STRUCTURAL TOPOLOGY OPTIMIZATION AS A TEACHING TOOL IN THE ARCHITECTURE*

OTIMIZAÇÃO DE TOPOLOGIA ESTRUTURAL COMO FERRAMENTA DE ENSINO NA ARQUITETURA

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ABSTRACT

The student of architecture with the basic knowledge of the disciplines of structural engineering can easily understand structural processes more technologically developed. The use of optimization starts from the understanding of how the structure behaves statically and aims to generate physical models that address structural criteria, such as supports and loads, and functional criteria, such as the type of material used, openings and solid areas. By knowing the variables required for optimization, the geometric shape of the structural element tends to change to an organic architecture configuration. Even with optimization processes presenting a lot of mathematical complexity, this paper approaches the essential stages for teaching the structural topological optimization in undergraduate architecture. Also, through examples, propose practical and simplified models that can be easily implemented in the classroom and in the professional life of the architect, conferring excellent results in terms of functionality and structurally efficient. To achieve these results, three software were used in free and educational versions, such as *ForcePad, Topostruct* and *MIDAS NFX*. The studies were applied in the structural design of a vertical building, a footbridge, a hammerhead bridge pier and a cantilevered building.

Keywords: Architecture; teaching; structural optimization; topological optimization.

RESUMO

O estudante de arquitetura com o conhecimento básico das disciplinas de Engenharia Estrutural pode facilmente entender os processos estruturais mais desenvolvidos tecnologicamente. O uso da otimização parte do entendimento de como a estrutura se comporta estaticamente, e tem como objetivo gerar modelos físicos que contemplem critérios estruturais, como suportes e cargas; e critérios funcionais, como o tipo de material utilizado, aberturas e áreas sólidas. Conhecendo as variáveis necessárias para a otimização, a forma geométrica do elemento estrutural tende a mudar para uma configuração de arquitetura orgânica. Este trabalho aborda as etapas essenciais para o ensino da otimização topológica estrutural na arquitetura de graduação. Além disso, através de exemplos, propõe modelos práticos e simplificados que podem ser facilmente implementados em sala de aula e na vida profissional do arquiteto, conferindo excelentes resultados em termos de funcionalidade e estruturalmente eficientes. Para alcançar esses resultados, três softwares foram utilizados em versões gratuitas e educacionais, *ForcePad*, *Topostruct* e *MIDAS NFX*. Os estudos foram aplicados no projeto estrutural de um edifício vertical, uma passarela, um píer de *hammerhead bridge* e um edifício em balanço.

Palavras-chave: Arquitetura; ensino; otimização estrutural; otimização topológica.

INTRODUCTION

This article proposes a methodology to be used in the conceptual structural design stage, both for professionals of the area, but mainly as a tool of assistance in teaching fundamentals of structures for students. It exposes the application possibilities of topological optimization of the structure, through technologies already available but not yet explored within the teaching of architecture, such as free optimization software. The basis for the feasibility of applying these tools is justified within the conditions and curricular qualifications already implemented in most of the architecture schools.   For a better understanding of this work, a bibliographic review is presented, from previous researchers until the current scenario, regarding both teaching structures in the Architecture and in the concept of optimization, focusing on the main points that support the objective of this work. Following for the demonstration of the methodology and in sequence its application in examples of structures in the teaching of students of architecture through optimization processes. The results of the optimization are presented in four architectural applications developed by undergraduate students of architecture, who applied the methodology within the creation process of the architectural project. These applications are: in the design of a vertical building, a footbridge, a hammerhead bridge pier and a cantilevered building.

The teaching of the conceptual structural design

In the architecture academic requirements, the student must understand the professional concepts related to the structures, it is worth noting the charter that recommends the technical aspect in architectural education: the ability of technological application which respects the social, cultural and aesthetic needs, and aware of the appropriate use of structure and construction materials in architecture and their initial and maintenance cost (UNESCO/ UIA, 2011).

In teaching structures to architecture students, they receive the first concepts by analyzing an isolated piece: a column, a slab, and a beam, to then analyze in a global way the behavior of the elements set. According to Silva (2000), the global view is important, starting from the synthesis: the introductory knowledge of all the structural forms, regarding their logical conceptions, considering the economic use of the materials; the origin and evolution of forms according to cultures and technological progress; its purpose and aesthetics. This process of teaching and learning starts from the study of resistant structures from the point of view of form, considering their origins (morphogenesis) and evolution.

Several authors approach the importance of study and the relation of architecture with the structure for the architect's full professional training. Criteria and methodologies have been developed to make the understanding of structure behavior increasingly clear, in both functional and mathematical aspects. Schlaich (2006), affirm that is a challenge, teach theory and design in parallel.

According to Larsen and Tyas (2003), the development of a structural concept should be a collaborative process whereby the contrasting requirements of structural necessity, aesthetics, and functional utility are synthesized into a workable and impressive whole. According to the author, many of the best examples of modern building design where the structure is part of the architecture are the result of a truly combined effort.

A classical view, from Torroja (1958), defends the discussion of form and structure, the structural design being prior to calculation. The author proposes an understanding of structures functioning, using physical models. In this way, the master's dissertation by Oliveira (2008) developed the MOLA model that elucidates the behavior of the structure in terms of stability, representing the influence of different types of supports and joints, as well as loads on the structure. That's an easy tool application for the Torroja's proposes.

Margarido (2001) and Rebello (1993) agree that students feel the need for learning that provides the visualization and understanding of physical phenomena, in addition to dimensioning strategies through quantitative and abstract models.

Lindemann et al. (2004) see that teaching to design students involves providing them the ability to go on from investigating specific properties to active articulation and expression of ideas in design, from inner structure to outer contour. They propose the use of simple physical sketches with the use of computational tools aiming at the adequate solution of conceptual structural design.

In this way, the authors Kripka et al. (2017) present the use of computational tools to aid in the understanding of problems in structures, and one of these tools is GeoGebra for the calculation of efforts in structures. They discuss the need for changes in teaching environments, since the student is subject to an excessive number of information and new technologies, nowadays.

