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Structurally unstable A^{III}BiO₃ perovskites are predicted to be topological insulators but their stable structural forms are trivial band insulators

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Abstract

The quest for broadening the materials base of topological insulators (TIs) beyond the handful of presently known examples has recently led to exploratory calculations of the topological Z_2 metric from the band structure of various candidate compounds in *assumed crystal structures*. However, structural transformations such as volume compression, lattice straining, or atom swaps that are introduced to instigate in a trivial insulator the band inversion underlying TI-ness, might also destabilize the system to the point that it either distorts into a more stable structure or it does not form at all. Whether the more stable form of the candidate material is a TI, it is to be determined. Yet, often TI discovery calculations do not assess whether the postulated structural forms predicted to be TIs are also the stable forms of these compounds. Here, we show that in the broad family of III-Bi-O₃ oxides (III=Al, Ga, In, Sc, Y, and La) the cubic $A^{III}BiO_3$ perovskite structure which has been recently predicted to be TI, is unstable, whereas the stable $A^{III}BiO_3$ structural forms are trivial band insulators.

Topological insulators as a material search problem with potentially mutually-contradicting design requirements. Topological insulators (TIs) offer the interesting case where intrinsic bulk electronic properties can be used as a predictor of certain surface properties of the same material, a rather unusual occurrence in standard surface science of semiconductors and insulators[1] where surface effects are explained by the structure of the surface itself. In a bulk TI, an energy band that is normally an unoccupied conduction state switches order with a band that is normally an occupied valence state; the relevant states must occur at a time-reversal invariant momentum (TRIM), such as the wave-vector Γ , X, or L in the Brillouin zone of a face-centered cubic lattice. The bulk-calculated topological metric[2] Z_2 then determines if the material is a TI (when $Z_2 = 1$) or a normal band insulator (with $Z_2 = 0$, sometimes called "trivial"). Then, $Z_2 = 1$ in the *bulk crystal* is a predictor of surface electronic structure of the same material: the material will have within the surface band gap two states that cross each other ("gapless") and are linearly-dispersed ("massless Dirac cones") with an interesting spin-orbit induced spin texture[3, 4] that is resilient to nonmagnetic surface passivation. The quest for broadening the materials base of TIs beyond the handful of presently known examples has recently led to exploratory calculations of the topological Z₂ metric from the band structure of various candidate compounds.[5–12]

Often the practice in the field of TI discovery is to propose hypothetical compounds constructed starting from previously established TIs postulating the same crystal structure and substituting the chemical elements, or by subjecting the structure to significant strain. These composition and structural perturbations aim at inducing the transformation from normal to topological insulator, but they might contribute to loss of structural stability, e. g., when the strain is too big to maintain mechanical stability or when antibonding states get populated and bonding states get depopulated. The system could then respond by transforming into a different, more stable structure that cannot be guessed. *Whether that structure is still a TI or not needs to be examined.* The stability of hypothetical materials in a family of compounds cannot be guessed, and it is also hard to predict by extrapolating the results of stability calculations on a subset of compounds.[13]

Here, using the III-Bi-O₃ oxides (III= Al, Ga, In, Sc, Y, and La) with members recently suggested to be new TIs[5], we show that to predict new topological insulating compounds it is necessary to establish their topological nature *and* predict their stable phase. Jin *et al.* [5] predicted through band structure calculations that YBiO₃ in the assumed CaTiO₃-type perovskite structure (cubic $Pm\bar{3}m$ space-group symmetry) with Bi atoms located at the O_h (1*a* Wyckoff position) is a TI with an insulating indirect gap of 0.183 eV (and a gap of 0.33 eV at the *R* point, a TRIM, where a band inversion is found). Their result is reproduced in the upper right corner of Fig. 1. However, if one instead allows Bi to be located at the cavity site (1*b* Wyckoff position) of the same $Pm\bar{3}m$ perovskite structure, the total energy drops (see Fig. 1); relaxing the lattice constant of YBiO₃ from the previously used 5.428 Å to the density-functional equilibrium value for the same cubic $Pm\bar{3}m$ structure (a_{lat} =4.405 Å) further lowers the total energy by ~ 1 eV, whereas relaxing the condition that the structure is of the $Pm\bar{3}m$ perovskite type lowers the total energy by another ~ 1 eV and the system is not a TI (Fig. 1). We find indeed here that the cubic $Pm\bar{3}m$ perovskite with Bi at the O_h site is never the ground-state structure, but it shows a metallic band structure with a band inversion between the Bi-*s* and Bi-*p* orbitals (see Fig. 2) induced by spin-orbit coupling (SOC). In contrast, the alternative competing structure-types include the stable structures that nevertheless are trivial band insulators ($Z_2 = 0$).

