The former aims to predict exact change to the structure's dynamic properties when known modifications are made at a given location [1, 2]. The latter mainly studies how to modify an existing structure in order to achieve the expected dynamic characteristics (such as natural frequency and mode shape) [3–6]. The forward structural dynamic modification is also called structural reanalysis in early studies. In structural reanalysis, the dynamic effects of a modification on a structure are treated as an analysis problem involving the known dynamic properties of the original structure rather than a complete reanalysis of the modified structure [7]. This approach could avoid the blindness of the design, which reduces the design cost and has practical engineering application value.

Many works and literatures had conducted a comprehensive analysis of reanalysis methods. Reanalysis methods can be generally divided into two categories: direct methods and approximate methods. The approximate methods can be divided into three categories [8]: global approximation, local approximation, and combined approximation.

For the development of direct methods, early studies of structural reanalysis were reviewed and summarized by Baldwin and Hutton [7]. Several approaches such as Rayleigh quotient [9], sensitivity analysis [10], and perturbation approach [11] were used to address forward modification problems without a complete reanalysis of the whole structure.

This important issue has also been extensively discussed in recent years, and part of the relevant literature is summarized as follows. This problem was explored in [12] using developed successive matrix inverse method based on symmetry of corresponding stiffness matrix after constraint modification of the boundary. The numerical examples show that this method could quickly give accurate reanalysis results. In the same year, Liu et al. [13] proposed an approach for structural static reanalysis with unchanged number of degrees of freedom. This approach was based on a new preconditioner constructed by updating the Cholesky factorization of the original stiffness matrix, which could achieve fast convergence and accurate results. After two years, Song et al. [14] suggested using a direct reanalysis algorithm based on finding updated triangular factorization in sparse matrix solution to solve this problem. This algorithm is suitable for local modification, and the examples show that the algorithm improves reanalysis efficiency significantly, especially for high-rank structural modification. Later on, a "cheap" algorithm, named independent coefficients method, was put forward to reanalyze structures with local modification, which leads to a low-rank change in the stiffness matrix [15]. Considering that previous work [13] suffers limitation of a structure as added degrees of freedom, a new and efficient reanalysis method [16] had been proposed by the same author. Another advantage of this method is that the Cholesky factorization of the stiffness matrix of the modified structure can be used as the initial information for reanalysis when the structure is further modified. In the work of [17], the issue was suggested using a nonlinear reanalysis method based on structural modification of residual incremental approximations. In contrast to other existing nonlinear reanalysis methods, which were based on the evaluation of changed stiffness matrices, only residual vectors need to be computed and stored. Kim and Eun [18] studied coupling and recoupling reanalysis methods. These methods were performed by using the concept of compatibility conditions at interface nodes between the substructures or between the original structure and the substructures. The most recent theory proposed in [19] offered a new method for free-vibration reanalysis after structural topological modifications with added degrees of freedom. The implementation of this approach involved only LDL^T factorization of shifted substiffness matrices corresponding to the newly added DOFs, and the proposed method consists of matrix-matrix operations.

As for approximate methods, the combined approximation method is an effective solution method that combines the high efficiency of the local approximation method with the high quality of the global approximation method. The solution process of the method is based on the results of an accurate single-point analysis, and it is also a reanalysis method that has developed very fast in recent years. The original purpose of studying the CA method is to accelerate the optimization design. CA method has been widely used in many fields such as linear or nonlinear static analysis, dynamic analysis, modal analysis, and sensitivity analysis [8].

For the combined approximation method of reanalysis, Kirsch [20] first used forward and reverse substitution calculations to calculate the terms used as the basis vector binomial sequence in the CA method solving process. A new set of uncoupled basis vectors are generated and normalized by using the Gram–Schmidt orthogonalization process. This method can achieve an effective and accurate approximation for very large design changes. By 2006, Kirsch et al. [21] used combined approximation method to overcome the repeated eigenproblem solution of nonlinear dynamic reanalysis and solved its main problem. The method is based on the integration of several concepts and methods, including the basis of matrix factorization, series expansion, and reduction. In order to solve the frequency-constrained structural optimization problem, Zuo et al. [22] proposed an adaptive eigenvalue reanalysis method based on genetic algorithm for structural optimization. The modified impulse analysis method is a combination approximation method from Kirsch, and it has a high level for repeated eigenvalue problems accuracy. Considered to integrate the Kirsch's method into the result optimization process, a new adaptive method [23] that used the K condition number to determine the minimum number of basis vectors was proposed. Besides, on the reanalysis of sensitivity, Zuo et al. [24] proposes a new method for arbitrarily changing static displacement sensitive design variables. This method uses Taylor series expansion to approximate the current displacement of the modified sensitivity equation and then solves the direct sensitivity equation by a combined approximation method. One year later, the same author [25] conducted a sensitivity analysis of eigenvalues and eigenvectors using a combination approximation method, and the eigenvectors were solved by the Nelson method. This method can greatly improve the efficiency of sensitivity analysis and can accelerate the gradient-based structural optimization constraints with frequency and mode shape.

All these researches mentioned above are primarily based on the physical model, which requires the knowledge of mass, stiffness, and damping matrices. In practical engineering, however, these parameters matrices of vibration system structures are not easy to obtain. This is because the structure to be modified is usually a complex structure with multiple DOFs. Furthermore, in the process of reanalysis calculation, it is usually not very difficult to solve the single-element change problem. When it comes to multiple-element change structures, however, the calculation becomes more complicated. Therefore, these problems limit the application of the above methods to some extent.

