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RESEARCH ARTICLE

Structural Topology Optimization with Stress Constraint Considering Loading Uncertainties

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Abstract

This paper deals with the consideration of loading uncertainties in topology optimization via a fundamental optimization problem setting. Variability of loading in engineering design is realized e.g. in the action of various load combinations. In this study this phenomenon is modelled by the application of two mutually excluding (i.e. alternating) forces such that the magnitudes and directions are varied parametrically in a range. The optimization problem is stated as to find the minimum volume (i.e. the minimum weight) load-bearing elastic truss structure that transfers such loads acting at a fix point of application to a given line of support provided that stress limits are set. The aim of this paper is to numerically determine the layout, size, and volume of the optimal truss and to support the numerical results by appropriate analytical derivations. We also show that the optimum solution is non-unique, which affects the static determinacy of the structure as well. In this paper we also create a truss-like structure with rigid connections based on the results of the truss optimization and analyse it both as a bar structure (frame model) and a planar continuum (disk) structure to compare with the truss model. The comparative investigation assesses the validity of computational models and proves that the choice affects design negatively since rigidity of connections resulted by usual construction technologies involve extra stresses leading to significant undersizing.

Keywords

topology optimization · truss · continuum · stress constraints

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1 Introduction and literature review

Topology optimization has become a popoular and developing field of research in the last few decades. Vast majority of publications refer to Michell's works in 1904 [1] as the foundation of this branch of mechanics, missing the fact that the first contributions were made by James Clerk Maxwell [2] as early as 1870, of whom even Michell made reference.

Optimization has since gained widespread application because efficiency and economical design are essential requirements in modern engineering. Continuous advance can be observed in several areas and applications in the industry not only in terms of practical design but also of fundamental research. The determination of the optimal structural topology is a branch of fundamental research, where the optimal topology is constructed in a design space based on given boundary conditions, serving as a decision support for actual design.

The topology design historically started with the problem class of layout optimization of trusses and the works were called 'minimum volume design of frames' where the term 'frame' was historically used for what we now call a truss. The first important achievement in truss optimization was made by Maxwell [2], which deserves presentation here not only due to its significance but also because it is still unknown to many in the field. He considered the problem of attracting and repulsive forces between points set in the plane and proved a theorem regarding the equilibrium of the force system: '*In any system of points in equilibrium in a plane under the action of repulsions and attractions, the sum of the products of each attraction multiplied by the distance of the points between which it acts, is equal to the sum of the products of the repulsions multiplied each by the distance of the points between which it acts.*' This statement can be formulated as $\sum_t T_t L_t - \sum_c T_c L_c = 0$ where T and L denote the force and the distance between two points, respectively, and indices t and c refer to tension (attraction) and compression (repulsion), respectively. For the derivation of this statement the principle of virtual work was applied with a uniform virtual deformation field.

The far-reaching consequences of this theorem were not missed by Maxwell as he added therein: '*The importance of* *this theorem to the engineer arises from the circumstance that the strength of a piece is in general proportional to its section, so that if the strength of each piece is proportional to the stress which it has to bear, its weight will be proportional to the product of the stress multiplied by the length of the piece. Hence these sums of products* give an estimate of the total quantity of material which must be used *in sustaining tension and pressure respectively.*' [no emphasis in original] Note that the term 'stress' was then used for 'load' today.

Michell in the early 20th century recognised the importance of Maxwell's result and applied to the calculation of the optimum structural weight. This led him to sufficient conditions for a structure to be an optimum. He proved the geometric restriction which determines the classes of orthogonal sets of curves along which the members of an optimum structure must lie. Michell's theorem is based on the volume of the framework (truss) complying with a particular condition of virtual deformation field chosen by him (purely $\pm e$ uniform deformations) is the minimum.