Teaching structural optimization inspired in nature

Within the teaching of structures in architecture, optimization processes are rarely discussed. One of the few relations with optimization processes is discussed when comparing the structure with natural elements. The structural forms developed in living beings are initially applied in teaching: nature teaching optimization processes in all their manifestations (SILVA, 2000). This concept, called *natürliches Tragwerk* (Natural Structure), was formulated by Otto (1958) and deals with the search for optimization of the resistant form, in which the structural systems arouse interest by constituting means of building more, however, expending fewer material resources and energy, and that as a product, originates more natural structures, despite the technical process to obtain the form. The authors Cassie and Napper (1958) analyze the relevance of the study of natural structures, because nature itself is optimized and uses a minimal amount of material in its structural form can achieve high resistance, thus, signing the idea of economy of means.

There are criteria for possible variations of forms correlated with variations in the effort, well addressed in Rebello (2000), in the chapter *Analogies between structural systems of Nature and buildings*, which exposes the observation of existing examples and natural phenomena. An example of this can be observed in a branch of a tree, where the inertial variation becomes possible as a function of the bending moment variation (there is a relief of stresses), allowing structures with variable section. As well in Mattheck's (1990) work, the evolution of trees, which is guided by the necessity to withstand wind and to maximize their capacity to collect nutrients, or in the process under which bones change their shape and constitution, according to the loads they must bear (WOLFF, 2012).

Such relation involving nature merge with the concepts of *Biomimicry* (is an approach to innovation that seeks sustainable solutions to human challenges by emulating nature's time-tested patterns and strategies), which is the search for structural efficiency in the natural elements, through the optimization of the form seeking a reduction of material use. This relation becomes clear with the following quotation: Shape is cheap but the material is expensive (BENYUS, 1997). Figure 1 shows this application, in which the ribs of the Victoria Cruziana (a flower of the family of water lilies) constitute a natural process of stiffening the flat surface by varying thicknesses and irradiations throughout its length, also inspiring Pier Luigi Nervi (PAWLYN, 2011).

Figure 1. Victoria cruziana and Palazzetto dello Sport by Pier Luigi Nervi.
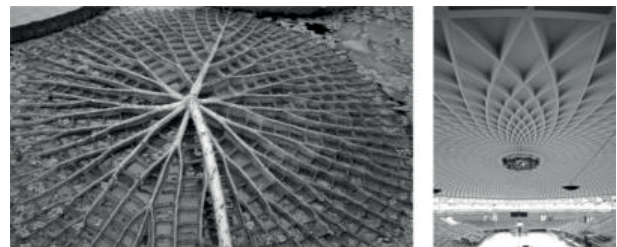
Source: Pawlyn (2011).

In a current scenario, researchers seek to understand the dynamic behavior of elements of nature. In the work of Schleicher et al. (2015) they developed a methodology for transferring principles of plant movements to elastic systems in architecture, as showed in Figure 2 the elastic deformation in the *Strelitzia reginae* flower applied to a facade shading system.

Figure 2. Elastic deformation in the Strelitzia reginae flower applied to a facade shading system
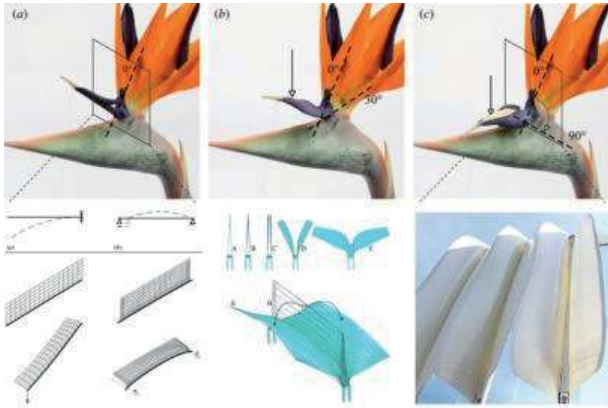
Source: Adapted from Schleicher et al. (2015).

Topological optimization

The first applications of structural optimization concepts date back to 1866 in Culmann's work and in 1870 in Maxwell's work. Maxwell (1870) considered that the optimal structure would be the one that used less material, and based on the theory of elasticity, he proposed trusses with elements aligned exactly with the directions of the principal stresses. Studies of the 90s converge with Maxwell's theory, as is the case of structures with maximum rigidity and lower weight for only one load, obtaining a similar geometric configuration (ROZVANY et al., 1995).

Michell, in 1904, in his work "The Limits of the Economy of Material in Frame-structures", presented one of his first studies on optimization, giving continuity to Maxwell's (1872) work. At the beginning of the study of structural optimization, the famous Michell beams became known as Figure 3. Due to their high degree of mathematical complexity as well as difficulty for implementation for the time, such processes were stopped.

Figure 3. Michell Optimizations: (a) cantilever; (b) Simply Supported beam; (c) torsion bar.
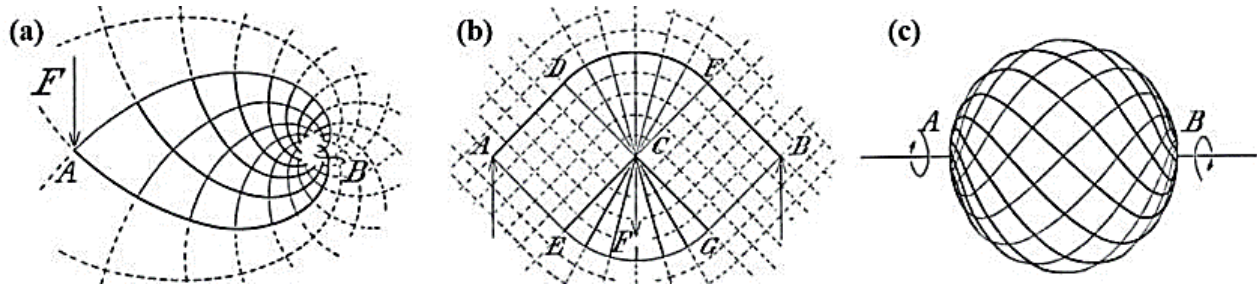

Source: Michell (1904).

After 1904, the studies did not have practical applications for six decades, until the release of the computational tool and mainly the finite element method in the mid-60s the aeronautical industry was the main beneficiary. In the 70s, the theory developed previously was then implemented with the emergence of algorithms in programming language. In the 1980s, commercial software for finite elements started to include some structural optimization modules (ROZVANY et al., 1995). At the end of the 1980s, the topological optimization method began to emerge.