This paper proposed a method for reanalyzing FRFs of the modified structure. This method is developed by employing SMW formula [26, 27] based on the FRFs related to modifications coordinates of the original system. The advantage of this FRFs-based method is that the FRFs can be directly measured by model testing, without knowledge of the system matrices M, C, K, which are usually unavailable in practical engineering. Another salient feature of this proposed strategy is that the FRFs of a structure with multiple-elements changing can be calculated in only one step, which improves the efficiency of reanalysis.

2. Theoretical Development

The equation of motion of a free-vibration damping multi-degree-of-freedom system can be expressed as

$$M\ddot{x} + C\dot{x} + K = 0. \tag{1}$$

The dynamic stiffness matrix of original structure can be given by

$$Z = K - M\omega^2 + j\omega C, \tag{2}$$

where Z is the dynamic stiffness matrix of original structure; ω represents the frequency variable and $j = \sqrt{-1}$.

2.1. Adding Masses, Stiffness, and Damping to the Original Model Structure. It is assumed that the local modification of the structure involves n coordinate points, labeled $1, 2, \ldots, n$. The additional masses, stiffnesses, and damping at these points are denoted as $\Delta m_1, \Delta m_2, \ldots, \Delta m_n$; $\Delta k_1, \Delta k_2, \ldots, \Delta k_n$; $\Delta c_1, \Delta c_2, \ldots, \Delta c_n$, respectively. The values of these additional stiffnesses, additional masses, and additional damping can be positive or negative. If the value is positive, then additional parameters are added to the original structure, while a negative value means that additional parameters are reduced from the original structure.

The above additional masses, stiffnesses, and damping can be expressed as the diagonal matrix of (3), (4), and (5), respectively.

$$\Delta M = \begin{bmatrix} \Delta m_1 & & \\ & \ddots & \\ & & \Delta m_n \end{bmatrix}, \tag{3}$$

$$\Delta K = \begin{bmatrix} \Delta k_1 & & \\ & \ddots & \\ & & \Delta k_n \end{bmatrix}, \tag{4}$$

$$\Delta C = \begin{bmatrix} \Delta c_1 & & \\ & \ddots & \\ & & \Delta c_n \end{bmatrix}. \tag{5}$$

After adding additional masses ΔM, additional stiffnesses ΔK, and additional damping ΔC,

$$Z^* = K + \Delta K - (M + \Delta M)\omega^2 + j\omega(C + \Delta C) = Z + \Delta Z, \tag{6}$$

where Z^* is the structure dynamic stiffness matrix after adding additional parameters to original structure.

$$\Delta Z = \Delta K - \Delta M\omega^2 + j\omega\Delta C. \tag{7}$$

To make ΔZ more intuitive, it is expressed by the following formula:

$$\Delta Z = \sum_{k=1}^{n} U_k V_k^T, \tag{8}$$

where U_k is a column vector of $n \times 1$ in which element of the k_{th} row is 1, and the other elements are zero. V_{k} represents a column vector of $n \times 1$ in which element of the k_{th} row is and the other elements are zero. Then the column vectors U and V can be given by

$$U_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots, U_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} V_1 = \begin{bmatrix} \Delta k_1 - \Delta m_1\omega^2 + j\omega\Delta c_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots, V_N = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Delta k_n - \Delta m_n\omega^2 + j\omega\Delta c_n \end{bmatrix}. \tag{9}$$

According to SMW formula [28], the dynamic stiffness Z^{*-1} of modified structure with multielement change is simply written as

$$Z^{*-1} = (Z + \Delta Z)^{-1} = Z^{-1} - Z^{-1}\left[U_1 \ \cdots \ U_N\right]W^{-1} \cdot \left[V_1^T \ \cdots \ V_N^T\right]^T Z^{-1}, \tag{10}$$

where

$$W = \begin{bmatrix} 1 + V_1^T H U_1 & V_1^T H U_2 & \cdots & V_1^T H U_n \\ V_2^T H U_1 & \ddots & & \vdots \\ \vdots & & \ddots & \\ V_n^T H U_1 & \cdots & & 1 + V_n^T H U_n \end{bmatrix}. \tag{11}$$

Since the dynamic stiffness matrix of structure and the FRFs matrix are inverse matrices of each other, then

$$Z^{*-1} = H^* = H - H\left[U_1 \ \cdots \ U_N\right]W^{-1}\left[V_1^T \ \cdots \ V_N^T\right]^T H, \tag{12}$$

where H is FRFs matrix of original structure; H^* is FRFs matrix of the structure after adding additional masses, stiffnesses, and damping.

It is obvious that the FRFs matrix H^* can be calculated according to (12) when the FRFs H of original structure and added masses matrix ΔM, stiffnesses matrix ΔK, and damping matrix ΔC are known.

It should be noted that the FRFs matrices H and H^* mentioned above are displacement FRFs matrices. In the practical engineering cases, acceleration sensors are usually used to measure the response, so accelerance Ha is directly obtained. The relationship between H and Ha can be described as

$$Ha = -\omega^2 H. \tag{13}$$

Therefore, once accelerance matrix Ha of the original structure is obtained in practical application, the receptances H can be calculated according to (13), then substituting it into (10) and (11) to calculate receptances H^* of the modified structure, where H^* could be expressed as

$$H^* = H - H\left[U_1 \ \cdots \ U_N\right] W^{-1} \left[V_1^T \ \cdots \ V_1^T\right]^T H. \tag{14}$$

Finally, the obtained H^* is substituted into (13) to calculate accelerances Ha^* of the modified structure.

2.2. Adding Spring-Mass Substructures to the Original Structure. In many engineering cases, the vibration system structures are designed to be immutable. When original structure does not satisfy the dynamic characteristics, adding spring-mass substructures to the original structural system is not a bad idea. The added spring-mass substructure is shown in Figure 1.