For the case of external loads $\overline{F_i}$ acting at positions $\overline{r_i}$, Maxwell's theorem is reformulated by Michell as $f_t V_t - f_c V_c = \sum_i \overline{F_i}\overline{r_i}$ where f_t and f_c denote the elastic stress limits for tension and compression, respectively, and V_t is the volume of all the tension members and V_c is the volume of all the compression members. Now the total volume of the truss can be expressed in two ways as

$$V = V_c\left(1 + \frac{f_c}{f_t}\right) + \frac{1}{f_t}\sum_i \bar{F}_i\bar{r}_i \tag{1}$$

and

$$V = V_t\left(1 + \frac{f_t}{f_c}\right) - \frac{1}{f_c}\sum_i \bar{F}_i\bar{r}_i \tag{2}$$

from which it follows that the volume is minimum if the volume of the tension or compression bars is minimum.

Michell's key invention is the so-called Michell structure consisting of members aligned with the principal axes of stresses, in accordance with the optimality condition mentioned above.

Michell's results did not induce further advance in this field at that time and it was not until the 1950's that works on minimum volume design restarted at various schools. Within the diverse literature we only focus on achievements regarding trusses and truss-like structures, which relate to the topic of this study. For example, Sved [3] worked on frictionless joint truss design, and suggested a method for determining the minimum weight structures belonging to a specific configuration but he dealt with the case of a single fixed load only. He showed that the minimum weight structure is always statically determinate. A similar theorem but of more general validity was proven by Barta [4]. The optimum of symmetric three-bar trusses with given geometry reduces to structures with one or two bars in the case of different stress constraints and only one vertical load [5].

Applications of the theorems of Maxwell and Michell to simple design problems have been made by Cox [6]. Some of his layouts are given in Fig. 1 and we have to note that all these structures have equal weight. One can recognise the non-uniqueness of the optimal layout and we can have an infinity of solutions ranging from mechanisms through simple stiff structures to structures of any degree of redundancy.

Shield [7] noted that Michell's design fails if kinematic constraints are considered and presented an alternative approach, which does not have the limitation of the Michell method. Like Cox, he also emphasized and illustrated with examples that solutions are not always unique.

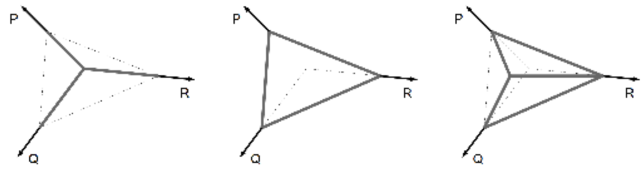

Fig. 1. Simple tension structures by Cox.

The abovementioned results imply that it is to be examined whether the solution is unique or there exist more than one optimal solutions for the same problem.

Continuum modelling as a new branch of research in topology optimization became into focus in the 1970's by the advance of computational tools, see e.g. [8]. The modern continuum type optimization is derived from the works of Bendsoe and Kikuchi [9], and later in the 1990's the previous optimality conditions (OC) algorithm has become well known by the acronym SIMP.

The consideration of stochastics in topology optimization has become an emerging approach, which may relate to many aspects of loading. One possible option to model variability of loading on the structure is to apply resultants of different combinations of loads, for instance variable loads (i.e. wind) can be potentially active simultaneously with the constant dead load.

A simplified model of this concept consisting of two alternative loads is investigated in this study. The term 'alternative' means that the two fixed forces are not acting on the structure simultaneously, but alternately, as two independent load cases.

The aim of this paper is to analyse and compare the validity of the optimal truss and truss-like structures under stochastic loading through the example of a fundamental and popular optimization problem, which was stated e.g. by Nagtegaal and Prager [10]. Furthermore, we aim to demonstrate that non-uniqueness of solutions can also be found in the case of uncertain loading.

2 General problem statement

The fundamental question arises whether the optimum structure is statically determinate or indeterminate. Consider for example the case of a horizontal beam supported at three points (Fig. 2). Depending on whether the beam is continuous or divided into two separate simply supported parts, the mechanical behaviour changes. It is also an important question to consider whether the design domain should be a solid continuum (e.g. a solid beam) or a ground structure containing the potential structural elements to chose from, see Fig. 3, and how it affects the optimal solution.