The optimization process can occur in three ways: sizing optimization, shape optimization, and topology optimization. According to Bendsøe and Sigmund (2003), sizing optimization considers a structure that can be separated into a finite number of components. Each component is parameterized so that only one variable defines it. The optimization will try to find the optimal value of the parameter to match the requirements of the problem, for example, cross-sectional area (Figure 4a). Shape optimization is an extension of parametric optimization which allows more freedom of connections between components within the framework. The allowed design is limited to a fixed topology and the number of optimal variables can be limited, this number is larger than the parametric optimization, as shown in Figure 4b. Topological optimization is a mathematical approach to finding the optimal material distribution for a given design domain and does not give lim-

itations to the structure that must be optimized. In structural problems, topology optimization includes the determination of position, shape and number of holes and type of supports in the project's domain (Figure 4c).

Figure 4. a) Sizing optimization, b) Shape optimization, c) Topology optimization
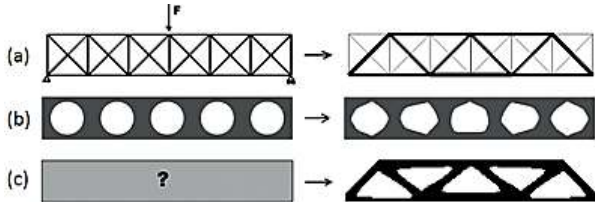
Source: Bendsøe and Sigmund (2003).

Figure 5. Standard form of an optimization problem.

Maximization or minimization problem: $f(x_1, x_2, \ldots, x_n)$	(objective function)
Subject to: $h_i(x) = 0, \quad i = 1, 2, \ldots, m$	(equality constraints)
$g_j(x) \leq 0, \quad j = 1, 2, \ldots, r$	(inequality constraints)
$x_1, x_2, \ldots, x_n$	(variable)

The topological optimization of the structure has an appeal mainly related to sustainability, due to the rational consumption of the material. Nowadays, the manufacturing process often impedes the application of optimized solutions due to the manufacturing process (ELEFTHERIADIS, 2015), as the case of structural models in profile I with varying depths (Figure 6). To counter such impediments, one of the solutions is the use of 3D printers, extremely linked to topological optimization (POPRAWE, 2017).

Figure 6. Examples of Variable Depth Beams.

Source: Carruth & Allwood (2012).

Another significant and more recent importance of topological optimization is within the BIM (building modeling information) scenario, which has become a step in the Interactive Abilities during Design Processes (CHI, 2015; ELEFTHERIADIS, 2015).

Optimization in architecture, applications and teaching

Topology optimization has been used randomly in the structural design process and has no clear defined role, as described by Kingman et al. (2014). According to Bendsøe and Sigmund (2003), optimization was defined as "an intellectual sparring partner" during the preliminary design phases. The results of a topological optimization study were directly translated into the geometry of the final structure in a process called Computational Morphogenesis (OHMORI, 2010).

The results of many applications often show a strong similarity with structures that are found in nature (Xie et al. 2011; Frattari et al. 2010) and are generally structurally efficient as well as aesthetically pleasing within organic solutions. It is interesting to highlight Dombernowsky and Sondergaard's (2009) work, which analyzed three-dimensional compositions for application in slab elements and their direct application in manufacturing processes in CNC machines.

Within the most recent scenario, Beghini's (2013) work presents a solution for structural stabilization applications of multi-floor

buildings. Stromberg et al. (2012) used the theoretical work on the stabilization project of multi-floor buildings to develop conceptual projects for buildings that are aesthetically pleasing and structurally efficient. In Japan, the execution of a building was presented, with walls that were modeled as rectangular plates and optimized for vertical and horizontal loading combinations (KINGMAN et al., 2014 apud OHMORI et al., 2005). Topological optimization was also used in Arata Isozaki's work, an architect, at the Qatar National Convention Center in a structural support application for access coverage of the convention center (Figure 7).

Figure 7. Qatar National Convention Center.

Source: Isozaki (2017)

The possibility of educational implementation of topological optimization within the architecture began in 2001, with Tcherniak and Sigmund's work, who developed an online platform (Figure 8a) for easy implementation of structural problems, with conditions of loads, supports and domains (Figure 8b).

Figure 8. a) Online software user interface; b) Domains and constraints for cantilever structure and the result of topological optimization
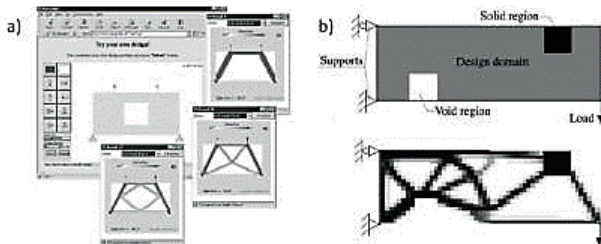
Source: Tcherniak & Sigmund (2001).

An educational implementation record can be found in the work developed in the Lund University classroom, where the application of the free software ForcePad was made in the development of tripod structures and tested in the classroom through cardboard cutouts (LINDEMANN et al., 2004).

Lindemann and Damkilde (2009) developed implementations in the ForcePad software for two-dimensional problem solving with finite element analysis and an enrichment in the interactive process, aiming for educational use and free access with a simple and practical interface (Figure 9a).

Mac Namara and Guest (2012) presented in their article the result of an exercise developed within the University of Syracuse for master's degree students in architecture. With several practical problems, students should use optimization software and, at the end of the work, answer a questionnaire. At last, with very interesting solutions, the result of the questionnaire stands out in the interest of its students, in which 90% judged it to be interesting. More than 80% answered that the optimization applications increased the level of structural knowledge understanding.

Nobel (2016) presents an interactive solution for developing 2D and 3D problems through free applications for smartphones and tablets, called TopOpt 2D and TopOpt 3D (Figure 9b). Its use requires only knowledge regarding support and connection types, loads and domains. More recent works stimulate structural optimization through application games, such as TopOpt Game, published in the article by Nobel et al. (2016), where the goal of the player is to find the ideal material distribution, which will give the highest score meeting the minimum compliance (Figure 9c).