For the reason of adding a spring-mass substructure, the DOFs of original system has been changed, and one DOF is added to original system. The above calculated method is not suitable for this kind of condition anymore, and thus transforming the DOFs of modified structure to ones of original structure is needed [29].

The equation of motion of a linear undamped multi-degree-of-freedom system can be expressed as

$$M\ddot{x} + Kx = f. \tag{15}$$

Assuming that the mass and stiffness modification of original system are ΔM, ΔK, then (15) can be written as

$$(M + \Delta M)\ddot{x} + (K + \Delta K)x = f. \tag{16}$$

Assuming harmonic response $x = ue^{i\omega t}$ and substituting it into (15), dynamic stiffness matrix Z and FRFs H of the system can be written as follows:

$$\begin{aligned} Z(\omega) &= -\omega^2 M + K, \\ H(\omega) &= \left(-\omega^2 M + K\right)^{-1}. \end{aligned} \tag{17}$$

Then (16) yields

$$H^{-1}u = \left(\omega^2 \Delta M - \Delta K\right)u + f. \tag{18}$$

It is assumed that a spring-mass substructure is added at the i_{th} freedom of original structure. The mass and stiffness are Δm, Δk and the relative amplitude of vibration is Δu. Since one substructure is added to the original system structure, an extra freedom is added. The DOFs of original structure are changed from n to $n+1$, and the matrices in (18) are enlarged by one row and column. Then, the equation of motion of the modified system is described by

$$\begin{pmatrix} H_{n\times n}^{-1} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ \Delta u \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -\Delta k & \cdots & 0 & \Delta k \\ \vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \Delta k & \cdots & 0 & -\Delta k + \omega^2 \Delta m \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ \Delta u \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \\ 0 \end{pmatrix}. \tag{19}$$

The last row of above equation is

$$\Delta k u_i + \left(-\Delta k + \omega^2 \Delta m\right)\Delta u = 0. \tag{20}$$

Substitute Δu with u_i

$$\Delta u = \left(\frac{\Delta k}{\Delta k - \omega^2 \Delta m}\right) u_i. \tag{21}$$

Substituting (21) into (19), the right matrix of (19) can be written as follows:

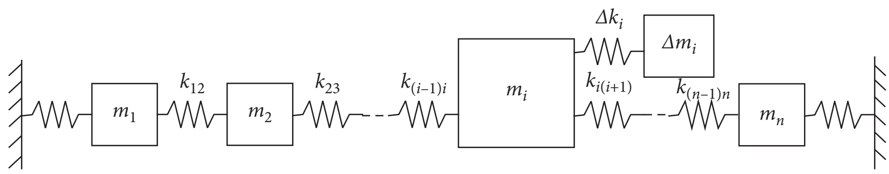

Figure 1: Adding a spring-mass substructure on the i_{th} coordinate of the n DOF system.

$$\begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -\Delta k & \cdots & 0 & \Delta k \\ \vdots & \vdots & & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \Delta k & \cdots & 0 & -\Delta k + \omega^2 \Delta m \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \\ \left(\dfrac{\Delta k}{\Delta k - \omega^2 \Delta m}\right) u_i \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \\ 0 \end{pmatrix}. \tag{22}$$

Considering the i_{th} and $(n+1)_{\text{th}}$ row of (22), one can obtain

$$-\Delta k u_i + \left(\frac{(\Delta k)^2}{\Delta k - \omega^2 \Delta m}\right) u_i + f_i = \left(\frac{\omega^2 \Delta m \Delta k}{\Delta k - \omega^2 \Delta m}\right) u_i + f_i$$

$$\Delta k u_i + \left(\frac{\omega^2 \Delta m - \Delta k}{\Delta k - \omega^2 \Delta m}\right) \Delta k u_i = 0. \tag{23}$$

Then (19) can be written as follows:

$$H_{n\times n}^{-1} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & 0 & \cdots & \dfrac{\omega^2 \Delta m \Delta k}{\Delta k - \omega^2 \Delta m} & \cdots & 0 \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_i \\ \vdots \\ u_n \end{pmatrix} + \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{pmatrix}. \tag{24}$$

It is easy to observe from (24) that when adding one spring-mass substructure at the i_{th} freedom of the original structure, element in the i_{th} row represented by V_k in the (8) is $-(\omega^2 \Delta m_k \Delta k_k / \Delta k_k - \omega^2 \Delta m_k)$ in this situation, and the other elements of V_{k} are zero; namely,

$$V_1 = \begin{pmatrix} -\dfrac{\omega^2 \Delta m_1 \Delta k_1}{\Delta k_1 - \omega^2 \Delta m_1} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, V_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ -\dfrac{\omega^2 \Delta m_n \Delta k_n}{\Delta k_n - \omega^2 \Delta m_n} \end{pmatrix}. \tag{25}$$

Without changing original structure of the system and adding one substructure to a certain coordinate of the system, after added one substructure, the DOFs of original system structure are changed from n to $n+1$. According to a series of numerical operations, the DOFs of the system structure after adding one substructure are transformed from $n+1$ to n, which could meet the calculation approach proposed in this paper.

3. Verification of the Method

In this section, three simulated examples are analyzed by the proposed method.

3.1. Numerical Experiment Setup. To verify the accuracy of the above method, a cantilever beam modal test model is constructed as shown in Figure 2. The physical parameters are shown in Table 1. The cantilever beam is discretely divided into 6 equal parts along the length direction, and 6 measuring points are evenly distributed.

As shown in Figure 3. *Ha-ori-lp* are the accelerances curves of original structure, where Ha_{lp} are calculated accelerances relating points l and p.

Two kinds of examples are given to prove this approach proposed in this paper: one is to directly increase or decrease masses, stiffnesses, or damping on original structure. The other is to add a spring-mass substructure to a certain coordinate or multiple coordinates to the original structure. What should be pointed out is that the substructures are added in the vertical direction of the cantilever. For comparison purpose, exact accelerances Ha' corresponding with modified structure are also numerically calculated.