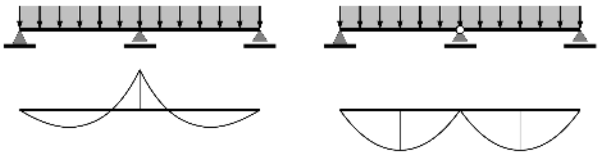

Fig. 2. Modelling of a beam as statically indeterminate and determinate.

Fig. 3. Modelling of a beam as a solid continuum and a truss-like structure.

A typical structural problem to investigate [10] is to define a line support and a point of application of the loads, and the task is to design a structure (truss, frame, etc.) to transfer the loads to the supports, see Fig. 4.

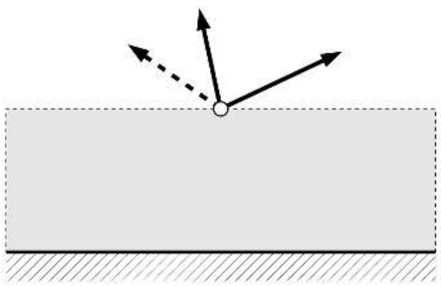

Fig. 4. Typical basic optimization problem.

In this paper we deal with a problem setting with a straight horizontal line of support and a fix point of application of the loads at a certain height above the line based on the above mentioned example taken from the literature. (The details are given in Section 3.1.)

We intend to design a skeleton structure opposed to continuum structures to connect the point of application to points on the support line. The joints of the framework are considered frictionless in accordance with problems usually analysed in the literature, thus the optimization model is a truss.

The number of bars applied in the model is three in order to account for both statically indeterminate and determinate structures.

Uncertainties of loading may relate to the magnitude, direction or position of the loads, of which here the former two are taken into considerations while the latter one is obviously not as a fix point of application is set. In our problem these uncertainties are included to provide a more realistic description of real loading behaviour.

The optimization task in this problem is to find a truss, which is able to carry any of the alternating loads such that the stresses in the structure do not exceed a prescribed stress limit, and is optimal with respect to the total volume, that is with respect to the total weight, which is an equivalent condition. We apply a numerical computational algorithm (elaborated in Section 3) to determine the optimal layout and size of the structural elements. We also deal with special cases and perform analytical investigations to support numerical calculations (Section 3.4).

Parts of the results have been presented at conferences by the authors, see [11, 12]. In this paper we extend our investigation in the continuum analysis of the framework and draw conclusions regarding the optimality and applicability of the results. For this purpose, one of the investigated cases is chosen and remodelled with rigid connections to form a truss-like structure, which now has shearing and bending further to the axial forces (Section 4). Both a bar structure (frame) and a planar continuum model (disk) are analysed. We examine these two models to compare the behaviour with that of the optimal truss. Its relevance lies in the fact that realistic construction technologies do not typically build perfectly moment-free joints. Our aim is to demonstrate that optimal trusses might fail requirements in such cases and to establish the significance of the choice of models on the optimal solution.

3 Truss optimization problem

3.1 Truss layout

A point and a straight line are given in the plane. In this point two forces with given directions and given magnitudes can be applied alternately. The aim is to determine the minimum volume elastic structure which can carry the loads, its supports are in the given line, and its tension and compression stresses do not exceed the prescribed limit.

In the case of truss, number n bars connect the point of application and the support line with hinged connections. Fig. 5 shows a schematic sketch of the problem.