Figure 9. Available apps

a) ForcePad   b) TopOpt 2D   c) TopOpt Game
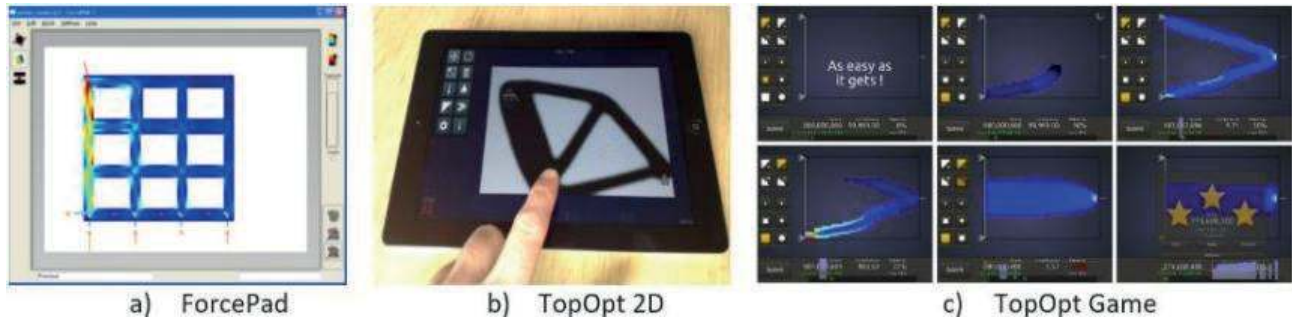
Source: Nobel et al. (2016).

MATERIALS AND METHODS

The general principle of topological optimization problem is to determine the optimal distribution of material within a domain designed to meet a given goal. In general, the problem considered is to obtain the structure with maximum rigidity and lower weight in a domain containing constraints, supports, and subject to a loading system (Figure 10).

Figure 10. Designable domain and boundary conditions.
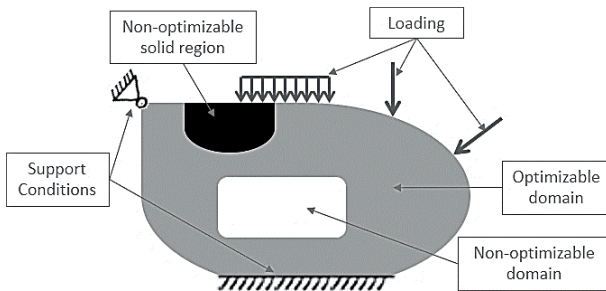
For the development of this study, the following steps were proposed for undergraduate students in architecture to analyzed: design's goal, mass study, support conditions, loading conditions, topology optimization and geometric interpretation, as Figure 11.

Figure 11. Methodology for application of topology optimization in architecture.
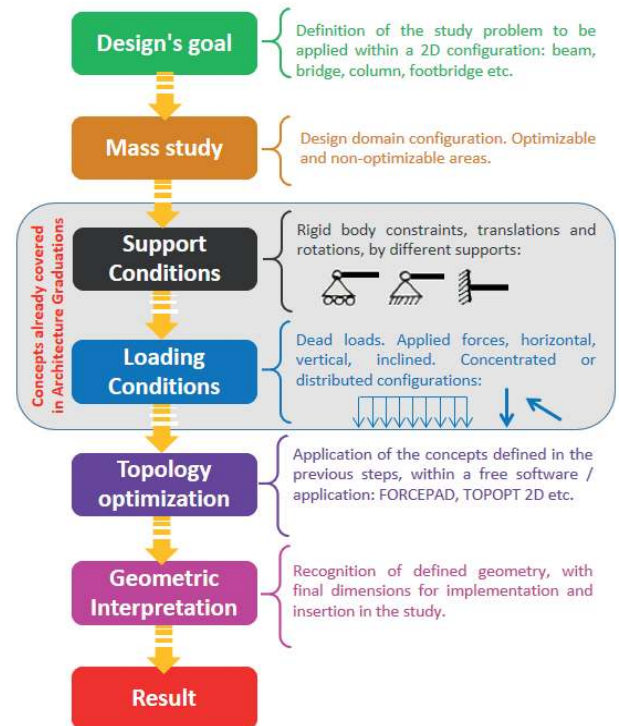
The first step is the definition of a goal to which the application of the methodology will be inserted. For the present work, this methodological proposal was implemented in four general objectives of projects, being:

I. Apply in a multi-floor building project;
II. Apply in the design of a footbridge structure;
III. Design a hammerhead bridge pier in the city of Curitiba.
IV. Cantilever building.

In the mass study criterion, we approach the geometric delimitation, also called the definition of the domain. Here is the choice of where the optimization will occur, by means of

optimizable regions or not. The possibility of imposing non-optimizable areas allows a greater control over the personalization shape, for example, the imposition of holes in the work domain (that is, areas where we do not have optimization because there are no masses) is quite simple and allows to explore different designs, using different strategies to bear the imposed loads (DAPOGNY, 2017).

The boundary conditions of the problem are the next procedure, which is divided into conditions of support (rigid body movement restrictions) and loading conditions (concentrated loads, distributed loads) in the most varied configurations of directions. The concepts required for the practical application of optimized problems within undergraduate architecture courses are already widely covered in the disciplines of structural systems, namely stability of buildings. The basic principles required for this step are: concept of loads in structures (concentrated and distributed forces) and the concept of structural supports (restrictions of rigid body movements).

The shape to be generated will be based on the load configuration and restrictions imposed by the user, that is, the geometric response created will be for the defined input problem. Non-applied loading cases will not be analyzed, so every occurrence of loading as combinations (permanent and variable loads) must be studied by the user. This is a point of extreme fragility for the interpretation of problems that are structurally viable, or not. The user must define the relevance of the movement restrictions and load application as the principle that generates the form.

Once all the constraints have been defined, the student must implement these definitions in an educational software of optimization and of recognized technical scientific value as Force-Pad, Topostruct, TopOpt 2D, BESO2D and MIDAS NFX. The basic configurations required are those described in the previous steps, not requiring deepening, since the modeling environment is two-dimensional and with easily understood icons (as a symbol of the structural supports) (Figure 8a and Figure 8b). In Force-Pad as in TopOpt 2D, there is a simplification of properties relative to materials. The modulus of elasticity is constant, as well as the Poisson coefficient. The loads are assumed to be of the same magnitude, but the user can apply multiple force vectors on the same point when there is a need to represent a higher intensity.

Finally, with the optimized solution, the student must interpret the generated geometry to configure a digital model (Google Sketchup) that can insert the dimensions (lengths and thicknesses), resulting in the final model of the structure to be implemented in the specific objective of each project. The physical interpretation and justification of the obtained results always generate sums of extreme importance in the generated learning, being the user's responsibility to understand what took the final configuration to assume this form.