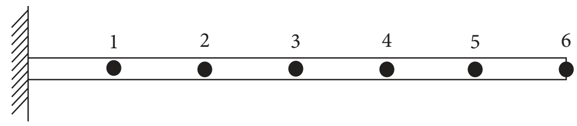

Figure 2: Cantilever beam modal test model.

Table 1: Cantilever beam physical parameters.

Parameter	l (m)	b (m)	h (m)	ρ (kg m^{-3})	E (MPa)
Value	1.5	0.05	0.012	7547	$2.07 * 10^5$

3.2. Adding Mass, Stiffness, and Damping to the Original Model Structure. As shown in Figure 4, the additional stiffness and damping are added at coordinates 2 and 4, stiffnesses Δk_2 and Δk_4 are 6000 N/m and 8000 N/m, respectively, and damping Δc_2 and Δc_4 are 30 Ns/m and 20 Ns/m, respectively. Additional masses are added at coordinates 2, 4, and 6, respectively, and the additional masses Δm_2, Δm_4 and Δm_6 are 0.38 Kg, 0.42 Kg, and 0.4 Kg, respectively. Response points are chosen at coordinates 2, 4, and 6. Hammer impact is moving sequentially from points 2, 4 to 6 for "measuring" accelerances $Ha = \begin{bmatrix} Ha_{22} & Ha_{24} & Ha_{26} \\ Ha_{42} & Ha_{44} & Ha_{46} \\ Ha_{62} & Ha_{64} & Ha_{66} \end{bmatrix}$. The goal of this example is to calculate the accelerances $Ha^* = \begin{bmatrix} Ha^*_{22} & Ha^*_{24} & Ha^*_{26} \\ Ha^*_{42} & Ha^*_{44} & Ha^*_{46} \\ Ha^*_{62} & Ha^*_{64} & Ha^*_{66} \end{bmatrix}$.

According to (3), (4), and (5), the additional mass matrix, stiffness matrix, and damping matrix in this example are

$$\Delta M = \begin{bmatrix} m_2 & & \\ & m_4 & \\ & & m_6 \end{bmatrix} = \begin{bmatrix} 0.38 & & \\ & 0.42 & \\ & & 0.4 \end{bmatrix},$$
$$\Delta K = \begin{bmatrix} k_2 & & \\ & k_4 & \\ & & k_6 \end{bmatrix} = \begin{bmatrix} 6000 & & \\ & 8000 & \\ & & 0 \end{bmatrix}, \qquad (26)$$
$$\Delta C = \begin{bmatrix} c_2 & & \\ & c_4 & \\ & & c_6 \end{bmatrix} = \begin{bmatrix} 30 & & \\ & 20 & \\ & & 0 \end{bmatrix}.$$

Expression of vector V obtained by (9) can be given as follows:

$$U_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} -\omega^2 \Delta m_2 + j\omega \Delta c_2 + \Delta k_2 \\ 0 \\ 0 \end{pmatrix},$$
$$U_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad V_4 = \begin{pmatrix} 0 \\ -\omega^2 \Delta m_4 + j\omega \Delta c_4 + \Delta k_4 \\ 0 \end{pmatrix}, \qquad (27)$$
$$U_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad V_6 = \begin{pmatrix} 0 \\ 0 \\ -\omega^2 \Delta m_6 + j\omega \Delta c_6 + \Delta k_6 \end{pmatrix}.$$

According to (13), receptances H of original structure can be obtained.

$$H = -\frac{Ha}{\omega^2} = \begin{bmatrix} H_{22} & H_{24} & H_{26} \\ H_{42} & H_{44} & H_{46} \\ H_{62} & H_{64} & H_{66} \end{bmatrix}. \qquad (28)$$

The accelerances H^* of modified structure can be calculated by (11) and (14).

$$H^* = H - H[U_2 \ U_4 \ U_6]W^{-1}[V_2 \ V_4 \ V_6]^T H,$$
$$W = \begin{bmatrix} 1 + V_2^T H U_2 & V_2^T H U_4 & V_2^T H U_6 \\ V_4^T H U_2 & 1 + V_4^T H U_4 & V_4^T H U_6 \\ V_6^T H U_2 & V_6^T H U_4 & 1 + V_6^T H U_6 \end{bmatrix}. \qquad (29)$$

It can be seen from the above that all the FRFs H (corresponding to original structure) required for