Notation used hereafter are as follows:

(x, y)	coordinate system; the origin is in the point of application of the forces
H	distance of the point of application and the support line (support line: $y = -H$)
F_j	the j. loading force ($j = 1;2$)
β_j	the angle of the j. load vector (measured clockwise from $+y$ axis)
α_i	the inclination of the i. bar (measured clockwise from $-y$ axis)
A_i	the cross-section area of the i. bar
σ_{ij}	the stress in the the i. bar due to the j. load
σ_L	stress limit (the same absolute value for tension and compression)
E	elastic modulus
V	volume

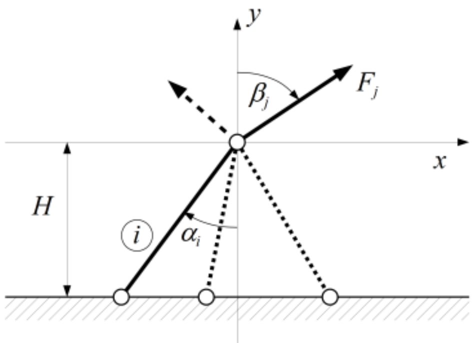

Fig. 5. Sketch of the optimality problem in the case of truss model.

3.2 Method of calculation

The optimization is performed on a statically indeterminate three-bar structure, where $-\pi/2 < \alpha_3 < \alpha_2 < \alpha_1 < \pi/2$ and $A_1, A_2, A_3 > 0$ without limiting the generality. The length of the bars are $L_i = H / \cos\alpha_i$ with the notations mentioned above. In the case of given loads the response of the structure is obtained with matrix analysis as follows.

The unit direction vectors of the bars are $e_i = \begin{bmatrix} -\sin\alpha_i & -\cos\alpha_i \end{bmatrix}^T$, from which the equilibrium matrix $G = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}$ is constructed as well as the diagonal flexibility matrix $F = \langle L_i / EA_i \rangle$.

For the two loads the load vectors of the structure are $q_j = \begin{bmatrix} F_j \sin\beta_j & F_j \cos\beta_j \end{bmatrix}^T$ $(j = 1, 2)$, and the total stiffness matrix is $K = GF^{-1}G^T$.

By the solution of the equation system of the structure the displacement vector can be expressed as $v_j = K^{-1} q_j$, then the vectors of bar forces from the compatibility equations as $s_j = F^{-1} G^T v_j$. The normal stresses are $\sigma_{ij} = S_{ij} / A_i$ where S_{ij} is the force in bar i due to load j. Note that from the calculation above it follows that the bar forces and stresses do not depend on the elastic modulus (provided that the material is homogeneous). The volume of the structure is $V = \sum_i L_i A_i$.

With stress constraints the optimization problem can be stated as follows:

$$\begin{aligned} &\text{Let } V = \sum_i L_i A_i = \min! \\ &\text{subject to } s_j = -F^{-1}G^T K^{-1} q_j \\ &\text{and } -\sigma_L < \frac{s_{ij}}{A_i} < \sigma_L. \end{aligned} \tag{3}$$

We can solve the problem for various load conditions (regarding magnitude and direction) using an approximate numerical iterative procedure. At fixed directions of the bars the values of the cross-section areas in the optimal structure are obtained on the condition that the stress has to be $\pm\sigma_L$ in each bar due to at least one load. The minimum of the volume is found by varying the inclination angles of the bars in the range given above. During the process the cross-sectional area of a bar or bars may converge to zero; in this case a small minimum value is used to avoid numerical singularity. Futhermore, during the procedure the inclination angles of any two bars may converge to the same value; in this case, the two bars are going to be merged and the process continues as a determinate structure.

3.3 Numerical calculations

3.3.1 Parameter domain

The geometrical (H) and material (E; $\pm\sigma_L$) parameters of the problem have no qualitative effect on the optimal topology due to the linearity. In the calculation $H = 100$ [cm] and $\sigma_L = 20$ [kN/cm^2] values are used. Value of one of the loads was fixed at $F_1 = 100$ [kN] and the other load value is chosen as $F_2 = 25; 50; 75; 100; 133{,}33; 200; 400$ [kN]. The inclination of the first force (F_1) is varied between (β_1) $\pm\pi/2$ with increments $\pi/12$ and the inclination of the other force (F_2) is varied between 0 and $\pi/2$ with the same increments. In the case of β_1 it defines the half-plane which does not cross the support line. (Note that the other half-plane is not necessary because central symmetry leads to results of the same magnitude and opposite sign). In the case of β_2 it is sufficient to consider the positive part of this half-plane because appropriate mirroring of the results provides solutions for the missing domain.