We show that performing Z_2 calculations on a guessed structure can deliver the prediction of a TI; but, the guessed structure could be unstable and the system could be topologically trivial in its stable structure, as in the case of the $A^{III}BiO_3$ compounds. Other theoretical works seeking new TIs[7, 10–12] have not always examined the stability of the predicted compounds in the structures that make them TIs. We recommend the stability analysis to avoid predictions of new TIs that are unrealizable synthetically.

Methods to screen the stability of hypothetical structures via first principles approaches. To explore the stable structures of the $A^{III}BiO_3$ compounds we use two approaches. The simplest, conventional method is to remove the symmetry restrictions present in the CaTiO₃-type structure to allow tilting of the octahedra and perform a standard relaxion of the cell vectors and atom positions to the nearest minimum-energy structure. The local minimization might produce a daughter structure that is globally stable or metastable. We then determine whether the topological character of the wavefunctions in the assumed $Pm\bar{3}m$ perovskite structure is preserved or destroyed in the distorted perovskite phases. This approach provides a bridge with solid-state chemistry constructs that predict the degree of octahedral tilting from intuitive, heuristic concepts.[14, 15]

A second, more general, but computationally more demanding, approach to predict the lowenergy structures of the $A^{III}BiO_3$ compounds is assumption-free global energy minimization, starting from randomly chosen lattice vectors and atom positions. To this end we apply the global space-group optimization (GSGO)[16–20] method, which uses an evolutionary algorithm with variable cell shape and atom coordinates to predict the global minimum-energy structure of a material modeled by density functional theory (see, e.g., Ref. 17, 19–24 for some applications). This algorithmn predicts also a range of structures close in energy to the ground-state one that might be targeted by synthesis. The evolutionary GSGO algorithm searches for the global energy-minimum structure by replacing at each generation a subset of the population including the highest energy structure (e.g., 25 % of the whole population) with new ones generated by the cut-and-splice crossover[16, 19, 20] or mutation operations and then fully relaxed and included in the next generation of the population. We note that ternary Bi oxides of A^{III} species have been synthesized with the $A^{III}BiO_3$ composition,[25–27] thus here we focus on comparing the relative stability of each $A^{III}BiO_3$ compound in various ABX_3 structure-types.

Method of total energy calculations. We applied density functional theory as implemented in the VASP[28, 29] package to calculate total energies and band structures. The lattice vectors and atom positions of each structure were fully optimized using the PBE generalized gradient-corrected approximation. On the relaxed geometry, we included spin-orbit coupling to calculate the energy bands and the Z_2 invariant.

Method of Calculation of the Z_2 *topological invariant.* To determine the topological character of the compounds with centro-symmetric structures, we calculated the Z_2 topological invariant using the formula derived by Fu and Kane,[2]

$$(-1)^{Z_2} = \prod_{n=1}^{N_e/2} \prod_{\mathbf{k_i}} P(n, \mathbf{k_i}),$$
(1)

in which N_e is the number of electrons and $P(n, \mathbf{k_i})$ is the parity of the Bloch states $\psi_{n,\mathbf{k_i}}(\mathbf{r})$ at the time-reversal invariant momenta $\mathbf{k_i}$ in the Brillouin zone calculated for each occupied band n by comparing $\psi_{n,\mathbf{k_i}}(\mathbf{r})$ to $\psi_{n,\mathbf{k_i}}(-\mathbf{r})$. Materials with $Z_2 = 0$ are topologically trivial band insulators, while those with $Z_2 = 1$ have a non-trivial topological phase characterized by properties such as topologically protected dispersion-less surface states.