RESULTS AND DISCUSSION

The results of the three objects of the proposed methodology application are presented in the following items. In all analyzed case studies, the proposed methodological steps were implemented to turn their use didactic.

Case study 01

The purpose of this application was to design a multi-floor building (Figure 12). It was based on the definition of the vertical and horizontal dimensions of the building, followed by a mass study that defined the region to be optimized, represented by the black region.

After the mass study, the support of the structure was then considered, where the base was fixed in the foundation. The next step was the definition of loads. Considering its verticality, it was studied the structure that would be necessary for the building not to tumble. Then horizontal forces were applied representing the wind pressure (both leeward and windward), according to Figure 12. With the definition of optimizable regions, supports and loads, the process followed by its optimization implementation in a free educational software mentioned in the methodology.

Figure 12. Case 1 (development process)
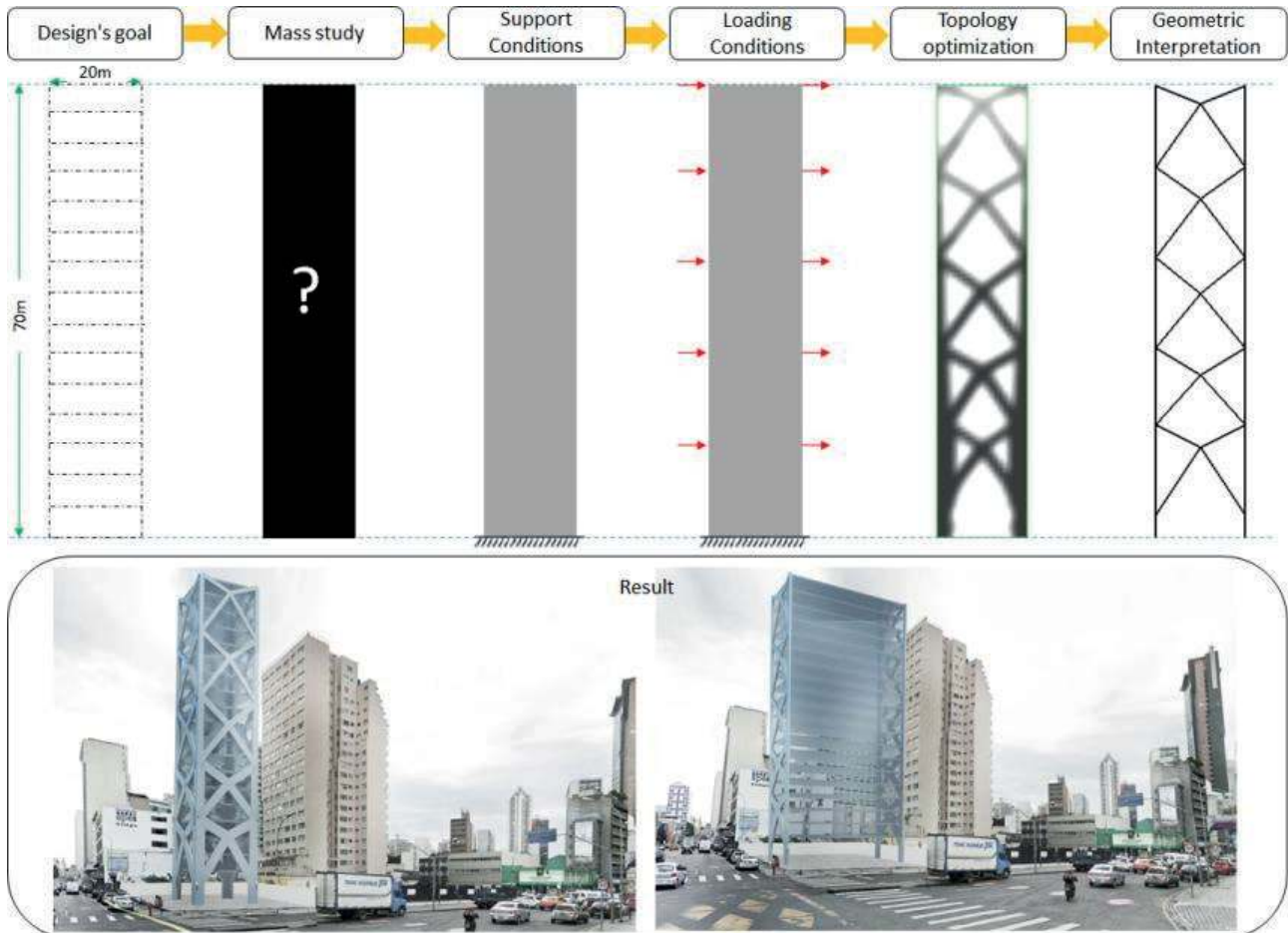
The final product was a truss structure, like some architectural models with cross bracing using exoskeleton, as shown the figure 13, the John Hancock Center in Chicago.

Figure 13. John Hancock Center in Chicago, IL

Source: Adapted from Beghini et al. (2014)

The geometric interpretation of the structure for modeling in 3D CAD software was carried out, with the practical objective of visualizing and applying such a solution in design and with the proper geometric definition of lengths and thicknesses defined in the optimization. As a result, we present an image insertion effect for the final practical visualization of the proposed problem. Two final interpretations of the problem emerged, one with unidirectional locking of the structure, and the other bidirectional locking. The case studied has similarity with a cantilever structure, in which the effect of the moment is greater in the support, the final response obtained with the optimization of the shape was a structure clearly with greater stiffness of the base of the building, with slight reduction of the stiffness from the elements to the top of the building within a concept of bracing. The gravitational loads were not applied, and new analysis and verification of their combined effects were carried out.

Case study 02

In this case, the goal was a footbridge crossing on a highway in the city of Curitiba, with spans between supports of 20 meters and cantilevers of 5 meters at each end. The proposal was based on the curiosity to explore new structural possibilities in a flat system, different from the typologies of flat trusses and Vierendeel girder existing on the studied highway itself (Figure 14).

Figure 14. Typologies of existing footbridge on the highway under study.

a) Truss Footbridge

b) Vierendeel girder footbridge

In mass study step, according to Figure 15, there is the establishment of non-optimizable regions, which are supporting columns of the footbridge. The next step was to insert the supports at the bottom of the optimizable region and loads both pedestrians and a cover system on it. These loads were considered the main ones for the search of the optimized solution and the wind action and other loads required by standards were not considered on the structure. The load presented was configured by means of concentration of forces relative to the transverse reactions of the structural system.