3.3.2 Results

In the optimal truss design obtained with the method we eliminate bars with cross-section area equal to the numerical minimum value, and thus regarded as zero. Among optimal designs obtained for different load cases there are structures consisting of one, two, and three bars. These cases correspond to statically overdeterminate, determinate, and indeterminate trusses, respectively. In the investigated domain of the three-dimensional parameter space the optimal truss typically is a statically determinate two-bar truss except for some null subsets. The overdeterminate and indeterminate structures can be optimal only in special cases.

In a two-dimensional subspace of the three-dimensional parameter space a statically overdeterminate degenerate one-bar truss is obtained; this occurs when the two alternating loads have equal angles and the direction of the common angle is between $\pm\pi/4$. A statically indeterminate three-bar structure is obtained in a one-dimensional subspace when the two loads have the same magnitude, their directions are symmetric to the line which is perpendicular to the support line and the angle is less than a certain value. In these two special cases the numerical results are verified analytically as well.

3.4 Analytical calculations

3.4.1 Degenerate case

If the two forces have the same direction, the optimal solution is the one which is obtained by the greater force. Consider a two-bar structure in which the inclinations of the bars are α_1 and α_2 and the inclination of the load F is β. The bar forces can be calculated from the equilibrium of the hinge and then the stresses are:

$$\sigma_1 = \frac{F}{A_1}\frac{c_1\sin(\alpha_2-\beta)}{\sin(\alpha_2-\alpha_1)},$$
$$\sigma_2 = \frac{F}{A_2}\frac{c_2\sin(\alpha_1-\beta)}{\sin(\alpha_2-\alpha_1)}, \tag{4}$$

where c_1 and c_2 are constants with the value ± 1 depending on the sign of angles in the numerator. In the case of total utilization of the structure the stresses are equal to the limit stress, from which the cross-sectional areas can be calculated.

The volume of the structure in terms of the inclination of the bars is $V(\alpha_1, \alpha_2) = \sum_i L_i A_i$.

The optimal topology is obtained from the solution of the equation system $\partial V / \partial \alpha_i = 0$ $(i = 1, 2)$. A number of cases need to be examined depending on the load and the relative value of the inclinations of the bars. If β is in the (α_2, α_1) interval, the system of equations does not have any solution. If β is outside the (α_2, α_1) interval, the solution will be either the $\alpha_1 = \pi/4$, $\alpha_2 = -\pi/4$ two-bar structure or a degenerated case in which one of the bars is parallel to the direction of the load and the other bar has zero force. This latter solution is valid in the case in which β is on the boundary of the interval.

From the solution set of the equation system it follows that the optimum is a one-bar structure with inclination $\alpha = \beta$ in the case of $-\pi/4 \le \beta \le \pi/4$, whereas in the case of load directions outside this domain it is the two-bar structure with inclination angles $\pm\pi/4$ because in this case the one-bar solution corresponds to larger volume.

Note that it is recommended to apply a mathematical program using symbolic description (i.e. Maple, etc.) to solve the equation system analytically.

3.4.2 Indeterminate solution

In the case of two loads of equal magnitude and symmetrical layout ($F_1 = F_2 = F$, $\beta_1 = -\beta_2 = \beta > 0$) the optimal structure can be a symmetrical three-bar statically indeterminate structure ($\alpha_1 = -\alpha_3 = \alpha$, $\alpha_2 = 0$, $A_1 = A_3$), see Fig. 6(a).