Key results. Figure 1 compares the total energies of the $A^{III}BiO_3$ compounds with $A^{III} = AI$, Ga, In, Sc, Y, and La in the following structures: (i) the undistorted cubic $Pm\bar{3}m$ CaTiO₃-type perovskite (see Figure 1, inset, left-hand side) which has two possible sites for Bi, the octahedral O_h site (1*a* Wyckoff position) and the cavity site (1*b*), at the corner of the cubic unit cell; (ii) the distorted orthorombic Pnma GdFeO₃-type perovskite (see Figure 1, inset, right-hand side); (iii) the GSGO lowest-energy structures (see Supplemental Material (SM)); and (iv) the experimentally observed structures (see SM). The $A^{III}BiO_3$ compounds based on Al, Ga, In and Sc have been synthesized[25–27] at high pressure (6 GPa), and the obtained phases are observed to be metastable at ambient pressure and temperature. Here, we performed total energy calculations and

GSGO searches at zero pressure (P = 0). In Figure 1, we also show the Z₂ value for the insulating structures that are centro-symmetric which include the CaTiO₃-type and GdFeO₃-type perovskites, while not all of the GSGO and experimental $A^{III}BiO_3$ structures are centro-symmetric. The total energies of the $A^{III}BiO_3$ structures are referred to the total energy of the experimentally observed $A^{III}BiO_3$ structures (or the model defective fluorite structure for YBiO₃ and LaBiO₃). We find that the cubic $Pm\bar{3}m$ perovskite with Bi at the O_h site is never the ground-state structure, but it shows a metallic band dispersion with a SOC-induced band inversion between the Bi-*s* and Bi-*p* orbitals (see Figure 2). In contrast, phases (ii)-(iv) include the stable structures that nevertheless are trivial band insulators (Z₂ =0). In what follows we discuss the results in more detail.

The two configurations of the unstable $Pm\bar{3}m$ CaTiO₃-type perovskite structure. In the undistorted $Pm\bar{3}m$ perovskite structure for the $A^{III}BiO_3$ compounds, Bi could occupy the O_h (1*a*) site and A^{III} the cavity (1*b*) site, a configuration we denote as $A^{\text{III}}_{1b}\text{Bi}_{1a}$. Alternatively, one can also construct the anti-site phase $Bi_{1b}A_{1a}^{III}$ with Bi at the cavity site. Figure 1 shows that the $Pm\bar{3}m$ perovskite structure of the $A^{III}BiO_3$ systems with Bi at the cavity site is more stable than the one with Bi at the O_h site, except when A^{III} is La. This reflects the fact that the optimal atom packing in the perovskite structure is achieved when Bi, which is larger than the $A^{\rm III}$ cations except La, is at the cavity site, thus maximizing the perovksite tolerance factor (see the SM). Moreover, the two cubic perovskite configurations become closer in energy as the difference between the Bi and A^{III} ionic radii decreases going from Al to In and from Sc to La. However, the $A^{\text{III}}\text{BiO}_3 Pm\bar{3}m$ perovskite form is never stable irrespective of the cation arrangement. YBiO₃, predicted to be a TI by Jin et al., [5] has an equilibrium lattice parameter in the $Y_{1b}Bi_{1a}O_3$ and $Bi_{1b}Y_{1a}O_3$ configuration respectively of 4.405 Å and 4.349 Å, much smaller than the 5.428 Å value reported by Li et al. [30] and assumed by Jin et al. in their calculations. YBiO₃ in the cubic perovskite structure with this expanded lattice parameter and Bi at the octahedral site is about 2.50 eV/atom above the lowest-energy structure and it is unlikely to be stable unless.

Electronic structure of the two variants of the unstable $Pm\bar{3}m$ structure. We find that all the $A^{III}BiO_3$ systems are metallic in the $Pm\bar{3}m$ perovskite structure irrespective of the Bi position in the lattice, and we illustrate this taking AlBiO₃ as an example. Fig. 2(a) and 2(b) display the GGA+SOC band dispersion of AlBiO₃ calculated in the $Pm\bar{3}m$ perovskite structure in the configuration with Bi at the O_h (1*a*) site and Al at the cavity (1*b*) site. The size of the red and blue symbols is proportional to the projections of the Bloch wave-functions on the Bi *s* and *p* atomic orbitals, respectively. AlBiO₃, as well as the Ga and In systems, have metallic band structures

with the Bi *s* states with anti-bonding character at a lower energy than the Bi *p* ones that have bonding character except at the time-reversal-invariant *R* point, where the ordering is inverted (see Fig. 2(a)). Switching the positions of Bi and A^{III} and putting Bi at the cavity site (see Fig. 2(b)), AlBiO₃ and the other $A^{III}BiO_3$ systems become insulators with fully occupied Bi *s* states, but without *s*-*p* inversion, making the $Pm\bar{3}m A^{III}BiO_3$ perovskites in this configuration trivial insulators (with Z₂ =0).