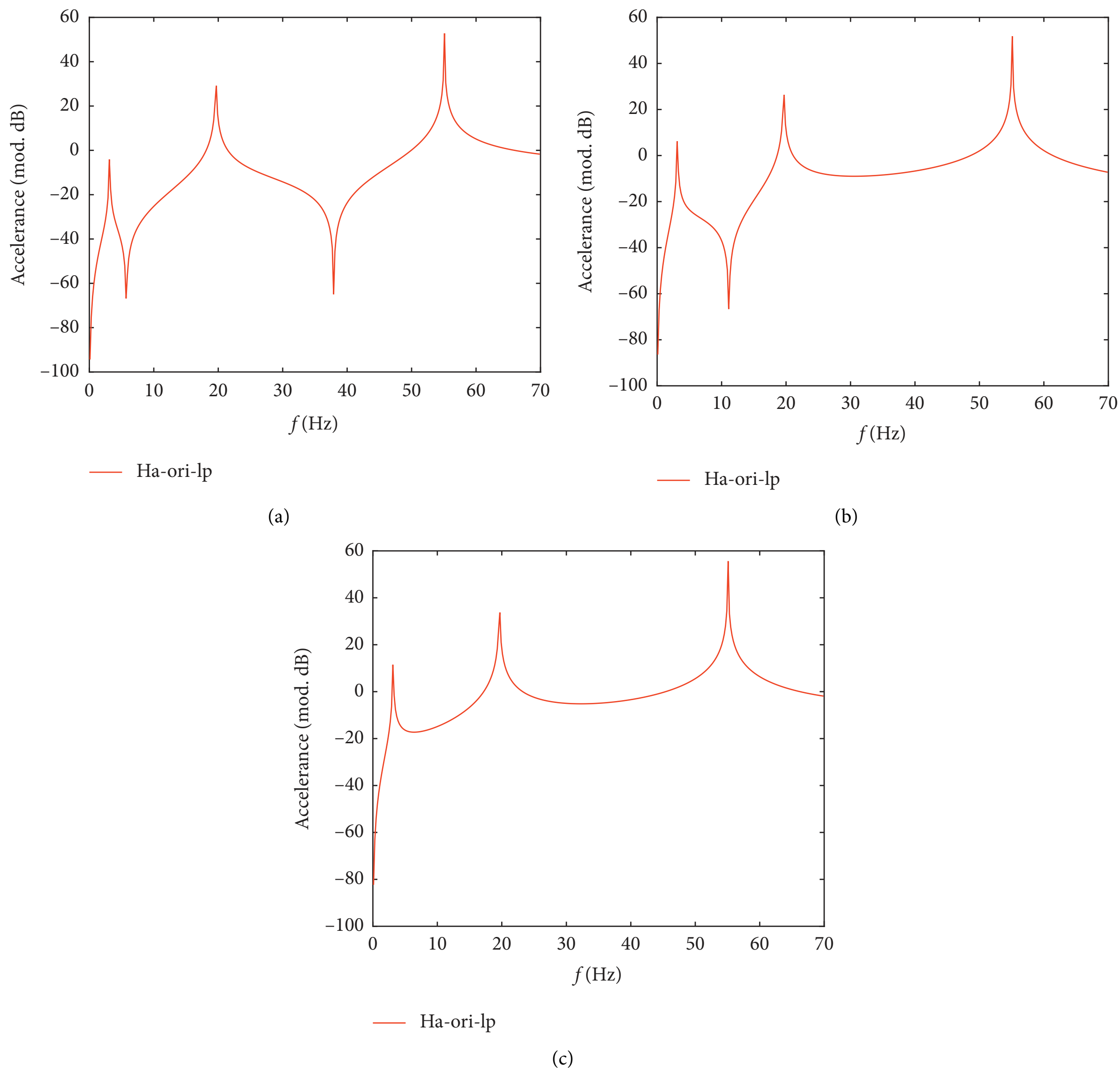

FIGURE 3: Accelerances of original structural, (a) original accelerance Ha_{22}, (b) original accelerance Ha_{24}, and (c) original accelerance Ha_{26}.

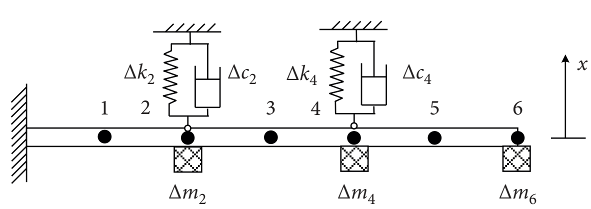

FIGURE 4: Adding mass, stiffness, and damping.

calculating H^* (corresponding to modified structure) are related to the modification coordinates. Then, accelerances Ha^* of modified structure can be directly calculated by (13).

$$Ha^* = -\omega^2 H^* = \begin{bmatrix} Ha^*_{22} & Ha^*_{24} & Ha^*_{26} \\ Ha^*_{42} & Ha^*_{44} & Ha^*_{46} \\ Ha^*_{62} & Ha^*_{64} & Ha^*_{66} \end{bmatrix}. \tag{30}$$

Although nine FRFs are calculated, only three of them are shown for the sake of brevity, as can be seen in Figure 5.

As shown in Figure 5, "exact" represents the exact accelerances curves obtained directly by numerical calculation. "Measured" represents the accelerances curves obtained by the method this paper proposed. "Original" represents the accelerances curves of original structure.