**Figure 15. Case study 02 (development process)**
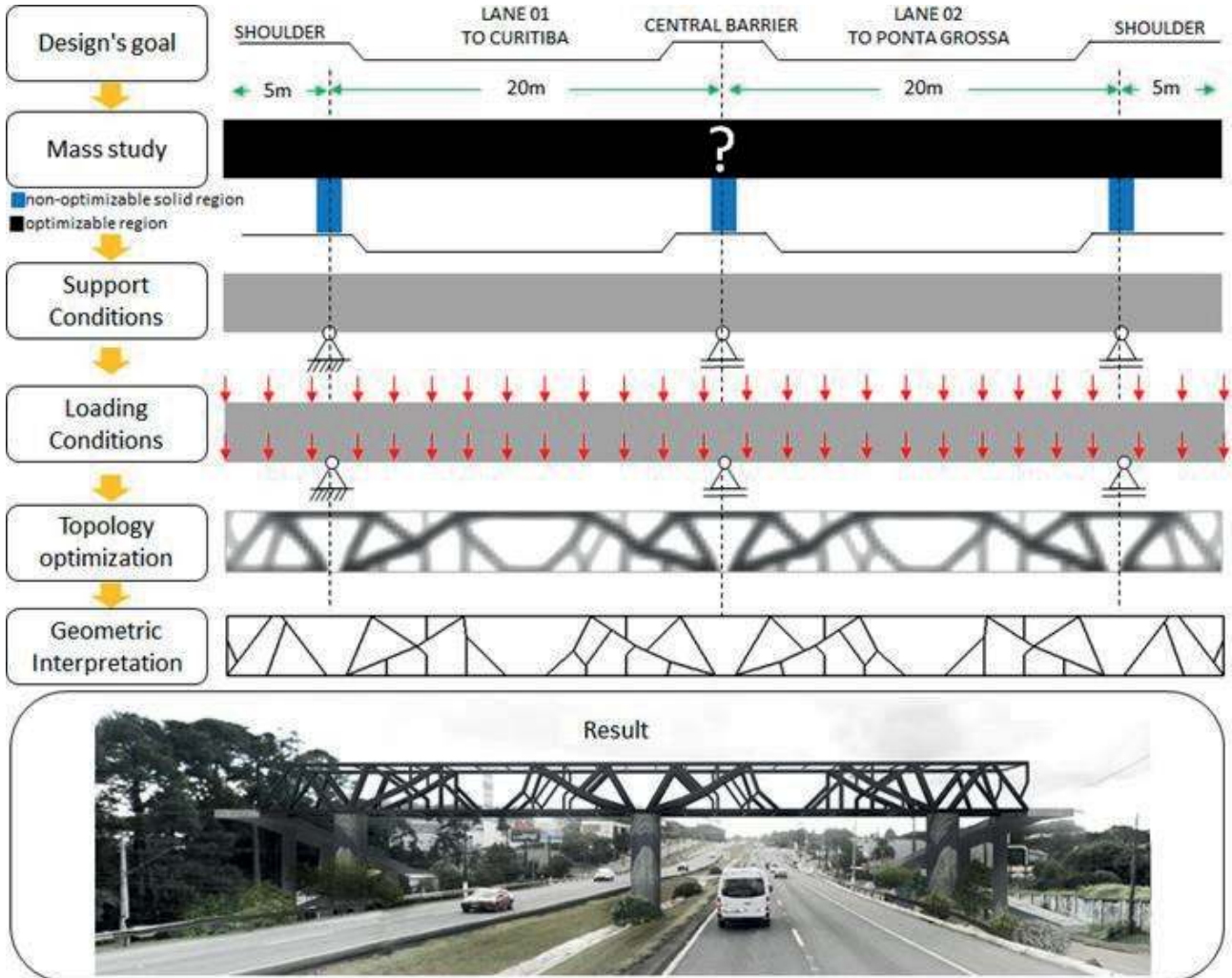
The optimization resulted in an organic geometry, implemented in a flat truss configuration and arranged in parallel, according to the result. There was a stiffening in the bars that receive the loads for the supports, visible in the optimization and the result images (Figure 15).

After the solution of the footbridge problem, there was a curiosity to modify the position of the pedestrian deck and so change the position of the structural supports. The ease of modelling the problem, and especially the understanding that the constraints and actions loads have a significant role in the generation of the form, allowed the configuration of a new hypothesis to explore the result obtained. Figure 16 presents this solution, in which in the step of support conditions, the movement restrictions were inserted in the optimizable domain of the structure (in the longitudinal axis). Similarly, the loading conditions were implemented in the central axis of the domain, referencing the board and the passage of the pedestrians.

Figure 16. Case study 02 (development process for a new concept of load and support)
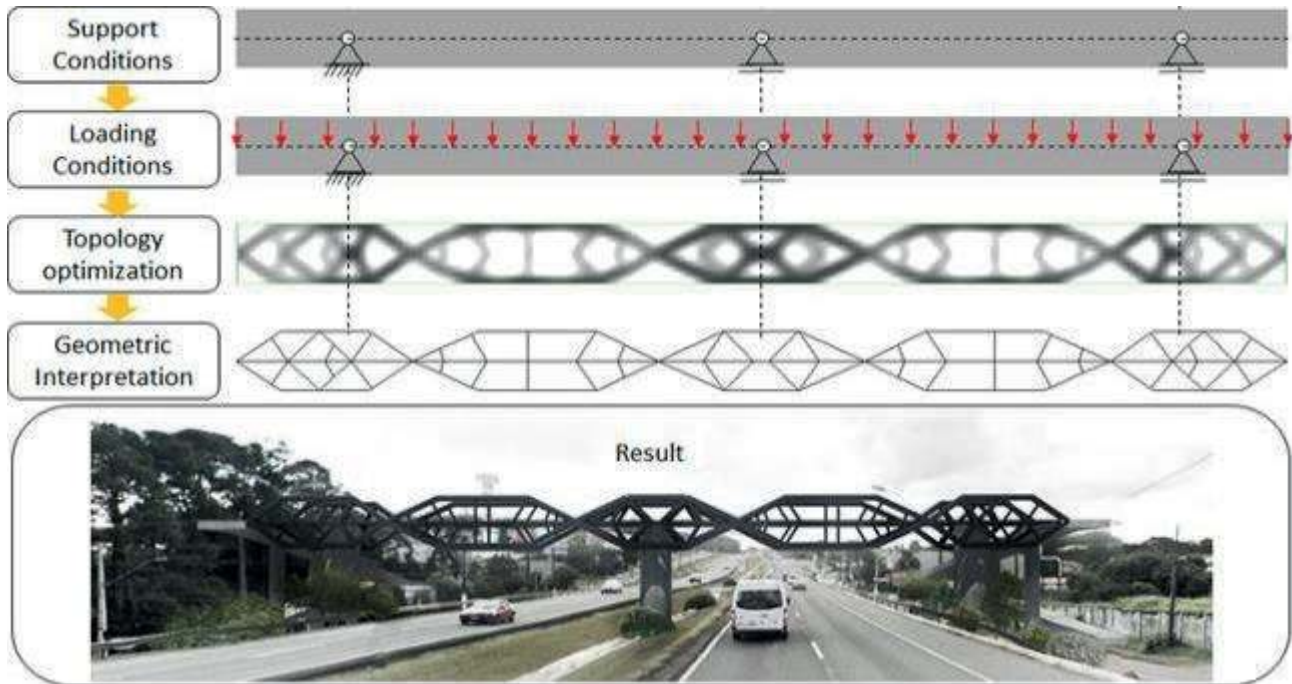
The simple modification in the handling of supports and loads resulted in a solution with a very interesting and acceptable form to be implemented in design compositions, according to the result of Figure 16. Students should explain the generated geometric difference, mainly the strangulation (reduction of the useful height of the footbridge). The static provides such justification, using the diagram of bending moment in a continuous beam, positions of zero bending moment in the central spans of the footbridge are noticed (Figure 17).