The stresses from the F_1 force are

$$\sigma_1 = \frac{F\sin\beta}{2A_1\sin\alpha} + \frac{F\cos^2\alpha\cos\beta}{2A_1\cos^3\alpha + A_2},$$
$$\sigma_2 = \frac{F\cos\beta}{2A_1\cos^3\alpha + A_2}, \tag{5}$$
$$\sigma_3 = -\frac{F\sin\beta}{2A_1\sin\alpha} + \frac{F\cos^2\alpha\cos\beta}{2A_1\cos^3\alpha + A_2}.$$

In the optimal structure $\sigma_1 = \sigma_L$ and $\sigma_2 = \sigma_L$ (and $\sigma_3 < \sigma_L$) from which A_1 and A_2 can be calculated. The volume of the structure is

$$V_3(\alpha) = \frac{HF\cos\beta}{\sigma_L\sin^3\alpha}\cdot$$
$$\cdot\left(\frac{\tan\beta}{\cos\alpha} + \cos\alpha\tan\beta\sin^2\alpha - \cos\alpha\tan\beta + \sin^3\alpha\right). \tag{6}$$

The solution of the $\partial V_3(\alpha)/\partial\alpha = 0$ equation is independent from β: $\alpha_0 = \arctan\sqrt{2}$. In the optimal structure the cross-section areas of the bars are

$$A_1 = A_3 = \frac{F}{\sigma_L}\left(\frac{3\sqrt{6}}{8}\sin\beta\right),$$
$$A_2 = \frac{F}{\sigma_L}\left(\cos\beta - \frac{\sqrt{2}}{4}\sin\beta\right), \tag{7}$$

thus the volume of the structure is

$$V_{opt,3} = \frac{HF}{\sigma_L}\left(2\sqrt{2}\sin\beta + \cos\beta\right). \tag{8}$$

It is remarkable to note that the numerical calculation also resulted in a statically determinate two-bar solution which is equivalent with the three-bar solution regarding volume (Fig. 6(b)). In this case $\alpha_1 > 0$ and $\alpha_2 = 0$. After calculating the bar forces, stresses and cross-sections, the volume can be written in the form

$$V_2(\alpha) = \frac{HF}{\sigma_L}\left(\frac{\sin\beta}{\sin\alpha\cos\alpha} + \cos\beta + \frac{\sin\beta}{\tan\alpha}\right). \tag{9}$$

The solution of the $\partial V_3(\alpha)/\partial\alpha = 0$ equation is the same as the angle obtained in the case of the three-bar structure: $\alpha_0 = \arctan\sqrt{2}$. In the optimal structure the cross-section areas of the bars are

$$A_1 = \frac{F}{\sigma_L}\left(\frac{\sqrt{6}}{2}\sin\beta\right),$$
$$A_2 = \frac{F}{\sigma_L}\left(\cos\beta + \frac{\sqrt{2}}{2}\sin\beta\right), \tag{10}$$

then after substituting back for the volume of the structure an expression is obtained, which is identical to the previous case:

$$V_{opt,2} = \frac{HF}{\sigma_L}\left(2\sqrt{2}\sin\beta + \cos\beta\right), \tag{11}$$

that is, the two optimum cases, which are topologically different, are equivalent.

The numerical calculations show that this double optimum is valid in a certain domain of angle β (β<0,569612851...) and above this limit value a statically determinate symmetric structure is the optimal solution (Fig. 6(c)).

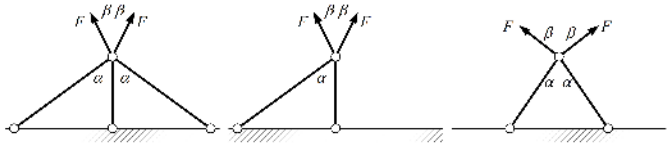

Fig. 6. Optimal topologies in the special cases of two symmetrical alternative loads: (a) statically indeterminate symmetrical structure, (b) statically determinate asymmetrical structure which is equivalent with the previous one, (c) statically determinate symmetrical structure.

4 Truss-Like Design

The main difference between optimization of structures modelled as a truss or a truss-like frame or disk is that in the first case uniaxial stress state is formed in all members, while in the other cases the stress state is two-dimensional. The real design of the connections in the optimal structure determines the stress state and it necessarily affects the optimality.