Perovskite structures where the octahedra are allowed to tilt: the $Pnma \ GdFeO_3$ -type structure. Relative stability of the two $GdFeO_3$ -type perovskite variants. A whole range of distorted perovskite structure types can be obtained starting from the ideal cubic perovskite by octahedra rotations and cation shifts. These distorted structures can lower the energy by optimizing the atom packing. The $Pnma \ GdFeO_3$ structure-type in particular is the perovskite in which the octahedra can tilt around a generic axis and distort from the ideal regular shape. As shown in Figure 1, the energy of the $A^{III}BiO_3$ compounds decreases considerably going from the $Pm\bar{3}m$ to the Pnmaperovskite structure, regardless of the Bi position. The configuration with Bi at the cavity site is still energetically preferred, but, with an energy distance from the configuration with Bi at the O_h site much reduced with respect to the $Pm\bar{3}m$ perovskite and that becomes as low as ~ 0.1 eV/at. Moreover, the GdFeO_3-type structure with the A^{III} cation at the octahedral site, is close in energy to the experimentally observed structure of AlBiO_3 and BiInO_3 and degenerate with this in the case of ScBiO_3.

Electronic structure of the Pnma orthorombic perovskites. The $A^{III}BiO_3$ systems in the Pnma perovksite structure are insulators but without the *s*-*p* band inversion observed in the $Pm\bar{3}m$ form. Figure 3 compares the GGA+SOC band structure of InBiO₃ in the $Pm\bar{3}m$ and Pnma perovskite structure with Bi at the octahedral site. While InBiO₃ in the $Pm\bar{3}m$ structure is a metal with the Bi *s*-*p* band inversion at the *R* point at about 3 eV above the Fermi level, the octahedral tilting and internal distortions that produce the GdFeO₃-type structure open a band gap but make the *s*-*p* inversion disappear. We find that all $A^{III}BiO_3$ systems in this *Pnma* perovskite are trivial insulators with Z₂ =0 irrespective of the Bi site.

GSGO-predicted and experimental structures. The structural models of the experimental and GSGO structures are shown in the SM. $InBiO_3[26]$ and $ScBiO_3[27]$ have been synthesized under pressure and form in distorted perovskite structures with Bi between the tilted octahedra. The observed phases are near-degenerate and structurally similar (or directly related, as in the case of $InBiO_3$) to the GdFeO₃-type structure, and are, thus, unlikely to exhibit a topological phase.

GaBiO₃[25] forms in a structure with Ga tetrahedrally coordinated to O that we found to have $Z_2 = 0$. The GSGO structure of GaBiO₃ is a variant of the experimental structure with a similar tetrahedral GaO₄ pattern and is also expected to be topologically trivial. AlBiO₃[25] forms in the LiNbO₃ (non-centrosymmetric) structure; the GSGO structure of AlBiO₃ is slightly lower in energy than the LiNbO₃-type at P = 0 and shows a tetrahedral bonding for Al similar to that in the GaBiO₃ phase, and is also expected to be a trivial band insulator.

YBiO₃[31–33] and LaBiO₃[34, 35] form in oxygen-defective fluorite phases in which the Bi and Y (La) atoms randomly occupy the cation sub-lattice. We used anion-defective fluorite structures with randomized cation configurations (a special quasi-random structure[36] (SQS) in the case of YBiO₃) to model the random cation distribution on these lattices. Interestingly, the *Pnma* perovskite phases are found to be close in energy to (and even slightly lower than) the random defective fluorite models probably due to the octahedral coordination which approximates the near-to-octahedral six-fold packing that is locally possible in an anion-defective fluorite lattice.