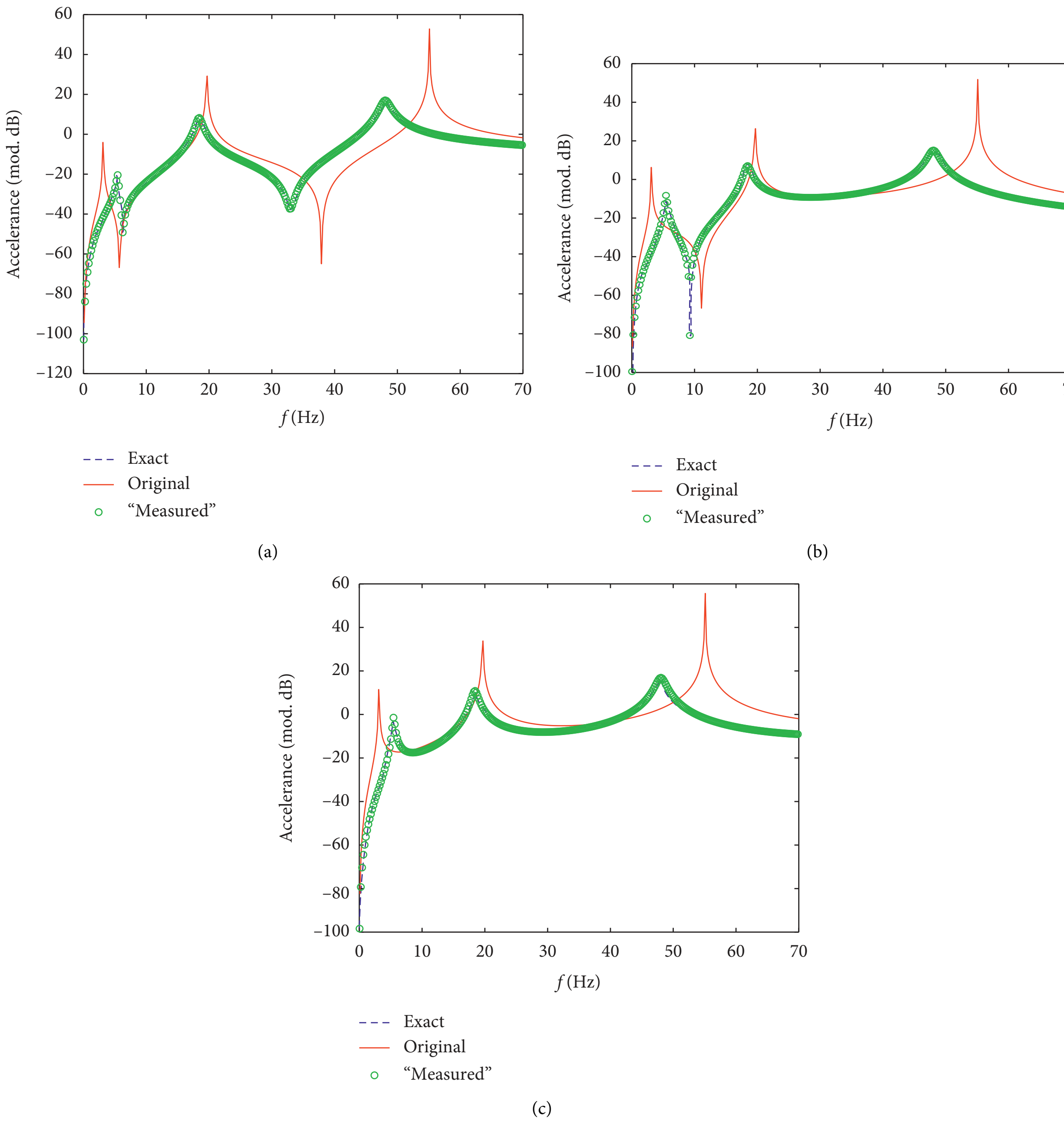

FIGURE 5: Comparison of exact, original, and "measured" accelerations, (a) accelerations Ha'_{22}, Ha_{22} and Ha^*_{22}, (b) accelerations Ha'_{24}, Ha_{24} and Ha^*_{24}, and (c) accelerations Ha'_{26}, Ha_{26} and Ha^*_{26}.

It is obvious from Figure 5 that, after dynamic modification to the original structure of vibration system, the accelerances curves "Measured" as a whole are in quite good agreement with those of target accelerances curve "exact", which indicates that the results of these two methods are consistent and accuracy of the method is verified.

3.3. Adding Spring-Mass Substructure to the Original Structure. One and multiple spring-mass substructures are, respectively, added to the original structure to calculate the FRFs of the modified structure for verification of the proposed method.

3.3.1. Adding One Spring-Mass Substructure to the Original Structure. As shown in Figure 6, a spring-mass substructure is added to the original structure at coordinate 1, where the stiffness Δk_1 of the substructure is 6000 N/m, and the mass Δm_1 of the substructure is 0.5 kg. For the accelerances of the structure after adding this spring-mass substructure, 4 natural frequencies are obtained within the range of 0–70 Hz. Response points are chosen at coordinates 1, 3, and 5. Hammer impact is moving sequentially from points 1, 3 to 5 for "measuring" accelerances $Ha = \begin{bmatrix} Ha_{11} & Ha_{13} & Ha_{15} \\ Ha_{31} & Ha_{33} & Ha_{35} \\ Ha_{51} & Ha_{35} & Ha_{55} \end{bmatrix}$. The goal of this example is to calculate the nine accelerances

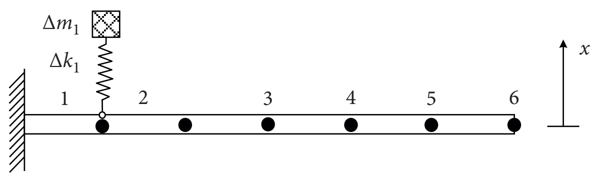

FIGURE 6: Adding one spring-mass substructure to the original structure.