In the following a comparative analysis of the two systems is performed through the example of the structure shown in Fig. 6(a). Due to the symmetry of the structure and the load it is sufficient to consider only one of the loads. Fig. 7(a) shows the frame structure, in which the cross-sectional dimensions can be determined by forming rectangular cross-sections, such that they have unit (1 cm) thickness and areas equal to the values calculated by the algorithm for a chosen value of angle β. Typical normal force and moment diagrams are shown in Figs. 7(b) and (c). Although the bending moment values are less than those of beams designed for bending but their effect is significant because the structure is optimal with respect to normal forces. The maximum bending moment and the maximum normal stress (in the bottom cross-section of the central bar) are affected by the length of the bars. Fig. 8 shows the ratio (σ / σ_L) of the maximum stress and the stress limit in terms of the ratio of the beam length and height (in the case of $\beta = \pi / 6$).

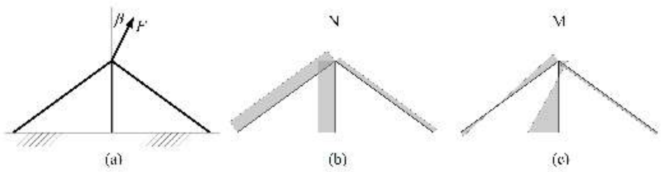

Fig. 7. Analysis of a symmetric three-bar structure: (a) model, (b) normal force diagram, (c) bending moment diagram.

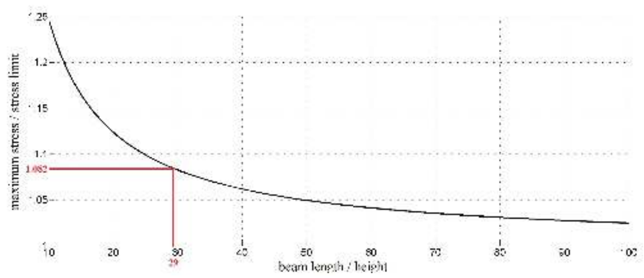

Fig. 8. The maximum normal stresses scaled by the stress limit in terms of the length-to-height ratio of the frame structure.

It can be seen in Fig. 8 that in the case of ratio 10:1 the maximum normal stress is higher than the allowable limit stress by nearly 25%. Increasing the ratio the excess stress is reduced but it is still more than 12% in the case of ratio 20:1 and it is 5% even at ratio 50:1. Therefore the optimal structures designed as trusses will be undersized significantly when the joints are not perfectly moment-free hinges. Thus a procedure using continuum model is necessary for optimization of structures with moment bearing connections.

Stresses calculated in the frame model can be verified by finite element stress analysis. Fig. 9 shows the distribution of the major principal stresses in the structure. Dark orange and red colours in bars 1 and 2 illustrate well the maximum stresses corresponding to bending moments shown in Fig. 7(c). Fig. 10 shows the larger principal stresses at the bottom part of the central bar, where the stresses are maximal. The difference between the maximum stresses calculated in the frame structure and in the continuum model are within 1%. The finite element model shown in Fig. 9 is highlighted in red in Fig. 8.

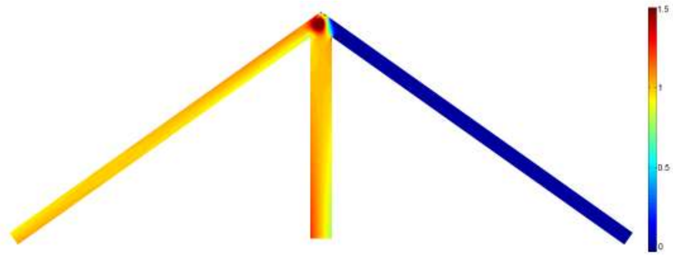

Fig. 9. The larger principal stresses of the continuum model scaled by the stress limit. Load and layout as in Fig. 7(a). (Note that that around the point of application of the concentrated force high stress peak is generated thus the values above 1.5 are not marked with separate colours for better visualization only.)