Discussion and Summary. The $A^{III}BiO_3$ perovskites were recently proposed[5] as systems with a valence configuration analogous to that of the Pb and Sn halide perovskites TIs, with the hope that also these oxides would show a topological band structure but with a larger gap than these halides. From this set of oxides, using band structure calculations, YBiO₃ was predicted to be a TI[5], while ScBiO₃ and LaBiO₃ were predicted to be topological metals[6]. Here, we calculated the Z₂ invariant of these three oxides and of the $A^{III}BiO_3$ oxides based on Al, Ga, and In in the previously assumed cubic perovskite structure and in the *Pnma* GdFeO₃-type perovskite structure with tilted octahedra. We fully relaxed all phases to the equilibrium structure. We find that all $A^{III}BiO_3$ systems are, depending on the Bi position, metallic or trivial insulators in the cubic perovskite relaxes into distorted perovskite phases of much lower energy. However, we find that the distorted perovskites along with the experimental structures and the low-energy structures predicted by GSGO are trivial insulators. Here, we show that to predict new TIs one must establish both the topological character and the stable phase of the compound applying "firstprinciples thermodynamics."

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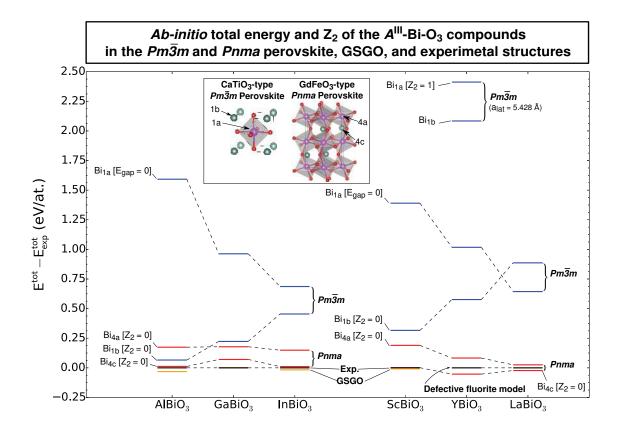


FIG. 1. GGA total energies of the $A^{\text{III}}\text{BiO}_3$ systems, where $A^{\text{III}} = \text{Al}$, Ga, In, Sc, Y, and La. The blue and red lines, respectively, correspond to the assumed CaTiO₃-type $(Pm\bar{3}m)$ and GdFeO₃-type (Pnma)perovskite structures, shown in the inset. The subscript of the Bi symbol indicates the Wyckoff site taken by this element in the two perovskite structures. The orange lines correspond to the GSGO structures, while the black lines correspond to the experimental structures (or the random configurations in the case of YBiO₃ and LaBiO₃) taken as the zero of the energy for each $A^{\text{III}}\text{BiO}_3$ system.

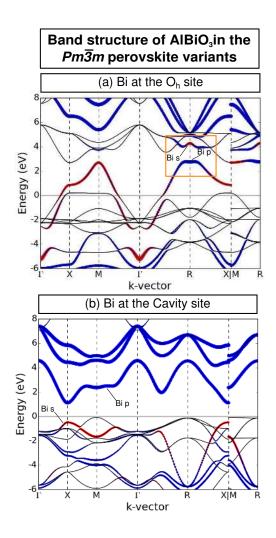


FIG. 2. Electron bands of AlBiO₃ in the $Pm\bar{3}m$ perovskite structure with Bi at the O_h (a) and cavity site (b). The radius of the markers is proportional to the projection of the wavefunctions on the Bi-s (red) and p (blue) atomic orbitals.

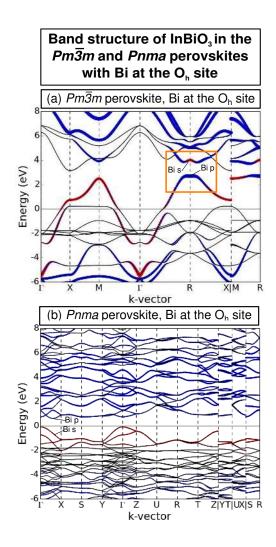


FIG. 3. Electron bands of $InBiO_3$ in the $Pm\bar{3}m$ (a) and Pnma (b) perovskite structure with Bi at the O_h site. As in Figure 2 the radius of the markers is proportional to the projection of the wavefunctions on the Bi-s (red) and Bi-p (blue) atomic orbitals.