**Figure 17. Bending moment diagram**
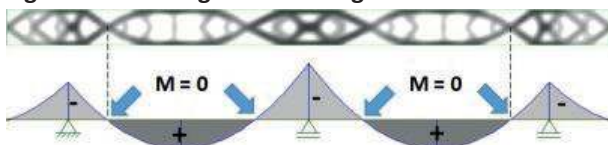
The association of the topological optimization and the justifications of the developed final form must be explored so that the application does not become simply procedural but rather an instructive tool of the structural logical behavior, as shown in Figures 16 and 17. It also allows the evolution of developed analysis, such as the implementation of cable elements, bars or other structural systems based on the geometry obtained.

Case study 03

Another study developed in this article was related to the replacement of a conventional solution with a new configuration for the hammerhead bridge pier. The chosen bridge has 4 girders that support the bridge deck system, and for its new proposal, the girders would continue to unload on the hammerhead bridge pier. Such

constraint was respected when implementing the four concentrated loads on the optimizable domains in their respective positions. Two possibilities were explored to understand the handling of the mass study concept (Figure 18), the optimizable regions (black) and non-optimizable regions (blank).

Figure 18. Case study 03 (development process)
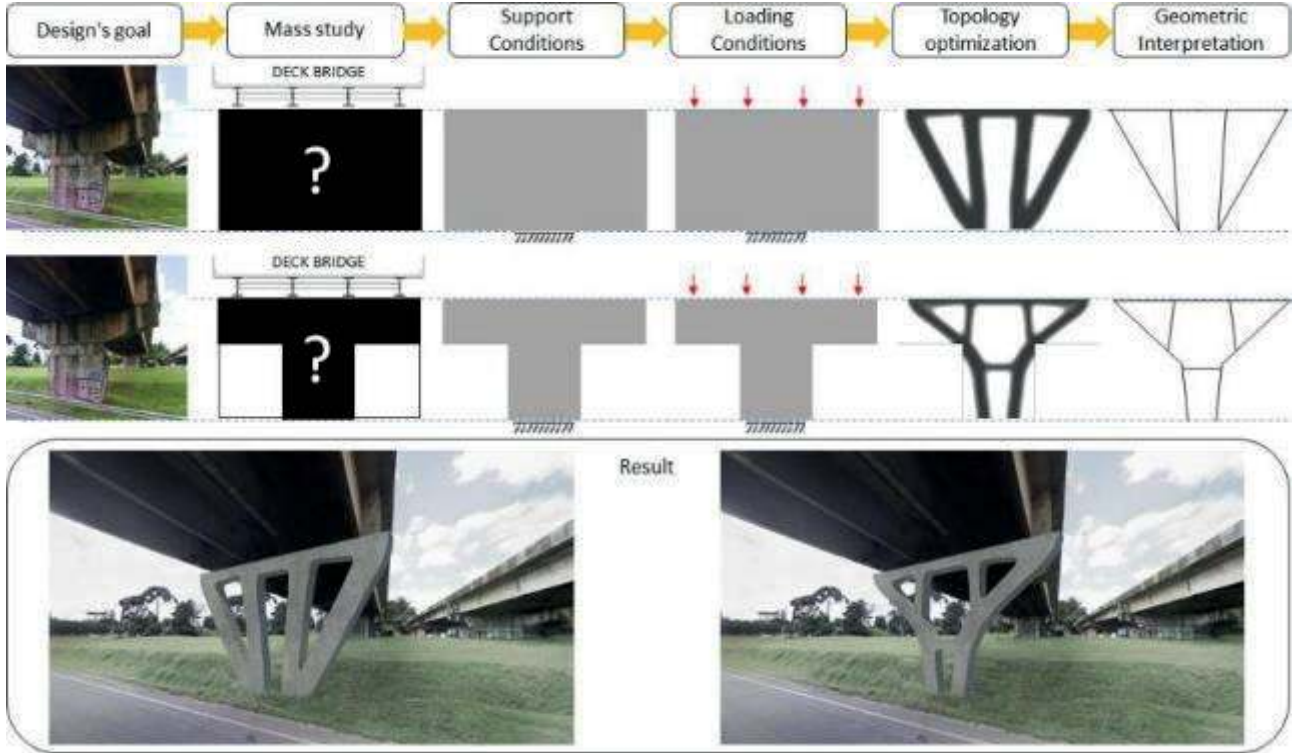
Case study 04

In this last application, the principles of cantilever structures are applied. The understanding of this typology involves the three configurations showed in Figure 19, to which the solution of the first structure involves a simple cantilever, which requires a continuous element to capture the generated bending moment, or a good stiffness in the bearing, like a fixed support; the second configuration adopts a fully drawn upper support, and in the latter configuration the solution becomes a fully compressed lower support.