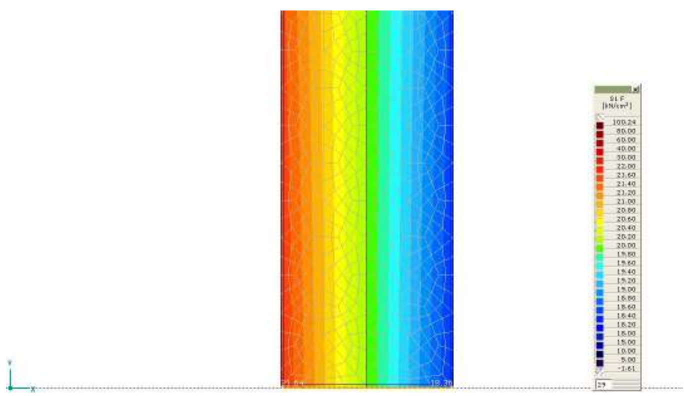

Fig. 10. Fig. 10: The larger principal stresses of the continuum model, focus on the bottom cross-section of the central bar. Large stresses develop on the tension side.

Note that, around the point of application of the concentrated force high stress peak is generated which is not to be taken into consideration in the comparison (Fig. 11). Also note that its local effect is limited to a narrow domain, and the stresses of the bars show a very good agreement with the results of the frame model.

5 Conclusions

In the case of topology optimization on a structure where the loading is defined by stochastic variables, one possible way to create the equivalent mechanical model is to use a number of mutually excluding (alternating) loads. In this study optimal truss structure topologies were determined in the case of two alternative loads subject to stress limit and elastic behaviour in a wide range of described system parameters. We determined the

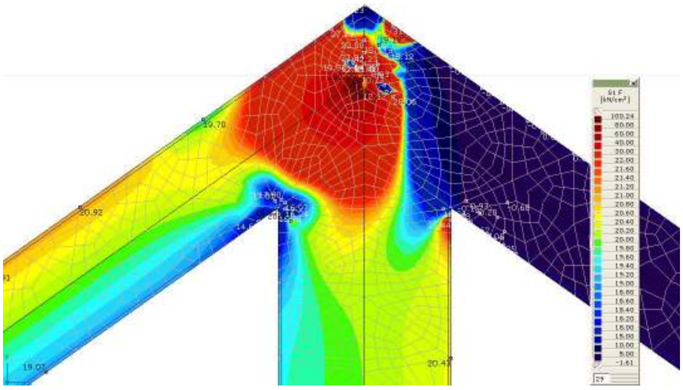

Fig. 11. The larger principal stresses of the continuum model, focus on the point of application of the load. Large stresses develop on the tension side of the central bar and of the left bar.

topology and size of the optimal structure that minimizes the volume (that is the amount of material) for various force ratios and force vector directions using a numerical iterative procedure. It is found that the optimum topology is typically statically determinate, however, in a certain domain of parameters a statically overdeterminate degenerate structure represents the minimum volume and in another domain the solution can be statically indeterminate. In these special cases the numerical results were confirmed by analytical calculations as well. Our numerical and analytical investigations have proved that the solution in certain cases is non-unique.

Furthermore, it is found that optimal designs calculated by the truss model (that is structures with hinges) are valid only for this type of structures. In structures where the joints are not constructed perfectly free of moments, even the small bending moments occuring simultaneously with the dominant normal forces lead to significant excess normal stresses which involve the undersizing of the structure. To verify this, an analysis of a chosen structure was performed modelled as a frame construction and a two-dimensional (plane stress state) continuum model by finite elements methods. The two analyses had the same results (with negligible difference) both showing the significant excess stresses. The investigated example demonstrates that for the determination of optimum frame structures an optimization procedure using continuum model is required. The model chosen has to be justified sufficiently.

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