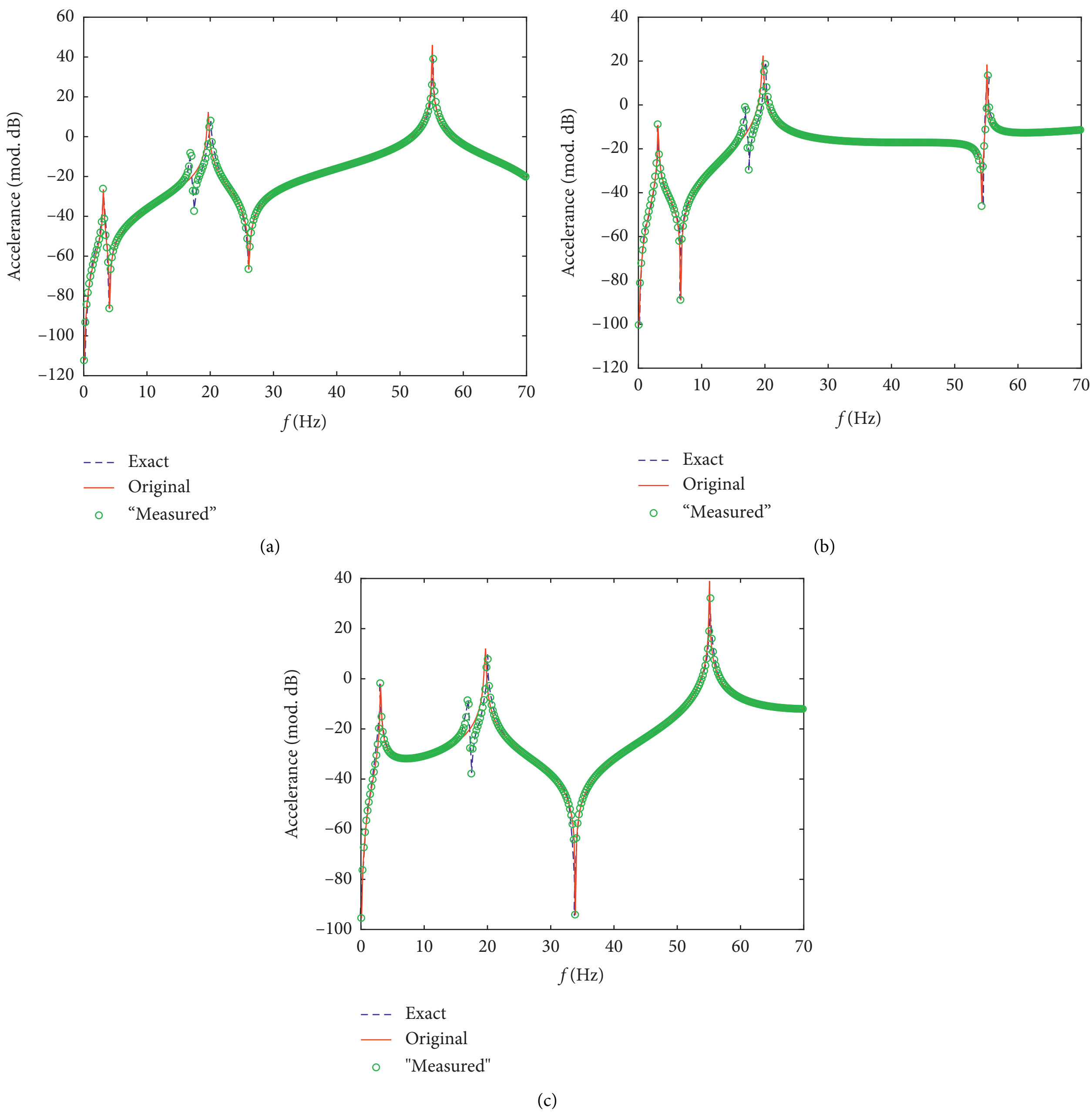

FIGURE 7: Comparison of exact, original, and "measured" accelerances, (a) accelerances Ha'_{11}, Ha_{11} and Ha^*_{11}, (b) accelerances Ha'_{13}, Ha_{13} and Ha^*_{13}, and (c) accelerances Ha'_{15}, Ha_{15} and Ha^*_{15}.

$Ha^* = \begin{bmatrix} Ha^*_{11} & Ha^*_{13} & Ha^*_{15} \\ Ha^*_{31} & Ha^*_{33} & Ha^*_{35} \\ Ha^*_{51} & Ha^*_{35} & Ha^*_{55} \end{bmatrix}$. Similarly, only three of them are shown for the sake of simplicity, as can be seen in Figure 7.

As shown in Figure 7, as one spring-mass substructure is added to the original structure, the exactly calculated accelerances curves of the modified structure and the ones calculated by the proposed method completely coincide.

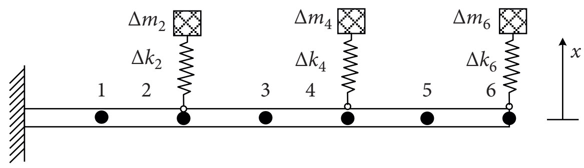

Figure 8: Adding three spring-mass substructures to the original structure.