Figure 19. Draft of cantilever structures
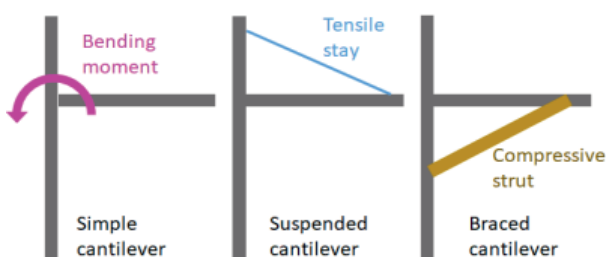
This theory is sufficient to contemplate the application of different solutions of the optimized structure, as proposed in the 3 initial configurations according to Figure 20. The objective of the project is to optimize a building that is one-third supported and two thirds in the cantilever.

The first case is based on the principle of existing continuity, and the idea is to use a flat solution that will be doubled in parallel to configure the support of the whole building. The second solution has as the volumetric objective of the same building, but it takes advantage of the premise of suspended structure, so the area to be optimized involves only the stretch in cantilever. In the third solution, the creation of a volume below the cantilever region is adopted, in order to create a compressed support structure. All the solutions considered a load of cover and the floor, considering its different intensities in the problems.

Analyzing the result, applied to an architectural solution, it is observed that all development of the topological optimization followed the basic principles of a structure in the cantilever, in which in the first analysis the continuity of the support served as an element to support the generated bending moment and to transmit the loads to the foundation. This case is similar to the principle of a crane suspending a weight, as shown in Figure 21. To absorb the bending moment in the column is necessary to put the cable in a higher level than the cantilever brace.

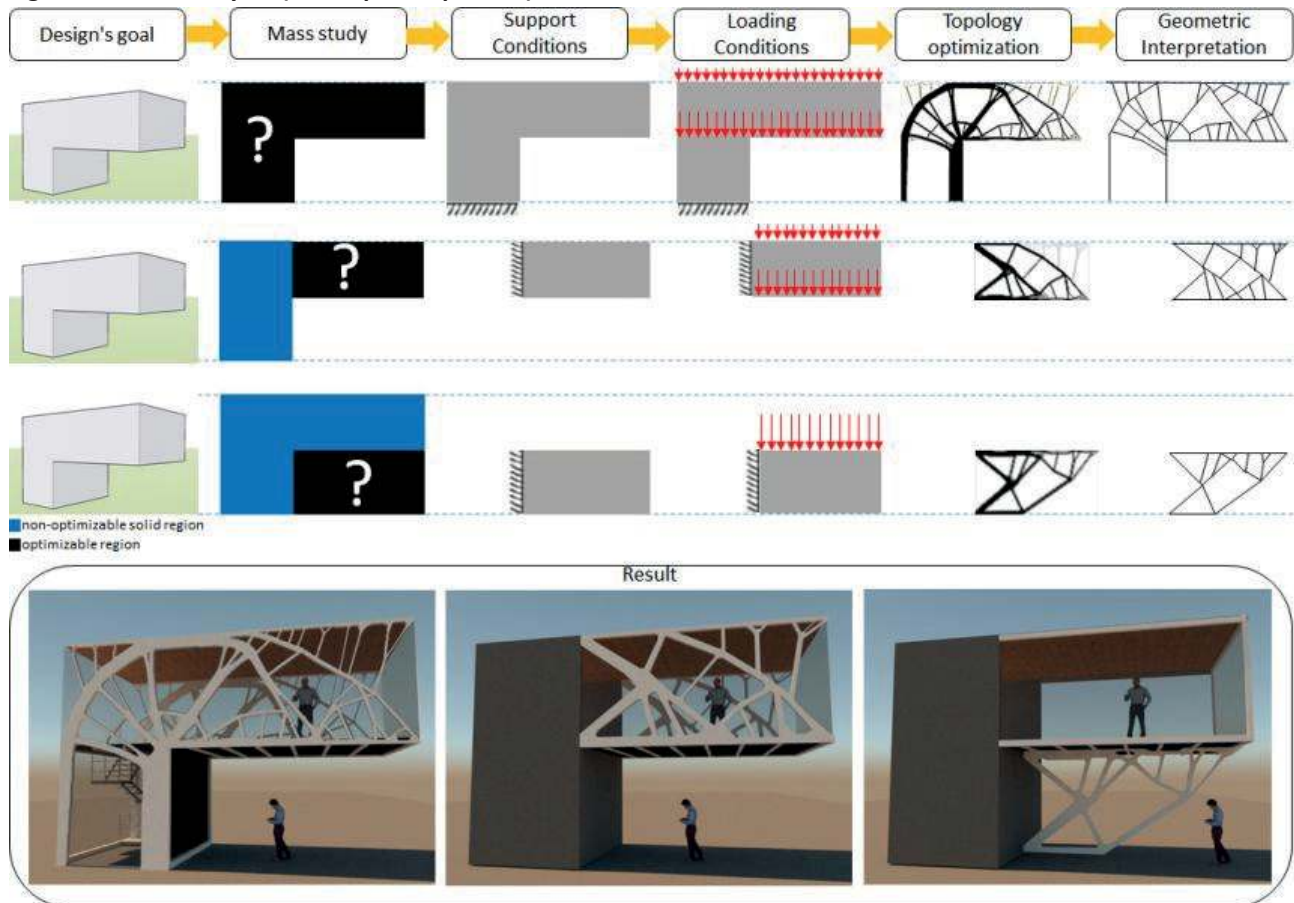**Figure 20. Case study 04 (development process)**

Figure 21. Crane application (Cabo and Bernardo, 2015).

In the other two solutions, both the suspended structure and the support arm became evident. It is also important to highlight the space freedom that the suspended structure generated in relation to the last case when analyzing the ground space.

CONCLUSIONS

The research presented in this paper shown that the use of this methodology is capable of significant for the result of a design, proving to be considered as an integral or partial way in the creation process. The application of the topological optimization become a mechanism of the orientation of the structural geometry to be defined. The composition of the new form found may be a basis for choosing new solutions.

On the other hand, the decision-making about the form found should consider structural performance, not discarding the basic knowledge of the fundamentals of stability of structures.

Finally, this optimization process can be shown as a guide through the architectural and

structural conceptual design, increasing number of the solutions from the traditional shapes. Its use within the teaching process can become a great tool to expand understanding of structural behavior.

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