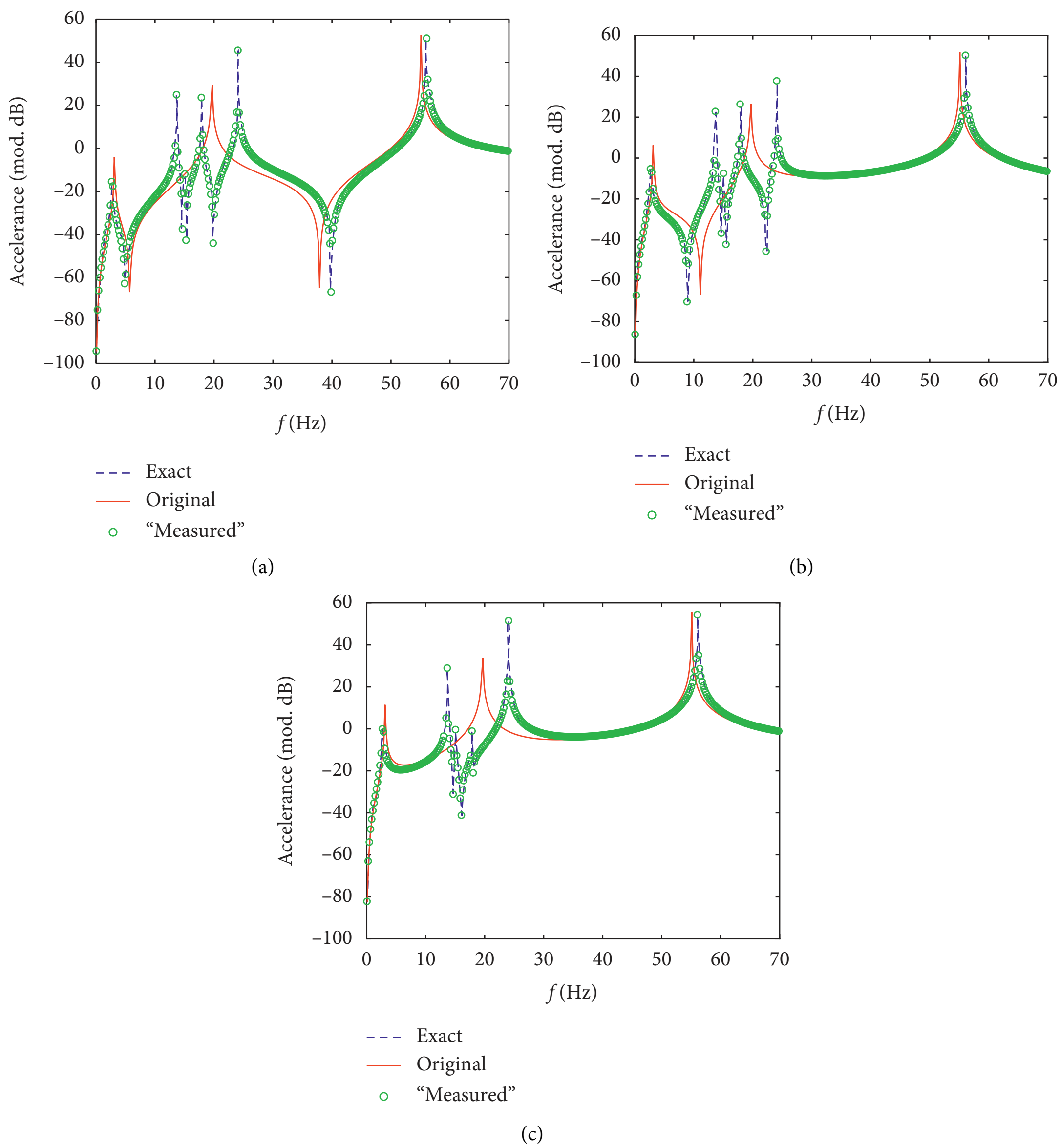

Figure 9: Comparison of exact, original, and "measured" accelerations, (a) accelerations Ha'_{22}, Ha_{22} and Ha^*_{22}, (b) accelerations Ha'_{24}, Ha_{24} and Ha^*_{24}, and (c) accelerations Ha'_{26}, Ha_{26} and Ha^*_{26}.

This result indicates that, in the condition of adding one spring-mass substructure, modified accelerations can be accurately calculated by the proposed method.

3.3.2. Adding Three Spring-Mass Substructures to the Original Structure. Figure 8 is a cantilever beam model after adding substructures. Three spring-mass substructures are added at coordinate points 2, 4, and 6, respectively. The masses Δm_2, Δm_4, and Δm_6 of these substructures are 0.35 kg, 0.42 kg, and 0.39 kg, respectively, and stiffness Δk_2, Δk_4 and Δk_6 of these substructures are 3000 N/m, 4000 N/m, and 5000 N/m, respectively.

According to (24) and (25), the vector U and V can be written as follows.

$$U_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

$$V_2 = \begin{pmatrix} -\dfrac{\omega^2 \Delta m_2 \Delta k_2}{\Delta k_2 - \omega^2 \Delta m_2} \\ 0 \\ 0 \end{pmatrix},$$

$$U_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$V_4 = \begin{pmatrix} 0 \\ -\dfrac{\omega^2 \Delta m_4 \Delta k_4}{\Delta k_4 - \omega^2 \Delta m_4} \\ 0 \end{pmatrix}, \quad (31)$$

$$U_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$V_6 = \begin{pmatrix} 0 \\ 0 \\ -\dfrac{\omega^2 \Delta m_6 \Delta k_6}{\Delta k_6 - \omega^2 \Delta m_6} \end{pmatrix}.$$

Response points are chosen at coordinates 2, 4, and 6. Hammer impact is moving sequentially from points 2, 4 to 6. Then accelerances $Ha = \begin{bmatrix} Ha_{22} & Ha_{24} & Ha_{26} \\ Ha_{42} & Ha_{44} & Ha_{46} \\ Ha_{62} & Ha_{64} & Ha_{66} \end{bmatrix}$ can be "measured". The accelerances to be calculated are $Ha^* = \begin{bmatrix} Ha^*_{22} & Ha^*_{24} & Ha^*_{26} \\ Ha^*_{42} & Ha^*_{44} & Ha^*_{46} \\ Ha^*_{62} & Ha^*_{64} & Ha^*_{66} \end{bmatrix}$. Only the curves of Ha^*_{22}, Ha^*_{22} and Ha^*_{22} are shown for the sake of brevity, as can be seen in Figure 9.

As shown in Figure 9, after adding three spring-mass substructures to the original structure, accelerances curves obtained by the proposed method coincide with the exactly values, which proves the accuracy of this method.

4. Conclusion

The work presented in this paper dealt with the problem of reanalyzing FRFs of a modified structure. A method is developed by employing SMW formula based on the FRFs related to the modification coordinates of original system. This method can solve the calculation problem of FRFs of modified structure in only one step. The accuracy and efficiency of this method are proved according to the simulated modal testing of cantilever beam model.

Two kinds of examples are used in verification. One is directly adding additional masses, stiffnesses, and damping to the cantilever beam. This kind of modification is mainly for the case that internal structural parameters matrices are changed. The other is adding spring-mass substructures to the cantilever beam. This condition is suitable for models whose original structure is immutable. Both numerical examples demonstrated good performance in the simulation verification. The results show that this approach is suitable for multiple-element change situation of structural modification and could improve the efficiency of structural reanalysis.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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